Observer-Based Decentralized Event-triggered Consensus for Leader-following Linear Multi-Agent Systems

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***Abstract*—This paper studies the consensus problem for general linear MASs under directed graphs that full-state measurements are not available. Based on output feedback, we propose an observer-based event-triggered controller to achieve consensus. Each agent only needs to monitor its own output feedback by its observer continuously and updates its controllers and broadcast it to its out-neighbors when it triggers an event or receives new information from its in-neighbors. It is proved that the MAS reaches consensus for all initial conditions and no Zeno behavior exhibits. Furthermore, when there exists a leader, the proposed control protocol can also lead all the followers to track the leader's state. At last, simulation is given to show the theoretical analysis and illustrate the effectiveness of the proposed controll methods.**

***Index Terms*—Consensus, Decentralized event-triggering, Multi-Agent System, General linear dynamics, Observer-based output feedback**

I. Introduction

In recent years, multi-agent systems (MAS) have received considerable attention due to its wide practical applications such as control of autonomous mobile robots [1], [2], unman aerial vehicle (UAV) formation [3], satellite formation flying [4], air traffic control [5] and so on. However, designing a distributed approach to solve the consensus is one of the most critical issues. Due to limited bandwidth and energy, each agent could not transmit the too much information, which motivates the development of event-triggered control schemes. In event-triggered control, single-integrator dynamics for MAS was first considered in [6], [7]. General linear MAS for event-triggered average-consensus problem is considered in [8]. In the research above, each agent still need to communicate with their neighbors and monitor its own states continuously to check the triggering function, which will cause lots of unnecessary communication and wastee energy. To solve this problem, researchers have begun to study the self-triggered consensus and quantized consensus problem for single-integrator agents [9] and general linear dynamics under a general directed graph without continuous communication [10].

It is easily to see that most of the event-triggered controllers of MASs available recently are based on static state-feedback with an assumption that all the agent states can be measured. However, in many real applications, the agent states are not available to be measured, which means that all the event-triggered state feedback controllers cannot be used. Hence, researchers begin to working on event-triggered controllers based on output feedback, which is more challenging in general.

Motivated by the above discussion, this paper is concerned with the observer-based event-triggered output feedback controllers to solve consensus problem of MAS with general linear dynamics under a general directed graph, which is assumed to contain a directed spanning tree. The contribution of the work can be summarized as follows. Firstly, with a general linear system that states are unavailable to be measured, we propose a distributed observer-based event-trigger consensus controller and prove that it can asymptotically converges to zero when communication is discrete. Secondly, we prove that we can avoid Zeno behavior in this controller, which means that the event would not be triggered continuously.

The rest of this paper is organized as follows. Section II introduces some basic notation and some useful results of algebraic graph theory. Section III investigates a new event-triggered controller based on output feedback. Section IV gives the simulation examples and analysis. Section V concludes the paper.

II. Preliminaries

Let $\mathbf{R}^{m\times n}$ and $\mathbf{C}^{m\times n}$ be the set of $m\times n$ real and complex matrices. respectively. Let $\mathbf{1}_m$ and $\mathbf{1}_0$ denote the $m\times 1$ column vector of all ones and all zeros, respectively. Let $\mathbf{0}_{m\times n}$ denote the $m\times n$ matrix with all zeros and I_m denote the $m\times m$ identity matrix. The superscript T means the transpose for real matrices. $\lambda_i(\cdot)$ denote s the ith eigenvalue of a matrix. By $\operatorname{diag}(A_1,\ldots,A_n)$, we denote a block-diagonal matrix with matrices A_i, $i=1,\ldots,n$ on its diagonal. A matrix $A\in\mathbf{C}^{m\times m}$ is Hurwitz if all of its eigenvalues have strictly

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negative real parts. The matrix $A \otimes B$ denotes the Kronecker product of matrices A and B. Let $\|\cdot\|$ denote, respectively, the Euclidean norm for vectors and the induced 2-norm for matrices. Let dim$(\cdot)$ describe the dimension of a square matrix. For a complex number, Re$(\cdot)$ denotes its real part.

