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Finite-time Distributed Event-triggered Consensus Control for Leader-following General Linear Multi-Agent Systems

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***Abstract*—In this paper, we study the finite-time containment consensus problem for multi-agent systems with general linear dynamics. Finite time consensus can reach consensus faster and have better disturbance rejection properties compared with conventional asymptotic consensus. Hence, we propose a distributed, independent and asynchronous event-triggered control protocol with leaders. We demonstrate that each follower agent can achieve consensus to the convex hull spanned by the leaders' agents in a certain time regardless of the initial condition under this event-triggered control strategy.**

***Index Terms*—Consensus, Decentralized Event-triggering, Multi-Agent System, General linear dynamics, Finite-time**

I. Introduction

Recent years, we have witnessed that multi-agent systems have been paid considerable and growing attention due to its wide practical applications such as control of unmanned aerial vehicle [1], satellite formation flying [2], flocking [3], [4], formation [5], swarming [6], [7] and so on. Consensus problems, as a fundamental of distributed coordination control, refers to the group behaviors that all agents asymptotically reach a certain common agreement through a local distributed protocol. If there exists a single leader, the goal of the control protocol is to let all the followers to track the dynamic leader's state [8]. However, when there is a multi-leader case, the problem becomes the containment control problem and the objective is to make all the followers come into the leaders' convex hull [9]–[11].

In real applications, the multi-agent system usually works under the environment of limited energy and communication band width. Hence, it is shown that the event-triggered control strategy is an effective way to reduce energy consumption of the agents [12], [13]. What's more, the convergence speed are also greatly significant for the consensus of MASs. Many researchers have studied to increase the convergence rate by enlarging the coupling strength, optimizing communication weights, and designing optimal network topology [14], [15]. The paper [16] studied finite-time distributed event-triggered consensus control for multi-agent systems and proposed a nonlinear distributed control protocol. Furthermore, in paer [17], they considered the multi-agent systems with leaders based on the controller proposed in [16]. The paper [18] extended the multi-agent systems to one with general linear dynamics, proposed a new event-triggered control strategy and demonstrated that it can achieve consensus in a certain time.

Motivated by the above discussion, this paper is concerned with the finite-time distributed event-triggered consensus containment control problem of general linear multi-agent systems. The contribution of the work can be summarized as follows. Given a multi-agent systems with general linear dynamics, we propose a distributed event-triggered consensus controller and prove that the system can asymptotically converges to zero in a certain time when communication is discrete.

The rest of this paper is organized as follows. Section II introduces some basic notation and some useful results of algebraic graph theory. Section III investigates a new finite-time event-triggered controller for general linear multi-agent systems. Section IV concludes the paper.

II. Preliminaries and Notations

A. Algebraic graph theory

Let $\mathbf{R}^{m\times n}$ and $\mathbf{C}^{m\times n}$ be the set of $m\times n$ real and complex matrices. respectively. Let $\mathbf{1}_m$ and $\mathbf{1}_0$ denote the $m\times 1$ column vector of all ones and all zeros, respectively. Let $\mathbf{0}_{m\times n}$ denote the $m\times n$ matrix with all zeros and I_m denote the $m\times m$ identity matrix. The superscript T means the transpose for real matrices. $\lambda_i(\cdot)$ denote s the ith eigenvalue of a matrix. By $\mathrm{diag}\,(A_1,\ldots,A_n)$, we denote a block-diagonal matrix with matrices A_i, $i=1,\ldots,n$ on its diagonal. A matrix $A\in\mathbf{C}^{m\times m}$ is Hurwitz if all of its eigenvalues have strictly negative real parts. The matrix $A\otimes B$ denotes the Kronecker product of matrices A and B. Let $\|\cdot\|$ denote, respectively, the Euclidean norm for vectors and the induced 2-norm for matrices. Let dim$(\cdot)$ describe the dimension of a square matrix. For a complex number, Re$(\cdot)$ denotes its real part.

