$\boxed{\text{MEAN — Average — Expectativa .}}$

$\boxed{\text{QUEUE}}$ OR

Arrival

Expected Time = Average = MEAN

## MEAN & VARIANCE (ST. DEV)

$$\mu = E(X).$$

| | | |
|---|---|---|
| $X$ | | |
| $f(x)$ freq | . | |
| ~~$f(x)$~~ $fx$ | | $\sum fx$ |

$$\mu = \frac{\sum fx}{\sum f} = \frac{\sum fx}{N} = \sum f\left(\frac{x}{N}\right)$$

$$= \sum x \cdot \left(\frac{f}{N}\right) = \sum x \cdot \left(\frac{\text{freq}}{\text{Total No}}\right)$$

$$P(X=x) = \frac{n(\text{outcome})}{n(\text{Total no})}$$

$$= \sum x \cdot P(X=x) = \sum x\, f(x)$$

p.m.f.

$$\mu = E(X) = \sum_{x} x \cdot f(x)$$

$$x = \{x_1, x_2, x_3, \dots, x_N\}.$$

Variance

| $x$ | freq, $f$ | $(x-\mu)$ | $(x-\mu)^2$ | $f(x-\mu)^2$ |
|---|---|---|---|---|
| | | | | |

$$\mu = \frac{\sum f x}{N}$$

$$\sigma^2 = \frac{\sum f(x-\mu)^2}{N} = \sum (x-\mu)^2 \cdot \left(\frac{f}{N}\right).$$

$$\sigma^2 = \sum (x-\mu)^2 \cdot P(X=x) = \sum (x-\mu)^2 \cdot f(x)$$

is pmf.

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$$\mu = E(X) = \sum x \cdot f(x)$$

$$\sigma^2 = \sum \underbrace{(x-\mu)^2}_{} \cdot f(x) = E(X-\mu)^2$$

$$\boxed{\sigma^2 = E(X-\mu)^2}$$

Scanned with CamScanner

| $x$ | $(x-\mu)^2$ |
|---|---|
| | |

$\mu$ = mean

$$\sigma^2 = \sum (x-\mu)^2 \cdot f(x)$$

$$= \sum (x^2 - 2x\mu + \mu^2) f(x)$$

$$= \sum \left(x^2 f(x) - 2x\mu f(x) + \mu^2 f(x)\right)$$

$$= \sum x^2 f(x) - \sum 2x\mu f(x) + \sum \mu^2 f(x)$$

Properties of Expectations

1. $E(c) = c$ — constant, constant
2. $E(1) = 1$
3. $E(cX) = c\,E(X)$

$$= \sum x^2 f(x) - 2\mu \sum x f(x) + \mu^2 \sum 1 \cdot f(x)$$

Scanned with CamScanner

$$\mu = E(X) = \sum x \cdot f(x) \,;$$

$$* \sum x^2 f(x) = E(X^2).$$

$$* \sum x f(x) = E(X) \,;$$

$$* \sum 1 \cdot f(x) = E(1) = 1,$$

$$= E(X^2) - 2\mu E(X) + \mu^2 (1)$$

$$= E(X^2) - 2E(X) \cdot E(X) + (E(X))^2$$

$$= E(X^2) - 2(E(X))^2 + (E(X))^2$$

$$= E(X^2) - (E(X))^2$$

$$\sigma^2 = \sum (x-\mu)^2 f(x) = E(X^2) - (E(X))^2$$

| X | $X^2$ | |
|---|---|---|
| | | |

2.02

Coin

| X | 0 | 1 | 2 |
|---|---|---|---|
| $P(X=x)$ | 1/4 | 1/2 | 1/4 |

Expectation

$$E(X) = \sum x \cdot f(x)$$

$$= 0\left(\tfrac{1}{4}\right) + 1\cdot\left(\tfrac{1}{2}\right) + 2\left(\tfrac{1}{4}\right)$$

$$= \tfrac{1}{2} + \tfrac{1}{2} = 1;$$

Variance,

$$\sigma^2 = \sum (x-\mu)^2 f(x)$$

$$= (0-1)^2\cdot\tfrac{1}{4} + (1-1)^2\cdot\tfrac{1}{2} + (2-1)^2\cdot\tfrac{1}{4}$$

$$\sigma^2 = \tfrac{1}{4} + 0 + \tfrac{1}{4} = 2\left(\tfrac{1}{4}\right) = \tfrac{1}{2} = 0.5$$

OR

$$\sigma^2 = E(x-\mu)^2 = E(x^2) - (E(x))^2$$

$$E(x^2) = \sum x^2 f(x)$$

$$= 0^2\cdot\left(\tfrac{1}{4}\right) + 1^2\left(\tfrac{1}{2}\right) + 2^2\cdot\left(\tfrac{1}{4}\right)$$

$$= 0 + \tfrac{1}{2} + 1 = 3/2;$$

$$\sigma^2 = E(x^2) - (E(x))^2 = \tfrac{3}{2} - (1^2) = \tfrac{3}{2} - 1 = 1/2 = 0.5$$