MECA 482
Furuta Pendulum
Spring 22’

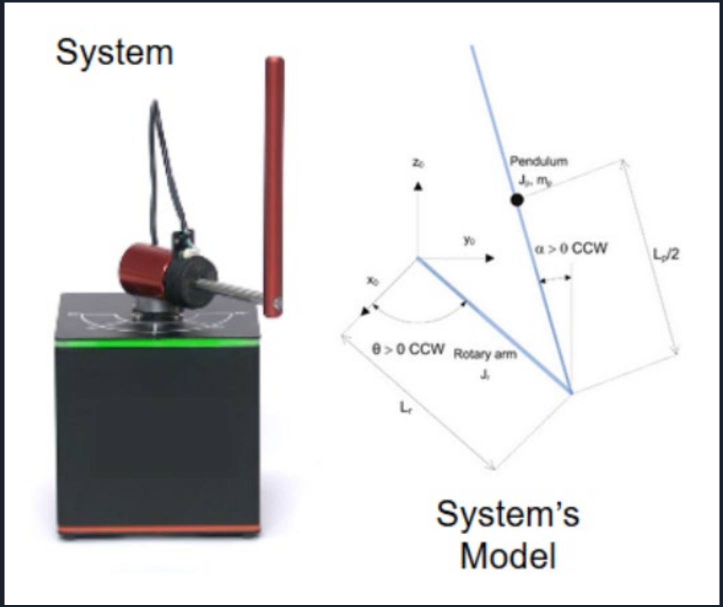
Project Team:

- Tyler Chesney
- Keegan Penso
- Quincy Owyang
- Mojit Bhardwaj
- Jack Russo

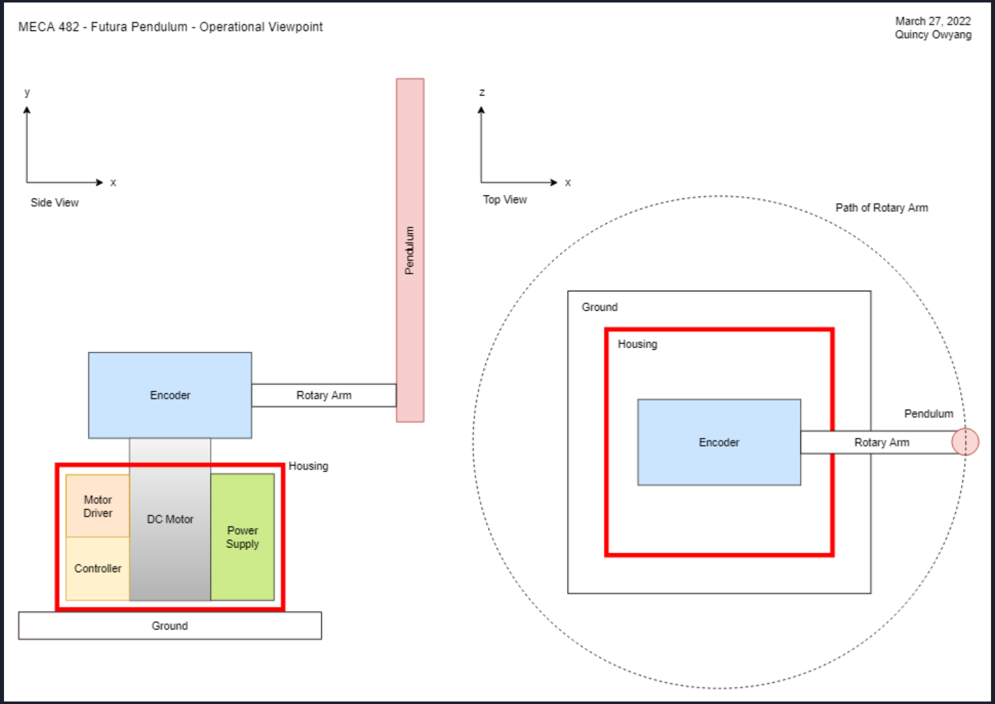
Introduction

The Furuta pendulum, or rotational inverted pendulum, consists of a driven arm which rotates in the horizontal plane and a pendulum attached to that arm which is free to rotate in the vertical plane.

Our goal is to design a system that is able to stabilize itself in the upright-vertical position.

