GEG 412: STATISTICAL DISTRIBUTIONS

Probability — Uncertainty — Randomness

SURE

Die — {1, 2, 3, 4, 5, 6}

Coin — {H T}

Match — {W, L, D}

I – Sample space: A set of all possible val in a probability experiment

Coin $\underbrace{\{H, T\}}$

Universal set

ξ U

Vowels $\underbrace{\{a, e, i, o, u\}}$

– Experiment / Events

Tossing of coin

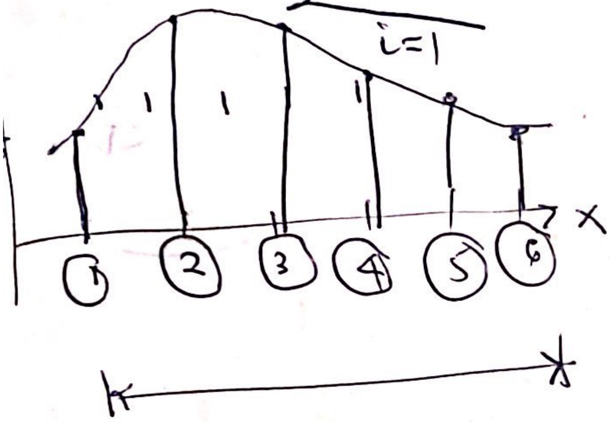
Picking of ball in a basket/defined domain.

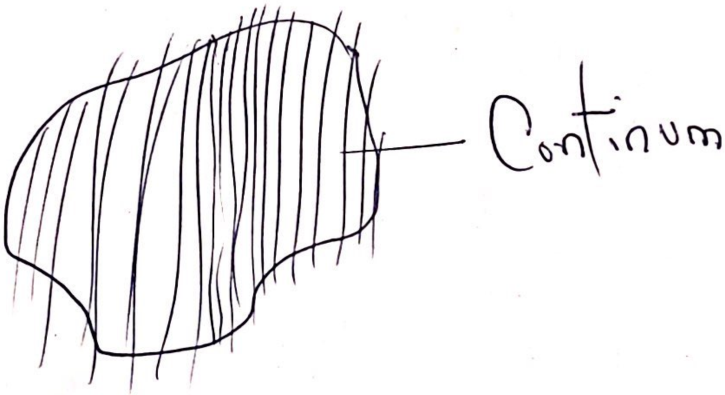
Random Variable

Discrete — $\sum_{i=1}^{\infty}$

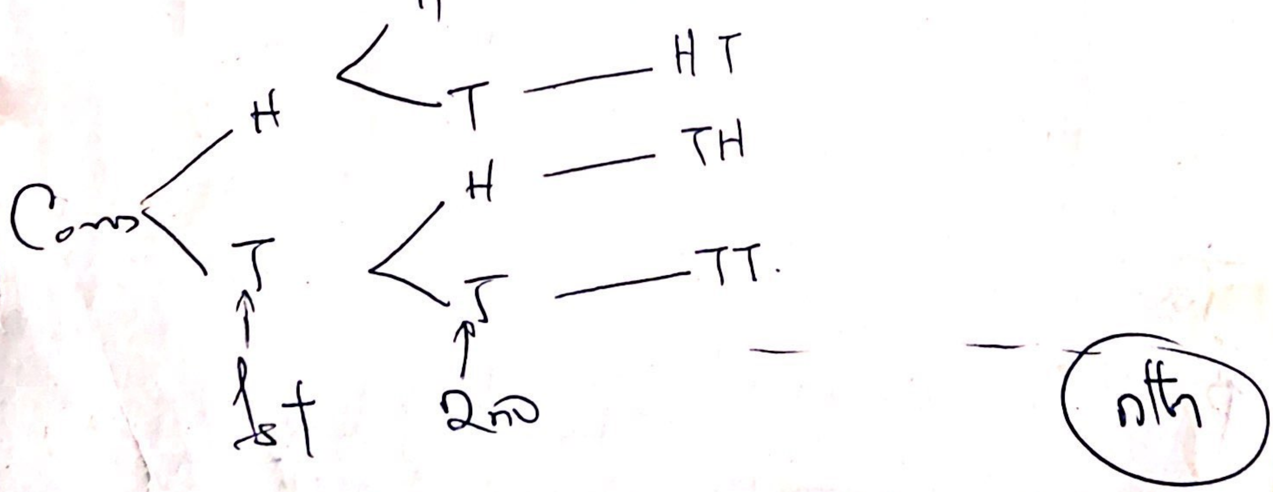
Continuous — $\int_a^b$

$P(x)$





If coin is tossed twice



Sample space, $S = \{HH, HT, TH, TT\}$

Let X represent the no of heads. Random Variable

$X = 0 =$ No Head

$X = 1 = 1$ Head

$X = 2 = 2$ Heads.

