# SEMI-IMPLICIT PARTICLE-IN-CELL METHOD WITH ADAPTIVE MESH REFINEMENT FOR KINETIC PLASMA SIMULATION IN SPACE WEATHER


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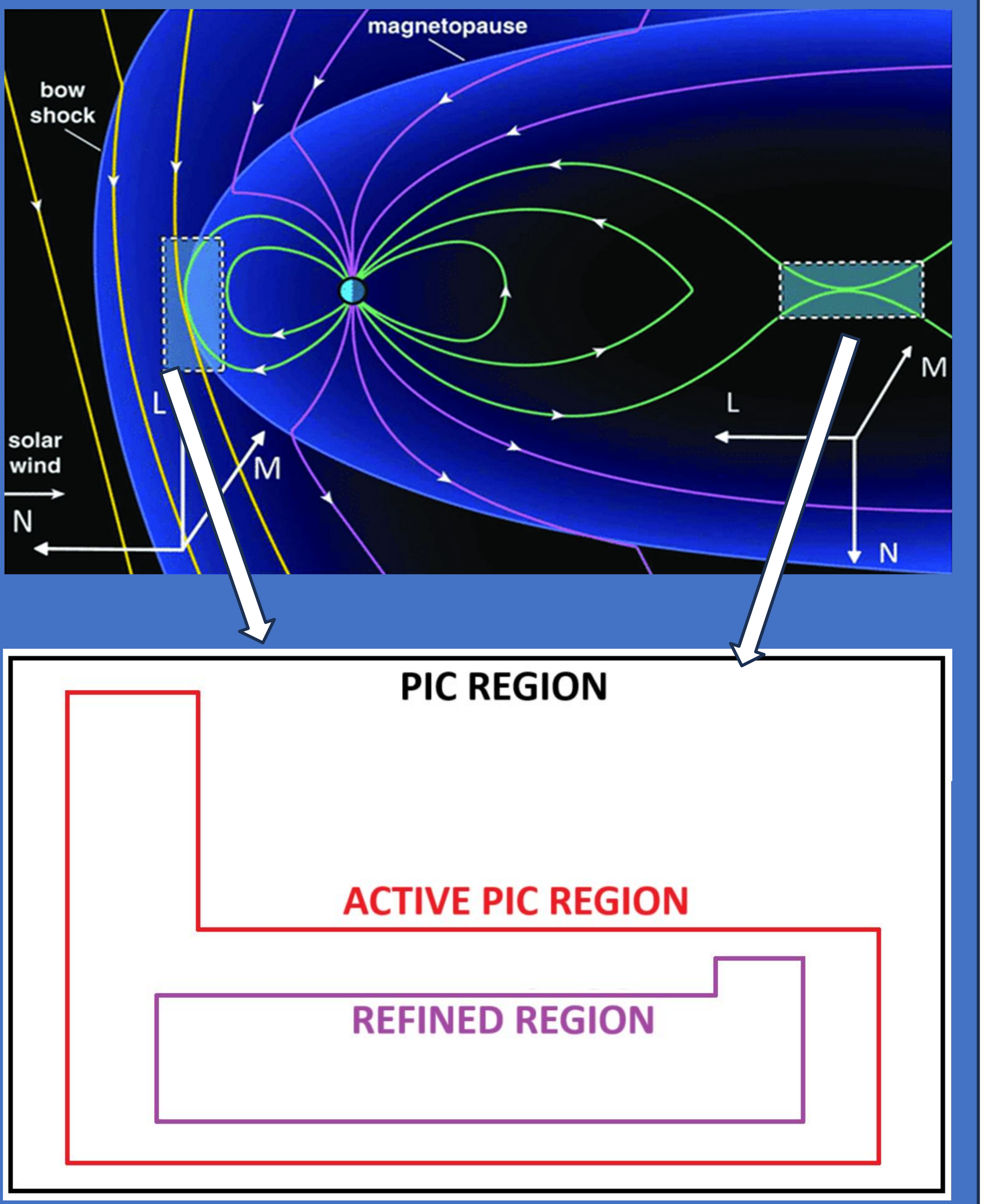
## ABSTRACT

Magnetic reconnection that involves the electron scales plays a crucial role in space physical systems. We develop an efficient numerical model to capture electron scales and global scales at the same time. We introduce adaptive mesh refinement (AMR) into our semi-implicit particle-in-cell (PIC) code FLEKS (Flexible Exa-scale Kinetic Simulator). Since resolving electron skin depth across the entire PIC domain is computationally very expensive, the grid will be refined around the electron diffusion region to resolve the electron skin depth, while only the ion inertial length is fully resolved elsewhere.


