Chapter 12: Hyypperbolic and Parabo REJECT ic Partial Differential Equations

12. REJECT 1: EXAMPLES AND CONCEPTS OF HYPERBOLIC PDE'S

In the last chapter, we discussed in some detail the heat and Laplace s equations which are prototypes for parabolic and elliptic PDEs, respectively. We would like now to introduce some concepts and theory for the wave equation, which is the prototype for hyperbolic equations. The wave equation models many natural phenomena, including gas dynamics (in particular, acoustics), vibrating solids and electromagnetism. It was first studied in the eighteenth century to model vibrations of strings and columns of air in organ pipes. Several mathematicians contributed to these initial studies, including Taylor, Euler, and Jean DAlembert, about whom we will say more shortly. Subsequently in the nineteenth century, the wave equation was used to model elasticity as well as sound and light waves, and in the entieth century, it has been used in quantum mechanics and relativi$^{\mathrm{ty}}$ and most recently in such fields as supercondducttilvi1tty and string theory. In general, the ave equation has a time vari1abble t and any number of space var$_1$i$^{\mathrm{a}}$bblles JC, y, zz, and takes the fo

$\mathrm{w}_{lll}u\equiv \mathrm{c}^{C^2}\mathrm{Aiici}\#_{\mathrm{e}} + \mathrm{i}^{\mathrm{u}}+)$, (])

(0])

where c is a positive constant and the Laplace operator on the right is with respect to all of the space variables. Modifications of this equation have been successfully used to model numerous physical waves and wavelike phenomena. In two space variables for example allowing for a variable wave speed due to dep differences in an ocean, the PDE:$u_{tt}\equiv$V$''$[[$//(x > y >, /)$)Vw] + $//$ has been used to model large destructive ocean waves. In such an appli$^{\text{cat}}$i$^{\text{on}}$, the function HH is the

depth ofthe ocean at space coordinates (longitude and l$^{\text{at}}$i$^{\text{tu}}$d$^{\text{e}}$)(JC, y)and at time $/t$.

The latter term corresponds to the changes in depth due to underwater l$^{\text{an}}$d$^{\text{sl}}$id$^{\text{es}}$.

For more on this and other applications ofthis variable media wave equation, we

mention the text [Lan-99].

1 The symbol V , read as nabla' or ddeel/ is used to represent the gradient operator, which is the

vector of all P$^{\text{a}}$ti ldivative of f nction Thus frfu ctio of to va ibl s$f(f, y1,$

$f = \nabla f(x, y) \equiv$ (r$(xy), f_\nu(x, y))$ The I$_{\text{lar}}$e dot re resents the vector dot product so 1in$^{\text{I}}$lon fo

$\nabla < \{HH(x, y, t)\nabla]] - -((d_y H)H(_t uHu_y^{H)} y)((Hu_{y)}\}$. In particular, when $H//$s-il we ha

V $\cdot$ [∇u$\boxed{\texttt{REJECT}}$ $= \partial(u) + \partial_v(u_\nu) = u + u. = \Delta u,$, another way tto write the Laplacian of u. Such notations

are very common in the literature for partial differential equations involving several space variables.

5233