3022 classification notebook

April 13, 2022

```
[1]: %matplotlib inline
    import calendar
    from sklearn.preprocessing import Normalizer
    import pandas as pd
    import seaborn as sns
    import matplotlib.pyplot as plt
    import numpy as np
    import scipy
    import scipy.stats as stats
    import math
    # Jupyter notebook settings
    import warnings
    warnings.filterwarnings('ignore')
    plt.rcParams['figure.figsize'] = [24, 8]
    # Dataset feature constants
    TARGET_CLASS = ["DEP_DEL15"]
    CATEGORICAL = ["CARRIER_NAME", "DEPARTING_AIRPORT", "PREVIOUS_AIRPORT",
     →"DEP_TIME_BLK"]
    CONTINUOUS = ["FLT_ATTENDANTS_PER_PASS", "GROUND_SERV_PER_PASS", "LATITUDE", |
     "PRCP", "SNOW", "SNWD", "TMAX", "AWND"]
    DISCRETE = ["MONTH", "DAY_OF_WEEK", "DISTANCE_GROUP", "SEGMENT_NUMBER",
                "CONCURRENT_FLIGHTS", "NUMBER_OF_SEATS", "AIRPORT_FLIGHTS_MONTH", "
     "AIRLINE_AIRPORT_FLIGHTS_MONTH", "AVG_MONTHLY_PASS_AIRPORT", "
      ⇔"AVG_MONTHLY_PASS_AIRLINE", "PLANE_AGE"]
```

1 Semester Project - Part 2

CSPB 3022 Data Science Algorithms - Spring 2022

- Author: Thomas Cochran
- Github: https://github.com/t-cochran/CSPB-3022-Data-Science-Project

1.1 Project Topic

Project goal:

The goal of this project is to create a binary classifier that predicts whether a domestic flight will be delayed or not.

Type of problem:

This is a binary classification problem. The target class DEP_DEL15 labels flights as on-time 0 or delayed 1 if the departure time exceeds 15 minutes.

Project motivation:

The motivation for this project is to identify when flight delays might not occur given a set of weather and transportation conditions. If we anticipate a delay, perhaps we could plan around it ahead of time. Or, at the very least, we could mentally prepare ourselves and hope we get an aisle seat.

1.2 Dataset

Dataset Source:

The dataset used in this project is from kaggle (source) and is located in file: dataset/full_data_flightdelay.csv

Each row of the dataset corresponds to a domestic airline departure flight and its associated weather for the year of 2019. This dataset has been wrangled and merged by the kaggle source from the following primary sources:

- 1. Bureau of Transportation statistics: Link
- 2. National Centers for Environmental Information (NOAA): Link

The departure data from the Bureau of Transportation statistics consists of monthly performance reports that contain a plethora of features for domestic departure flights. This is merged with the NOAA data which adds some interesting features that may contribute to flight delays, such as departure wind, snowfall, and precipitation.

Dataset size and features:

The dataset is quite large. Each row corresponds to a single domestic departure and the dataset contains over 6 million domestic departures. Here is a summary of the dataset's contents:

- 1.27 GB size
- 6,489,062 rows
- 26 features

There are 5 categorical and 21 numeric features. A description of each feature can be found in the file dataset/documentation.md.

```
[2]: # Load the dataset

df = pd.read_csv('.../dataset/full_data_flightdelay.csv', dtype='unicode')

# A brief look at a random sample of departure flights and some of their_

ofeatures
```

```
MONTH DEP_DEL15
                                    CARRIER_NAME NUMBER_OF_SEATS
3111605
            6
                      0
                             Mesa Airlines Inc.
                                                              76
            7
3658787
                      1
                              Endeavor Air Inc.
                                                              76
6143797
           12
                         SkyWest Airlines Inc.
                                                              66
1572326
            4
                      O Southwest Airlines Co.
                                                              143
1860569
            4
                      O Frontier Airlines Inc.
                                                              180
1525081
                          United Air Lines Inc.
                      1
                                                              120
3032008
                      0
                                     Comair Inc.
                                                              70
            6
                         American Airlines Inc.
5248182
                      0
                                                              128
           10
                         DEPARTING AIRPORT PLANE AGE FLT ATTENDANTS PER PASS
3111605
                     Norfolk International
                                                    5
                                                                           0.0
3658787
             John F. Kennedy International
                                                    6
                                                                           0.0
6143797
                    General Mitchell Field
                                                   14
                                                        3.419267401443636e-05
1572326
          Austin - Bergstrom International
                                                    8
                                                        6.178236301460919e-05
1860569
                   Stapleton International
                                                    8 0.00011572564827677376
              Chicago O'Hare International
                                                   18 0.00025380424062159367
1525081
3032008
         Ronald Reagan Washington National
                                                   17
                                                                           0.0
5248182
                         Douglas Municipal
                                                   19
                                                         9.82082928995461e-05
         PRCP SNOW
          0.0 0.0
3111605
3658787
          0.0 0.0
          0.0 0.0
6143797
          0.0 0.0
1572326
1860569
          0.0 0.0
1525081
          1.3 0.0
3032008 0.74 0.0
5248182 0.19 0.0
```

