Applied Finance,
Path-dependent option pricing
with Monte Carlo and Rcpp package
Individual home project for:
Szymon Socha (411462)

deadline: 2023-01-29, 23:59:59

PROBLEM

You are considering a European style **up-and-out call option** with a barrier active between the moment of pricing and the option expiry.

Specifically, you are interested in:

1. the theoretical price of this option,
2. relation between the theoretical price of the option and two factors (simultaneously):
   a) volatility of the underlying instrument returns, and
   b) time to maturity of the option.

Your task is to:

1. create an `Rcpp` package which provides a function returning the Monte Carlo approximation of the option theoretical price,
2. create an `Rmd` document, where you load the package, and then call the function to produce a plot which represents the relation described above. Do not install the package inside the `Rmd` document. Instead, you may include the code which would do this, however, please comment it out.

You may use the option pricer and all other code we have developed during the labs.

Your code should be ready to price the option for any values of its characteristics, nevertheless please find the theoretical price for the following values:

- price of the underyling at the moment of option pricing: $S_0 = 95$,
- strike price $K = 100$,
- annualized volatility rate $\sigma = 0.24$
- annualized risk-free rate $r = 0.07$
- time to maturity $t = 0.75$

As far as the barrier level b is concerned, it's up to you. Choose a reasonable value, that will slightly influence the price of a corresponding non-barrier option. Try to avoid the barrier that will make the option price to be zero.

RULES

1. The project is to be prepared **individually**. My hint is: use class materials and codes.
2. Use necessary explanations in your `Rmd` document as much as possible.
3. Solution of the project should consist of two files:
   (a) a `zip` file with the `Rcpp` source package (not a binary package!)
   (b) a `Rmd` file with a short report (see below)
4. Alternatively, you can push the source code of your package to the `github.com` and use the `install_github()` function in your `Rmd` document.
5. A good report (the `Rmd` file) should include a short information about:
   - objective of the project,
   - assumptions (for example: dynamics of the prices of underyling, number of iterations in Monte Carlo, etc.),
   - destription of the option to be priced and its characteristics,
   - information about the results of the simulation.
6. Send your project solution (two files: `*.Rmd` and `*.zip`) by replying to the email with this attachment. You should get a confirmation of this.
7. You must agree to abide by the following Honor Code:
   (a) My solutions of home project will be my own work.
   (b) I will not make solutions to homeworks available to anyone else.
   (c) I will not engage in any other activities that will dishonestly improve my results or dishonestly improve/hurt the results of others.
8. Cheating is a very serious offence. Please keep in mind that any evidence of copying of your home project solutions (the report as well as the code) will be treated as cheating and will be reported immediately to the Dean. Please take is seriously.
9. Additionally, you are asked to include in your report a following declaration:

   > In accordance with the Honor Code, I certify that my answers here are my own work, and I did not make my solutions available to anyone else.

Good luck!

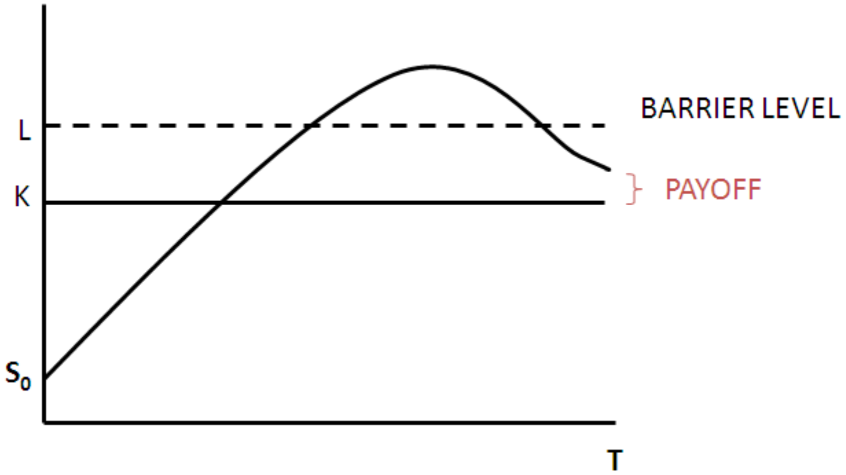
Paweł

A short refresher: payoffs of barrier options with continuous barriers

1. CALL UP-and-IN

- the price has to increase to reach the barrier
- activated when asset price reaches the barrier