$$Pr(X = x) = \frac{\text{no of } x}{\text{Total no}}$$

$$Pr(X = 0) = \frac{TT}{S} = \frac{1}{4};$$

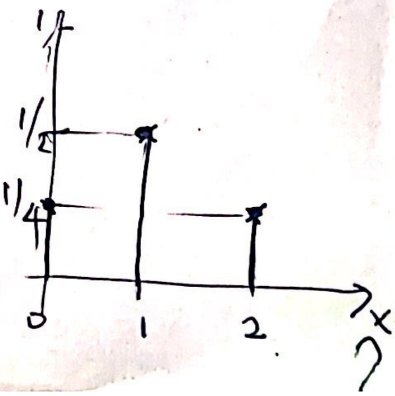
$$\begin{aligned} Pr(X = 1) &= Pr(HT \text{ or } TH) \\ &= Pr(HT) + Pr(TH) \\ &= 1/4 + 1/4 = 2(1/4) = 1/2 \end{aligned}$$

$$Pr(X = 2) = Pr(HH) = 1/4$$

The distribution for the no of heads that would show up:

Sample space

X	0	1	2
$P(X)$	1/4	1/2	1/4



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DRV

$\boxed{X \sim \text{discrete terms}}$, — $x_1, x_2, x_3, \ldots, x_k$.

$$P(X = x_k) = f(x_k) \quad \{ k = 1, 2, 3, 4, 5, \ldots \}.$$

$$P(X = x) = f(x)$$

Axioms

1. $P(A) \geq 0$, $f(x) \geq 0$

2. $P(S) = 1$, $\sum_x f(x) = 1$

Coin $\{H \ T\}$

$P(H) = 1/2$; $P(T) = 1/2$;

$$P(S) = P(H) + P(T)$$
$$= \frac{1}{2} + \frac{1}{2} = 2(1/2)$$
$$= 1.$$

Ex:

Cummulative distribution function (PDF) F(x)

mass (Density)
Probability function — $f(x)$.
p.m.f / p.d.f

$$F(x) = P(X \le x)$$

freq.		C.F
X	2	2
	3	$2+3=5$ — $F(X=3) = P(X \le 3)$
	4	$2+3+4=9$ $F(X=4) = P(X \le 4)$.

$$\boxed{F(X=4) = P(X \le 4) = P(X=4) + P(X<4)}$$

$$F(X=x) = P(X \le x).$$

$$= \sum_{X \le x}^{-\infty} f(x).$$

$=$

X	0	1	2
f(x)	1/4	1/2	1/4

(i) Cummulative distribution function cdf

$$F(x) = \begin{cases} 0, & -\infty < x < 0 \\ F(0) \rightarrow 1/4, & 0 \le x < 1 \quad [x=0 \\ F(1) \quad 1/4 + 1/2 = 3/4, & 1 \le x < 2 \quad [x=0,1 \\ F(2) \quad 3/4 + 1/4 = 1, & 2 \le x < \infty \quad [x=0,1,2] \end{cases}$$

