Continuous Random Variable

$$\sum \text{ — } \int.$$

Axioms:

1. $f(x) \geq 0$ — $f(x) \geq 0$

2. $\sum_x f(x) = 1$ — $\int_{-\infty}^{\infty} f(x)\,dx = 1$

3. $P(X=x) = f(x)$. — $P(X=x)$

$$P(a < X < b) \quad (-\infty < X < \infty) = \int_{-\infty}^{\infty} f(x)\,dx$$

$$= \int_a^b f(x)\,dx,$$

= Area under the curve

Schaums :-

(a) find the constant c such that the function,

$$f(x) = \begin{cases} cx^2, & 0 < x < 3 \\ 0, & \text{otherwise.} \end{cases}$$

is a density function, and (b) compute $P(1 < X < 2)$.

II $\sum f(x) = 1$

$$\int_{-\infty}^{\infty} \gamma(x)\,dx = 1$$

$$\int_{-\infty}^{0} \gamma(x)\,dx + \int_{0}^{3} \gamma(x)\,dx + \int_{3}^{+\infty} \gamma(x)\,dx = 1$$

$$= \int_{-\infty}^{0} \cancel{0\,dx} + \int_{0}^{3} cx^2\,dx + \cancel{\int_{3}^{\infty} 0\,dx} = 1$$

$$c\int_{0}^{3} x^2\,dx = 1$$

$$c \cdot \frac{x^3}{3}\Big|_0^3 = 1$$

$$\frac{c}{3}\left(3^3 - 0^3\right) = 1$$

$$\frac{c}{3}(27) = 1$$

$$9c = 1$$

$$c = 1/9;$$

$$f(x) = \begin{cases} cx^2 = x^2/9, & 0 < x < 3 \\ 0, & \text{otherwise}, \end{cases}$$

(b) $P(1 < X < 2)$

$$P(a < X < b) = P(a \le X < b) = P(a < X \le b)$$
$$= P(a \le X \le b);$$
$$= \int_a^b f(x)\,dx;$$

$P(1 < X < 2)$
$$= \int_1^2 f(x)\,dx = \int_1^2 \frac{x^2}{9}\,dx = \frac{1}{9} \cdot \frac{x^3}{3}\Big|_1^2$$
$$= \frac{1}{27}(2^3 - 1^3) = \frac{1}{27}(8-1) = \frac{7}{27}$$
$$P(1 < X < 2) = 7/27$$

Cummulative distribution function.
$$F(x) = P(X \le x) = \sum_{i=1}^{k} f(x)$$

$$F(x) = P(X \le x) = \int_{-\infty}^{x} f(x)\,dx$$

$$F(x) = \int_{-\infty}^{0} \cancel{f(x)\,dx} + \int_0^x f(x)\,dx + \cancel{\int_3^{\infty} f(x)\,dx}$$
$$= \int_{-\infty}^{0} 0\,dx + \int_0^x f(x)\,dx$$

$$= \int_0^x \frac{x^2}{9}\,dx$$

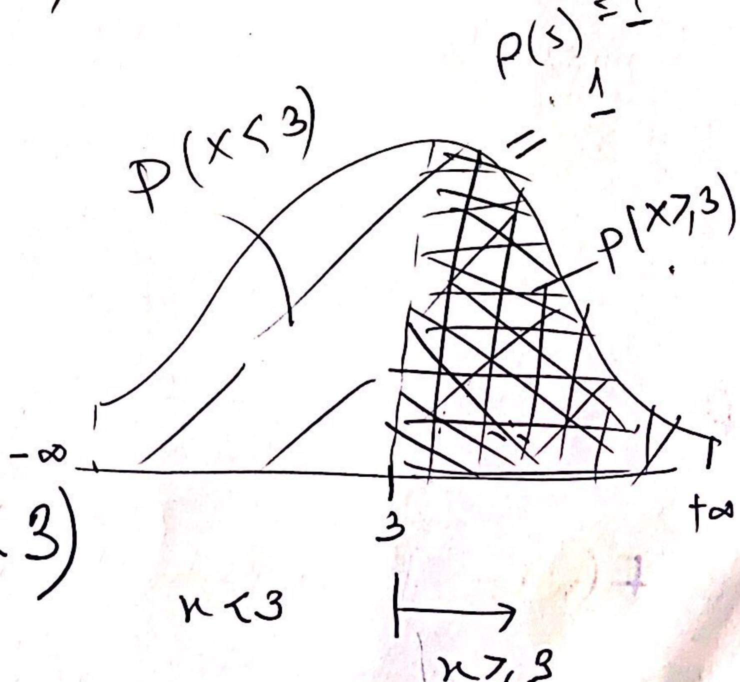
$$= \left.\frac{x^3}{27}\right|_0^x = \frac{x^3}{27} - \frac{0^3}{27}$$