## INTRODUCTION

In our previous work MHD models are used to simulate the bulk of the domain while kinetic models like PIC are used only in small regions. MHD with Embedded PIC (MHD-EPIC), is one such coupled model, but the PIC region can only be rectangular. MHD with Adaptively Embedded PIC (MHD-AEPIC) improved this and allows the active PIC region to be any shape and adapt to the region of interest. Our current effort further improves on MHD-AEPIC and adds AMR to the active PIC region and is therefore called MHD with Adaptively Embedded Adaptive PIC (MHD-AEAPIC). Multilevel iPIC3D by Innocenti+ (2013) shows that this is feasible.

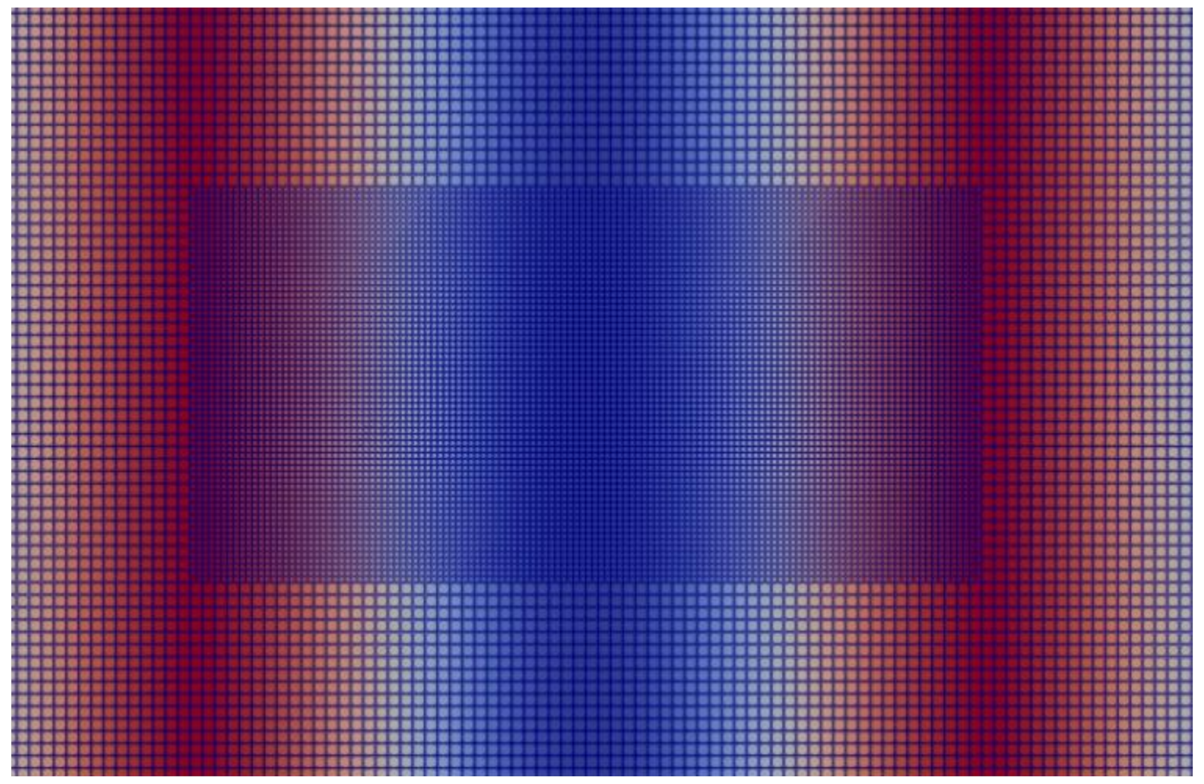



## SEMI-IMPLICIT PARTICLE IN CELL METHOD

Particle-in-cell methods involve tracking particles in continuous phase space while the moments (charge density, current density, pressure) and electromagnetic fields are computed on a grid. Explicit PIC, while simple and accurate, has stringent temporal and spatial resolution restrictions, which makes it inefficient for large 3D simulations. Fully implicit PIC relaxes the resolution restrictions, but it is complex and still computationally expensive. Semi-Implicit PIC solves the electric/magnetic fields implicitly, but particle locations and velocities are evolved explicitly. This enables us to relax the strict resolution criteria of explicit PIC, maintain simplicity and reduce computational cost by orders of magnitude.

