Miscellaneous

@t-34400

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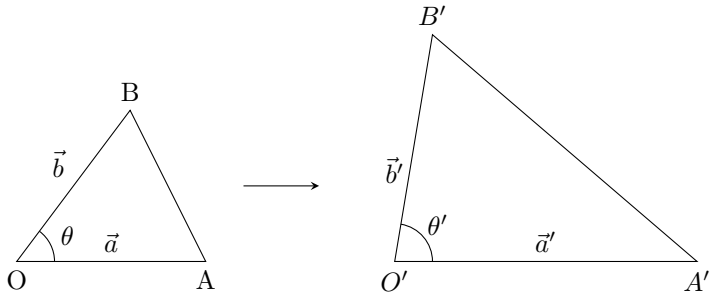
1 Strain

Definition 1.1. strain matrix

1.1 Triangle mesh (2D strain)

Let the infinitesimal triangular surface OAB be deformed into an infinitesimal triangular surface $O'A'B'$.



When considering strain, the rigid body transformation part can be ignored, so it can be assumed that O and O' are the same point, and O, A and A' are colinear, and OAB and $OA'B'$ are in the same plane. Then we define the x_0-axis as the OA direction and the x_1-axis so that the x_0x_1 plane contains the triangles OAB and $OA'B'$. In this case, the normal strains and shear strains

are described as follows:

$$\begin{aligned} \varepsilon_{00} &= \frac{|\vec{a}'| - |\vec{a}|}{|\vec{a}|} \\ \varepsilon_{11} &= \frac{b_1' - b_1}{b_1} \\ \varepsilon_{10} = \varepsilon_{01} &= \frac{b_0' - (1 + \varepsilon_{00})b_0}{2b_1} \end{aligned} \tag{1}$$

The principal strains are defined as the eigenvalues of the strain tensor, so the principal strains are as follows:

$$\varepsilon_{\max}, \varepsilon_{\min} = \frac{1}{2}\left(\varepsilon_{00} + \varepsilon_{11} \pm \sqrt{(\varepsilon_x + \varepsilon_y)^2 - 4(\varepsilon_{00}\varepsilon_{11} - \varepsilon_{01}\varepsilon_{10})}\right) \tag{2}$$

1.1.1 Shader program

Please refer to **SampleCode/PrincipalStrainShader** for a sample code of shaders that calculates the 2D principal strains from the original and current vertex positions, and color meshes based on their principal strains.