I. Discrete Uniform distribution

i. probability function, $f(x) = P(X=x)$

ii. Expectation

ⓐ finite number of outcomes

ⓑ Equal probabilities.

$\frac{1}{n}$

The probability function is defined as

$$f(x) = \frac{1}{n}$$

WAS: pg 70, Ex 3-13

$\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$

first digit of a part's serial Number

0xx1, 1xx

>> one is selected

$X[\quad]$

n = No of digits

$$f(x) = \frac{1}{10} = 0.1$$

a, $a+1, a+2, \ldots, b$ $a \leq b$.

Range = Highest − Lowest
$= b - (a+1)$

$$\boxed{n = b - a + 1}$$

Probability $= \dfrac{1}{b-a+1} = \dfrac{1}{n}$

Mean = Expectation

$$E(X) = \sum x f(x)$$

$$= \sum_{x=a}^{b} x \cdot \frac{1}{(b-a+1)}$$

=

$$E(X) = \mu = \frac{b+a}{2}$$

$$\sum_{x=a}^{b} x = \frac{b(b+1)-(a-1)a}{2}$$

$$\boxed{\sigma^2 = \frac{(b-a+1)^2-1}{12}}$$

Ex 3-14 : $X \sim$ No of 48 variations

range 0 to 48.

$$E(X) = \frac{b+a}{2} = \frac{48+0}{2} = 24$$

$$\sigma^2 = E(X-\mu)^2 = \frac{(b-a+1)^2-1}{12}$$

$$= \frac{(48-0+1)^2-1}{12} = 14.14$$

Binomial distribution

(i)

$^{n}C_{r}$

$^{n}C_{r}$

= success / failure

(i) Two outcomes in a single trial
(ii) n-trials
(iii) Equal probabilities per trial
$(P(S) = \text{constant})$