1.3 Data cleaning:

Setting data types

The data types for each feature are listed as object which means they are encoded as a string of varying length. The first cleaning step will be to cast each feature to an appropriate type that we can work with.

```
[3]: # Cast groups of features from strings to an appropriate data type
    df[TARGET_CLASS] = df[TARGET_CLASS].astype(int)
    df[CATEGORICAL] = df[CATEGORICAL].astype(str)
    df[CONTINUOUS] = df[CONTINUOUS].astype(float)
    df[DISCRETE] = df[DISCRETE].astype(int)
```

```
# List the features of the first 8 columns and their datatypes display(df.iloc[:, 0:7].dtypes)
```

MONTH	int32
DAY_OF_WEEK	int32
DEP_DEL15	int32
DEP_TIME_BLK	object
DISTANCE_GROUP	int32
SEGMENT_NUMBER	int32
CONCURRENT_FLIGHTS	int32
dtype: object	

Checking for missing values

Next I will look for any missing values by looking for features with np.nan values:

```
[4]: # Sum NaN values in all columns
missing_vals = df.isnull().sum()
print(f"Total missing values in dataset: {missing_vals.sum()}")
```

Total missing values in dataset: 0

There don't appear to be any missing values in the form of np.nan, however there are some empty categorical features listed with NONE:

```
[5]: # Search the dataframe for "NONE" string values
none_vals = df[df.eq("NONE").any('columns')]

# Print the name of columns with NONE value and the percentage of the dataset
with NONE values
print(f"Columns with NONE values: {df.columns[df.eq('NONE').any('rows')]}")
print(f"Percent of dataset with NONE values: {(len(none_vals) / len(df) * 100):.

□2f}%")
```

```
Columns with NONE values: Index(['PREVIOUS_AIRPORT'], dtype='object')
Percent of dataset with NONE values: 22.33%
```

22.3% of the dataset has rows with NONE values, so it would be costly to drop all rows with this missing value. However, this affects only one feature: PREVIOUS_AIRPORT. Given these circumstances, I will keep these rows and note that the feature PREVIOUS_AIRPORT is imbalanced.

1.4 Exploratory Data Analysis (EDA):

Overview: Why EDA?

Exploratory data analysis is a collection of methods that investigate characteristics and patterns in the dataset which can improve our understanding of the features and their relationships.

How EDA will be performed:

I will be using EDA to visualize a variety of relationships between features and the target class DEP_DEL15, such as:

- 1. What numeric features are highly correlated? Are any redundant?
- 2. How does the frequency of flight delays vary by month or day of the week?
- 3. Are there certain airlines that experience more frequent delays?

I will begin by looking at the distribution of the target class DEP_DEL15.

Target class distribution

The distribution of the target class is important because it can affect model selection. If the target class is imbalanced, then the predictive model could be more sensitive to biases and errors.

```
[110]: # Create a countplot of our target class
sns.set_theme(style="darkgrid")
ax = sns.countplot(x="DEP_DEL15", edgecolor='black', data=df)
ax.set_xticks([0, 1], ["on-time", "delayed"], size=20)
ax.set_ylabel("count", fontsize = 20)
ax.set_title("Total on-time and delayed flights", fontdict={'fontsize':26})
ax.set_xlabel(None)

# Print the proportion of each target
class_targets = df.groupby(['DEP_DEL15']).size()
print(f"On-time flights in the dataset: {class_targets[0]/len(df)*100:.2f}%")
print(f"Delayed flights in the dataset: {class_targets[1]/len(df)*100:.2f}%")
```

On-time flights in the dataset: 81.09% Delayed flights in the dataset: 18.91%



There is a clear bias towards on-time flights, whose proportion is more than 4-times the proportion of delayed flights. This imbalance will need to be considered when we select our model.