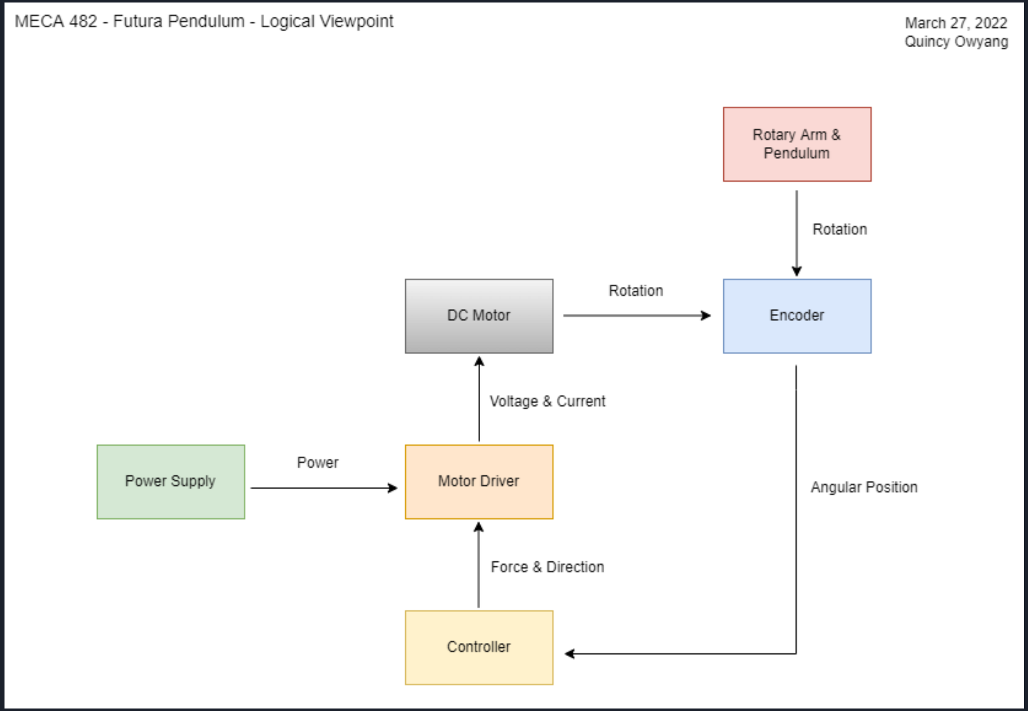


Operational Viewpoint



This viewpoint shows the physical space and movements of components of the pendulum

Operational Viewpoint



This logical viewpoint shows the interactions between components in the system.

System Model

Lr - Arm length

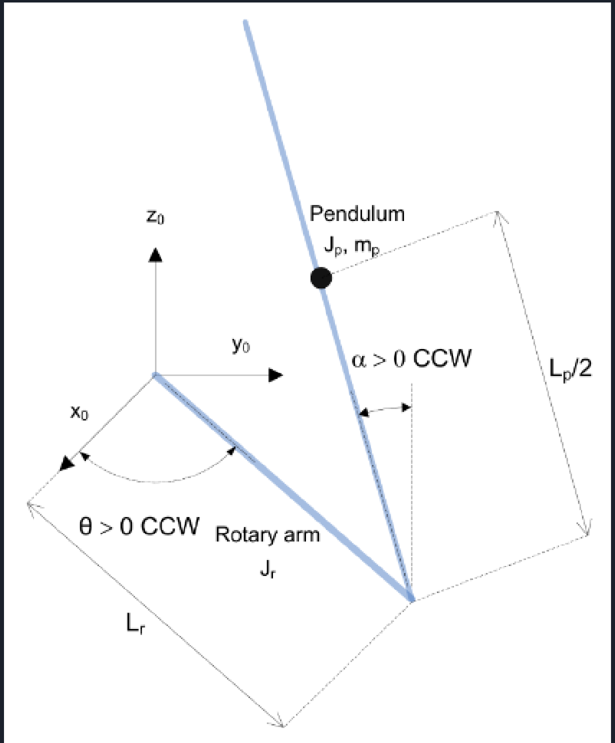
Jr - Arm Moment of Inertia

θ - Arm Angle

Lp - Pendulum Length

Jp - Pendulum Moment of Inertia

α - Pendulum angle



Linearized Equations of Motion

$$\left(m_p {L_r}^2 + J_r\right)\ddot{\theta} - \frac{1}{2} m_p L_p L_r \ddot{\alpha} = \tau - B_r \dot{\theta}$$

$$-\frac{1}{2} m_p L_p L_r \ddot{\theta} + \left(J_p + \frac{1}{4} m_p {L_p}^2\right)\ddot{\alpha} - \frac{1}{2} m_p L_p g \alpha = -B_p \dot{\alpha}$$