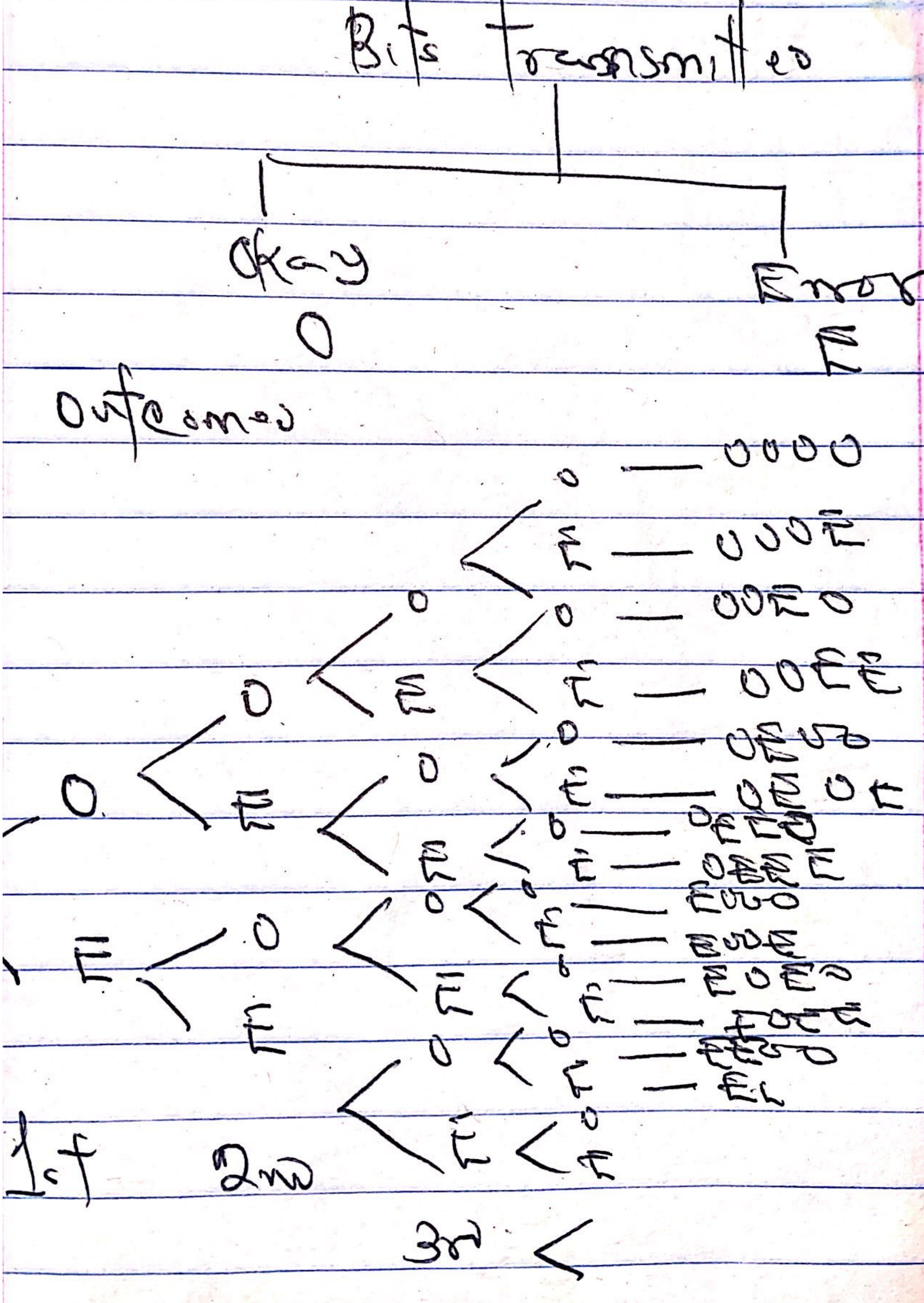
Bernoulli Experiment / Trial

$$^{n}C_{x} = \binom{n}{x} = \frac{n!}{(n-x)!\,x!}$$

$$\boxed{\binom{n}{x} p^{x} q^{n-x}} \longrightarrow \text{Bernoulli Experiment}$$

$X \sim$ No of bits in error for next 4 transmitted bits.

$P(X = 2)$.



x - no. of Er

OOOO	0	EOOO
OOOE	1	EOOE
OOEO	1	EOEO
OOEE	2	EOEE
OEOO	1	

$P_r(X=2)$

$p=0.1, \quad \bar{p}=0.9$

$P(E)=0.1, \quad P(O)=0.9$

$P(OOEE) = P(O) \cdot P(O) \cdot P(E)(PE)$

$$\boxed{{}^4C_2} = \frac{4!}{(4-2)!\,2!} = \frac{4!}{2!\,2!} = \frac{4\times3!}{3!\,2!}$$

$= 2$

$$(x+y)^n$$

OOOO
OOOE $= {}^nC_0 x^n y^0 + {}^nC_1 x^{n-1} y^1 +$
OOEO ${}^nC_2 x^{n-2} y^2 + \cdots + {}^nC_n x^0 y^n$
OO

$$P(X=x) =$$

$$(p+q)^n = \boxed{{}^nC_x p^x q^{n-x}}$$

success failure

$x = 0, 1, 2, 3, \ldots, n;$

$$(p+q)^4 = {}^4C_2 (0.1)^2 (0.9)^{4-2}$$

Bernoulli experiment/trial

(i) The trials are independent
(ii) The outcomes are just two.
(iii) constant probability

Bernoulli RV p q

$$f(x) = P(X=x) = \binom{n}{x} p^x q^{n-x}$$

$$p + q = 1$$

$$q = 1 - p$$

$$f(x) = \binom{n}{x} p^x (1-p)^{n-x}$$

$^nC_0 p^0 q^n$ . . . $^nC_n p^n q^0$

single trial, $n = 1$ $1 \cdot p^n \cdot q^0$

(p) $x \cdot n$ $p(n)$

$S\{B, G\}$

$P(B) = 0.5$; $P(G) = 0.5$

$n = 3$

$X \sim$ No of girls boys

$$P(X=2) = \binom{3}{2}\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^{3-2} =$$

$n = 3$, $x = 2$, $p = \frac{1}{2}$

$$= \frac{3!}{(3-1)}$$

Expectation $= np$;

Variance $= npq$;

3-18

=18) 10% chance of containing water pipputant

-independent,

$n = 18, \quad x = 2$.