Mean and standard deviation of numeric features

Next I will have a quick look at some descriptive statistics for numeric features to get a handle on where the averages are. It may be interesting to see where some differences exist for delayed flights, however we cannot say whether any differences are statistically significant without further analysis. First I will compute the mean and standard deviation for delayed flights:

```
[7]: # Select numeric features, exluding month and day of week
     numeric = df[TARGET_CLASS + CONTINUOUS + DISCRETE]
     numeric = numeric.loc[:, ~numeric.columns.isin(['MONTH', 'DAY_OF_WEEK'])]
     # Select delayed flights only
     delayed_numeric = numeric[numeric['DEP_DEL15'] == 1]
     # Compute the mean and standard deviation for numeric features of delayed,
      \hookrightarrow flights
     print("DELAYED FLIGHTS:")
     display(delayed_numeric.describe().loc[['mean', 'std']].applymap('{:.2f}'.
      →format))
    DELAYED FLIGHTS:
         DEP DEL15 FLT ATTENDANTS PER PASS GROUND SERV PER PASS LATITUDE \
                                        0.00
               1.00
                                                              0.00
                                                                      36.71
    mean
              0.00
                                        0.00
                                                                       5.25
                                                              0.00
    std
         LONGITUDE PRCP
                           SNOW SNWD
                                        TMAX AWND DISTANCE_GROUP SEGMENT_NUMBER \
             -93.25 0.16 0.06 0.13 71.13 8.72
                                                               3.90
                                                                              3.47
    mean
             16.81 0.43 0.49 0.86 19.24 3.78
                                                               2.38
                                                                              1.77
    std
         CONCURRENT FLIGHTS NUMBER OF SEATS AIRPORT FLIGHTS MONTH \
    mean
                       28.24
                                       134.88
                                                            13174.01
                       21.30
                                        46.48
                                                            8731.56
    std
         AIRLINE_FLIGHTS_MONTH AIRLINE_AIRPORT_FLIGHTS_MONTH
                       63211.75
                                                       3579.94
    mean
                       35100.32
                                                       4094.56
    std
         AVG_MONTHLY_PASS_AIRPORT AVG_MONTHLY_PASS_AIRLINE PLANE_AGE
                        1645375.69
                                                  7828102.68
                                                                  11.62
    mean
    std
                        1095573.89
                                                  5043404.98
                                                                   6.77
    Next, I will compute the same for on-time flights:
[8]: # Select on-time flights only
     ontime_numeric = numeric[numeric['DEP_DEL15'] == 0]
     # Compute the mean and standard deviation for numeric features of on-time_{\sqcup}
      \hookrightarrow flights
     print("ON-TIME FLIGHTS:")
     display(ontime_numeric.describe().loc[['mean', 'std']].applymap('{:.2f}'.
      →format))
```

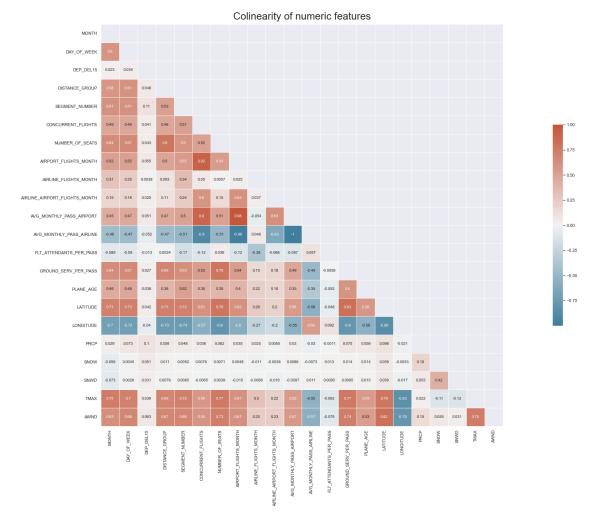
```
DEP_DEL15 FLT_ATTENDANTS_PER_PASS_GROUND_SERV_PER_PASS_LATITUDE \
          0.00
                                    0.00
                                                          0.00
                                                                  36.70
mean
          0.00
                                    0.00
                                                          0.00
std
                                                                   5.56
                                           AWND DISTANCE GROUP SEGMENT NUMBER \
     LONGITUDE
                PRCP
                       SNOW
                             SNWD
                                     TMAX
        -94.49
                       0.02
                             0.08
                                   71.55
                                                           3.80
                                                                           2.95
                0.09
                                           8.25
mean
std
         18.15
                0.32
                       0.26
                             0.69
                                   18.14
                                           3.56
                                                           2.38
                                                                           1.74
     CONCURRENT FLIGHTS NUMBER OF SEATS AIRPORT FLIGHTS MONTH
                                  133.47
mean
                   27.74
                                                        12570.41
                   21.56
                                   46.44
                                                         8860.97
std
     AIRLINE_FLIGHTS_MONTH AIRLINE_AIRPORT_FLIGHTS_MONTH
                   62901.99
                                                    3431.10
mean
                   34212.30
                                                    4286.35
std
     AVG_MONTHLY_PASS_AIRPORT AVG_MONTHLY_PASS_AIRLINE PLANE_AGE
                    1575403.75
                                              7811906.61
                                                              11.51
mean
                    1129931.10
                                              5047687.73
                                                               6.97
std
```