State Space Equations

$$A = \frac{1}{J_T}\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & \frac{1}{4}m_p^2 L_p^2 L_r g & -\left(J_p + \frac{1}{4}m_p L_p^2\right) B_r & -\frac{1}{2}m_p L_p L_r B_p \\ 0 & \frac{1}{2}m_p L_p g \left(J_r + m_p L_r^2\right) & \frac{1}{2}m_p L_p L_r B_r & -\left(J_r + m_p L_r^2\right) B_p \end{bmatrix}$$

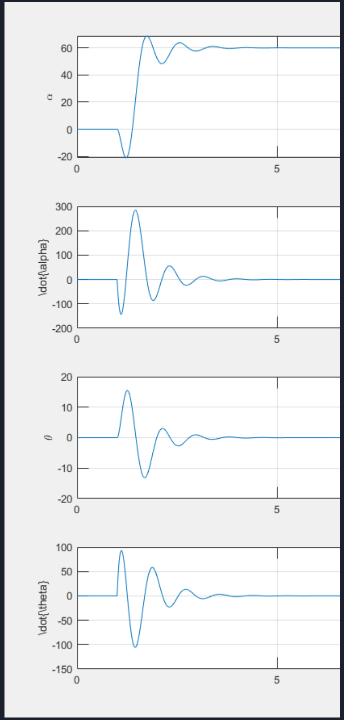
$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

$$B = \frac{1}{J_T}\begin{bmatrix} 0 \\ 0 \\ J_p + \frac{1}{4}m_p L_p^2 \\ \frac{1}{2}m_p L_p L_r \end{bmatrix}$$

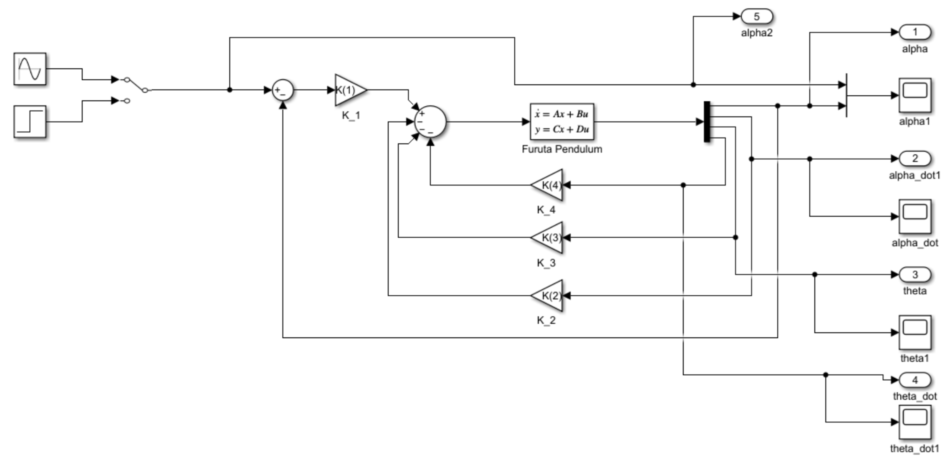
$$D = \begin{bmatrix} 0 \\ 0 \end{bmatrix}.$$

MATLAB Code

```
%% List of parameters

L_r = 0.36; %length of radial arm
L_p = 0.58; %length of the pendulum
m_p = 0.31; %mass of pendulum (kg)
m_r = 0.53; %mass of the radial arm.
B_p = 0.025; %damping of the pendulum.
B_r = 0.01; %damping of the radial arm.
fre = 0.1; %the frequency of the sin wave input

J_r=((m_r+m_p)*L_r^2)/3;
J_p=(m_p*L_p^2)/3;
J_T=J_p*m_p*L_r^2+J_r*J_p+0.25*J_r*m_p*L_p^2;
g=9.81; % gravity

%% Matrices

A=[0 0 1 0;
   0 0 0 1;
   0 0.25*m_p^2*L_p^2*L_r*g -(J_p+0.25*m_p*L_p^2)*B_r -0.5*m_p*L_p*L_r*B_p;
   0 0.5*m_p*L_p*g*(J_r+m_p*L_r^2) 0.5*m_p*L_p*L_r*B_r -(J_r+m_p*L_r^2)*B_p];

B=1/J_T*[0;0;J_p+0.25*m_p*L_p^2;0.5*m_p*L_p*L_r];

C=[1 0 0 0;
    0 1 0 0];
D=[0;0;0;0];

P=[-17.1 8.34 -2.87 0];
```