A directed graph $\mathcal{G}$ is a pair $(\mathcal{V}, \mathcal{E})$, where $\mathcal{V} = \{v_1, \ldots, v_N\}$ is a nonempty finite set of nodes and $\mathcal{E} \in \mathcal{V} \times \mathcal{V}$ is a set of edges, in which an edge is represented by an ordered pair of distinct nodes. An edge (v_i, v_j) means that node v_j can receive information from node v_i or equivalently node v_i can broadcast information to node v_j . Here we call v_i an in-neighbor of v_j and v_j an out-neighbor of v_i . A directed path from node $v_{i_k}$to node $v_{i_{k+1}}$ is a sequence of ordered edges of the form $\left(v_{i_k}, v_{i_{k+1}}\right), k = 1, \ldots, l-1$. A directed graph contains a directed spanning tree if there exists a node called the root such that there exist directed paths from this node to every other node. The adjacency matrix $\mathcal{A} = [a_{ij}] \in \mathbf{R}^{N \times N}$ associated with the directed graph $\mathcal{G}$ is defined by $a_{ii} = 0, a_{ij} > 0$ if $(v_j, v_i) \in E$ and $a_{ij} = 0$ otherwise. The Laplacian matrix $\mathcal{L} = [l_{ij}] \in \mathbf{R}^{N \times N}$ is defined as $l_{ii} = \sum_{j=1, j \neq i}^{N} a_{ij}$ and $l_j = -a_j, i \neq j$. The graph $\mathcal{G}$ is undirected if$a_{ij} = a_{ji}, \forall i, j = 1, \ldots, N$ and directed otherwise.

Consider a group of N identical agents with general linear dynamics. The dynamics of the ith agent are described by

$$\dot{x}_i = Ax_i + Bu_i; \quad y_i = Cx_i \tag{1}$$

where $x_i = [x_i, 1, \ldots, x_{i,n}]^T \in \mathbf{R}^n$ is the state,$u_i \in \mathbf{R}^p$ is the control input, and $y_i \in \mathbf{R}^q$ is the measured output.

Assumption 1 The matrix pair (A, B) is stabilizable and (A, C) is detectable. The directed graph $\mathcal{G}$ contains a directed spanning tree.

Lemma 1 Suppose that $A \in \mathbf{R}^{n \times n}$ is Hurwitz. Then, for all $t \geq 0$, it holds that $\|e^{At}\| \leq \|P_A\| \|P_A^{-1}\| c_A e^{a_A t}$, where P_A is a nonsingular matrix such that $P_A^{-1} A P_A = J_A$ with J_A being the Jordan canonical form of A, $c_A > 0$ is a positive constant determined by A, and $\max_i \operatorname{Re}\left(\lambda_i(A)\right) < a_A < 0$ [10].

Lemma 2 If $\mathcal{G}$ contains a directed spanning tree, 0 is a simple eigenvalue of the Laplacian matrix $\mathcal{L}$ and all the other eigenvalues have positive real parts. $r^T = [r_1, \ldots, r_N] \in \mathbf{R}^{1 \times N}$ be the non-negative left eigenvector of $\mathcal{L}$ associated with zero eigenvalue, satisfying $r^T \mathbf{1} = 1$ and $\mathbf{1}_N$ is a right eigenvector associated with the zero eigenvalue [11].

III. Observer-based Event-triggered Consensus Control

In this section, firstly, we will propose an observer-based event-triggered consensus controller for MAS with general linear dynamics based on output feedback under a general directed graph with a direct spanning tree. Secondly, we prove the consensus and convergence of the system and that no Zeno behavior is exhibited.

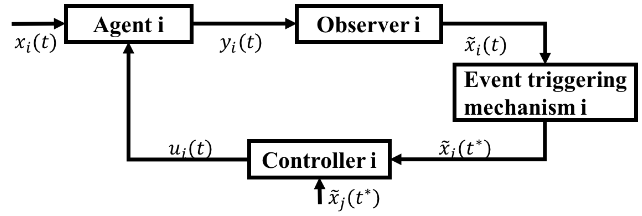

Fig. 1: Observer-based event-triggered control schematic.

The consensus controller proposed in [12] as follows are widely used

$$u_i(t) = cK \sum_{j=1}^{N} a_{ij} \left(x_i(t) - x_j(t)\right) \tag{2}$$

where $c > 0$ is the coupling gain, $K \in R^{p \times n}$ is the feedback gain matrix to be determined, and a_{ij} is the ijth entry of the adjacency matrix $\mathbf{A}$. It was proved in reference [12] that if the directed graph $\mathcal{G}$ contains a directed spanning tree, the controller (2) solves the consensus problem if and only if all matrices $A + c\lambda_i(\mathcal{C})BK$ are Hurwitz, where $\lambda_i(\mathcal{L}) \neq 0$.