The average daily percipitation and snow (PRCP, SNOW; in inches) is lower for on-time flights, but both feature means have a relatively large variance. There is a difference between the average monthly passengers at the airport (AVG_MONTHLY_PASS_AIRPORT), which is larger for delayed flights.

Correlation matrix

A correlation matrix is a collection of correlation coefficients between features in a dataset. This is useful because it can identify strong positive or negative linear relationships between features. If a pair of features have a very strong linear relationship, then they may be redundant; that is, one can be substituted for the other.

In this section, I will pair numeric features and then create a correlation matrix. Using the correlation matrix, I will identify which features are highly correlated and then remove redundant features using a simple heuristic: If two features have $|\rho| >= 0.90$, I will remove the feature with a lower correlation to our target class.



The heatmap shows some highly correlated numeric features. For example, the average airport flights per month (AIRPORT_FLIGHTS_MONTH) is highly correlated with the average airport departing passengers per month (AVG_MONTHLY_PASS_AIRPORT), which we may expect.

The following algorithm should target and remove highly correlated features from the dataset using the heuristic above.

```
[10]: # Select the upper triangle of correlation matrix using our mask
      corr_matrix = corr.where(cond=mask)
      # Remove redundant features according to heuristic:
      # If |p| \ge 0.90 remove the feature less correlated to target class 'DEP_DEL15'
      redundant_features = []
      for col in corr matrix:
          current_column = corr_matrix[col] # Iterate over columns in the
       \hookrightarrow correlation matrix
          if current_column.name == 'DEP_DEL15': # Ignore our target class
              continue
          else:
              for row in current_column.index:
                                                  # Select each row in the current
       \hookrightarrow column
                  if current_column.name == row: # Ignore correlation between the_
       ⇒same row and column
                      continue
                  if np.abs(current_column[row]) >= 0.90:
                                                                             # Select
       ⇔highly correlated |p|>=0.90 features
                      target_class_row = corr_matrix.loc['DEP_DEL15'][row] # Check_
       ⇔the correlation with our target class
                      target_class_col = corr_matrix.loc['DEP_DEL15'][col]
                      if target_class_row >= target_class_col:
                                                                             # Remove
       → feature with lower correlation to target
                          redundant_features.append(current_column.name)
                      else:
                          redundant_features.append(row)
      # Print and drop redundant features
      df.drop(set(redundant_features), axis=1, inplace=True)
      print(f"Dropped redundant features:\n{set(redundant_features)}")
```

```
Dropped redundant features:
{'AVG_MONTHLY_PASS_AIRLINE', 'AVG_MONTHLY_PASS_AIRPORT', 'CONCURRENT_FLIGHTS'}
```

Flight delays by month and weekday

In this section, I will be investigating how delays vary over months and days of the week. My intuition tells me that holiday months and Mondays should have a higher incidence of delays due to more congestion at the airport.