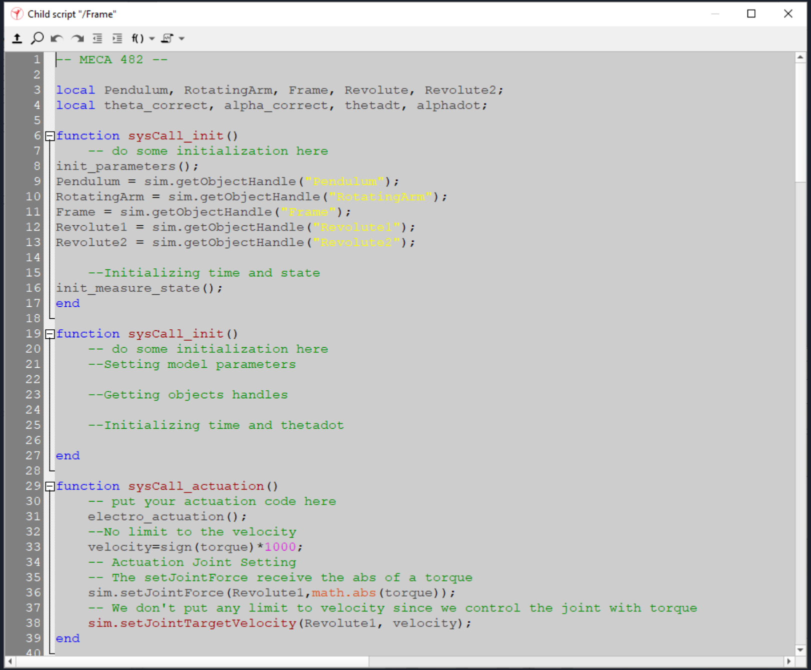


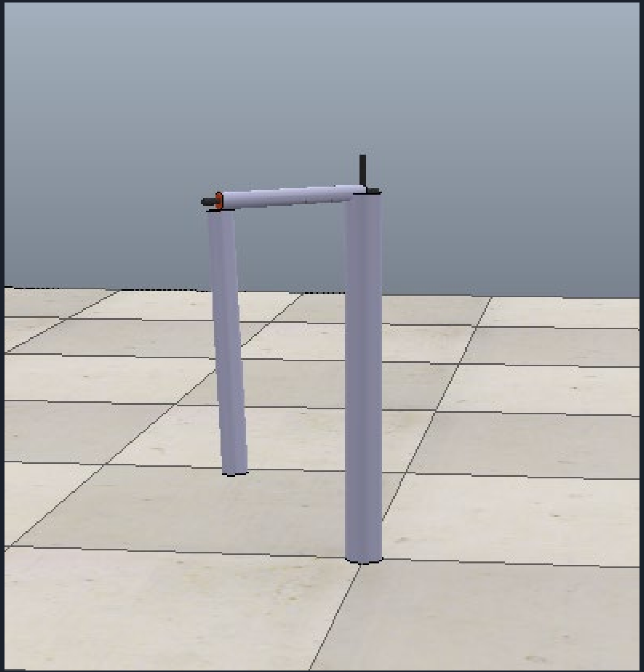
SIMULINK Diagram



Vikash Gupta (2022). Full State Feedback of Furuta Pendulum (https://www.mathworks.com/matlabcentral/fileexchange/25585-full-state-feedback-of-furuta-pendulum), MATLAB Central File Exchange. Retrieved May 19, 2022.

Simulation Results





Due to complications interfacing MATLAB and CoppeliaSim, an accurate simulation could not be produced

References

[1] Vikash Gupta (2019). Full State Feedback of Furuta Pendulum (https://www.mathworks.com/matlabcentral/fileexchange/25585-full-state-feedback-of-furuta-pendulum), MATLAB Central File Exchange. Retrieved May 19, 2022.

[2] Hernández-Guzmán Victor Manuel and Silva-Ortigoza Ramón, Automatic control with experiments. Cham, Switzerland: Springer, 2019.

[3] Jacob Apkarian, Michel Lévis and Hakan Gurocak, SRV02 Rotary Servo Base Unit User Manual. Ontario, Canada: Quanser, 2011.