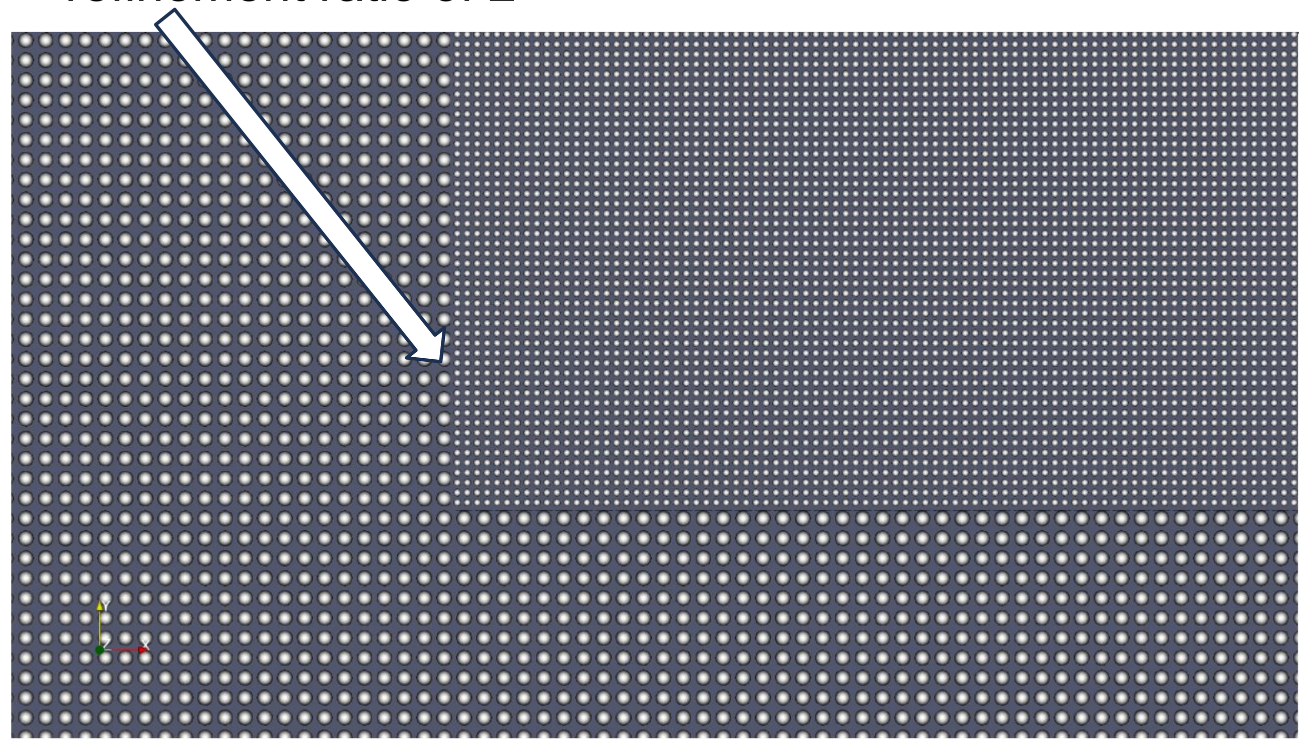
### 1 Initial Conditions

On the base grid the initial plasma parameters and electromagnetic fields are filled in from the MHD solution. For refined regions, the initial conditions are bilinearly interpolated from the coarse region.



### 2 Particle Generation

Particles are generated in each cell based on a (bi-)Maxwellian distribution. Total number of particles per cell is kept constant across all levels. Number of particles per unit volume jumps by a factor of 8 for refinement ratio of 2