In (2) the state of each agent is required to be measured, which is sometimes unavailble. In order to solve this problem, we introduce an observer and propose an observer-based ETCC that relies on output feedback

$$\begin{aligned} \dot{\hat{x}}_i(t) &= A\hat{x}_i(t) + Bu_i(t) + F\left(\hat{y}_i(t) - y_i(t)\right) \\ \hat{y}_i(t) &= C\hat{x}_i(t) \\ u_i(t) &= cK \textstyle\sum_{j=1}^{N} a_{ij} \left(\hat{x}_i(t) - \hat{x}_j(t)\right) \end{aligned} \tag{3}$$

where $\hat{x}_i(t) \in \mathbf{R}^n$ is the observer state, $\hat{y}_i(t) \in \mathbf{R}^q$ is the output of the observer, F is the feedback gain matrix to be determined, c, K and a_{ij} are defined in (2).

For each agent i and $t \geq 0$, we define the observer-based measurement error

$$\hat{e}_i(t) = \hat{x}_i\left(t^*\right) - \hat{x}_i(t) \tag{4}$$

where t^* is the latest triggering time for agent i.

For each agent i, the observed-based triggering function is given by

$$f_i\left(t, \hat{e}_i(t)\right) = \|\hat{e}_i(t)\| - c_1 e^{-\alpha t} \tag{5}$$

where $c_1 > 0$ and α is a positive constant to be determined. The Obeserver-based ETCC controller of agent i continuously monitor its own states. An event is triggered for this agent with either the following conditions: **1)** the observer-based measurement error of agent i exceed a given threshold, that is, $f_i\left(t, \hat{e}_i(t)\right) \geq 0$. **2)** One of the in-neighbors of agent i are triggered. When triggered, agent i uses its current observed state to update its controller and broadcast it to its out-neighbors. It is easy to see that no communication needed if the observed-based measurement error is less than the threshold.

However, to use the observer-based measurement error in the triggering function, we should prove that $\hat{e}_i(t) \rightarrow e_i(t), \quad i = 1, 2, \ldots, N$, as $t \rightarrow \infty$, which means to prove $\hat{x}_i(t) \rightarrow x_i(t)$. Hence, we introduce the following vectors $\xi(t)$ to prove it.

With the stack vectors $x(t) = \left[x_1^T(t), \ldots, x_N^T(t)\right]^T$, $\hat{x}(t) = \left[\hat{x}_1^T(t), \ldots, \hat{x}_N^T(t)\right]^T$ and $\hat{x}\left(t^*\right) = \left[\hat{x}_1^T\left(t^*\right), \cdots, \hat{x}_N^T\left(t^*\right)\right]^T$, let $\xi_i = \begin{bmatrix} x_i \\ \hat{x}_i \end{bmatrix}$, the closed-loop system (1) using (3) can be written as

$$\dot{\xi}(t) = \begin{bmatrix} \dot{x}(t) \\ \dot{\hat{x}}(t) \end{bmatrix} = \mathcal{P} \begin{bmatrix} x(t) \\ \hat{x}(t) \end{bmatrix} = \mathcal{P}\xi(t) \tag{6}$$

where $\mathcal{P} \triangleq \begin{bmatrix} I_N \otimes A & c\mathcal{L} \otimes BK \\ -I_N \otimes FC & I_N \otimes (A+FC) + c\mathcal{L} \otimes BK \end{bmatrix}$, $\xi = \left[\xi_1^T, \ldots, \xi_N^T\right]^T \in \mathbf{R}^{2Nn \times 2Nn}$. Let $r^T = [r_1, \ldots, r_N] \in \mathbf{R}^{1 \times N}$ be the left eigenvector of $\mathcal{L}$ associated with zero eigenvalue, satisfying $r^T \mathbf{1} = 1$. Introduce the disagreement vector

$$\delta(t) = \xi(t) - \left(\left(\mathbf{1} r^T\right) \otimes I_{2n}\right) \xi(t) \tag{7}$$

where $\delta \in \mathbf{R}^{2Nn \times 2Nn}$ satisfies $\left(r^T \otimes I_{2n}\right)\delta = 0$ and $\dot{\delta}(t) = \mathcal{P}\delta(t)$.