$= P(X=0) +$

$= 1/4 +$

$F(x) = \sum f(x)$

$F(1) = \frac{1}{4} + \frac{1}{2} = P(X=0) + P(X=1) = f(0) + f(1)$

$F(1) =$

$F(X=x) = P(X \le x) = \sum_{x=0}^{1} f(x)$

$F(X=1) = P(X \le 1) = 3/4$

$F(2) =$

$$F(2) = \sum_{x=0}^{2} f(x)$$

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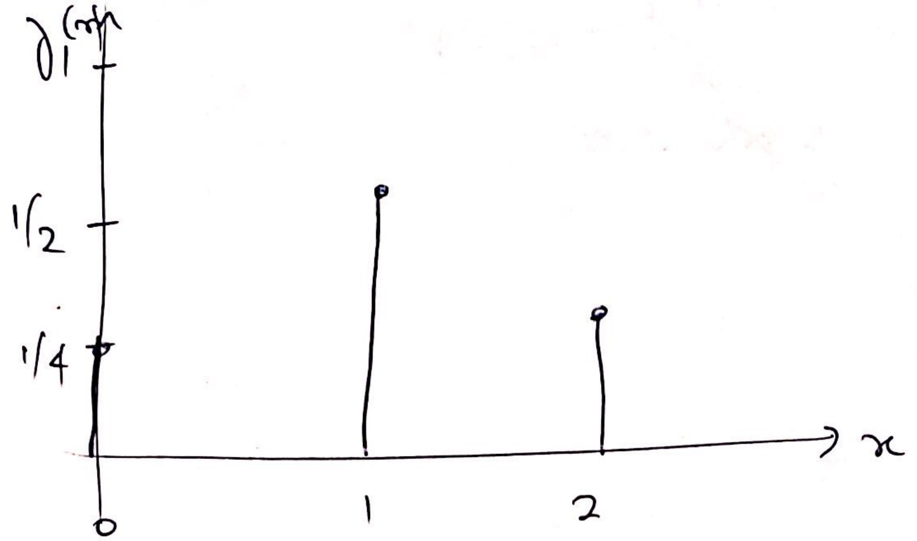
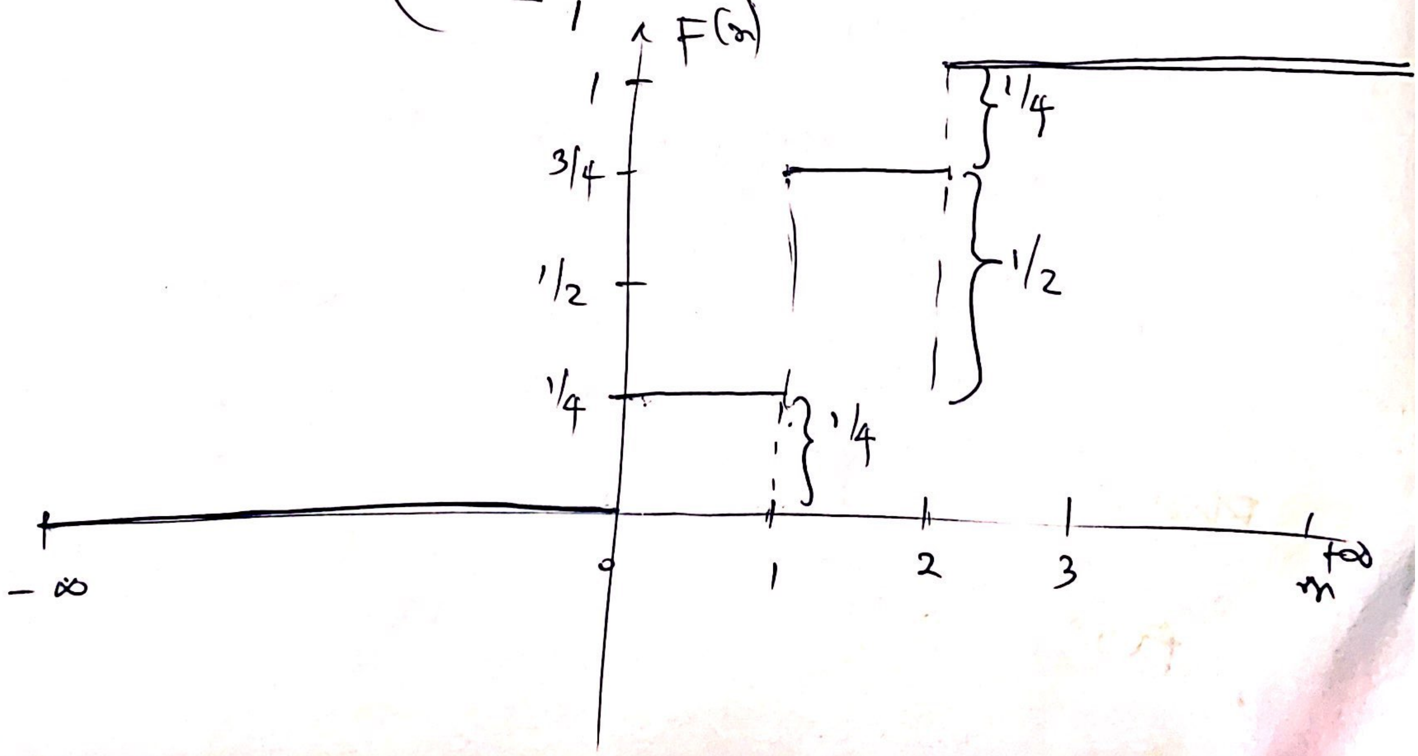
p.m.f