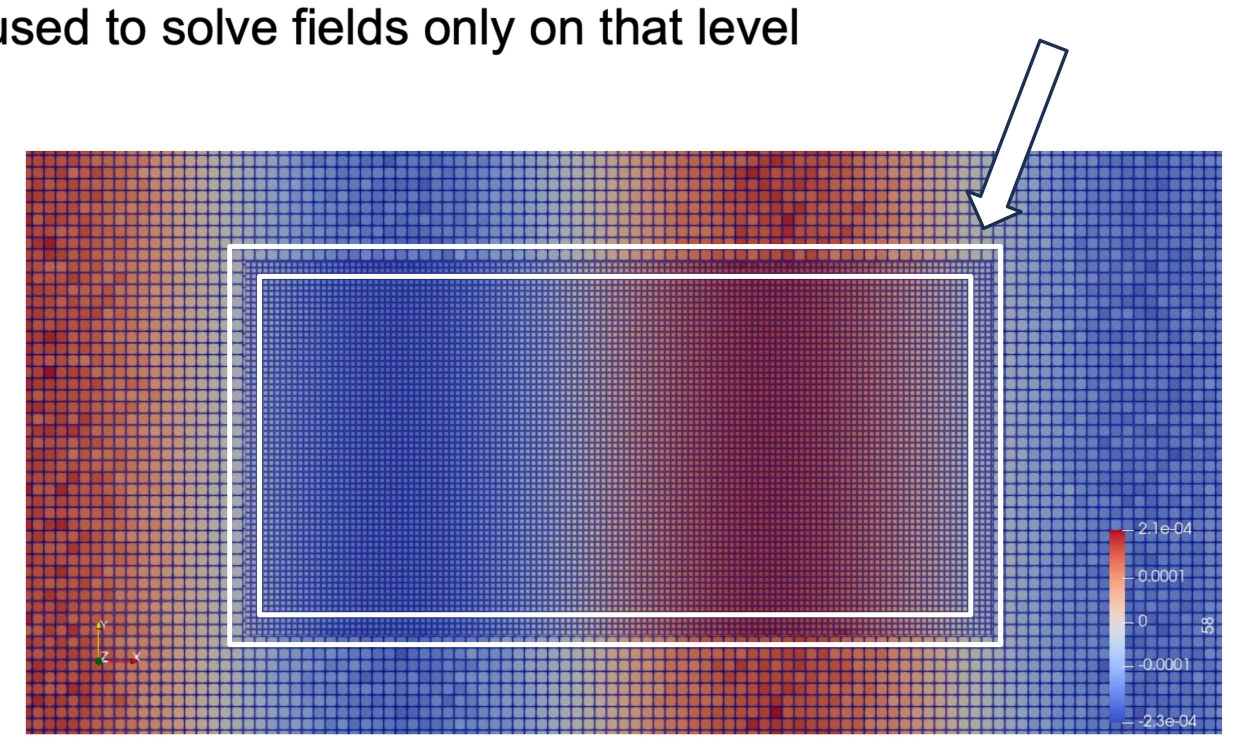
### 3 Field Solver

Electric field and current density are solved implicitly:

$$\mathbf{E}^{n+\theta} + \delta^2\left[\nabla\left(\nabla\cdot\mathbf{E}^{n+\theta}\right) - \nabla^2\mathbf{E}^{n+\theta}\right] = \mathbf{E}^n + \delta\left(\nabla\times\mathbf{B}^n - \frac{4\pi}{c}\bar{\mathbf{J}}\right)$$

$$\bar{\mathbf{J}}_{sg} = \frac{1}{V_g}\sum_{\mathrm{p}} q_{\mathrm{p}}\hat{\mathbf{v}}_p W_{pg} + \frac{\beta_s}{V_g}\sum_{g'} M_{s,gg'}\ \mathbf{E}_{g'}^{\mathrm{n}+\theta}$$

Mass Matrix for each level is calculated independently and used to solve fields only on that level



### 4 Particle Mover

Particle locations and velocities are evolved in time explicitly. One particle can only exist on either the refined or coarse grid and is transferred over as it crosses the coarse-fine boundary.

$$\mathbf{x}_p^{n+\frac{1}{2}} = \mathbf{x}_p^{n-\frac{1}{2}} + \Delta t\mathbf{v}_p^n$$

$$\mathbf{v}_p^{n+1} = \mathbf{v}_p^n + \frac{q_p\Delta t}{m_p}\left(\mathbf{E}^{n+\theta}\left(\mathbf{x}_p^{n+\frac{1}{2}}\right) + \bar{\mathbf{v}}_p\times\mathbf{B}^n\left(\mathbf{x}_p^{n+\frac{1}{2}}\right)\right)$$

### 5 Particle Merging / Splitting

The total number of particles per cell is kept within a desired range. This is achieved by merging particles in cells that have too many particles and splitting in cells with too few particles. Both merging and splitting conserve momentum and energy. The splitting/merging reduces statistical noise while keeping computational cost low, and it also improves parallel load balancing.

## DISCUSSION

- The initial conditions for plasma parameters and fields have been interpolated to the refined region successfully.
- Number and weight calculation for particles in the fine region is implemented correctly. The particle velocities are also sampled appropriately using (bi-)Maxwellian distribution
- Particle mover has also been implemented completely. The particle mover also keeps track of the particles and determines which refinement region they belong to.
- The mass matrix in the field solver has been calculated at both levels independently and is used to solve fields for that particular refinement level.
- The implicit AMR electric field solver has been developed and tested
- Code is currently in debugging and testing stage.

## REFERENCES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