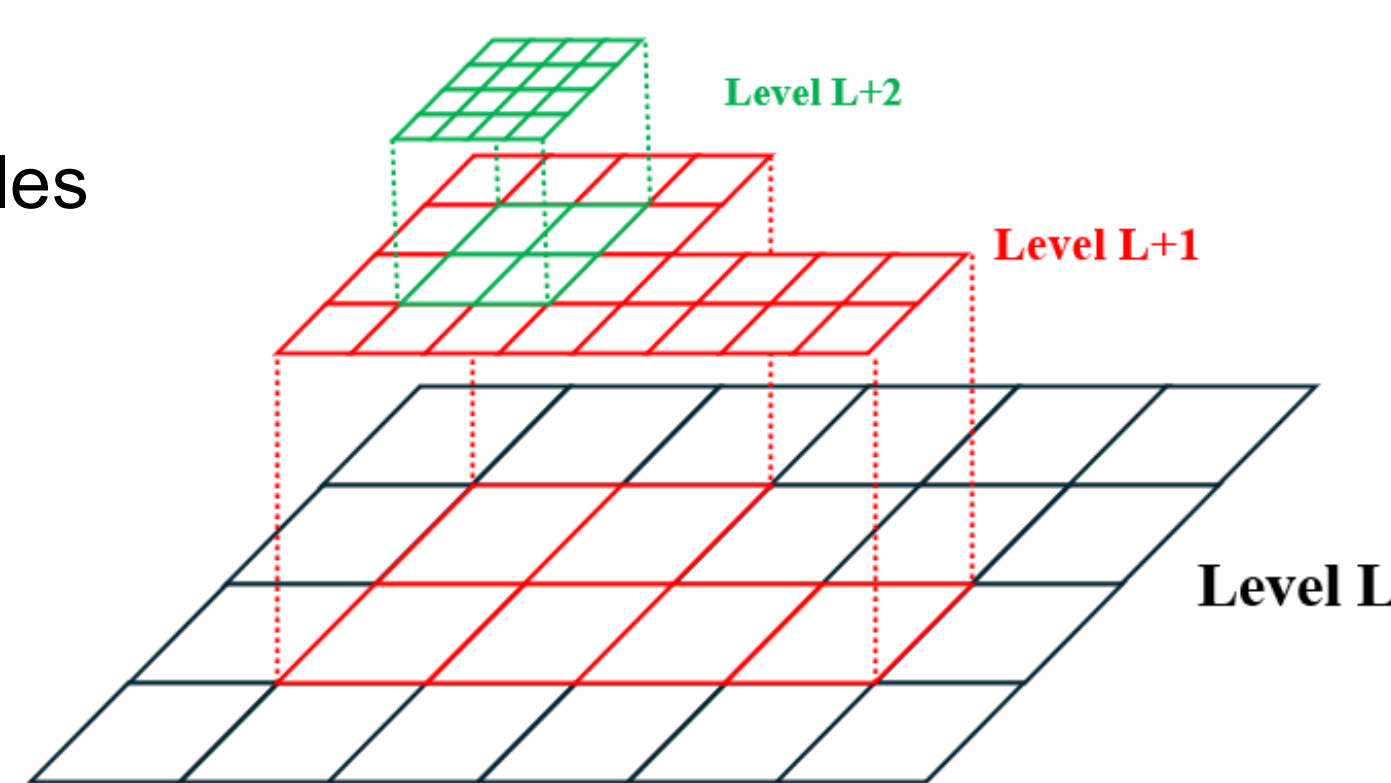


## ABSTRACT

We have developed a new Adaptive Mesh Refinement (AMR) version of the Gauss-Law satisfying Energy Conserving Semi-Implicit Model (GL-ECSIM) and implemented it into the Flexible Exascale Kinetic Simulator (FLEKS). Unlike the earlier Multi-Level-Multi-Domain method by Innocenti+, the new algorithm uses a single set of particles over the whole domain. Particles are split and merged as needed by efficient and accurate methods. The coarser level receives both the field information and the phase space distribution (through the particles) from the fine level. The fine level uses the coarse level as boundary condition. The new algorithm satisfies Gauss Law on the entire domain, including grid resolution changes. We show various tests confirming the accuracy and robustness of the new algorithm. In particular, we simulate magnetic reconnection with an ion-electron mass ratio of 25. The two extra levels of AMR resolve the electron scale near the reconnection site, while the grid is four times coarser elsewhere matching the ion scales.