The following theorem presents a necessary condition for the observer-based ETCC controller to reach consensus.

Theorem 1: For a network of agents with communication topology $\mathcal{G}$ that has a directed spanning tree, the observer-based ETCC (3) and the triggering function (5), the disagreement vector (7) of the closed-loop system asymptotically converges to zero for all initial conditions if and only if all matrices $A + c\lambda_i(\mathcal{L})BK$ and $A + FC$ are Hurwitz, where $\lambda_i(\mathcal{L})$ are the nonzero eigenvalues of the Laplacian matrix $\mathcal{L}$ of $\mathcal{G}$.

Proof: First, we prove that the consensus problem of system (6) is equivalent to the asymptotical stability problem of the disagreement vector (7). Let Let $\hat{M} = I_N - 1r^T$, we can rewrite equation (6) as $\delta = \left(\hat{M} \otimes I_{2n}\right)\xi$

From lemma 2, the definition of r, it is easy to observe that zero is a simple eigenvalue of $\hat{M}$, with $\mathbf{1}$ as the corresponding right eigenvector, and one is another eigenvalue with multiplicity $N-1$. We can get the condition that $\delta = 0$ if and only if $\xi_1 = \cdots = \xi_N$, which means the consensus problem is solved if and only if $\delta(t) \to 0$, as $t \to \infty$.

Next, to solve the consensus problem, the stability of disagreement dynamics is investigated. Let $Y \in \mathbf{R}^{N \times (N-1)}, W \in \mathbf{R}^{(N-1) \times N}, T \in \mathbf{R}^{N \times N}$, and upper triangular $\Delta \in \mathbf{R}^{(N-1) \times (N-1)}$ be such that

$$T = \begin{bmatrix} 1 & Y \end{bmatrix} \; T^{-1} = \begin{bmatrix} r^T \\ W \end{bmatrix} \; T^{-1}\mathcal{L}T = J = \begin{bmatrix} 0 & 0 \\ 0 & \Delta \end{bmatrix} \tag{8}$$

where the diagonal entries of Δ are the nonzero eigenvalues of $\mathcal{L}$. Introduce a new vector

$$\epsilon(t) = \left(T^{-1} \otimes I_n\right)\delta(t) = \left[\epsilon_1^T(t), \epsilon_{2-N}^T(t)\right]^T \tag{9}$$

where $\epsilon_1(t) \in \mathbf{C}^{2n}, \quad \epsilon_{2-N}(t) \in \mathbf{C}^{2(N-1)n}$ Then, $\epsilon(t)$ can be represented as follows

$$\dot{\epsilon} = \left(I_N \otimes \mathcal{A} + cJ \otimes \mathcal{H}\right)\epsilon \tag{10}$$

For $i = 1$, it can be observed from (7) that

$$\epsilon_1 = \left(r^T \otimes I_{2n}\right)\delta \equiv \mathbf{0}_{2n} \tag{11}$$

For $i = 2, \ldots, N$

$$\dot{\epsilon}_{2-N}(t) = \overline{A}\epsilon_{2-N}(t) \tag{12}$$

$$\epsilon_{2-N}(t) = e^{\overline{A}t}\epsilon_{2-N}(0) \tag{13}$$

$$\overline{A} \triangleq \begin{bmatrix} I_{N-1} \otimes A & c\Delta \otimes BK \\ -I_{N-1} \otimes FC & I_{N-1} \otimes (A+FC) + c\Delta \otimes BK \end{bmatrix}$$

Note that $\overline{A}$ is similar to

$$\overline{\Pi} \triangleq \begin{bmatrix} I_{N-1} \otimes A + c\Delta \otimes BK & c\Delta \otimes BK \\ 0 & I_{N-1} \otimes (A+FC) \end{bmatrix}$$

Because the elements of matrix (10) are either block upper triangular or block diagonal, $\epsilon_i, i = 2, \ldots, N$ converge asymptotically to zero if and only if the $N-1$ subsystems along the diagonal. Because $\overline{\Pi}$ is a block upper triangular matrix, if allif all matrices $A + c\lambda_i(\mathcal{L})BK$ where $\lambda_i(\mathcal{L}) \neq 0$ and $A + FC$ are Hurwitz, $\overline{\Pi}$ is Hurwitz. Note that when $t \to \infty$, we have $\|e_{2-N}(t)\| \to 0$, so from equation (13) $\epsilon_{2-N}(t)$ asymptotically converges to zero. Hence, for a closed-loop system in any initial conditions, disagreement vector $\delta(t)$ converges to zero. That is to say, the observer-based ETCC (3) solve the consensus problem of MAS by using output feedback instead of state feedback.