X	0	1	2
$f(x)$	1/4	1/2	1/4



PDF — $F(x)$

$$F(x) = \begin{cases} 0, & -\infty \leq x < 0 \\ 1/4, & 0 \leq x < 1 \\ 3/4, & 1 \leq x < 2 \\ 1, & 2 \leq x < +\infty \end{cases}$$



$$F(X=x) = F(X) = \overset{①}{P}(X \leq x_k) = \overset{②}{f(x_1) + f(x_2)} + \dots f(x_k)$$

$$\overset{③}{=} \sum_{x_k} f(x_k)$$

probability function = probability (mass/density) function

= p.m.f or p.d.f

= $f(x)$.

distribution function = ~~probability~~ (cummulative) distribution function

= PDF

= $F(x)$

$$\boxed{F(x) = \sum f(x)}$$

AS : Ex 3-4, pg 61

$X \sim$ No of bits in error

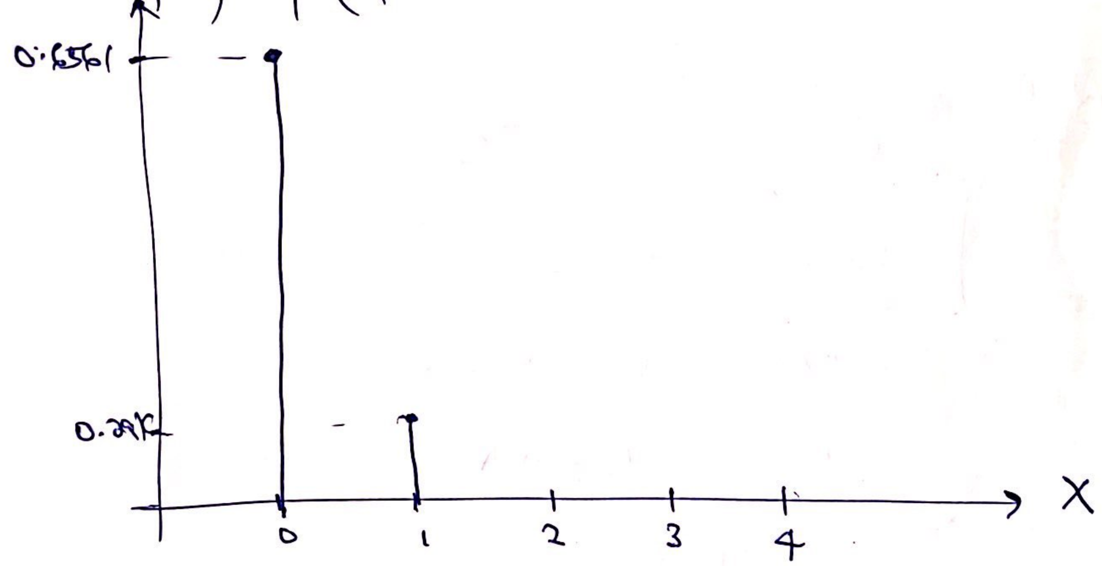
$X - \{0, 1, 2, 3, 4\}$ — DRV.

$P(X=0) = 0.6561$, $P(X=1) = 0.2916$, $P(X=2) = 0.0486$

No bit in error — 1 bit in error

$P(X=3) = 0.0036$, $P(X=4) = 0.0001$;

$f(x)$ $P(X=x_k) = f(x_k)$.