$$F(x) = x^3/27$$

$$\begin{cases} 0 & , \; x < 0 \\ x^3/27 & , \; 0 \le x < 3 \\ 1 & , \; x \geq 3 \end{cases}$$

$x \geq 3 = \int$

$$P(X \geq 3) = 1 - P(X < 3)$$



$$P(x<3) + P(x \geq 3) = 1$$

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$$F(x) = P(X \le x) = \int_a^b f(x)\,dx$$

Cummulative distribution function

probability mass function

$$\underline{F(x)} = \int_a^b f(x)\,dx$$

$$f(x) = \frac{dF(x)}{dx}$$

$$F(x) = \begin{cases} 0, & x < 1 \quad - f(x) = 0 \\ x/2, & 1 \le x < 2 \quad - f(x) = \frac{dF}{dx} = 1/2 \\ 1, & x \ge 2. \quad f(x) = 0 \end{cases}$$

$$f(x) = \begin{cases} 1/2, & 1 < x < 2 \\ 0, & \text{otherwise}. \end{cases}$$

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Very Successful , V ;   3-19

Moderately " , M ;

Unsuccessful , U

$S = \{V, M, U\}$
0.3 0.6 0.1

$p(S) = \sum f(x) = 1$

X	
f(x)	

Yearly Revenue $10 $5 $1

X ~ Yearly Revenue

X ($ million)	10	5	1
f(x)	0.3	0.6	0.1

$$F(x) = P(X \leq x)$$

3-20

1000GB = 1 Tb — 0.5 — $50
500 GB — 0.3 — Revenue — $25
100 GB — 0.2 — $10

$\sum = 1$

X ~ Revenue of storage devices

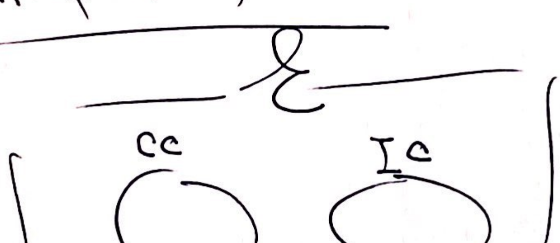
X ($million)	50	25	10
f(x)	0.5	0.3	0.2

3-21

Correct classification — 0.98

Incorrect classification = 1 - 0.98 = 0.02

Independent Events



Dependent Events

X ~ No of parts that are correctly classified.

3 parts are inspected

X	

$$P(X = \begin{cases} \text{All 3 are classified} & CCC \\ \text{2 are classified} & CCI \\ \text{1 is classified} & CII \\ \text{None is classified} & III \end{cases}$$

$X \sim$ No of classified

X	III 0	CII 1	CCI 2	CCC 3
$f(x)$	8×10^{-6}	3.92×10^{-4}	0.0912	0.941

$\sum = 1$

$P(X=0) =$ None is classified

$= P(III) = (0.02)\cdot(0.02)(0.02) = 8\times10^{-6}$

$$\boxed{P(ABC) = Pr(A)\cdot Pr(B)\cdot Pr(C).}$$

$Pr(X=1) =$ 1 is classified

$= P(CII) = 0.98\times0.02\times0.02$

$= 3.92\times10^{-4}$

$Pr(X=2) =$ 2 are classified

$= CCI = 0.98 \times 0.98 \times 0.02 = 0.0192$

$Pr(X=3) =$ All are classified

$= 0.98 \times 0.98 \times 0.98 = 0.941$

3-22

3 wafers are tested.

S: {Pass, fail}

Pass: 0.8, fail: 0.2

X ~ No that passes

select 3:
- 3 pass — PPP
- 2 pass — PPF
- 1 pass = PFF
- None pass = FFF.

