Joint Probability Distribution

$X \quad\quad Y$

$P(X=x) = f(x) \quad\quad P(Y=y) = f(y)$

$$P(X=x, Y=y) = f(x,y).$$

Discrete RV

1. $f(x,y) \geq 0$
2. $\sum_x \sum_y f(x,y) = 1$.
3. $P(X=x, Y=y) = f(x,y)$.

$X \backslash Y$	y_1	y_2	y_3	$\cdots$	y_n	
x_1	$f(x_1,y_1)$	$f(x_1,y_2)$	$f(x_1,y_3)$	$\cdots$	$f(x_1,y_n)$	
x_2	$f(x_2,y_1)$	$f(x_2,y_2)$		$\cdots$	$f(x_2,y_n)$	$f_1(x_2)$
x_3	$\vdots$	$\vdots$			$f(x_3,y_n)$	$f_1(x_3)$
$\vdots$	$\vdots$	$\vdots$			$\vdots$	
x_m	$f(x_m,y_1)$	$f(x_m,y_2)$		$\cdots$	$f(x_m,y_n)$	$f_1(x_m)$
	$f_2(y_1)$	$f_2(y_2)$			$f_2(y_n)$	

Row 1

$$\cancel{\sum_{x=} f(x,y)}$$

$Y = 1$

$X = 2$

$Z = 3$

$$\sum_{x=1}\sum_{y=1}^{n} f(x,y) = f(x_1,y_1) + f(x_R,y_2) + f(x_R,y_3) + + f(x_i, y_m)$$

marginal function of x.

$$= f_{1}(x_1).$$

$$f_1(x_1) = \sum_{x=1}\sum_{y=1}^{n} f(x,y).$$

$\sum$ Row $= 1$, $\sum$ colomn $= 1$;

Continuous RV

1. $f(x,y) \geq 0$

2. $\int_{-\infty}^{x}\int_{-\infty}^{y} f(x,y)\, \cancel{dx}\, dx\, dy = 1$;

3. $P(a < X < b,\ c < Y < d) = \int_{x=a}^{b}\int_{y=c}^{d} f(x,y)\, dx\, dy$

The joint probability function of two discrete random variables X and Y is given by $f(x, y) = c(2x + y)$, where x and y can assume all integers such that $0 \le x \le 2$, $0 \le y \le 3$, and $f(x, y) = 0$ otherwise.

$$f(x, y) \begin{cases} c(2x+y), & 0 \le x \le 2,\ 0 \le y \le 3. \\ 0, & \text{otherwise.} \end{cases}$$

X \ Y	0	1	2	3	
$x_1 = 0$	0	c	2c	3c	$= yc = f_1(x_1) = 6c$
$x_2 = 1$	2c	3c	4c	5c	$= c(2 + y) = f_1(x_2) = 14c$
$x_3 = 2$	4c	5c	6c	7c	$= c(4 + y) = f_1(x_3) = 22c$
	$2xc$	$c(2x+1)$	$c(2x+2)$	$c(2x+3)$	

$f_2(y_1) = 6c$, $f_2(y_2) = 9c$, $f_2(y_3) = 12c$, $f_2(y_4) = 15c$.

$$\sum_{i=1} f_i(x_i) = \sum_{i=0}^{2} f_1(x_i) = \sum_{i=0}^{2} \sum_{y} f(x, y)$$

$$= \sum^{2} \sum_{0}^{3} f(x, y) =$$

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$$\sum_{x_i=x_1}^{2(x_3)} f_1(x_i) = f_1(x_1)+f_1(x_2)+f_1(x_3) = 1.$$

$$= 6c+14c+22c$$

$$42c = 1$$

$$c = 1/42;$$

$$\sum_{y_i=y_1}^{y_3} f_2(y_i) = f_2(y_1)+f_2(y_2)+f_2(y_3)+f_2(y_4) = 1$$

$$= 6c+9c+12c+15c$$

$$42c = 1$$

$$c = 1/42$$

$$\sum_{x=0}^{2}\sum_{y=0}^{3} f(x,y) = 1$$

$$f(0,0)+f(0,1)+f(0,2)+f(0,3)+f(1,0)+f(1,1)$$
$$+f(1,2)+f(1,3)+\dots+f(2,3) = 1$$

$$0+c+2c+3c+2c+3c+4c+5c+4c+$$
$$5c+6c+7c = 1$$

$$42c = 1$$

$$c = 1/42;$$

X \ Y	0	1	2	3
0	0	1/42	$\frac{2}{42} = \frac{1}{21}$	$\frac{3}{42} = \frac{1}{14}$
1	$\frac{2}{42} = \frac{1}{21}$	$\frac{3}{42} = \frac{1}{14}$	$\frac{4}{42} = \frac{2}{21}$	5/42
2	$\frac{4}{42} = \frac{2}{21}$	5/42	$\frac{6}{42} = \frac{1}{7}$	

(b) $P(X=2, Y=1) = 5/42$

$$f(x, y) = \begin{cases} \frac{2x+y}{42}, & 0 \le x \le 2, \ 0 \le y \le 3; \\ 0, & \text{otherwise} \end{cases}$$