X = No of sample that contains pipul

$p = 10\% = \frac{10}{100} = 0.1$

$q = 0.9$

$P(X=x) = \binom{n}{x} p^x (q)^{n-x}$

$P(X = \text{exactly two}) = P(X=2)$

$= \binom{18}{2} (0.1)^2 (0.9)^{18-2}$

$(0.9)^{15}$

$= \binom{18}{2} (0.1)^2 (0.9)^{16}$

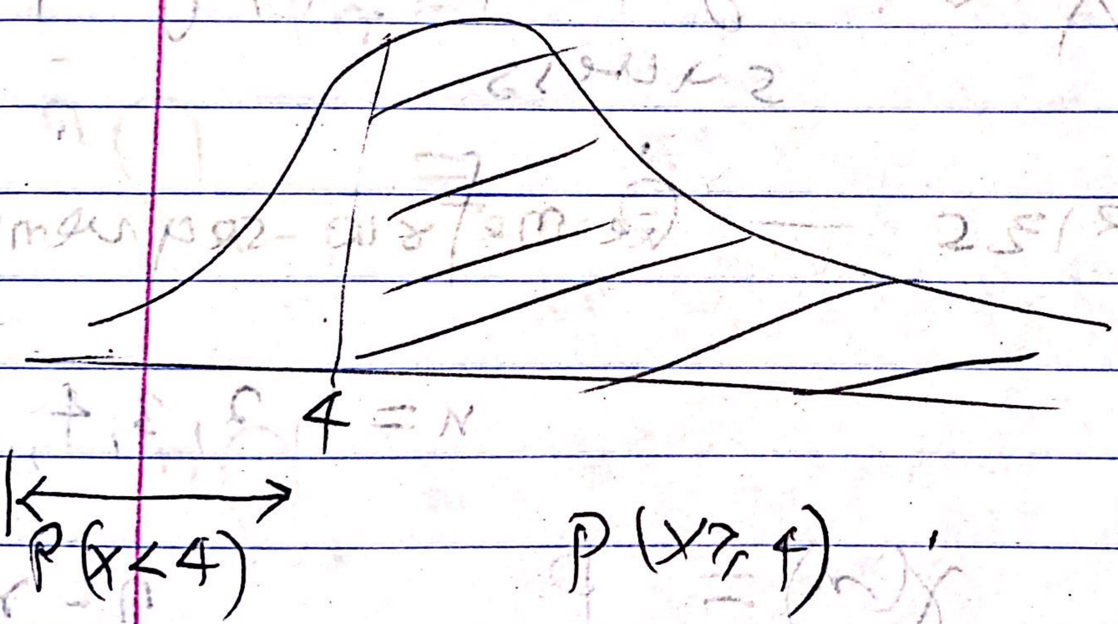
$P(X=2)$ 0.2835

P(at least 4 samples contains pollution)

$$P(X \geq 4) = P(X=4) + P(X=5) + \dots + P($$

$$P(X<4) + P(X \geq 4) = 1$$

$$P(X \geq 4) = 1 - P(X<4)$$



$$P(X<4) = P(X=0,1,2,3)$$

$$P(X=0) + P(X=1) + P(X=2) + P(X=3)$$

$$\binom{18}{0}(0.1)^0(0.9)^{18} + \binom{18}{1}(0.1)^1(0.9)^{17} + \binom{18}{2}(0.1)^2(0.9)^{16} + \binom{18}{3}(0.1$$

$$=$$

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GEOMETRIC & NEGATIVE

Bernoulli

$$f(x) = \binom{n}{x} p^x q^{n-x} = \binom{n}{x} p^x (1-p)^{n-x}$$

$X \sim$ No of trials until first success

SERIES — Geometric sequence

$x = 1, 2, 3, 4, \ldots,$

$$f(x) = p \qquad n = x$$

$$\boxed{f(x) = p(1-p)} \quad \underbrace{p^x q^{n-x}}$$

$${}^nC_0 p^n q^0 + \ldots + \boxed{{}^nC_{n-1} p^k q^{n-1}} + \boxed{{}^nC_n p^0 q^n}$$