X	0	1	2	3
f(x)				

$Pr(X=0) =$ None pass $= FFF = (0.2)^3$

$Pr(X=1) =$ 1 pass $= PFF = 0.8(0.2)^2$

$P(X=2) =$ 2 passes $= PPF = (0.8)^2(0.2)$

$P(X=3) =$ 3 passes $= PPP = (0.8)^3$

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B+PP :-

Schaums

2.1 :- $X \sim$ sum of points ;

a pair = 2 die

	{1	2	3	4	5	6}
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

$X \sim$ sum of points of a pair of die

Total = 36

$S \{2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$

X	2	3	4	5	6	7	8	9	10	11	12
$F(X)$	$1/36$	$2/36$ $1/18$	$3/36$ $1/12$	$4/36$ $2/18$	$5/36$	$6/36$ $1/6$	$5/36$	$4/36$ $1/9$	$3/36$ $1/12$	$2/36$ $1/18$	$1/36$

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2.2

$S\{B, G\}$

$P(B) = 1/2$; $P(G) = 1/2$; = Equal probabilities

Family of 3 children

3 boys	2 boys	1 boy	No boy
BBB	BBG	BGG	GGG

$X \sim$ No of boys

X	0	1	2	3

$Pr(X=0, = \text{No boys}) = P(GGG) = \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2}$.

$$\boxed{{}^{n}C_{x}\, p^{x} q^{n-x}}$$

* $(x+y)^n$
* $(p+q)^n$

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Q-5 X – random variable (continuous)

$$f(x) = \frac{c}{(x^2+1)}, \quad -\infty < x < \infty$$

(a) find the value of c

(b) $P(X^2)$ $\quad 1/3 < x < 1$

(a) $$P(S) = \sum_x f(x) = 1.$$

$$= \int_{-\infty}^{\infty} f(x)\,dx = 1$$

$$\int_{-\infty}^{\infty} \frac{c}{x^2+1}\,dx = 1$$

$$c\int_{-\infty}^{\infty} \frac{1}{x^2+1}\,dx = 1$$

$\tan x = \theta$

$x = \tan^{-1}(\theta)$.