An undirected graph $\mathcal{G}$ is a pair $(\mathcal{V}, \mathcal{E}, A)$, where $\mathcal{V} = \{v_1, \ldots, v_N\}$ is a nonempty finite set of nodes and $\mathcal{E} \in \mathcal{V} \times \mathcal{V}$ is a set of edges, in which an edge is represented by an ordered pair of distinct nodes. An edge (v_i, v_j) means that node v_j and v_i can receive and broadcast information to each other. The adjacency matrix $\mathcal{A} = [a_{ij}] \in \mathbf{R}^{N \times N}$ associated with the undirected graph $\mathcal{G}$ is defined by $a_{ii} = 0, a_{ij} > 0$ if $(v_j, v_i) \in E$ and $a_{ij} = 0$ otherwise. Agent i and j can communicate with each other, then they are called neighbors. The set of neighbors of agent i is denoted by $N_i = \{j : (v_i, v_j) \in \mathcal{E}, j \neq i\}$. A path from i to j is a sequence of $v_{i_1}, \ldots, v_{i_k}$ which starting from i and ending with j, such that $(v_{i_1}, v_{i_1+1}) \in \mathcal{E}$ for $i = 1, 2, \ldots, k-1$. If there is a path from i to j, then i and j are called connected. If all pairs of nodes in $\mathcal{G}$ are connected, then $\mathcal{G}$ is called connected. The degree matrix of $\mathcal{G}$ is given by $D = diag\{d_1, d_2, \ldots, d_n\}$ with the degree d_i of vertex i is dened as the number of its neighboring vertices, i.e., $d_i = \sum_{j \in N_i} a_{ij}, i \in I$. The Laplacian matrix $\mathcal{L} = [l_{ij}] \in \mathbf{R}^{N \times N}$ is defined as $l_{ii} = \sum_{j=1, j \neq i}^{N} a_{ij}$ and $l_j = -a_j, i \neq j$. The Laplacian matrix $\mathcal{L}$ has a simple eigenvalue 0 whose corresponding eigenvector if and only if an undirected graph $\mathcal{G}$ is connected. $\lambda_n(L) \geq \cdots \geq \lambda_2(L) > \lambda_1(L) = 0$ and $\lambda_2(L)$ is the second smallest eigenvalue which is called algebraic connectivity (or Fiedler eigenvalue), so if $\mathbf{1}_n^T x = 0$, then $x^T L x \geq \lambda_2 x^T x$. For a connected graph, L also satisfies:

$$\begin{aligned} x^T L x &= \frac{1}{2} \sum_{i,j=1}^{n} a_{ij} (x_j - x_i)^T (x_j - x_i) \\ &= \frac{1}{2} \sum_{i=1}^{n} \sum_{j \in N_i} a_{ij} (x_j - x_i)^T (x_j - x_i) \end{aligned} \quad (1)$$

Definition 1 [19]. In a group of n+m agents, leader is the agent who has no neighbor and follower is the one who has at least one neighbor.

Definition 2 [20]. The convex hull is composed of a finite set of points $x_1, x_2, \cdots, x_n \in R^m$ and is denoted by $\mathrm{co}\{x_1, x_2, \cdots, x_n\}$. Particularly, $\mathrm{co}\{x_1, x_2, \cdots, x_n\} = \{\sum_{i=1}^{n} \alpha_i x_i | \alpha_i \in R, \alpha_i \geq 0, \sum_{i=1}^{n} \alpha_i = 1\}$.

Denote the set of followers as $F = \{1, 2, \cdots, n\}$ and the set of leaders as $L = \{n+1, n+2, \cdots n+m\}$. From the definition 1, each leader agent doesn't exchange information with other agents.

Assumption 1. Each follower agent can obtain information from at least one leader agent. Hence, we can described the Laplacian matrix as:

$$L = \begin{bmatrix} L_{FF} & L_{FL} \\ 0_{mxn} & 0_{mxm} \end{bmatrix}$$

where $L_{FF} \in R^{n \times n}, L_{FL} \in R^{n \times m}$

Lemma 1 [21]. Under assumption 1, all the real parts of L_{FF}'s eigenvalues are positive, the entries of $-L_{FF}^{-1} L_{FL}$ are all nonnegative and the sum of each row of $-L_{FF}^{-1} L_{FL}$ is one.