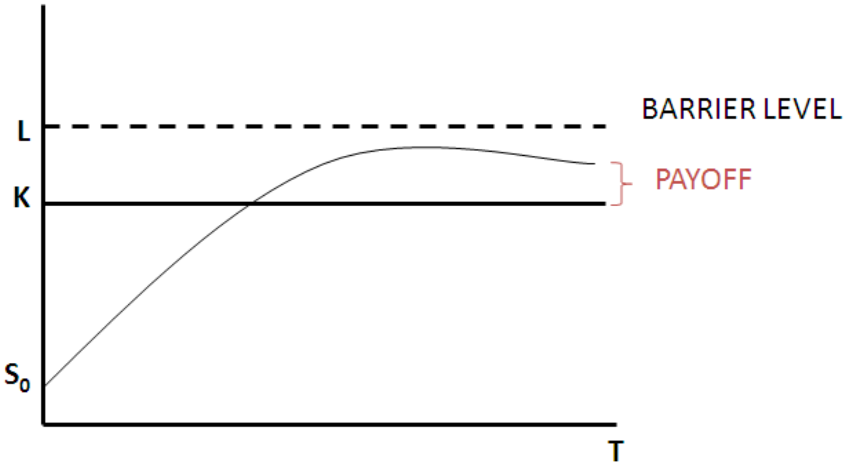
$$\Phi_t = \max(S_T - X, 0) \quad \text{if} \quad \max_{0 \le t \le T}(S_t) \ge L \tag{1}$$



2. CALL UP-and-OUT

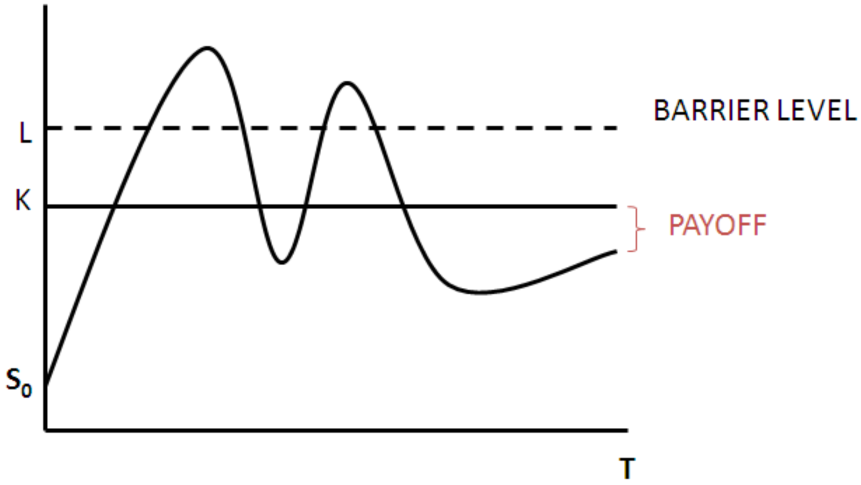
- the price has to increase to reach the barrier
- canceled when the asset price reaches the barrier

$$\Phi_t = \max(S_T - X, 0) \quad \text{if} \quad \max_{0 \le t \le T}(S_t) \le L \tag{2}$$



