

## DEFINITION

Variance is a measure of dispersion. It indicates how far a set of values is spread out from their average value. In other words, how much those values differ from the mean.

A variance of a random variable is the arithmetic mean of the squared deviations of individual values from their arithmetic mean (i.e. population mean or sample mean).

Variance takes values from 0 to plus infinity (there is no negative variance because there cannot be a negative variance, either there is or not. If not, then all the values are the same).

Remember that variance requires at least two elements of the set.

Standard deviation is the square root of the variance.

## FORMULAS

Depending on the type of data -- statistical population or sample -- separate formulas are used to calculate variance.

a) statistical sample:  $s^2 = \sum(x_i - \bar{x})^2 / n - 1$

b) statistical population:  $\sigma^2 = \sum(x_i - \bar{m})^2 / N$

$s^2$  - sample variance

$\sigma^2$  - population variance

$\Sigma$  - sum of...

$x_i$  - each value of the set

$\bar{x}$  - arithmetic mean of the variable

$n$  - number of values in the sample

$N$  - number of values in the population

The sequence of operations:

1. Calculate the arithmetic mean of the set.
2. Calculate the deviations (differences) of values from the arithmetic mean of the set.
3. Square the deviations.
4. Add the squares of the deviations of the variable's values.
5. Divide the sum of squared deviations by  $n-1$  (for the sample) or  $n$  (for the population).

## UNITS OF MEASUREMENTS

The variance of a variable is expressed in units that are the square of the units of the variable itself. For example, if some variable is measured in years, then the variance of that variable will be measured in years squared.

## WHY N-1?

Variance is the arithmetic mean of the sum of the squared deviations, so it might seem that the sum should be divided by  $n$ , the number of values of the variable. However, in the case of the variance calculated for the statistical sample, the sum is divided by  $n-1$ , not  $n$ .

A statistical sample is only a fragment of the population - not its entirety. Therefore, the arithmetic mean calculated for the sample is only an estimate of the real mean. This is

because the mean is based on a statistical sample that is not the whole population. In order to get the real mean, it is necessary to calculate it from the data of the whole population.

For a statistical sample, the squares of deviations are underestimated. Therefore, it is necessary to artificially increase the variance to compensate for too low estimates of the squared deviations. That is why their sum is divided by  $n-1$ .

## APPLICATIONS

Variance is widely used in finance. In this case, the variance is hidden under the term "risk". It is inherently associated with investing.

There are many factors to consider before purchasing financial instruments. In addition to the expected profit, the level of risk associated with a given investment is also important. While the risk can never be fully eliminated, it is at least possible to reduce it. And variance can help with that.

Variance can be used to calculate the volatility of stock prices. Looking at stock market quotations from the last few days, we can find a company that has recently been recording very large increases. Then one could get the impression that buying shares of this company would be a good idea. However, if we look more closely at the company's chart, it may turn out that the company is also recording significant drops as often. Hence, you can conclude that the company's stock is highly volatile and therefore unpredictable.

To check this, the variance should be calculated. The first step is to calculate the average share price over a sufficiently long period of time, e.g. 1 year. Then calculate the variance for the closing prices for the period under examination.

If the variance is high, it means that a company's stock prices are subject to large fluctuations. So it is difficult to predict what price a stock will reach in the future.

Another example of applying variance in everyday life is the analysis of exam results.

A teacher who has recently conducted an exam may be interested in how the class coped with this challenge. It is a good idea to start by counting the average grade or the average number of points.

Suppose you are also interested in the dispersion of grades/number of points, i.e. how much the students' scores differ. The point of reference is the arithmetic mean of the grades/number of points. The only thing left for us to do is to calculate the variance. Note that in this case, we are calculating the variance for the population, not the sample. This is because our analysis examines the performance of only one specific class. We have test results for all students in that class, and since there are relatively few of them, we can easily and quickly make all the necessary calculations.