Lemma 2 [22]. Finite-time lyapunov stability theorem: Consider the non-Lipschitz continuous nonlinear system $\dot{x} = f(x)$ with $f(0) = 0$. Suppose there exists a continuous function $V(x)$ dened on a neighborhood of the origin, and real numbers $c > 0$ and $0 < \alpha < 1$, such that the following conditions hold:

(i) $V(x) > 0$;
(ii) $\dot{V}(x) + cV^\alpha \leq 0$.

Lemma 3 [23]. Consider a linear system (A, B, C) ,if (A, B) is stabilizable and (C, A) is observable, then there exists a unique solution $P > 0$ to the following algebraic Riccati equation:

$$PA + A^T P - PBB^T P + C^T C = 0$$

Lemma 4 [24].

$$\left(\sum_{i=1}^{n} |x_i|\right)^p \leq \sum_{i=1}^{n} |x_i|^p \leq n^{1-p} \left(\sum_{i=1}^{n} |x_i|\right)^p$$

where $x_i \in R,$ and $0 < p \leq 1$.

III. Main Results

Let's consider a general linear multi-agent system including n followers and m leaders with the following dynamics:

$$\dot{x}_i = Ax_i(t) + Bu_i(t), i = 1, 2, \cdots, N + M \quad (2)$$

where $x_i(t) \in R^n$ is ith agents state , $u_i \in R^n$is the control input of agent i and A, B and suitable matrices that satisfy (A, B) is controllable. We denote $x_F = \left[x_1^T, x_2^T, \ldots, x_N^T\right]^T$ and $x_L = \left[x_{N+1}^T, x_{N+2}^T, \cdots, x_{N+M}^T\right]^T$.

The paper intends to illustrate a distributed event-based control protocol for all the followers to make all the followers' states come into the leader agents' convex hull in any given finite time, which equivalently show as:

$$\lim_{t \to T} x_F + L_{FF}^{-1} L_{FL} x_L = 0 \quad (3)$$

where x_F and x_L are the vector form of the followers' states and leaders' states.

Remark 1. The entries of $-L_{FF}^{-1} L_{FL}$ are all greater or equal than zero. Furthermore, based on Lemma 1, the sum of each row of $-L_{FF}^{-1} L_{FL}$ is one. Hence, (2) can guarantee that the followers' states reach the leaders' convex hull in a certain time T.

The event-triggered control portal for the consensus of general linear multi-agent system (2) is proposed as follow:

$$\begin{aligned} & u_i(t) = K q_i^\alpha \left(t_k^i\right), t \in \left[t_k^i, t_{k+1}^i\right), i \in F \\ & u_i(t) = 0, i \in L \\ & q_i(t) = \left(\sum_{j=1}^{N} a_{ij} \left(x_j(t) - x_i(t)\right)\right)^{\frac{1}{\alpha}} \end{aligned} \quad (4)$$

where $t \in \left[t_k^i, t_{k+1}^i\right)$, t_k^i is the kth triggering time for agent i, $K$is the feedback gain matrix to be designed.

For each agent i and $t \geq 0$ we define the measurement error as:

$$e_i(t) = \left(q_i^\alpha \left(t_k^i\right) - q_i^\alpha(t)\right)^{1/\beta} \quad (5)$$

where $\alpha = \frac{n}{m}$,n is positive odd numbers and $m > n$. $\beta \in (0,1]$ is the ratios of positive odd numbers. Let $\epsilon_i(t) = x_i\left(t_k^i\right) - x_i(t), t \in \left[t_k^i, t_{k+1}^i\right), i \in F$. Since the leader's control input is 0, we can get $\epsilon_i = 0,\ i \in L$. By the definition of ϵ_i, we can also get $e_i^\beta(t)$ as:

$$e_i^\beta(t) = q_i^\alpha\left(t_k^i\right) - q_i^\alpha(t) = \sum_{j\in N_i} a_{ij}\left(\epsilon_i(t) - \epsilon_j(t)\right) \quad (6)$$

The triggering function is given by:

$$h\left(e_i(t), q_i(t)\right) = \|e_i^\beta(t)\| - \eta_i\|q_i^\alpha(t)\| \quad (7)$$

where $0 < \eta_i < 1$ is a positive constant to be determined. The controller of agent i continuously monitor its own states. An event is triggered for this agent with either the following conditions: 1) the measurement error of agent i exceed a given threshold, that is, $h_i\left(t, e_i(t)\right) \geq 0$. 2) One of the neighbors of agent i are triggered. When triggered, agent i uses its current state to update its controller and broadcast it to its neighbors. It is easy to see that no communication needed if the observed-based measurement error is less than the threshold.