1

$1^{st}, 2^{nd}, 3^{rd}, 4^{th}, \ldots, x^{th}$ Trial

$P, \ldots$

Success

$$T_n = ar^{n-1}$$

$$T_n = a + (n-1)d.$$

$$^nC_{n-1} = \frac{n!}{(n-(n-1))!\,(n-1)!} = \frac{n(n-1)}{(n-1)!\,(n\ldots}$$

$$^nC_x = \frac{n!}{(n-1)!\,\times 1!}$$

$$= \frac{n}{(n-1)!}$$

$$7! = 7 \times 6 \times 5 = 7 \times 6!$$

$$n! = n(n-1)!$$

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$$\gamma(x) = 1 \cdot p^{1}(1-p)^{x-1}$$

$$\gamma(x) = p(1-p)^{x-1},$$

$x = 1, 2, 3, \ldots,$

Ex 3-21

$P(\text{contain contamination}) = 0.01$

$q = 1 - p = 0.99;$

$P(X = \underline{125}$ wayers need ~~to~~ to be analyzed before large particle $\underline{is}$ detected) (contamination)

fixed focal before success would be guaranteed.

$n = 125$

$$P(X=125) = p(1-p)^{x-1}$$

$$P(X=125) = (0.01)(0.99)^{125-1}$$

$$= (0.01)(0.99)^{124} = 0.002$$

Expectation:

$$E(X) = \sum_{x=1}^{N} x f(x)$$

$$= \underbrace{\sum_{x=1}^{N} x \cdot p(1-p)^{x-1}}_{\text{Convergence of series } (\forall r}$$ $= \frac{1}{p}$,

$x=0$, 0 $\leftarrow$ $x=3$, $3p(1-p)^2$

$x=1$, $p(\cancel{1-p})$ $x=4$, $4p(1-p)^3$

$x=2$, $2p(1-p)$

$$\sum x p(1-p)^{x-1}$$

$$= 0 + p + 2p(1-p) + 3p(1-p)^2 + 4p(1-p)^3 + \ldots$$

$$= \frac{1}{p}$$

$$\sigma^2 = E(X-\mu) = \sum (x-\mu)^2 f(x)$$

$$= \sum_{x=1}^{\infty} \left(x-\frac{1}{p}\right) \cdot p(1-p)^{x-1}$$

$$\sigma^2 = \frac{1-p}{p^2}$$

Negative Binomial distribution

$$\boxed{\gamma(x) = p^1(1-p)^{x-1}}$$

$\boxed{{}^nC_{n-1}}$

(before first success (single).

$\times$ ${}^nC_0$ ${}^nC_n$

r - successes

$$\gamma(x) = \binom{x-1}{r-1} p^r (1-p)^{x-r}$$

$$x = r, r+1, r+2, \ldots$$

$$E(X) = r/p \, ;$$

$$\sigma^2 = \frac{r(1-p)}{p^2} \, ;$$

Ex 3-25: P(failure) = 0.0005

$\boxed{X \sim \text{No of requests until all 3 servers fail.}}$

$p = 0.0005$, $q = 0.9995$

$\mu = E(X) = r/p$

$r = 3$ servers

$\mu = 3/0.0005$

$P(X \leq 5)$ = five requires

$= P(X=3) + P(X=4) + P(X=5)$

$= f(x) = \binom{x-1}{r-1} p^{r}(1-p)^{x-r}$

$P(X=5) =$ $x=5$, $r=3$

$$= \binom{5-1}{3-1}(0.0005)^3(0.9995)^{5-3}$$

$$= \binom{4}{2}(0.0005)^3(0.9995)^2 =$$

$P(X=4)$

$$= \binom{4-1}{3-1}(0.0005)^3(0.9995)^{4}$$

$$= \binom{3}{2}(0.0005)^3(0.9995)^1$$

$(X=3)$, $x=3$, $r=3$

$$= \binom{3-1}{3-1}(0.0005)^3(0.9995)^{3}$$

$$= \binom{2}{2}(0.0005)^3(0.9995)^0$$

Schaums outline pg 118.

Find The prob. that in tossing a fair coin three times, there will ~~be~~ appear
(a) 3 heads (b) 2 Tails & 1 Head
(c) at least 1 head (d) not more than 1 tail.