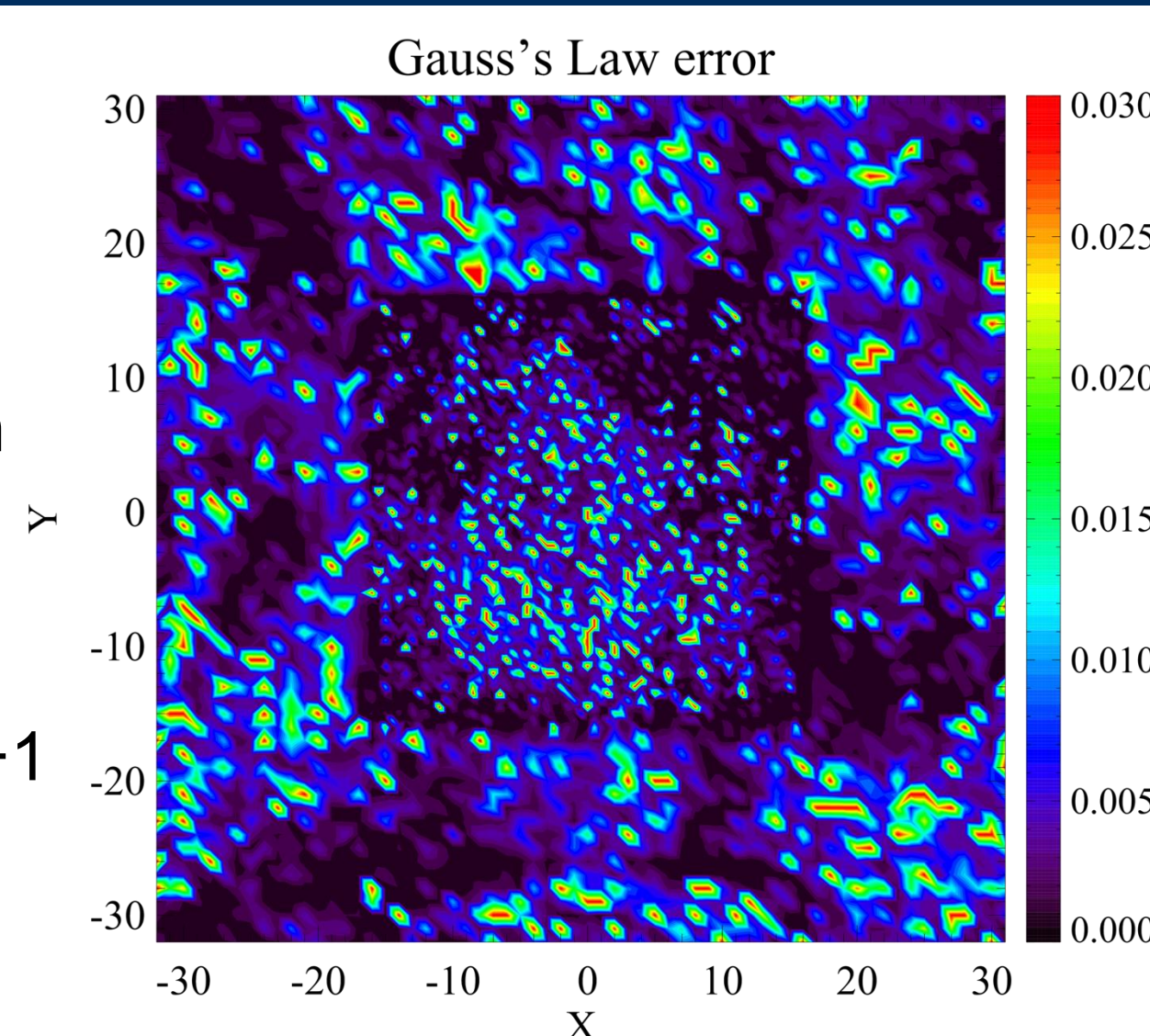
## ALGORITHM

- Grid is initialized with electric/magnetic fields and particles
- Particles exist on the finest level only
- Particles deposit current on all underlying grid levels using their respective shape functions
- After current deposition, electric and magnetic fields are updated on the coarsest level L
- The updated value of EM fields on level L act as boundary condition for level L+1
- After updating level L+1, EM fields from level L+1 are projected down back to level L, which ensures that both levels always have consistent solutions
- Particles are moved using the new field data
- Number of particles per cell is controlled by particle splitting and merging. A novel merging algorithm is used that ensures conservation of mass, momentum and energy while minimizing change in the weight of individual particles [Chen+ 2024]
- Gauss law error is corrected by introducing a small displacement to the particle positions. Using a **novel algorithm, we can correct Gauss law error** perfectly even around the refinement interface