Theorem 1. For a network of agents with communication topology $\mathcal{G}$ that is an connect undirected graph, considering the general linear multi-agent system (2) and control inputs (4) with $K = \mu B^T P$ and the triggering function (6) with $\eta_i = \left(\frac{\sigma_i\cdot\kappa(2-\kappa\rho)}{\rho}\right)^{(1/2)} < 1, \rho = \|PBB^TP\|, \sigma_i \in \left(0, \min\left(1, \rho^2\right)\right)$, and $0 < \kappa < 2/\rho$. The system asymptotically converges to zero in a finite time for all initial conditions and the settling time can be illustrated as follows:

$$T(x(0)) \leq \frac{V^{1-\alpha}(x(0))}{\bar{\bar{\varsigma}}(1-\alpha)}$$

$$\bar{\bar{\varsigma}} = \mu\left(1 - \frac{\kappa\rho}{2}\right)\left(1 - \overline{\sigma}_i\right)\|q_i^\alpha(t)\|_{\min}^{2-2\alpha}\frac{2^\alpha\lambda_2^\alpha}{\omega_{max}^\alpha}$$

where ω_{max} is the maximal eigenvalue of the matrix P, $\overline{\sigma_i} = \max\{\overline{\sigma_i}, i = 1,\ldots,N+M\}$, $\|q_i^\alpha(t)\|_{\min} = \min\{\|q_i^\alpha(t)\|, i = 1,\ldots,N+M\}$.

Proof: Under $K = \mu B^T P$ and control law (4), system (2) can be represented as:

$$\dot{x}_i = Ax_i + \mu BB^TP(e_i^\beta(t) + q_i^\alpha(t)), \forall i \in V \quad (8)$$

We have $q_i^\alpha(t) = \sum_{i=1}^N\left(L_{FF}\right)_y x_j + \sum_{j=N+1}^{N+M}\left(L_{F_L}\right)_y x_j$, so we can get

$$\begin{aligned}\dot{q_i}^\alpha(t)_i &= \sum_{j=1}^n\left(L_{FF}\right)_{ij}\dot{x}_j + \sum_{j=n+1}^{n+m}\left(L_{FL}\right)_{ij}\dot{x}_j\\ &= \sum_{j=1}^n\left(L_{FF}\right)_{ij}\dot{x}_j + 0\\ &= \sum_{j=1}^n\left(L_{FF}\right)_{ij}\left(Ax_i + \mu BB^TP(e_i^\beta(t) + q_i^\alpha(t))\right)\end{aligned} \quad (9)$$

Based on definition 1, we have $q_i^\alpha(t) = \sum_{j=1}^N a_{ij}\left(x_j(t) - x_i(t)\right) = 0$ and $e_i^\beta(t) = \sum_{j\in N_i} a_{ij}\left(\epsilon_i(t) - \epsilon_j(t)\right) = 0$ for $i \in L$.

Let $x = \left[x_1^T, x_2^T, \cdots, x_n^T, x_{n+1}^T, \cdots x_{n+m}^T\right]^T$, $x_F = \left[x_1^T, x_2^T, \cdots, x_n^T\right]^T$ and $x_L = \left[x_{n+1}^T, \cdots x_{n+m}^T\right]^T$, we have $x = \left[x_F^T, x_L^T\right]^T$. Also, Let $x(t) = \operatorname{col}\left(x_1(t), \ldots, x_{N+M}(t)\right)$, $e^\beta(t) = \operatorname{col}(e_1^\beta(t), \ldots, e_{N+M}^\beta(t))$ and $q^\alpha(t) = \operatorname{col}\left(q_1^\alpha(t), \ldots, q_{N+M}^\alpha(t)\right)$. We can get:

$$q^\alpha(t) = -\left(L \otimes I_n\right)x(t) \quad (10)$$

We can rewrite Eq. 8 as:

$$\begin{aligned}\dot{x}(t) &= \left(I_N \otimes A - L \otimes \mu BB^TP\right)x(t)\\ &+ \left(I_N \otimes \mu BB^TP\right)e^\beta(t)\end{aligned} \quad (11)$$