METHOD 1

No of Heads

X	0	1	2	3
$p(x)$				

$$Pr(X=0) = P(TTT) = P(T)\cdot P(T)\, P(T)$$
No Head
$$= \tfrac{1}{2}\cdot\tfrac{1}{2}\cdot\tfrac{1}{2} = \tfrac{1}{8}$$

$P(H) = 1/2$, $Pr(T) = 1/2$

$$S = \{H, T\}$$

$p = 1/2$

(a) 3 Heads $\quad {}^{n}C_{x}\, p^{x} q^{n-x}$

$n = 3 \quad x = 3$

$${}^{3}C_{3}\left(\tfrac{1}{2}\right)^{3}\cancel{\left(\tfrac{1}{2}\right)^{3-3}} = 1/8,$$

(b) Pr(2 tails 1 head) . $x = 2$

$$\binom{3}{2}\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^{3-2}$$

$$3 \cdot \frac{1}{4} \cdot \frac{1}{2} = 3/8$$

$= P(TTH \text{ or } THT \text{ or } HTT)$

$$\frac{1}{8} + \frac{1}{8} + \frac{1}{8} = 3/8$$

f: 2 find the prob that in 5 tosses of a fair die a 3 will appear

(a) twice (b) at most once (c) at least 2 times.

$X \sim$ No of times a 3 will appear

$n = 5$ (no of trials)

(a) twice, $x = 2$

$P(X=2) = \cancel{\binom{5}{2}\left(\frac{1}{2}\right)}$

p = probability that 3 will appear

$= 1/6$

$S = \{1, 2, 3, 4, 5, 6\}$

$P(\text{success}) = P(3 \text{ will show})$

$p = 1/6$

$q = 1 - p = 1 - 1/6 = 5/6$

(a) 3 will appear twice.

X – No of times 3 will appear

$P(X = \text{twice}) = P(X = 2)$

$$\binom{n}{x} p^x (1-p)^{n-x}$$

$$\binom{5}{2} \left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right)^3$$

(b) at most one. $(X \leq 1)$

$$P(X \leq 1) = P(X=1) + P(X=0)$$

$$= \binom{5}{1}\left(\frac{1}{2}\right)^1\left(\frac{5}{6}\right)^4 + \binom{5}{0}\left(\frac{1}{2}\right)^0\left(\frac{5}{6}\right)^5$$

P (at least two fumos),

$$P(X \geq 2) = P(X = 2, 3, 4, 5)$$

$$P(X < 2) + P(X \geq 2) = 1$$

$$P(X \leq 1) + P(X \geq 2) = 1$$

$$P(X \geq 2) = 1 - P(X \leq 1)$$

$X = 2 \quad 3 \quad 4 \quad 5$

$$\binom{5}{2}\left(\frac{1}{2}\right)^2\left(\frac{5}{6}\right)^3 + \binom{5}{3}\left(\frac{1}{2}\right)^3\left(\frac{5}{6}\right)^2 + \binom{5}{4}\left(\frac{1}{2}\right)^4\left(\frac{5}{6}\right)^1 +$$

$$\left({}^5C_5\right)\left(\frac{1}{2}\right)^5\left(\frac{5}{6}\right)^1,$$

$$\boxed{= \frac{1135}{432}}$$

Prove

$\mu = np, \quad \sigma^2 = npq$

Binomially distributed RV.

X	0	1
$f(x)$	q	p

$$E(X) = \sum x f(x)$$

$$= 0(q) + 1(p) = p$$

For n-trials

$$E(X) = np.$$

$$\sigma^2 = E(X-\mu) = \sum_x (x-\mu)^2 f$$

$$= (0-p)^2 \cdot q + (1-p)^2 \cdot p$$

$$= p^2 q + \cancel{(1+p^2-2p)}\, p$$

$$p^2 q + q^2 p$$

$$= pq(p+q)$$

$$\sigma^2 = pq(1)$$

$$\sigma^2 = pq$$

for n-trials

$$\sigma^2 = npq$$

$$\sigma^2 = E(X^2) - (E(X))^2$$

$$= E(X^2) - (p)^2$$

$$E(X^2) = \sum x^2 p(x)$$

$$= 0^2 \cdot q + 1^2 \cdot p$$

$$= p$$

$$\sigma^2 = p - p^2$$

$$= p(1-p)$$

$$\sigma^2 = pq$$

for n trials

$$\boxed{\sigma^2 = npq}$$

$0 < p < 1$ 0.1

4.9 If prob of a defective bolt is 0.1 ;

$p = 0.1$, $q = 0.9$,

$\mu = np$.   , $n = 400$

$n = 5$ trials

$x = 0, 1, 2, 3, 4, 5$

$\mu = 400 \times 0.1 = 40$,

$(X = n)$ $\sigma^2 = npq = 40 \times 0.9$

$= 36$

$\sigma = \sqrt{\sigma^2} = \sqrt{36}$

σ = standard deviation

σ^2 = Variance

n ≥ Extremely large

Poisson's distribution ≥ 100

$E(X)$ = mean = average = λ = np ;