Remark 1: The significance of this theorem lies in the fact that, by using observer-based event-triggered control, we can greatly reduce the communication load, though, it will influence the convergence speed. When the agents' states are not measurable, which happens in many real applications, we could still use this observer-based method to implement an ETCC for linear MAS to reach consensus.

Theorem 2: Think about the MAS with general linear dynamics satisfying Assumption 1. If the triggering function satisfies $c_1 > 0$ and $0 < \alpha < -max_i \mathrm{Re}\left(\lambda_i(\Pi)\right)$ and if all matrices $A + c\lambda_i(\mathcal{L})BK$ and $A + FC$ are Hurwitz, where $\lambda_i(\mathcal{L}) \neq 0$ are the nonzero eigenvalues of the Laplacian matrix $\mathcal{L}$ of $\mathcal{G}$, the closed-loop system does not exhibit Zeno behavior under the observer-based ETCC.

Proof: Let $P \in C^{(N-1)n \times (N-1)n}$ and $P^{-1} \in C^{(N-1)n \times (N-1)n}$ be the matrices such that $P^{-1}\Pi P = J_{\overline{\Pi}}$, where $J_{\overline{\Pi}}$ is the Jordan canonical form of the matrix Π. We have proved the consensus in theorem 1. Because the triggering function $f_i\left(t, e_i(t)\right)$ will be set to zero for agent i when an event is triggered, $f_i\left(t, e_i(t)\right)$ will not cross zero before the next triggered event. That is to say, $\|e_i(t)\| < c_1 e^{-\alpha t}$ is satisfied until the next event is triggered. Hence, $\|e_{2-N}(t)\| < \sqrt{N-1}c_1 e^{-\alpha t}$ and $\|e_{2-N}(t)\| \to 0$, as $t \to \infty$.

From $\epsilon_{2-N}(t) = e^{\overline{A}t}\epsilon_{2-N}(0)$ and Lemma 1, it is observed that for $0 \leq s \leq t$

$$\|\epsilon(t)\| = \|\epsilon_{2-N}(t)\| \leq a_1 e^{a_{\overline{\Pi}}t} \tag{14}$$

where $a_1 = c_{\overline{\Pi}}(2(N-1)n - 1)\|P\|\|P^{-1}\|\|\epsilon_{2-N}(0)\|$, $c_{\overline{\Pi}} > 0$ is a positive constant determined by $\overline{\Pi}$, and $\max_i \mathrm{Re}\left(\lambda_i(\overline{\Pi})\right) < a_{\overline{\Pi}} < 0$.

From equation (7), we conclude that $\|\delta(t)\|$ satisfies

$$\|\delta(t)\| \leq \|T \otimes I_n\|\|\mathcal{E}(t)\| \leq k_1 e^{a_{\overline{\Pi}}t} \tag{15}$$

where $k_1 = \|T\|a_1 > 0$.

Let $u(t)$ be the column stack vector of $u_i(t)$. Because the Laplacian matrix satisfies $\mathcal{L}\mathbf{1}_N \equiv \mathbf{0}_N$, we conclude that

$$\begin{aligned} \left(I_N \otimes B\right)u(t) &= (c\mathcal{L} \otimes BK)\hat{x}(t) \\ &\quad - (c\mathcal{L} \otimes BK)\left(\left(\mathbf{1}_N \mathbf{r}^T \otimes I_{2n}\right)\hat{x}(t)\right) \\ &= (c\mathcal{L} \otimes BK)\delta(t) \end{aligned} \tag{16}$$