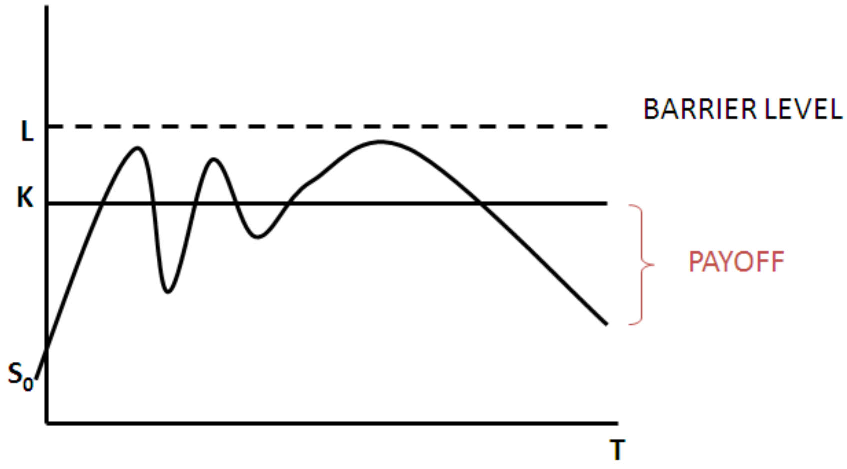
3. PUT UP-and-IN
   - the price has to increase to reach the barrier
   - activated when asset price reaches the barrier

$$\Phi_t = \max(X - S_T, 0) \quad \text{if} \quad \max_{0 \le t \le T}(S_t) \ge L \tag{3}$$



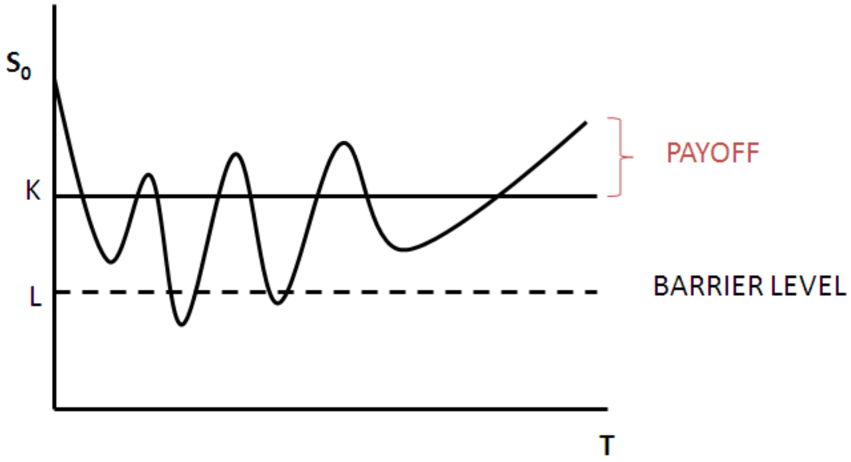
4. PUT UP-and-OUT
   - the price has to increase to reach the barrier
   - canceled when the asset price reaches the barrier

$$\Phi_t = \max(X - S_T, 0) \quad \text{if} \quad \max_{0 \le t \le T}(S_t) \le L \tag{4}$$



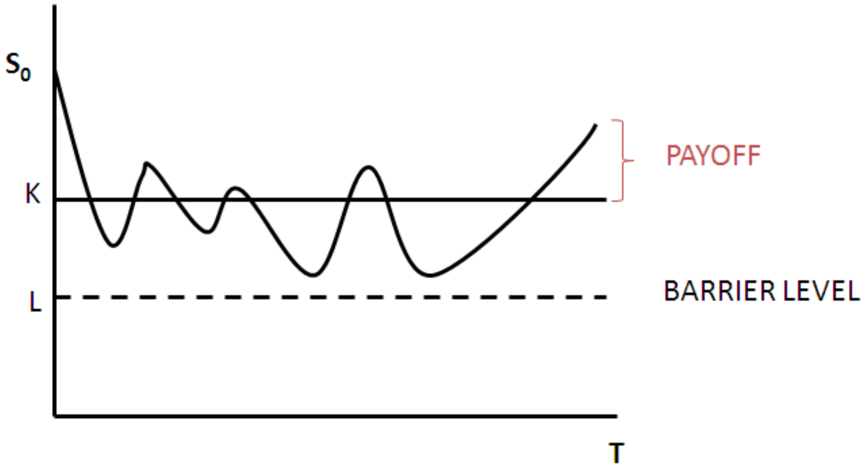
5. CALL DOWN-and-IN
   - the price has to decrease to reach the barrier
   - activated when asset price reaches the barrier

$$\Phi_t = \max(S_T - X, 0) \quad \text{if} \quad \min_{0 \leq t \leq T}(S_t) \leq L \tag{5}$$