The distribution of the poisson is :

$$P(X=x) = \frac{e^{-\lambda}\lambda^{x}}{x!}, \quad x = 1, 2, 3,$$

$$P(X=x) = \binom{n}{x} p^{x}(1-p)^{n-x}$$

$$\lambda = np \; ; \; p = \lambda/n$$

$$= \binom{n}{x}\left(\frac{\lambda}{n}\right)^{x}\left(\frac{1}{} - \frac{\lambda}{n}\right)^{n-x}$$

$$\lim_{n\to\infty} P(X=x)$$

Binomial

$$X \sim bin(n, p, x)$$

$$\frac{n!}{(n-1)!\,x!} \cdot \frac{\lambda^x}{n^x} \cdot \left(\frac{n-\lambda}{n}\right)^{n-x}$$

$$\frac{n\cancel{(n-1)!}}{\cancel{(n-1)!}\,x!} \cdot \frac{\lambda^x}{n^x} \cdot \frac{(n-\lambda)^{n-x}}{n^{n-x}} = \frac{n^n}{n^x}$$

$$\frac{n}{x!} \cdot \frac{\lambda^x}{\cancel{n^x}} \cdot \frac{(n-\lambda)^{n-x}}{n^n} \cdot \cancel{n^x}$$

$$\frac{\lambda^x}{x!} \cdot \frac{n(n-\lambda)^{n-x}}{n^n}$$

$$\frac{\lambda^x}{x!} \cdot \cancel{n^{1-n}} \frac{(n-\lambda)^{n-x}}{n^{n-1}}$$

$$\lim_{n \to \infty} \frac{(n-\lambda)^{n-x}}{n^{n-1}} = e^{-\lambda}$$

Series & sequences

$$= \frac{\lambda^{x}}{x!} e^{-\lambda}$$

$$P(X=x) = \frac{e^{-\lambda} \lambda^{x}}{x!}$$

3.99) Suppose that the number of customers that enter a bank in an hour is a poisson RV and $P(X=0) = 0.05$. Determine the mean and var.

$$\lambda = np;$$

$$p = P(X=0) = 0.05.$$

number = ^{one} hour = 3600 sec

or Customer

= 1hr

$$\lambda = E(X) = np = 1 \times 0.05$$

$$= 0.05 \text{ Per hour}$$

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$$^nC_r = \frac{n!}{(n-r)!\,r!}$$

$$= 60 \times 0.05$$
$$= 6 \times 0.5$$
$$= 3 \text{ per minute}$$

$$\binom{n}{x} p^x (1-p)^{n-x} \qquad \lambda = np$$

$$= \frac{n!}{(n-x)!\,x!} \left(\frac{\lambda}{n}\right)^x \left(1-\frac{\lambda}{n}\right)^{n-x}$$

$$= \cancel{n(n)}$$

$$\cancel{n(n-1)(n-2)\ldots(n-x+1)}!$$

$$\frac{n!}{(n-x)!} \cdot \frac{\lambda^x}{x!} \cdot \frac{1}{n^x} \cdot \left(1-\frac{\lambda}{n}\right)^{n-x}$$

$$n(n-1)(n-2)\ldots(n-x+1)(n-x)!$$

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$$\frac{n!}{(n-x)!} = \frac{n(n-1)(n-2)(n-3)\cdot(n-x+1)\cancel{(n-x)!}}{\cancel{(n-x)!}}$$

$$= n(n-1)(n-2)\ldots(n-x+1).$$

$$\Rightarrow \frac{n(n-1)(n-2)\ldots(n-x+1)}{x!}\cdot\frac{\lambda^x}{n^x}\left(1-\frac{\lambda}{n}\right)^{n-x}$$

$$\Rightarrow \frac{n(n-1)(n-2)\ldots(n-x+1)/n}{n^x/n}$$

$$= \frac{(n-1)(n-2)\ldots(n-x+1)}{n}$$

$$= \left(\frac{1}{1}-\frac{1}{n}\right)\left(1-\frac{2}{n}\right)\ldots\left(1-\frac{x-1}{n}\right)$$

$$= \left(\frac{n-1}{n}\right)\left(\frac{n-2}{n}\right)\ldots\left(\frac{n-x+1}{n}\right)$$