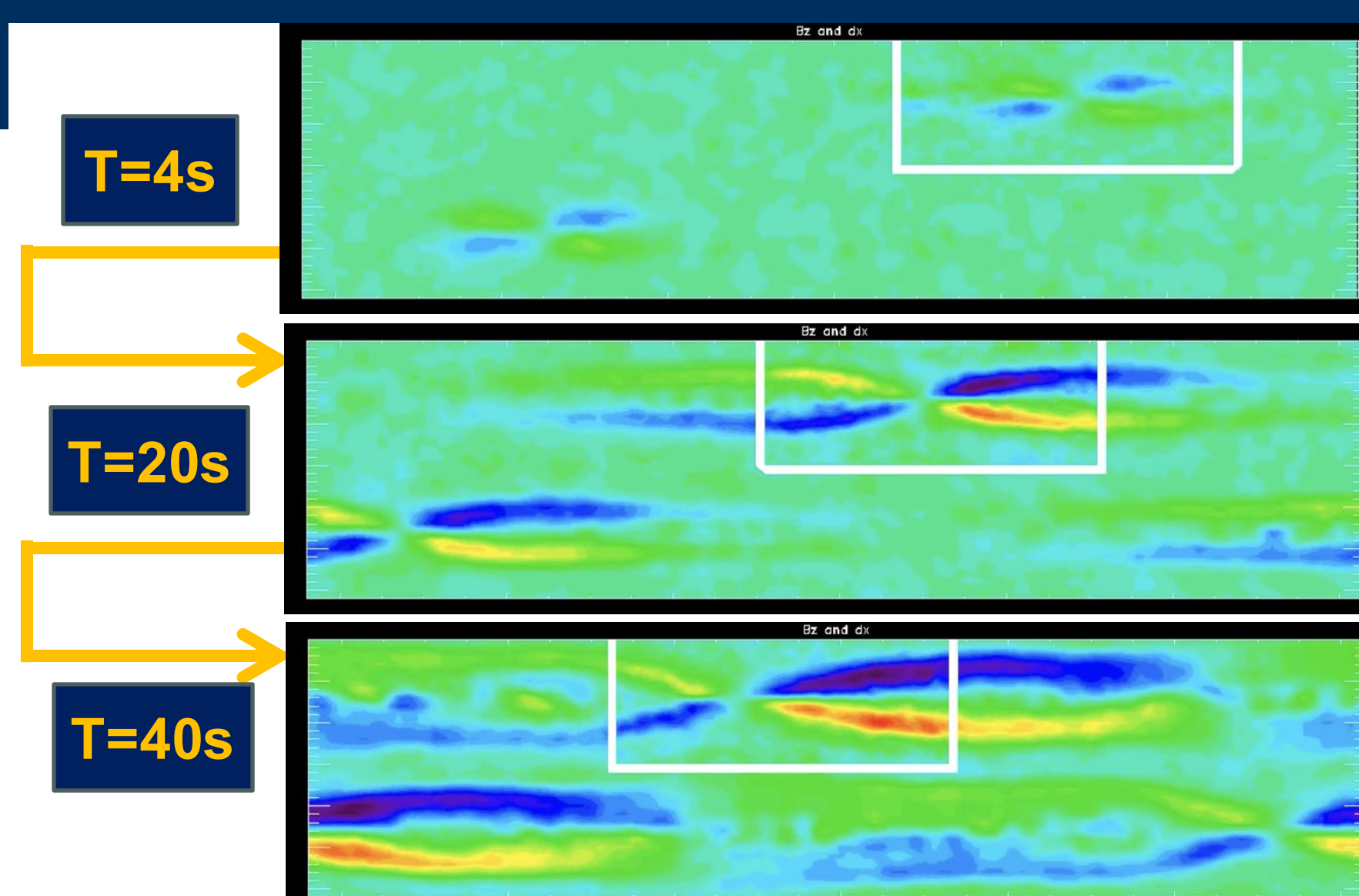
## Gauss Law / Charge Conservation

- Numerical inaccuracies lead to build up of errors in Gauss law satisfaction
- Our method applies a slight displacement to particles such that Gauss law error is eliminated/reduced [Chen+ 2019]
- The method is now generalized to multiple grid levels :
  - Particles at level N+1 are adjusted to satisfy Gauss law
  - At level N only the particles that don't influence level N+1 are adjusted