Similarly, $\|(I_N \otimes B)u(t)\|$ is upper bounded by

$$\begin{aligned} &\|\left(I_N \otimes B\right)u(t)\| \leq \|c\mathcal{L} \otimes BK\|\|\delta(t)\| = b_1 e^{a_{\overline{\Pi}}t} \\ &\|Bu_i(t)\| \leq \|\left(I_N \otimes B\right)u(t)\| \leq b_1 e^{a_{1\Pi}t} \end{aligned} \tag{17}$$

where $b_1 = \|c\mathcal{L} \otimes BK\| k_1$. It follows from equation (1) and (3) that the measurement error by the observer is $\dot{\hat{x}}_i(t) - \dot{x}_i(t) = (A + FC)(\hat{x}_i(t) - x_i(t))$. Hence, we can conclude that

$$\hat{x}_i(t) - x_i(t) = e^{(A+FC)t} (\hat{x}_i(0) - x_i(0)) \tag{18}$$

Because $A + FC$ is the sub-matrix of $\overline{\Pi}$, all the eigenvalues of $A + FC$ are the eigenvalues of matrix $\overline{\Pi}$. Since $A + FC$ is Hurwitz, there exits an postive constant b_3 that satisfies the following equation

$$\|FC (\hat{x}_i(t) - x_i(t))\| \leq b_3 e^{a_\Pi t} \tag{19}$$

Let $\hat{M} = I_N - 1r^T$ and from equation (7), we can get $\delta = \left(\hat{M} \otimes I_{2n}\right) \xi$. By calculating the inverse matrix, we can get equation $\xi = \left(\hat{M}^{-1} \otimes I_{2n}\right) \delta$. Moreover, we can conclude

$$\|\xi(t)\| \leq e_1 e^{a_{\overline{\Pi}} t} \tag{20}$$

where $e_1 = \|\hat{M}^{-1} \otimes I_{2n}\| k_1$, let $e_2 = \|A\| e_1$, so

$$\begin{aligned} &\|\hat{x}_i(t)\| \leq \|\xi_i(t)\| \leq \|\xi(t)\| \leq e_1 e^{a_{\overline{\Pi}} t} \\ &\|A\hat{x}_i(t)\| \leq e_2 e^{a_{\overline{\Pi}} t} \end{aligned} \tag{21}$$

Because we assume that agent i is triggered at the latest triggering instant t^*,if follows from equation (4) that

$$\begin{aligned} &\dot{\hat{e}}_i(t) = -\dot{\hat{x}}_i(t) = \\ &\quad - (A\hat{x}_i(t) + Bu_i(t) + FC(\hat{x}_i(t) - x_i(t))) \end{aligned} \tag{22}$$

We can get the upper bound of $\|\dot{e}_i(t)\|$ between the two triggered events for agent i as

$$\begin{aligned} \|\dot{\hat{e}}_i(t)\| \leq & \|A\hat{x}_i(t)\| + \|Bu_i(t)\| + \|FC(\hat{x}_i(t) - x_i(t))\| \\ & \leq d_1 e^{a_{\overline{\Pi}} t} \triangleq g(t) \end{aligned} \tag{23}$$

where $d_1 = b_1 + b_3 + e_2 > 0$. Because of the latest triggering instant t^*, it shows that $\|\hat{e}_i(t)\| = \| \int_{t^*}^t \dot{e}_i(s) ds\| \leq \int_{t^*}^t g(s) ds$. According to the triggering function, we know that agent i will not be triggered until $f_i(t, e_i(t)) = 0$, that is, $\|e_i(t)\| = c_1 e^{-\alpha t}$. Hence, the agent will not be triggered before $\int_{t^*}^t g(s) ds = c_1 e^{-\alpha t}$. Because $t \geq t^*$, $a_\Pi < 0$ and $-\alpha < 0$, we can conclude that $e^{-\alpha t} \leq e^{-\alpha t^*}$ and $e^{a_\Pi t} \leq e^{a_\Pi t^*}$. Let $\tau = t - t^*$ be the time-interval between the two triggered events. It is easy to be observed that τ is greater than or equal to the solution of the implicit equation $d_1 e^{a_{\overline{\Pi}} t^*} \tilde{\tau} = c_1 e^{-\alpha (t^* + \tilde{t})}$, and equivalently $d_1 e^{(\alpha + a_{\overline{\Pi}}) t^*} \tilde{\tau} = c_1 e^{-\alpha \tilde{\tau}}$. From Lemma 2, $\alpha < -\max_i \mathrm{Re}\left(\lambda_i(\overline{\Pi})\right)$, so a negative constant $a_{\overline{\Pi}}$ must exist that satisfies $\max_i \mathrm{Re}\left(\lambda_i(\overline{\Pi})\right) < a_{\overline{\Pi}} < -\alpha < 0$. Since $\alpha + a_{\overline{\Pi}} < 0$, we can conclude that $d_1 e^{(\alpha + a_{\overline{\Pi}}) t^*} \leq d_1$. So the solution of the implicit equation is greater than or equal to the solution of $d_1 \overline{\tau} = c_1 e^{-\alpha \overline{\tau}}$, which is strictly positive. It shows that if the coefficients c_1 and α satisfy $c_1 > 0$ and $0 < \alpha < -\mathrm{Re}\left(\lambda_1(\overline{\Pi})\right)$, for agent i, a positive bound $\tilde{\tau}$ exists between the two successive triggered events. As a result, there is no Zeno behavior when using this observer-based ETCC to solve consensus problem for general linear MAS.

