Expectation

$$E(X) = \mu = \sum_x x f(x)$$

$$= \int_{-\infty}^{\infty} x f(x)\, dx$$

$$\sigma^2 = E(X-\mu)^2 = \int_{-\infty}^{\infty} (\cancel{x}-\mu)^2 f(x)\, dx$$

$$= \int_{-\infty}^{\infty} (x^2 - 2\mu x + \mu^2) f(x)\, dx$$

$$= \int_{-\infty}^{\infty} x^2 f(x)\, dx - 2\mu \int x f(x)\, dx + \mu^2 \int 1 f(x) dx$$

$$= E(X^2) - 2\mu . E(X) + \mu^2 . E(1)$$

$$= E(X^2) - 2E(X)E(X) + (E(X))^2$$

$$= E(X^2) - 2(E(X))^2 + (E(X))^2$$

$$\boxed{\sigma^2 = E(X^2) - (E(X))^2}$$

$$F(x) = \begin{cases} 1 - e^{-2x}, & x \geq 0; \\ 0, & x < 0; \end{cases}$$

(a) density function, $f(x)$

(b) $P(X > 2)$

(c) $P(-3 < X \leq 4)$

$$P(X = x) = \int_{-\infty}^{\infty} f(x)\,dx = f(x)$$

(a)

$$F(x) = \int_{-\infty}^{\infty} f(x)\,dx$$

$$f(x) = \frac{dF(x)}{dx}$$

$$f(x) = \frac{d}{dx}(1 - e^{-2x}) = 2e^{-2x}, \quad x > 0$$

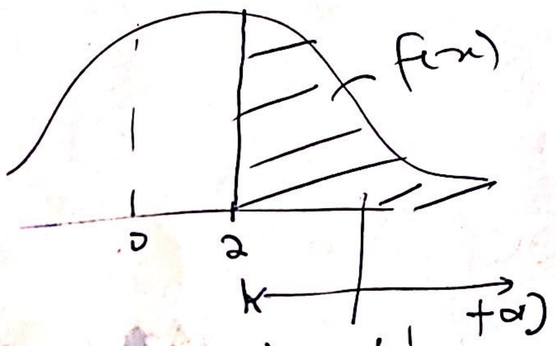
$$f(x) = \begin{cases} 2e^{-2x}, & x > 0 \\ 0, & \text{otherwise.} \end{cases}$$

(b) $$P(X > 2) = \int_{2}^{\infty} 2e^{-2x}\,dx = \cancel{2} \cdot \frac{e^{-2x}}{-\cancel{2}}\Bigg|_{2}^{\infty}$$

$$= -e^{-2x}\Big|_{2}^{\infty}$$

$$= -(\cancel{e^{-\infty}} - e^{-4})$$

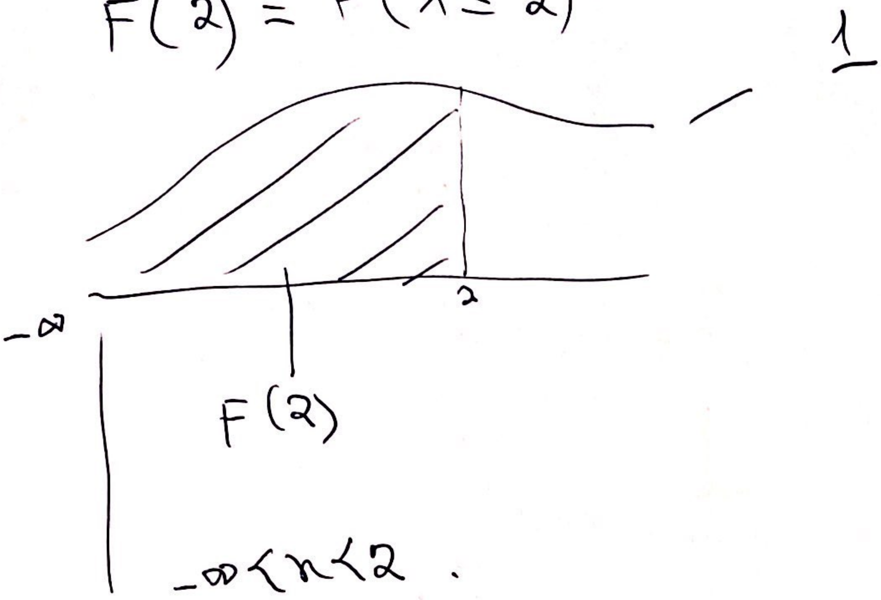
$$= e^{-4}$$



Area b/w $2 < x < +\infty$

$F(x) = 1 - e^{-2x}$

$F(2) = P(X \leq 2)$



$-\infty < x < 2$.