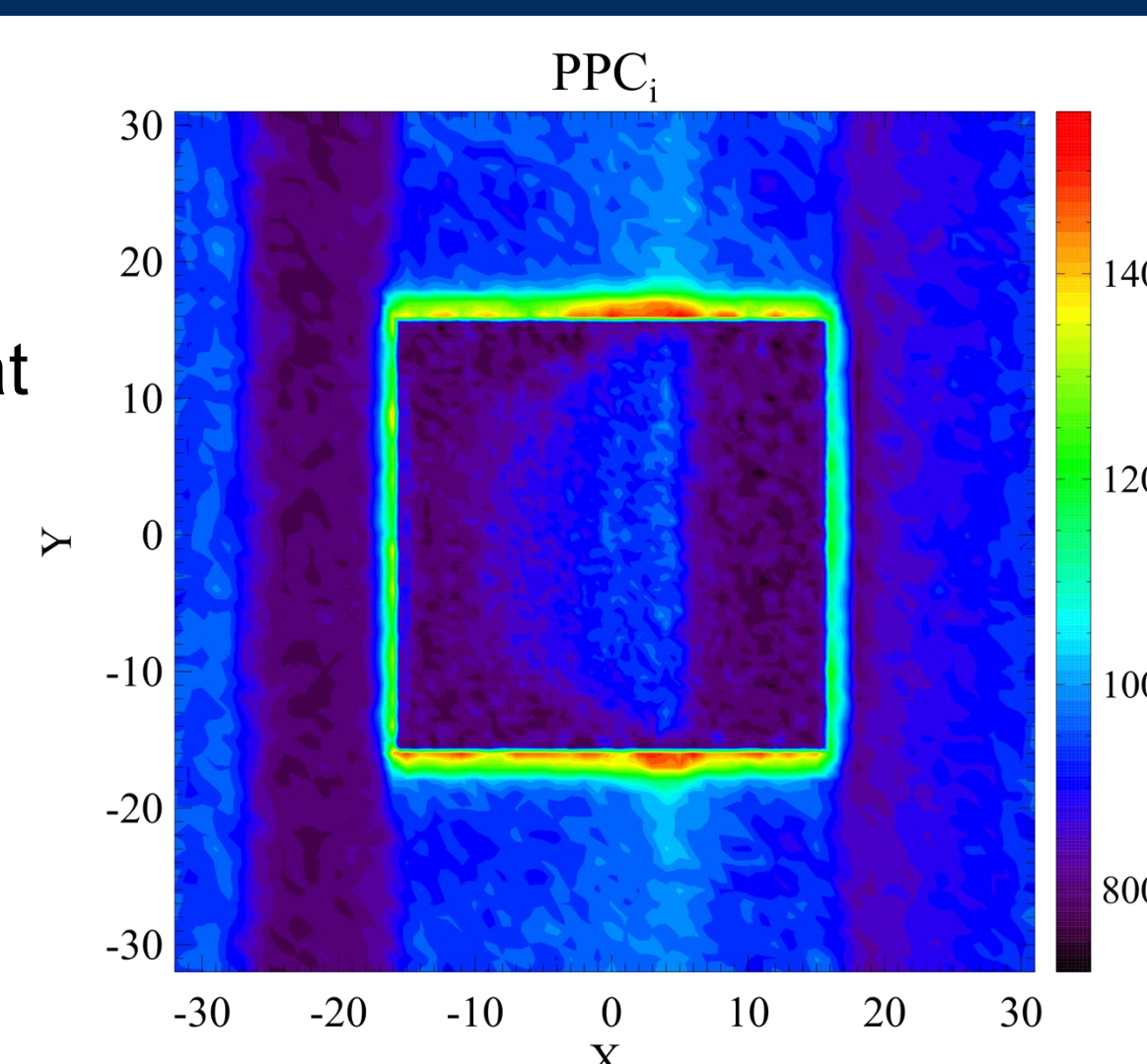
## Dynamic Mesh Refinement

- Refinement regions are dynamic and can be used to track structures of interest
- Left moving symmetric reconnection is shown. The refinement moves left and always keeps the reconnection site in focus