According to Eq. 11 We choose a Lyapunov function as follow:

$$V(t) = \frac{1}{2}x^T(t)(L \otimes P)x(t) \quad (12)$$

Then calculating $\dot{V}(t)$ we obtain:

$$\begin{aligned}\dot{V}(t) &= x^T(t)(L \otimes P)\left[(I_N \otimes A) - L \otimes \mu BB^TP\right)x(t)\\ &\quad + \left(I_N \otimes \mu PBB^TP\right)e^\beta(t)]\\ &= x^T(t)\left[L \otimes PA - L^2 \otimes \mu PBB^TP\right]x(t)\\ &\quad + x^T(t)\left(L \otimes PBB^TP\right)e^\beta(t)\end{aligned} \quad (13)$$

Let $\hat{A} = \left(PA + A^TP\right)/2, \hat{B} = PBB^TP$ and $\dot{V}(t)$ becomes:

$$\begin{aligned}\dot{V}(t) &= x^T(t)\left[L \otimes \hat{A} - L^2 \otimes \mu\hat{B}\right]x(t)\\ &+ x^T(t)(L \otimes \mu\hat{B})e^\beta(t)\end{aligned} \quad (14)$$

Because the communication graph of the general linear multi-agent system is an undirected graph, so the corresponding Laplacian matrix is a symmetric matrix. Hence, there exist an orthogonal matrix U that satisfies:

$$U^{-1}LU = U^TLU = J = \operatorname{diag}\left(\lambda_1, \ldots, \lambda_N\right)$$

where we can easily see that $U^TU = I_N$ and $L = UJU^T$. Thus we can define:

$$y(t) = \left(U^T \otimes I_N\right)x(t) = \operatorname{col}\left(y_1(t), \ldots, y_N(t)\right)$$

$$\hat{e}^\beta(t) = \left(U^T \otimes I_N\right)\hat{e}^\beta(t) = \operatorname{col}(\hat{e}_1^\beta(t), \ldots, \hat{e}_N^\beta(t))$$

Then, $\dot{V}(t)$ can be rewritten as:

$$\begin{aligned}\dot{V}(t) &= y^T(t)(J \otimes \hat{A} - J^2 \otimes \mu\hat{B})y(t) + y^T(t)(J \otimes \mu\hat{B})\hat{e}^\beta(t)\\ &= \sum_{i=2}^N y_i^T\lambda_i(\hat{A} - \lambda_i\mu\hat{B})y_i(t) + \sum_{i=2}^N y_i^T(t)(\lambda_i\mu\hat{B})\hat{e}^\beta(t)\end{aligned} \quad (15)$$

Because the communication graph $\mathcal{G}$ is connected, there always exists at least $P > 0$ one solution for the following inequality:

$$PA + A^TP - \gamma\mu PBBP + \delta\mu I_n \leq 0$$

where $0 < \gamma < 2\lambda_2$, $\delta \geq 2\lambda_N$ and $\mu > 0$, which λ_2 and λ_N are the Fiedler eigenvalue and the largest eigenvalue of the corresponding Laplacian matrix $\mathcal{L}$, separately.

Because $\hat{B} \geq 0$, for any $i \in \{2, \ldots, N\}$, we can obtain the inequality: $\hat{A} - \lambda_i \mu \hat{B} \leq \hat{A} - \frac{\gamma}{2}\mu\hat{B} \leq -\frac{\delta}{2}\mu I_n \leq -\lambda_i \mu I_n$ For any $\kappa > 0$, and any $\xi, \zeta \in R^n$, because of the inequality $\|\xi\| \cdot \|\zeta\| \leq \frac{\kappa}{2}\|\xi\|^2 + \frac{1}{2\xi}\|\zeta\|^2$, we can get:

$$\begin{aligned}
\dot{V}(t) \leq & \sum_{i=2}^{N} -\mu\lambda_i y_i^T(t) y_i(t) + \sum_{i=2}^{N} \mu\lambda_i y_i^T(t)\hat{B}\hat{e}_i^{\beta}(t) \\
\leq & -\mu \sum_{i=2}^{N} \lambda_i^2 \|y_i(t)\|^2 + \mu\|\hat{B}\|(\frac{\kappa}{2}\sum_{i=2}^{N}\lambda_i^2\|y_i(t)\|^2 \\
& + \frac{1}{2\kappa}\sum_{i=2}^{N}\|\hat{e}_i^{\beta}(t)\|^2) \\
\leq & -\mu\left(1 - \frac{\kappa\rho}{2}\right)\sum_{i=2}^{N}\lambda_i^2\|y_i(t)\|^2 + \frac{\mu\rho}{2\kappa}\sum_{i=2}^{N}\|\hat{e}_i^{\beta}(t)\|^2
\end{aligned} \tag{16}$$

where $\rho = \|\hat{B}\|$, and $\kappa > 0$. Since $q^{\alpha}(t) = -(L \otimes I_N)\, x(t)$, we can get:

$$\begin{aligned}
&\textstyle\sum_{i=1}^{N} \|\hat{e}_i^{\beta}(t)\|^2 = \left(e^{\beta}(t)\right)^T e^{\beta}(t) = \sum_{i=1}^{N}\|e_i^{\beta}(t)\|^2 \\
&\textstyle\sum_{i=1}^{N} \|q_i^{\alpha}(t)\|^2 = x^T(t)\left(L^2 \otimes I_N\right)x(t) = \sum_{i=2}^{N}\lambda_i^2\|y_i(t)\|^2
\end{aligned}$$

So we can rewrite inequality 16 as:

$$\dot{V}(t) \leq -\mu\left(1 - \frac{\kappa\rho}{2}\right)\sum_{i=1}^{N}\|q_i^{\alpha}(t)\|^2 + \frac{\mu\rho}{2\kappa}\sum_{i=1}^{N}\|e_i^{\beta}(t)\|^2$$

From the triggering function $h\left(e_i(t), q_i(t)\right)$, it is clear that

$$\|e_i^{\beta}(t)\| \leq \left(\frac{\sigma_i \cdot \kappa(2 - \kappa\rho)}{\rho}\right)^{\frac{1}{2}} \|q_i^{\alpha}(t)\|$$

Since for all $i \in N$, $\sigma_i \in \left(0, \min\left(1, \rho^2\right)\right)$, we can have:

$$\dot{V}(t) \leq -\mu\left(1 - \frac{\kappa\rho}{2}\right)\sum_{i=1}^{N}(1 - \sigma_i)\|q_i^{\alpha}(t)\|^2$$

Because of $\left(q^{\alpha}\right)^T q^{\alpha} = x^T(t)\left(L^2 \otimes I_N\right)x(t)$, we let $z(t) = \left(L^{1/2} \otimes I^N\right)$, which shows that $x(t)\left(L^{1/2} \cdot L \cdot L^{1/2} \otimes I_N\right) = z^T(t)\left(L \otimes I_N\right)z(t)$. Owing to $1_N^T \cdot L \cdot 1_N = \left(L^{1/2} \cdot 1_N\right)^T \cdot \left(L^{1/2} \cdot 1_N\right) = 0$, we can see that $L^{1/2} \cdot 1_N = 0$ or $1_N^T L^{1/2} = 0$, and $1_N^T \cdot z(t) = 1_N^T\left(L^{1/2} \otimes I_N\right)x(t) = 0$. Therefore, we can obtain:

$$\begin{aligned}
z^T(t)\left(L \otimes I_N\right)z(t) &\geq \lambda_2 \tau^T(t) z(t) \\
&= \lambda_2 x^T(t)\left(L \otimes I_N\right)x(t) \\
&\geq 2\lambda_2 V(t)/\omega_{max}
\end{aligned} \tag{17}$$

where ω_{max} is the maximal eigenvalue of the matrix P.