Remark 2: Zeno behavior is exclude, which the parameter α plays an important role in the convergence rate.

The decentralized event-triggered consensus problem for linear multi-agent systems is studied in [10]. Also, the paper [12] showed a distributed observer-type consensus protocol for general linear MASs. Motivated by the papers above, we proposed an observer-based decentralized event-triggered consensus controller for general linear MASs containing a leader.

Remark 3: Consider the MAS including a leader. Suppose the leader agent is the root node of the directed spanning tree in directed graph $\mathcal{G}$, and the follower agents do not have directed paths to the leader agent. In Eq. (7), for r the left eigenvector of $\mathcal{L}$, if and only if the r_i associated with root node is greater than zero and the r_i corresponding to other follower nodes is zero, we can easily conclude that the MASs reach consensus and all the followers asymptotically converges to the leader state.

Algorithm: Given the matrix pair (A, B) is stabilizable and (A, C) is detectable, an observer-based event-triggered algorithm in the form of (3) and (5) solving the consensus problem of general linear MAS can be constructed according to the following steps.

(1) Solve the following linear matrix inequality
$$AP + PA^T - 2BB^T < 0 \tag{24}$$
to get one symmetric positive-definite solution P. Next, choose the feedback gain matrix $K = -B^T P^{-1}$.

(2) Select the coupling gain c in (3) given by $c > \frac{1}{\min_i \mathrm{Re}(\lambda_i(\mathcal{L}))}$, where $\lambda_i(\mathcal{L}) \neq 0$ is the nonzero eigenvalues of Laplacian matrix $\mathcal{L}$.

(3) Choose the feedback gain matrix F such that $A + FC$ is Hurwitz.

(4) Choose the constants in the trigger function (5) to satisfy $c_1 > 0$ and $0 < \alpha < -\max_i \mathrm{Re}\left(\lambda_i(\overline{\Pi})\right)$, where $\overline{\Pi}$ is defined in (15).

IV. Simulation

In this section, we illustrate the simulation results of the above observer-based ETCC. Consider a group of 6 agents with general linear dynamics illustrated by, where $A = [-2, 2; 1; 1]$ and $B = [1; 0]$. We choose the feedback gain matrix $K = [-1, 2]$ so that $A + BK$ is Hurwitz. The communication topology among the agents is a directed graph containing a directed spanning tree. The Laplacian matrix of the communication graph is $\mathcal{L} = [0, 0, 0, 0, 0, 0; 1, 1, 0, 0, 0, 0; 1, 1, 2, 0, 0, 0; 1, 0, 0, 1, 0, 0; 0, 0, 0, 1, 1, 0; 0, 0, 0, 0, 1, 1]$. Obviously, the nonzero eigenvalues of the Laplacian matrix $\mathcal{L}$ are 1, $1.3376 \pm 0.5623i$, 2, 3.3247. Here by step 2 of Algorithm, we choose $c = 1.1$. The eigenvalues of $\overline{\Pi}$ defined after (13) are $-2.2, -1, -1.1,$ and -3. So, according to the conditions required in step 3 in Algorithm, we choose $c_1 = 0.6$ and $\alpha = 0.9$. The initial states are given by $x_1(0) = [0.4; 0.3], x_2(0) = [0.5; 0.2], x_3(0) = [0.6; 0.1], x_4(0) = [0.7; 0], x_5(0) = [0.8; 0.1],$ and $x_6(0) = [0.4; 0.2]$. The state trajectories are presented in Fig. 2. The observer-based output feedback measurement errors and their thresholds of agents are shown in Fig. 3.