6. CALL DOWN-and-OUT
   - the price has to decrease to reach the barrier
   - canceled when asset price reaches the barrier

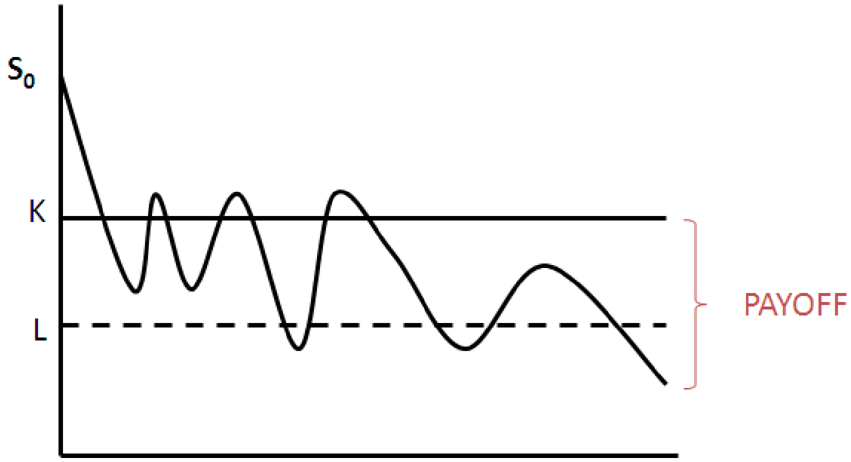
$$\Phi_t = \max(S_T - X, 0) \quad \text{if} \quad \min_{0 \leq t \leq T}(S_t) \geq L \tag{6}$$



7. PUT DOWN-and-IN

- the price has to decrease to reach the barrier
- activated when asset price reaches the barrier

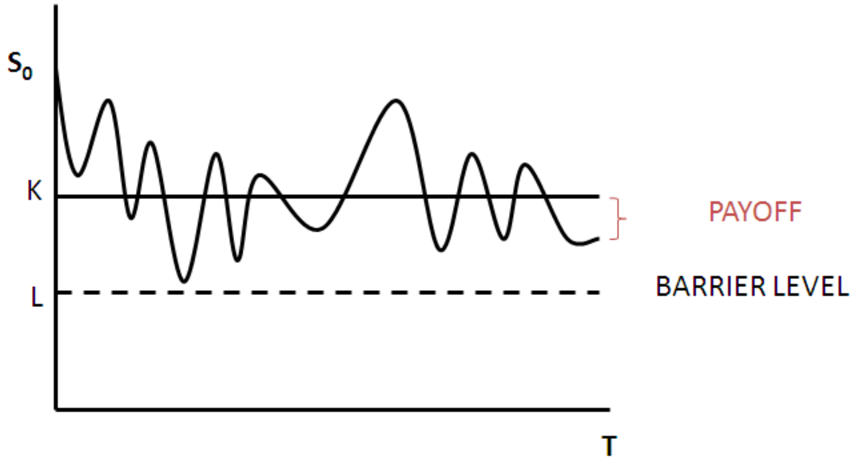
$$\Phi_t = \max(X - S_T, 0) \quad \text{if} \quad \min_{0 \leq t \leq T}(S_t) \leq L \tag{7}$$



8. PUT DOWN-and-OUT

- the price has to decrease to reach the barrier
- canceled when asset price reaches the barrier

$$\Phi_t = \max(X - S_T, 0) \quad \text{if} \quad \min_{0 \leq t \leq T}(S_t) \geq L \tag{8}$$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