Since $\|q_i^{\alpha}(t)\| \geq 0$, and we assume that every agent has own state, we can have $\|q_i^{\alpha}(t)\| > 0$, and $\lim_{x\to\infty}\|q_i^{\alpha}(t)\| \neq 0$. Hence, we can get:

$$\begin{aligned}
\dot{V}(t) \leq & -\mu\left(1 - \frac{\kappa\rho}{2}\right)\sum_{i=1}^{N}(1 - \sigma_i)\|q_i^{\alpha}(t)\|^2 \\
& = -\mu\left(1 - \frac{\kappa\rho}{2}\right)\sum_{i=1}^{N}(1 - \sigma_i)\|q_i^{\alpha}(t)\|^{2-2\alpha}\|q_i^{\alpha}(t)\|^{2\alpha} \\
& \leq -\mu\left(1 - \frac{\kappa\rho}{2}\right)\sum_{i=1}^{N}(1 - \sigma_i)\|q_i^{\alpha}(t)\|^{2-2\alpha}\left(\left(q_i^{\alpha}(t)\right)^T q_i^{\alpha}(t)\right)^{\alpha} \\
& \leq -\mu\left(1 - \frac{\kappa\rho}{2}\right)\sum_{i=1}^{N}(1 - \sigma_i)\|q_i^{\alpha}(t)\|^{2-2\alpha}\left(2\lambda_2 V(t)/\omega_{max}\right)^{\alpha} \\
& \leq -\mu\left(1 - \frac{\kappa\rho}{2}\right)(1 - \overline{\sigma}_i)\|q_i^{\alpha}(t)\|_{min}^{2-2\alpha}\frac{2^{\alpha}\lambda_2^{\alpha}}{\omega_{max}^{\alpha}}V^{\alpha}(t) \\
& \leq -\overline{\overline{\varsigma}}V^{\alpha}(t)
\end{aligned} \tag{18}$$

where ω_{max} is the maximal eigenvalue of the matrix P, $\overline{\sigma_i} = \max\{\overline{\sigma_i}, i = 1, \ldots, N + M\}$, $\|q_i^{\alpha}(t)\|_{\min} = \min\{\|q_i^{\alpha}(t)\|, i = 1, \ldots, N + M\}$ and

$$\overline{\overline{\varsigma}} = \mu\left(1 - \frac{\kappa\rho}{2}\right)(1 - \overline{\sigma}_i)\|q_i^{\alpha}(t)\|_{\min}^{2-2\alpha}\frac{2^{\alpha}\lambda_2^{\alpha}}{\omega_{max}^{\alpha}}$$

According to Lemma 2, we can conclude that $V(t) \to 0$ in finite time:

$$T(x(0)) \leq \frac{V^{1-\alpha}(x(0))}{\overline{s}(1 - \alpha)}$$

Since $q^{\alpha}(t) = L_{FF}x_F + L_{FL}x_L$ and we have $\lim_{T\to t} L_{FF}x_F + L_{FL}x_L = 0$, we can get

$$\lim_{t\to T} x_F = -L_{FF}^{-1}L_{FL}x_L \tag{19}$$

Because $-L_{FF}^{-1}L_{FL}1_{m\times 1} = 1_{n\times 1}$, we draw a conclusion that the followers' states come into the leaders' convex hull in finite time T. Hence, the event-based finite time containment consensus problem for general linear dynamics is solved. The proof is completed.

IV. Conclusion

In this paper, we considered the event-triggered finite-time containment control problem for multi-agent systems with general linear dynamics. We proposed an event-triggered control protocol with leaders, which decreases the times of the controller updates and lets the agents achieved consensus in a finite time. It is proved that the MAS reachs consensus for all initial conditions. In the future research, the event-triggered finite-time containment control problem for multi-agent systems with leaders having complex input and disturbance, switching topology and time delays would be investigated.

References

[1] B. Tang, G.-P. Liu, and W.-H. Gui, "Improvement of state feedback controller design for networked control systems," *IEEE Transactions on Circuits and Systems II: Express Briefs*, vol. 55, no. 5, pp. 464–468, 2008.

[2] J. Russell Carpenter, "Decentralized control of satellite formations," *International Journal of Robust and Nonlinear Control*, vol. 12, no. 2-3, pp. 141–161, 2002.

[3] H. G. Tanner, A. Jadbabaie, and G. J. Pappas, "Flocking in fixed and switching networks," *IEEE Transactions on Automatic control*, vol. 52, no. 5, pp. 863–868, 2007.

[4] W. Wenkai and P. Huanxin, "Flocking control with communication noise based on second-order distributed consensus algorithm," in *Power Engineering and Automation Conference (PEAM), 2012 IEEE*. IEEE, 2012, pp. 1–4.

