Miscellaneous

@t-34400

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Contents

1 Strain

Definition 1.1. strain matrix

1.1 Trianlge mesh (2D strain)

Let the infinitesimal triangular surface OAB be deformed into an infinitesimal triangular surface $OA'B'$. When considering strain, the rigid body transformation part can be ignored, so it can be assumed that O, A and A' are colinear and OAB and $OA'B'$ are in the same plane.

In this case, the normal strains are described as follows:

$$\begin{aligned} \varepsilon_{00} &= \frac{OA' - OA}{OA} \\ \varepsilon_{11} &= \frac{OB' - OB}{OB} \end{aligned} \tag{1}$$

and the shear strain is described as follows:

$$\gamma_{xy} = \gamma_{yx} = \frac{1}{2}\tan\left(\angle AOB - \angle A'OB'\right) \tag{2}$$

The principal strains are defined as the eigenvalues of the strain tensor:

$$\begin{pmatrix} \varepsilon_x & \gamma_{xy} \\ \gamma_{yx} & \varepsilon_y \end{pmatrix} \tag{3}$$

so the principal strains are as follows:

$$\varepsilon_{\max}, \varepsilon_{\min} = \frac{1}{2}\left(\varepsilon_x + \varepsilon_y \pm \sqrt{(\varepsilon_x + \varepsilon_y)^2 + 4(\varepsilon_x\varepsilon_y - \gamma_{xy}\gamma_{yx})}\right) \tag{4}$$

1.1.1 Shader program

Please refer to **SampleCode/PrincipalStrainShader** for a sample code of shaders that calculates the 2D principal strains from the original and current vertex positions, and colors them based on their magnitude.