X	0	1	2	3	4
$f(x)$	0.6561	0.2916	0.0486	0.0036	0.0001

AXIOMS

1. $f(x) \geq 0$

2. $\sum_x f(x) = 1$

3 $P(X=x) = f(x)$

(ii) Three or fewer bits in error

$$P(X \leq 3) = \sum_{i=0}^{3} f(x_i)$$

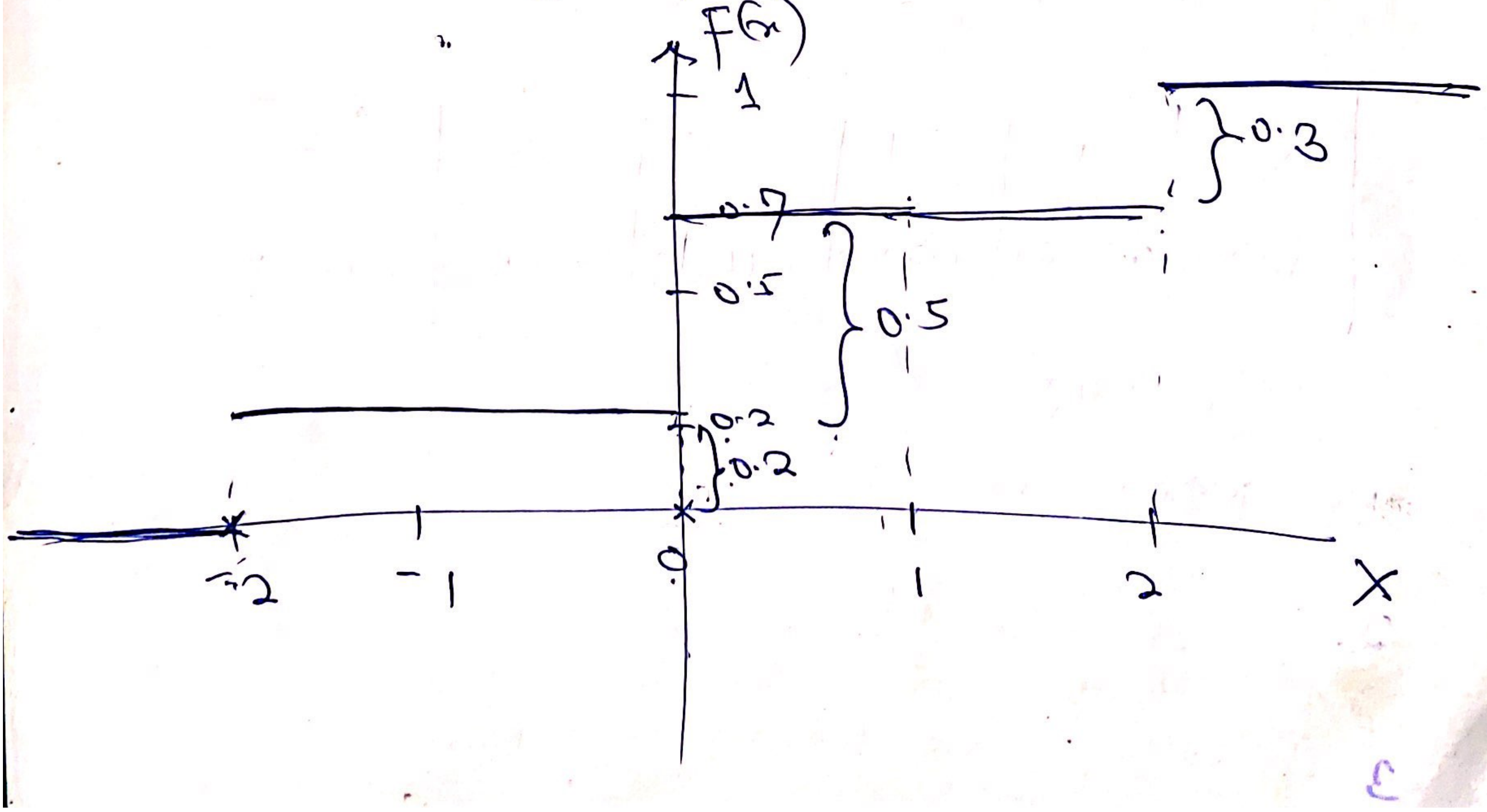
$$= f(0) + f(1) + f(2) + f(3)$$

$$= P(X=3) + P(X<3)$$

$$= P(X=3) + P(X=2) + P(X=1) + P(X=0)$$

Ex 3-7 : pg 65

$$F(x) = \begin{cases} 0, & x < -2 \\ 0.2, & -2 \leq x < 0 \\ 0.7, & 0 \leq x < 2 \\ 1 & 2 \leq x \end{cases}$$