$$P(X > 2) = 1 - P(X \leq 2)$$
$$= 1 - F(2)$$
$$= 1 - (1 - e^{-2(2)})$$
$$= 1 - 1 + e^{-4}$$
$$= e^{-4}$$

$$P(X \leq 2) = \int_{-\infty}^{2} f(x)\,dx = \int_{-\infty}^{2} 2e^{-(2)x}\,dx$$

$$= \int_{-\infty}^{0} f(x)\,dx + \int_{0}^{2} f(x)\,dx$$

$$= \int_{-\infty}^{0} 0\, f(x)\,dx + \int_{0}^{2} 2e^{-2x}\,dx$$

$$= 2 \cdot \frac{e^{-2x}}{-2} = -e^{-2x} = -(e^{-4} - e^{0})$$

$$= -e^{-4\cancel{x}} + 1.$$

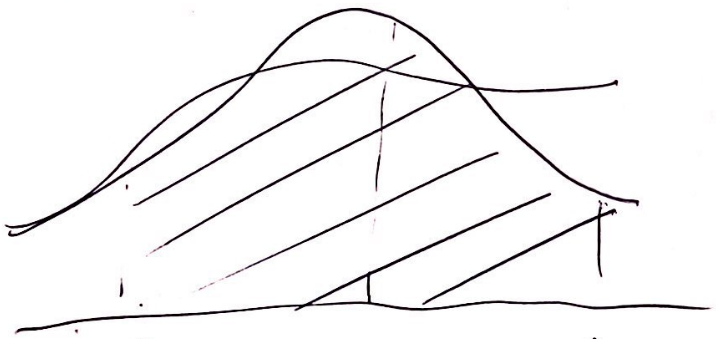
$$P(X > 2) = 1 - P(X \le 2)$$

$$= 1 - (-e^{-4\cancel{x}} + 1))$$

$$= 1 + e^{-4\cancel{x}} - 1$$

$$= e^{-4\cancel{x}} \ldots$$

(c) $P(-3 < X \le 4)$



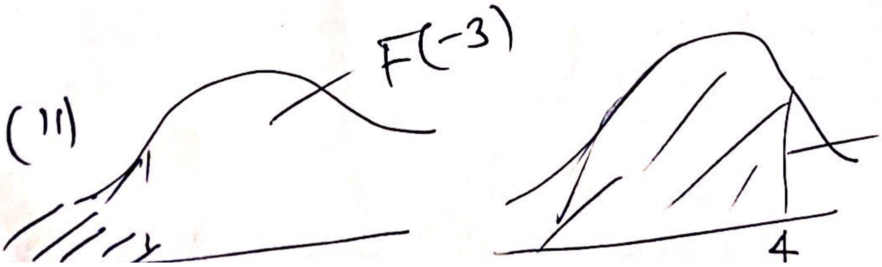
(c) $$\int_{-3}^{4} f(x)\,dx = \int_{-3}^{0} \overset{=0}{f(x)}\,dx + \int_{0}^{4} \overset{=2e^{-2x}}{f(x)}\,dx$$

$$= -e^{-2x}\Big|_0^4$$

$$= -(e^{-8} - 1)$$

$$= 1 - e^{-8} \ldots$$

(ii)



$$P(-3 < X \le 4) = F(4) - F(-3).$$

$$F(x) = \begin{cases} 1 - e^{-2x}, & x \geq 0 \\ 0, & x \leq 0 \end{cases}$$

$F(-3) = 0$

$F(4) = 1 - e^{-8}$

$$P(-3 \leq X \leq 4) = F(4) - F(-3)$$

$$= 1 - e^{-8} - 0$$

$$= 1 - e^{-8} \ldots$$