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$$\frac{n(n-1)(n-2)\ldots(n-x+1)}{n^x}$$

$$\frac{(n-1)(n-2)\ldots(n-x+1)}{n^{x-1}}$$

$$\frac{(n-1)(n-2)(n-3)\ldots\boxed{(n-(x-1))}}{n^{x-1}}$$

$x-1 = 1, 2, \ldots (x-1)$

$$\frac{n-1}{n} \quad \frac{n-2}{n} \quad \frac{n-3}{n} \quad \frac{n-(x-1)}{n^{x-1}}$$

$$1-\frac{1}{n} \quad 1-\frac{2}{n} \quad 1-\frac{3}{n} \quad 1-\frac{(x-1)}{n}$$

$n! = n(n-1)(n-2)(n-3)\ldots$

$5! = 5 \times 4 \times 3 \times 2 \times 1!$

$$\frac{\left(1-\frac{1}{n}\right)\left(1-\frac{2}{n}\right)\cdots\left(1-\frac{(x-1)}{n}\right)\lambda^{x}\left(1-\frac{\lambda}{n}\right)^{n-x}}{}$$

$$\lim_{n\to\infty} = \left(1-\cancel{\frac{1}{n}}^{0}\right)\left(1-\cancel{\frac{2}{n}}^{0}\right)\cdots\left(1-\frac{(x-1)}{n}\right)$$

$$\frac{1}{\infty}$$

$$= 1$$

$$\frac{\lambda^{x}}{x!}\left(1-\frac{\lambda}{n}\right)^{n-x}$$

$$\frac{\lambda^{x}}{x!}\cdot\left(1-\frac{\lambda}{n}\right)^{n^{\infty}}\cdot\left(1-\frac{\lambda}{n}\right)^{-x}$$

$$\cancel{\frac{\lambda}{n}}^{0}_{\infty}$$

$$\left(1-0\right)^{\infty}$$

$$= 1$$

1

1   1 — 1

1   2   1 — 2

1   3   3   1 — 3

1   4   6   4   1 — 4

$^4C_0$ $^4C_1$ $^4C_2$ $^4C_3$ $^4C_4$

$$^4C_2 = \frac{4!}{(4-1)!\,2!} = \frac{4!}{3!\,2!} = \frac{4\times \cancel{3!}}{\cancel{3!}\times 2} = 2$$

$$\frac{4!}{(4-2)!\,2!} = \frac{4!}{2!\,2!} = \frac{\cancel{4}^{2}\times 3\times \cancel{2}!}{\cancel{2}1\times \cancel{2}\times 1} = 6$$

$$= \frac{\lambda^x}{x!}\left(1-\frac{\lambda}{n}\right)^{-x}$$

$\boxed{1.\ e^{-\lambda}}$

$f(\lambda) = e^{-\lambda}$

$f'(\lambda) = -e^{-\lambda}$

$f''(\lambda) = e^{-\lambda}$

$e^{-\lambda} = f(0) + \frac{f'(0)}{1!} + \frac{f''(0)}{2!}$

$= e^{-0} +$

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$$\left(1-\frac{\lambda}{n}\right)^{-x} = e^{-\lambda}$$

$$\lim \left(1+\frac{u}{n}\right)^{n} = e^{u}$$

$$\boxed{P(X=x) = \frac{e^{-\lambda}\lambda^{x}}{x!}}$$

$\cancel{x}$

$$\frac{n-(x-1)}{n^{x-1}}$$

$$x=2 \qquad \frac{n-1}{n} = \left(1-\frac{1}{n}\right)$$

$$\left(1-\frac{1}{n}\right)\left(2-\frac{1}{n}\right)\cdots\left(n-\frac{x-1}{n}\right)$$