X	-2 (x_1)	0 (x_2)	2 (x_3)	
$f(x)$	0.2	0.5	0.3	

$f(-2) = 0.2 - 0$
$= 0.2$

$f(0) = 0.7 - 0.2$
$= 0.5$

$f(2) = 1 - 0.7$
$= 0.3$

Alternatively

$\boxed{f(-2)}$

$$F(X) = f(x_1) + f(x_2) +$$

$F(0..$

$$F(x_2) = f(x_1) + f(x_2)$$
$$= 0 + 0.2$$
$$= 0.2$$

$$f(x_2) = F(x_2) - F(x_1) = 0.2 - 0 = 0.2$$

$$F(x_3) = f(x_1) + f(x_2) + f(x_3)$$

$$F(x_3) = F(x_2) + f(x_3)$$

$$f(x_3) = F(x_3) - F(x_2)$$

$$\boxed{\therefore\ \underbrace{f(x_i)}_{\text{probability mass function}} = \underbrace{F(x_i) - F(x_{i-1})}_{\text{Cummulative}}}$$

Ex 3-8 :- 850 manufactured parts contains 50 parts that do not conform to cust. req.

$X \sim$ No of nonconforming parts.

$F(x)$ —— $\sum f(x_i)$

$f(x) = P(X=x) =$ 2 prob are selected $\begin{cases} =0 \\ =1 \\ =2 \end{cases}$

$P(X=0) = P(X =$ No nonconforming parts$)$

$P(X=1) = P(X = 1$ " " "$)$

$P(X=2) = P(X = 2$ " " "$)$.

Total 850 parts

No of nonconforming = 50

No of conforming = 800,

$P(X=0) = \dfrac{\text{No of conforming}}{\text{Total}}$ $\qquad P = \dfrac{n(T)}{n(S)}$

$= \dfrac{800}{850} \cdot \dfrac{799}{849} = 0.886$

850 800 50 $\qquad$ 849 799 50

C - Conforming

N - Non conformings

$P(X=1) = P(C\,N)$ or $P(N\,C)$

$= \dfrac{800}{850} \cdot \dfrac{50}{849} + \dfrac{50}{850} \cdot \dfrac{800}{849}$

$= 2\left(\dfrac{800}{850} \cdot \dfrac{50}{849}\right) = 0.111$

$P(X=2) = P(NN)$

$$= \frac{50}{850} \cdot \frac{49}{849} = 0.003.$$

X	0	1	2
$P=X=x=f(x)$	0.886	0.111	0.003

probability mass function

Cummolative distribution function

$$F(x) = \begin{cases} 0, & x < 0 \quad \text{--- } F(0) \\ 0.886, & 0 \le x < 1 \\ 0.997, & 1 \le x < 2 \\ 1, & 2 \le x \end{cases}$$

$F(X) = P(X \le x)$

$F(0) = P(X \le 0) = P(X=0) + \underbrace{P(X<0)}_{=0}$

$= 0.886$

$F(1) = P(X \le 1) = P(X=1) + P(X \le 0) = P(X=1) + F(0)$

$= 0.111 + 0.886 = 0.997$

$F(2) = P(X \le 2) = P(X=2) + F(1) = 0.003 + 0.997$

$= \cancel{0}\, 1.$

3-13

$S = \{a, b, c, d, e, f\}$;

outcome	a	b	c	d	e	f
x	0	0	1.5	1.5	2	3

$$P(X=x) = f(x) = \frac{n(\text{outcome})}{n(\text{total})}$$ → probability mass/density function.

$P(X=a) = 0/6$

$P(X=b) = 0/6$

$P(X=c) = 1.5/6$

$P(X=d) = 1.5/6$

$P(X=e) = 2/$

$P(X=f) = 3/$