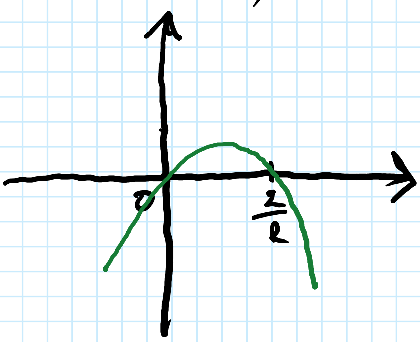
a) $f(x) = \frac{1}{x} - R$

$f'(x) = -\frac{1}{x^2}$

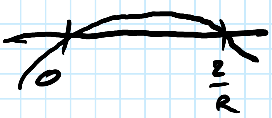
$$x_{n+1} = x_n - \frac{\frac{1}{x_n} - R}{-\frac{1}{x_n^2}} = x_n + (x_n - Rx_n^2) =$$

$$= x_n(2 - x_nR)$$

b)



c) $x_n(2-x_nR) < 0$



$x \in (-\infty, 0) \cup (\frac{2}{R}, +\infty)$

d) Załóżmy, że $x_n < 0$

$x_n(2 - x_nR) < x_n \quad /:x_n$

$2 - x_nR > 1$

$-x_nR > -1$

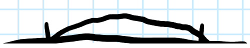
$x_n < \frac{1}{R}$

$x_n<0$, więc odwracamy znak nierówności

Skoro $x_n < 0$ i $R>0$ to nierówność zawsze prawdziwa

Zatem jeśli $x_n<0$ to $x_{n+1} < x_n$

e) $0 < x_n(2-x_nR)$



$x_n(2-x_nR) < \frac{1}{R}$

$-x_n^2R^2 + 2x_nR - 1 < 0$

e) $0 < x_n(2 - x_nR)$ $\qquad$ $x_n(2 - x_nR) < \frac{1}{R}$

$x \in (0, \frac{2}{R})$

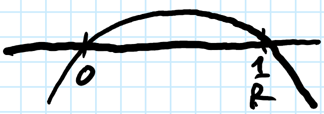
$$-x_n^2R^2 + 2x_nR - 1 < 0$$
$$-(x_nR - 1)^2 < 0$$

$x_n \in (-\infty, \frac{1}{R}) \cup (\frac{1}{R}, +\infty)$

Zatem $x \in (0, \frac{1}{R}) \cup (\frac{1}{R}, \frac{2}{R})$

f) Załóżmy, że $x_n \in (0, \frac{1}{R})$

$$x_n < 2x_n - x_n^2R$$
$$0 < x_n(1 - x_nR)$$



$x_n \in (0, \frac{1}{R})$

$2x_n - x_n^2R < \frac{1}{R}$

z e) wiemy, że $x_n \in (-\infty, \frac{1}{R}) \cup (\frac{1}{R}, +\infty)$

Zatem $x_n \in (0, \frac{1}{R})$

g)
$$g = \lim_{n\to\infty} x_{n+1} = \lim g(2 - x_nR) = 2g - g^2R$$
$$g = g(2 - gR) \quad /:g$$
$$1 = 2 - gR$$
$$gR = 1$$
$$g = \frac{1}{R}$$

Dla jakich x_0 metoda jest zbieżna?

1° $F(\frac{1}{R}) = \frac{1}{R}$

2° $|F'(x)| < 1$

Sprawdzamy dla jakich x te w są spełnione. $F = x_n(2 - x_nR)$

$2^\circ \ |F'(x)| < 1$ } są spełnione. $F = x_n(2 - x_n R)$

$1^\circ \ F\left(\frac{1}{R}\right) = \frac{1}{R} \cdot (2 - 1) = \frac{1}{R}$ [illegible]

$2^\circ \ F'(x) = 2 - 2xR$

$|2 - 2xR| < 1$

$|1 - xR| < \frac{1}{2}$

$|x - \frac{1}{R}| < \frac{1}{2R}$

czyli $x \in \left(\frac{1}{2R}, \frac{3}{2R}\right)$ – w tym przedziale metoda jest zbieżna