$$A = \begin{bmatrix} 780 & 563 \\ 913 & 659 \end{bmatrix} \qquad b = \begin{bmatrix} 217 \\ 254 \end{bmatrix}$$

$$Ax = b$$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$780x_1 + 563x_2 = 217$$

$$\begin{bmatrix} 780 & 563 \\ 913 & 659 \end{bmatrix} \begin{bmatrix} 780x_1 + 563x_2 \\ 913x_1 + 659x_2 \end{bmatrix}$$

$$\begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

Zad. 2

$$A = \begin{bmatrix} 2 & 0 & 2 & 1 \\ -4 & 2 & -4 & 0 \\ -6 & 8 & -4 & 5 \\ -10 & 12 & -24 & 9 \end{bmatrix}$$

$$u_{ij} = a_{ij} - \sum_{k=1}^{i-1} l_{ik} u_{kj}$$

$$l_{ji} = \frac{1}{u_{ii}} \left(a_{ji} - \sum_{k=1}^{i-1} l_{jk} u_{ki} \right)$$

Dla $j = 1, 2, \ldots, n$: wykonujemy kolejno dwie poniższe operacje:

Dla $i = 1, 2, \ldots, j$: wykorzystujemy wzór z punktu 1) do wyznaczenia elementów $u_{i,j}$.

Dla $i = j+1, j+2, \ldots, n$: wykorzystujemy wzór z punktu 2) do wyznaczenia elementów $l_{i,j}$.

$j = 1$

$$u_{11} = a_{11} = 2$$

$$l_{21} = \frac{1}{u_{11}} \cdot a_{21} = -2$$

$$l_{31} = \frac{1}{u_{11}} \cdot a_{31} = -3$$

$$l_{41} = \frac{1}{u_{11}} \cdot a_{41} = -5$$

$j = 2$

$$u_{12} = a_{12} = 0$$

$$u_{22} = a_{22} - l_{21} u_{12} = 2 + 2 \cdot 0 = 2$$

$$l_{32} = \frac{a_{32} - l_{31} u_{12}}{u_{22}} = \frac{8 + 3 \cdot 0}{2} = 4$$

$$l_{42} = \frac{a_{42} - l_{41} u_{12}}{u_{22}} = \frac{12 + 5 \cdot 0}{2} = 6$$

$j = 3$

$$u_{13} = a_{13} = 2$$

$$u_{23} = a_{23} - l_{21}u_{13} = -4 + 2\cdot 2 = 0$$
$$u_{33} = a_{33} - l_{31}u_{13} - l_{32}u_{23} = -4 + 3\cdot 2 - 4\cdot 0 = 2$$
$$l_{43} = \frac{a_{43} - l_{41}u_{13} - l_{42}u_{23}}{u_{33}} = \frac{-24 + 5\cdot 2 - 6\cdot 0}{2} = -7$$

$j = 4$

$$u_{14} = a_{14} = 1$$
$$u_{24} = a_{24} - l_{21}u_{14} = 0 + 2\cdot 1 = 2$$
$$u_{34} = a_{34} - l_{31}u_{14} - l_{32}u_{24} = 5 + 3\cdot 1 - 4\cdot 2 = 0$$
$$u_{44} = a_{44} - l_{41}u_{14} - l_{42}u_{24} - l_{43}u_{34} = 9 + 5\cdot 1 - 6\cdot 2 + 7\cdot 0 = 2$$

$$U = \begin{bmatrix} 2 & 0 & 2 & 1 \\ 0 & 2 & 0 & 2 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 2 \end{bmatrix} \qquad L = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -2 & 1 & 0 & 0 \\ -3 & 4 & 1 & 0 \\ -5 & 6 & -7 & 1 \end{bmatrix}$$

$$\det(U) = 2\cdot 2\cdot 2\cdot 2 = 16 \qquad \det(L) = 1\cdot 1\cdot 1\cdot 1 = 1$$

$$\det(A) = \det(U)\cdot\det(L) = 16$$

$$[U|I] = \left[\begin{array}{cccc|cccc} 2 & 0 & 2 & 1 & 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 2 & 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 2 & 0 & 0 & 0 & 1 \end{array}\right] \rightarrow \left[\right.$$

...

Zad. 3

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 & -10 \\ -1 & -3 & 0 & 11 \\ -2 & -10 & 5 & 25 \\ -3 & -13 & -16 & 25 \end{bmatrix} \begin{matrix} \\ W_2 - (-W_1) \\ W_3 - (-2W_1) \\ W_4 - (-3W_1) \end{matrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ -2 & 0 & 1 & 0 \\ -3 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 & -10 \\ 0 & -2 & 1 & 1 \\ 0 & -8 & 7 & 5 \\ 0 & -10 & -13 & -5 \end{bmatrix} \begin{matrix} \\ \\ w_3 - 4w_2 \\ w_4 - 5w_2 \end{matrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ -2 & 4 & 1 & 0 \\ -3 & 5 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 & -10 \\ 0 & -2 & 1 & 1 \\ 0 & 0 & 3 & 1 \\ 0 & 0 & -18 & -10 \end{bmatrix} \begin{matrix} \\ \\ \\ w_4 - (-6w_3) \end{matrix}$$

$$L = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ -2 & 4 & 1 & 0 \\ -3 & 5 & -6 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 & -10 \\ 0 & -2 & 1 & 1 \\ 0 & 0 & 3 & 1 \\ 0 & 0 & 0 & -4 \end{bmatrix} = U$$

$$Ly = b$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ -2 & 4 & 1 & 0 \\ -3 & 5 & -6 & 1 \end{bmatrix} \cdot \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix} = \begin{bmatrix} -6 \\ 9 \\ 31 \\ -13 \end{bmatrix}$$

$y_1 = -6$

$y_2 = 9 - 6 = 3$

$y_3 = 31 - 12 - 12 = 7$

$y_4 = -13 - 18 - 15 + 42 = -4$

$$Ux = y$$

$$\begin{bmatrix} 1 & 1 & 1 & -10 \\ 0 & -2 & 1 & 1 \\ 0 & 0 & 3 & 1 \\ 0 & 0 & 0 & -4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} -6 \\ 3 \\ 7 \\ -4 \end{bmatrix}$$

$x_4 = \frac{-4}{-4} = 1$

$x_3 = \frac{7-1}{3} = 2$

$x_2 = \frac{3-2-1}{-2} = 0$

$x_1 = -6+10-2 = 2$

$$x = \begin{bmatrix} 2 \\ 0 \\ 2 \\ 1 \end{bmatrix}$$