$x = 90^\circ = \pi/2$

$$c\left(\tan^{-1}x \Big|_{-\infty}^{\infty}\right) = 1$$

$$c\left(\tan^{-1}(\infty) - \tan^{-1}(-\infty)\right) = 1.$$

$$c\left(\frac{\pi}{2} - \left(-\frac{\pi}{2}\right)\right) = 1$$

$$c\left(\frac{\pi}{2}+\frac{\pi}{2}\right) = 1$$

$$c\left(2\cdot\frac{\pi}{2}\right) = 1$$

$$c\pi = 1$$

$$\therefore c = 1/\pi$$

$$\therefore f(x) = \frac{1}{\pi(x^2+1)} \quad -\infty < x < \infty.$$

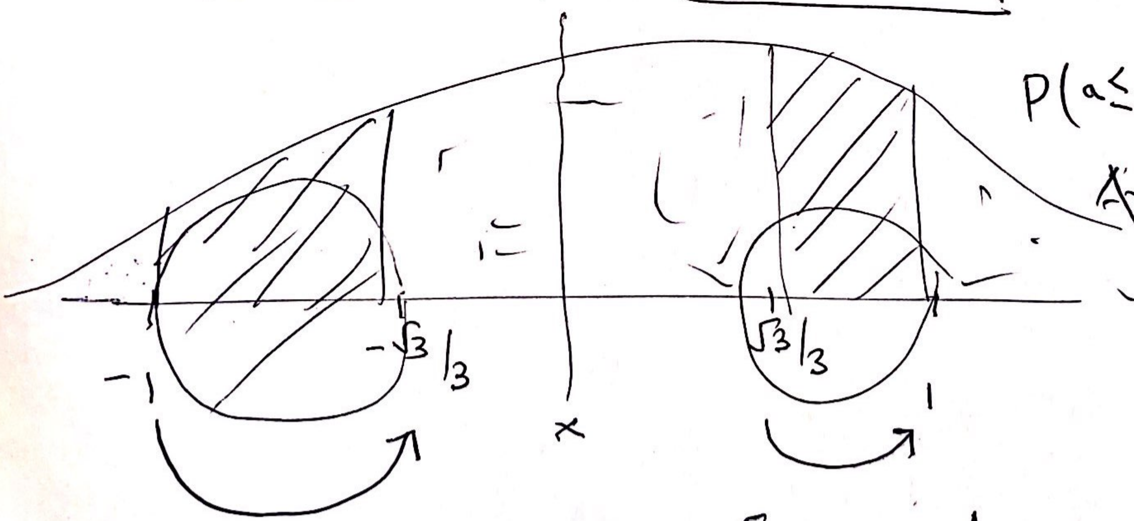
(b) $\frac{1}{3} \le x^2 \le 1$

$\sqrt{\frac{1}{3}} \le x \le \sqrt{1}$

$$\boxed{\pm\frac{1}{\sqrt{3}} \le x \le \pm 1}$$

$\sqrt{3}/3$

$$\boxed{\frac{1}{\sqrt{3}} \le x \le 1} \quad \text{or} \quad \boxed{-\frac{1}{\sqrt{3}} \le x - 1}$$



$P(a \le x \le b) =$ Area under the figure

$-1 \le x \le -\frac{\sqrt{3}}{3}$; $\quad \frac{\sqrt{3}}{3} \le x \le 1$;

$$\int_{-1}^{-\sqrt{3}/3} \frac{1}{\pi(x^2+1)}\,dx + \int_{\sqrt{3}/3}^{1} \frac{1}{\pi(x^2+1)}\,dx$$

$$= \frac{1}{\pi}\left[\tan^{-1} x \Big|_{-1}^{-\sqrt{3}/3} + \tan^{-1} x \Big|_{\sqrt{3}/3}^{1}\right] = \frac{1}{\pi}\left(2\tan^{-1} x \Big|_{\sqrt{3}/3}^{1}\right)$$

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$$\frac{2}{\pi}\left(\tan^{-1} 1 - \tan^{-1}\left(\sqrt{3}/3\right)\right)$$

$\tan 30 = \sqrt{3}/3$

$30° = \tan^{-1}(\sqrt{3}/3) = \pi/6;$

$\tan 45 = 1$

$45° = \tan^{-1}(1) = \pi/4;$

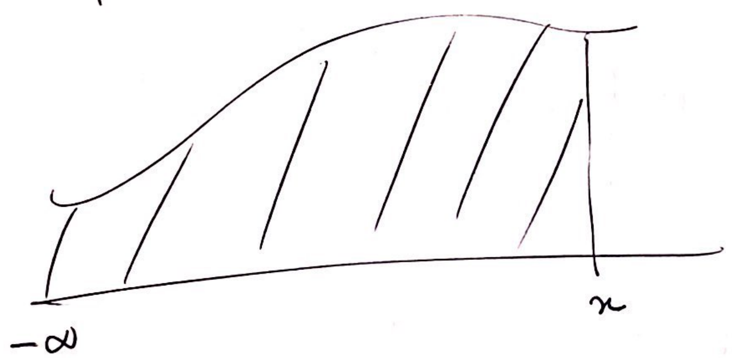
$$\frac{2}{\pi}\left(\frac{\pi}{4} - \frac{\pi}{6}\right) =$$

$$\frac{2}{\pi}\left(\frac{3\pi - 2\pi}{12}\right) = \frac{2}{\cancel{\pi}} \cdot \frac{\cancel{\pi}}{12} = \frac{1}{6}$$

$$P\left(\frac{1}{3} \le X^2 < 1\right) = 1/6;$$

(c) Find the distribution function.

$F(x) = P(X \le x)$ —— $-\infty < x < x$



$$F(x) = \int_{-\infty}^{x} f(x)\,dx = \int_{-\infty}^{x} \frac{1}{\pi(x^2+1)}\,dx = \frac{1}{\pi} \tan^{-1} x \Big|_{-\infty}^{x}$$

$$= \frac{1}{\pi}\left(\tan^{-1} x - \tan^{-1}(-\infty)\right)$$

$$= \frac{1}{\pi}\left(\tan^{-1} x - (-\pi/2)\right) = \frac{1}{\pi}\left(\tan^{-1} x + \pi/2\right)$$

$$\therefore F(x) = \frac{1}{2} + \frac{1}{\pi}\tan^{-1} x$$