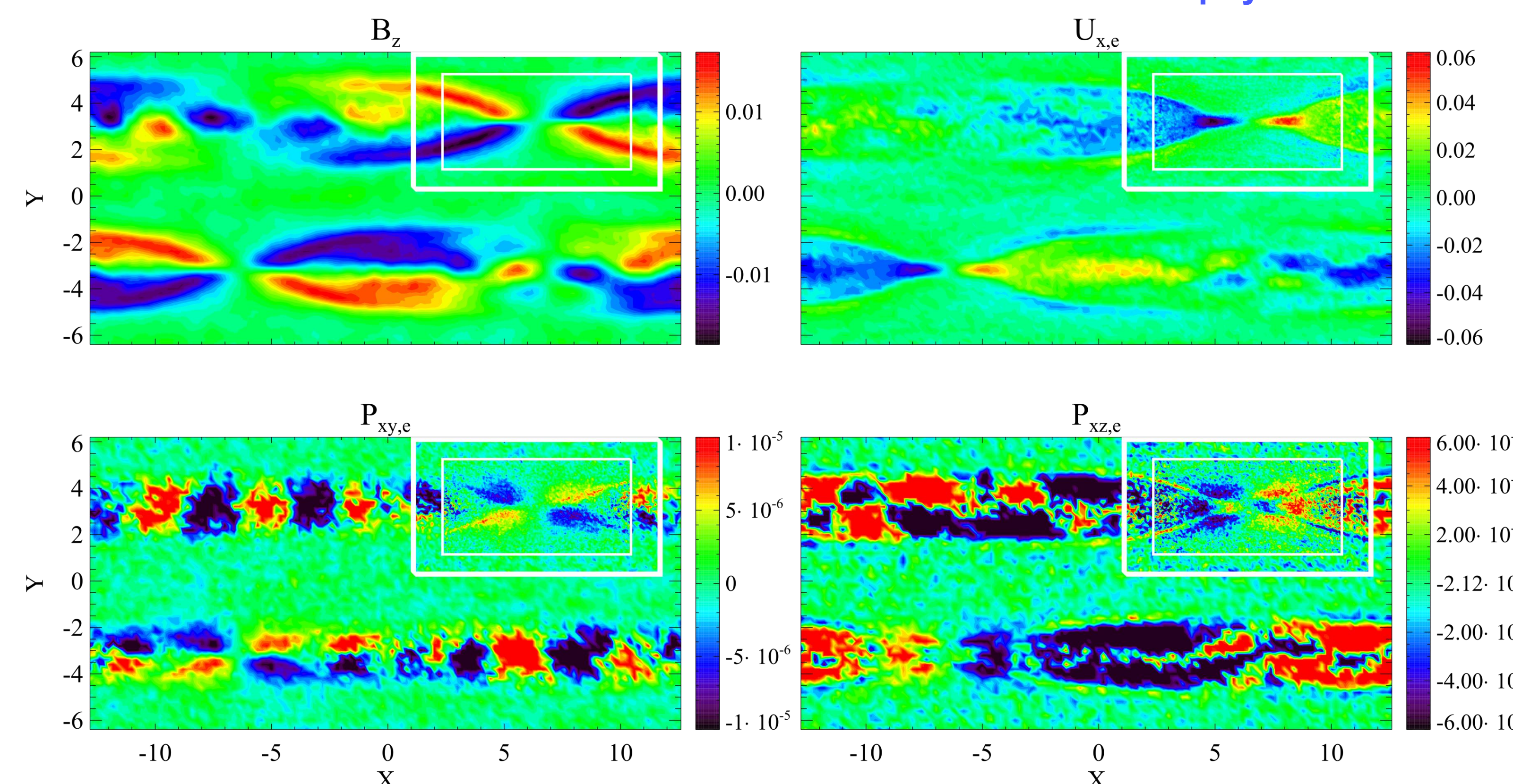
## Particle merging/splitting

- Particles are split to increase the number of particles per cell
- Merging involves converting N particles to M particles such that mass, momentum and energy are conserved.
- Merging uses a Lagrange multiplier method to minimize the relative change in the masses of the remaining M particles.
- Number of particles per cell increases drastically as fine level particles cross into the coarse region, which necessitates aggressive merging to keep the code efficient