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$$P(X=2, Y=1) = f(x=2, y=1) = f(2, 1)$$

$$= \frac{2(2)+1}{42} = 5/42$$

$\longrightarrow$

(c) $P(X \geq 1, Y \leq 2)$

$P(X = 1, 2;\ Y = 0, 1, 2$

$= \frac{2}{} + \frac{3}{} + \frac{4}{} + \frac{4}{} + 5/42 + 6/42 = \frac{\cancel{24}}{} = \quad = 4/$

(a) find The marginal probability function of X;

$$P(X=x) = f_1(x) = \sum f_1(x)$$

$$P(X=x, Y=y)$$

$$= f_1(x_1) + f_1(x_2) + f_1(x_3)$$

$$f_1(x) = \begin{cases} f_1(x_1) = 6c = 6\left(\frac{1}{42}\right) = 1/7, & x=0 \\ f_1(x_2) = 14c = 14/42 = 1/3, & x=1 \\ f_1(x_3) = 22c = \frac{22}{42} = 11/21, & x=2 \end{cases}$$

$$= 1/7 + 1/3 + 11/21 \qquad \left(\frac{6}{42} + \frac{14}{42} + \frac{22}{42}\right) = \frac{42}{42}$$

$$= 1$$

$$= 1$$

$$f(x,y) = \begin{cases} cxy, & 0 < x < 4,\ 1 < y < 5 \\ 0, & \text{otherwise} \end{cases}$$

(a) find c (b) find $P(1 < X < 2,\ 2 < Y < 3)$ (c) $P(X \geq 3,\ Y \leq 2)$

(a) $P(S) = \sum f(x) = 1$.

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\,dx\,dy = 1.$$

$$\int_0^4\int_1^5 cxy\,dx\,dy = 1.$$

$$c\int_0^4\left[\int_1^5 xy\,dy\right]dx = 1$$

$$c\int_0^4 \frac{xy^2}{2}\Big|_{y=1}^{5} dx = 1 \qquad F_2(y)$$

$$\int_0^4 \frac{cx}{2}\cdot(5^2-1^2)\,dx = 1$$

$$\int_0^4 12cx\,dx = 1$$

$$12c\cdot\frac{x^2}{2}\Big|_0^4 = 1;\quad 6cx^2\Big|_0^4 = 1$$

$$6c(4^2-0^2) = 1$$

$$c = \frac{1}{6\times 16} = \frac{1}{96}$$

$$f(x,y) = \begin{cases} \frac{xy}{96}, & 0 < x < 4,\ 1 < y < 5 \\ 0 & \text{otherwise} \end{cases}$$

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$P(1<X<2, 2<Y<3)$

$$= \int_{x=1}^{2} \int_{y=2}^{3} \frac{xy}{96}\, dy\, dx$$

$$= \frac{x}{96} \int_{x=1}^{2} \left. \frac{y^2}{2} \right|_2^3 dx = \int \frac{x}{96} \cdot \frac{1}{2} \cdot (3^2 - 2^2) = \int_{x=1}^{2} \frac{x}{192} \cdot 5$$

$$= \frac{5}{192} \cdot \left. \frac{x^2}{2} \right|_1^2 = \frac{5}{384} \cdot 2^2 - 1^2 = \frac{5 \cdot \cancel{3}}{\cancel{384}_{128}}$$

$$= 5/128$$

$P(1<X<2, 2<Y<3) = 5/128$;

$$\int_{-\infty}^{2} = \cancel{\int_{-\infty}^{1}} + \int_{1}^{2}$$

ⓒ $P(X \geq 3, Y \leq 2)$;

$$= \frac{1}{96} \int_{x=3}^{+\infty} \int_{y=\infty}^{+2} xy\, dy\, dx$$

$$= \frac{1}{96} \int_{x=3}^{4} \int_{y=1}^{2} xy\, dy\, dx = \frac{1}{96} \int_{x=3}^{4} x \cdot \int_{y=1}^{2} y\, dy\, dx$$

$$= \frac{1}{96} \int_{x=3}^{4} x \cdot \left. \frac{y^2}{2} \right|_1^2 = \int_{x=3}^{4} \frac{x}{192} \cdot 2^2 - 1^2\, dx$$

$$= \int_{x=3}^{4} \frac{x}{\cancel{192}_{64}} \cdot \cancel{3}\, dx = \frac{1}{64} \int_{x=3}^{4} x\, dx = \frac{1}{64} \int_{x=3} \left. \frac{x^2}{2} \right|_3^4$$

$$= \frac{1}{128} (4^2 - 3^2) = \frac{7}{128};$$

$P(X \geq 3, Y \leq 2) = 7/128$;

$$= \left.\frac{y^2}{24}\right|_1 = \frac{y^2 - 1^3}{24}$$

$$F_1(y) = \begin{cases} 0, & y < 1 \\ \frac{y^2 - 1^2}{24}, & 1 \leq y \leq 5 \\ 1, & y \geq 5 \end{cases}$$

SPECIAL DISTRIBUTIONS

DRV

- Binomial
- Bernoulli
- poisson
- Geometric
- Hypergeometric.

CRV

- Uniform
- standard
- Normal
- Exponential
- Erlang & Gamma
- Weibull
- Lognormal.

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