[5] J.-L. Wang and H.-N. Wu, "Leader-following formation control of multi-agent systems under fixed and switching topologies," *International Journal of Control*, vol. 85, no. 6, pp. 695–705, 2012.

[6] Z. Meng, Z. Lin, and W. Ren, "Leader–follower swarm tracking for networked lagrange systems," *Systems & Control Letters*, vol. 61, no. 1, pp. 117–126, 2012.

[7] V. Gazi, "Swarm aggregations using artificial potentials and sliding-mode control," *IEEE Transactions on Robotics*, vol. 21, no. 6, pp. 1208–1214, 2005.

[8] J. Hu, G. Chen, and H.-X. Li, "Distributed event-triggered tracking control of leader-follower multi-agent systems with communication delays," *Kybernetika*, vol. 47, no. 4, pp. 630–643, 2011.

[9] X. He, Q. Wang, and W. Yu, "Finite-time containment control for second-order multiagent systems under directed topology," *IEEE Transactions on Circuits and Systems II: Express Briefs*, vol. 61, no. 8, pp. 619–623, 2014.

[10] H. Liu, G. Xie, and L. Wang, "Necessary and sufficient conditions for containment control of networked multi-agent systems," *Automatica*, vol. 48, no. 7, pp. 1415–1422, 2012.

[11] X. Wang, S. Li, and P. Shi, "Distributed finite-time containment control for double-integrator multiagent systems," *IEEE Transactions on Cybernetics*, vol. 44, no. 9, pp. 1518–1528, 2014.

[12] Y. Fan, G. Feng, Y. Wang, and C. Song, "Distributed event-triggered control of multi-agent systems with combinational measurements," *Automatica*, vol. 49, no. 2, pp. 671–675, 2013.

[13] P. Tabuada, "Event-triggered real-time scheduling of stabilizing control tasks," *IEEE Transactions on Automatic Control*, vol. 52, no. 9, pp. 1680–1685, 2007.

[14] J. CortéS, "Finite-time convergent gradient flows with applications to network consensus," *Automatica*, vol. 42, no. 11, pp. 1993–2000, 2006.

[15] G. Chen, F. L. Lewis, and L. Xie, "Finite-time distributed consensus via binary control protocols," *Automatica*, vol. 47, no. 9, pp. 1962–1968, 2011.

[16] H. Zhang, D. Yue, X. Yin, S. Hu, and C. xia Dou, "Finite-time distributed event-triggered consensus control for multi-agent systems," *Information Sciences*, vol. 339, pp. 132–142, 2016.

[17] Y. Sun, Z. Li, and D. Ma, "Event-based finite time containment control for multi-agent systems," in *Chinese Association of Automation (YAC), Youth Academic Annual Conference of*. IEEE, 2016, pp. 129–134.

[18] Z. Cao, C. Li, X. Wang, and T. Huang, "Finite-time consensus of linear multi-agent system via distributed event-triggered strategy," *Journal of the Franklin Institute*, 2018.

[19] Y. Cao and W. Ren, "Containment control with multiple stationary or dynamic leaders under a directed interaction graph," in *Decision and Control, 2009 held jointly with the 2009 28th Chinese Control Conference. CDC/CCC 2009. Proceedings of the 48th IEEE Conference on*. IEEE, 2009, pp. 3014–3019.

[20] R. T. Rockafellar, *Convex analysis*. Princeton university press, 2015.

[21] Z. Meng, W. Ren, and Z. You, "Distributed finite-time attitude containment control for multiple rigid bodies," *Automatica*, vol. 46, no. 12, pp. 2092–2099, 2010.

[22] S. P. Bhat and D. S. Bernstein, "Finite-time stability of continuous autonomous systems," *SIAM Journal on Control and Optimization*, vol. 38, no. 3, pp. 751–766, 2000.

[23] V. Kucera, "A contribution to matrix quadratic equations," *IEEE Transactions on Automatic Control*, vol. 17, no. 3, pp. 344–347, 1972.

[24] S. Li, H. Du, and X. Lin, "Finite-time consensus algorithm for multi-agent systems with double-integrator dynamics," *Automatica*, vol. 47, no. 8, pp. 1706–1712, 2011.