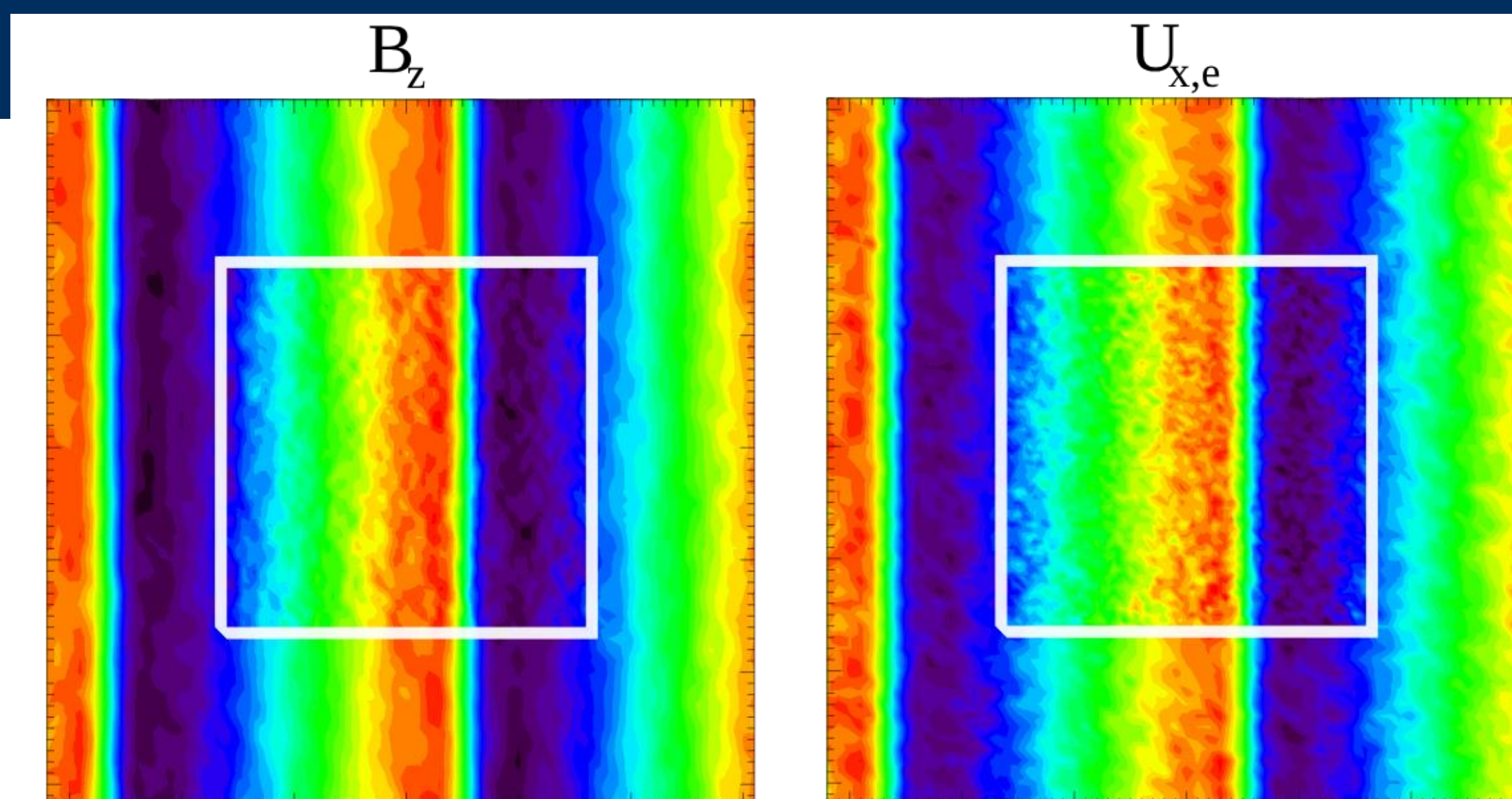
## 2D Reconnection Test

- 2D double current sheet reconnection test with only one reconnection site refined.
- White rectangular boxes show refinement level changes. The refinement ratio is 2.
- There are 2 levels of refinement so the grid resolution on the finest grid is 4 times higher than on the coarsest grid, matching the ratio of ion and electron inertial lengths for mass ratio 64.
- $B_z$ , electron velocity  $U_{x,e}$  and off-diagonal terms of electron pressure tensor are shown.
- The reconnection site with AMR shows well resolved electron-scale physics.**



## 2D Fast Wave Test

- 2D fast wave test with the center of domain refined.
- There is 1 refinement level with resolution 2 times greater than the base grid.
- The fast wave moves through the resolution changes without any artifacts.



## Summary

- A semi-implicit particle in cell code with adaptive mesh refinement has been developed including multi-level dynamic refinement.
- Particles can cross over between grid levels with minimal errors.
- Number of particles per cell is maintained with sophisticated merging and splitting algorithms.
- Gauss law is satisfied at all refinement levels including resolution changes.
- The method has been verified by 2D fast wave and reconnection tests.

## Future Steps

- Apply MHD with embedded AMR PIC to 3D magnetosphere simulations.

## Paper Preprint

\* Paper is under review



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