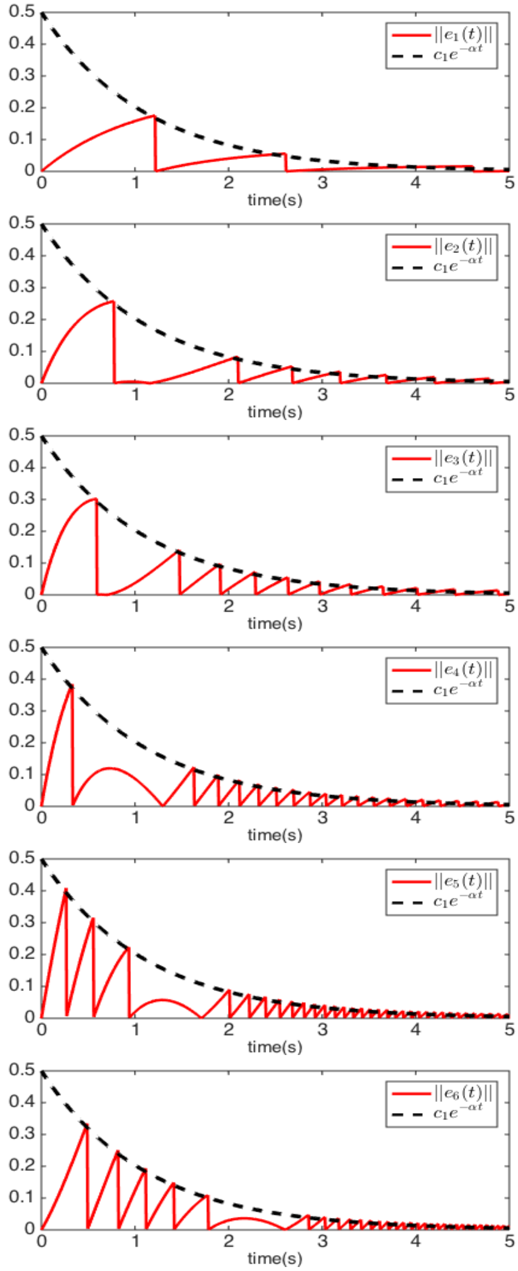

Fig. 2: The errors and the thresholds of the errors of each agent under the observer-based ETCC.

V. Conclusion

This paper considered the event-triggered consensus problem for MAS with general linear dynamics under general directed graphs and the agent state is unavailable to be measured. We introduce an observer for each agent and a distributed observer-based event-triggered algorithm based on output feedback. It is proved that the MAS reach consensus for all initial conditions and no Zeno behavior exhibits. Further research direction would include the investigation on the observer-based decentralized event-triggered control problem for MASs with nonlinear dynamics.

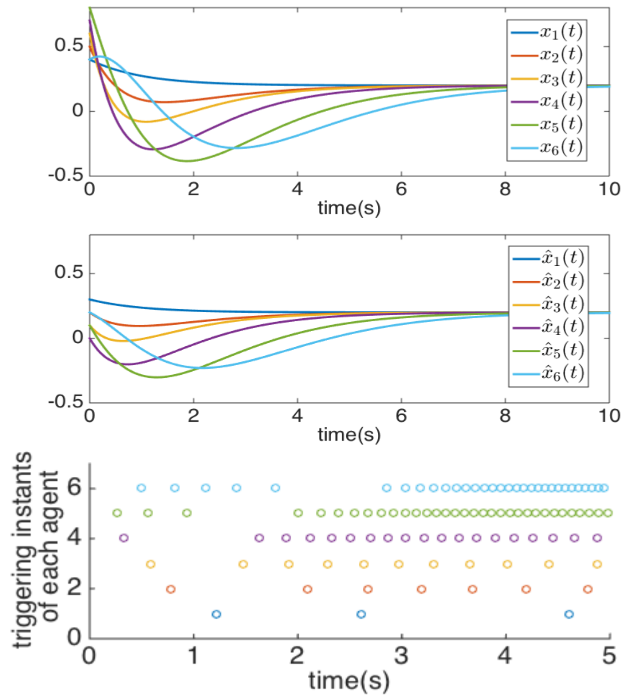

Fig. 3: The states, observed states and the triggering instants of each agent under the observer-based ETCC.

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