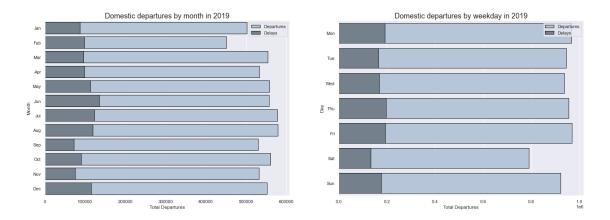
```
[121]: # Create dataframes to organize and sort the frequency data
month_df = pd.DataFrame({
    'MONTHLY_TOTAL' : df.groupby(['MONTH']).size(),
    'MONTHLY_DELAYS' : df[df['DEP_DEL15'] == 1].groupby(['MONTH']).size(),
```

```
})
day_df = pd.DataFrame({
    'WEEKDAY_TOTAL' : df.groupby(['DAY_OF_WEEK']).size(),
    'WEEKDAY_DELAYS' : df[df['DEP_DEL15'] == 1].groupby(['DAY_OF_WEEK']).size()
})
# Plot frequency of monthly delays
fig, (ax1, ax2) = plt.subplots(1, 2)
sns.barplot(x="MONTHLY_TOTAL", y=month_df.index, data=month_df, ax=ax1,_
 ⇔order=month_df.index,
            edgecolor='black', color="lightsteelblue", label="Departures", |
 ⇔orient="h")
sns.barplot(x="MONTHLY_DELAYS", y=month_df.index, data=month_df, ax=ax1,__

order=month_df.index,
            edgecolor='black', color="slategray", label="Delays", orient="h")
# Plot frequency of week day delays
sns.barplot(x="WEEKDAY_TOTAL", y=day_df.index, data=day_df, ax=ax2,_
 →order=day_df.index,
            edgecolor='black', color="lightsteelblue", label="Departures", |
sns.barplot(x="WEEKDAY DELAYS", y=day_df.index, data=day_df, ax=ax2,_

order=day_df.index,
            edgecolor='black', color="slategray", label="Delays", orient="h")
# Configure plot labels
ax1.set_title('Domestic departures by month in 2019', fontdict={'fontsize':18})
ax2.set_title('Domestic departures by weekday in 2019', fontdict={'fontsize':
→18})
ax1.set_yticklabels(['Jan', 'Feb', 'Mar', 'Apr', 'May', 'Jun', 'Jul', 'Aug', "
⇔'Sep', 'Oct', 'Nov', 'Dec'])
ax2.set yticklabels(['Mon', 'Tue', 'Wed', 'Thu', 'Fri', 'Sat', 'Sun'])
ax1.legend(ncol=1, loc="upper right", frameon=True), ax2.legend(ncol=1,__

¬loc="upper right", frameon=True)
ax1.set_xlabel("Total Departures"), ax1.set_ylabel("Month")
ax2.set_xlabel("Total Departures"), ax2.set_ylabel("Day");
```

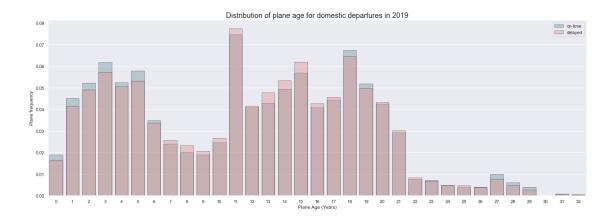


The month of August has the most departures, followed by July. The fewest departures occur on Saturday's. The largest number of delays occur on June and on Thursday. However, the all days of the week appear to have a comparable number of delays relative to their total departures.

Distribution of plane age

Next I will investigate whether delayed flights involve more older planes. In order to do this, I will plot the relative frequency of planes by age for both delayed and on-time flights.

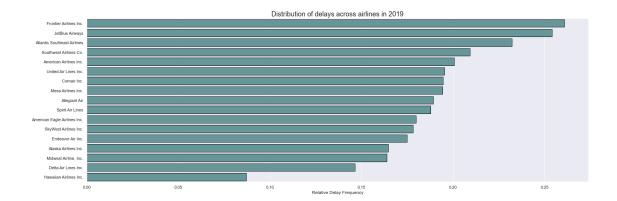
```
[129]: # Select delayed and ontime departures and sort by plane age
       ontime_df = df[df['DEP_DEL15'] == 0].sort_values(['PLANE_AGE'], ascending=True)
       delay_df = df[df['DEP_DEL15'] == 1].sort_values(['PLANE AGE'], ascending=True)
       # Compute the relative frequency of planes for each age group
       ontime_df['AGE_PCT'] = ontime_df['PLANE_AGE'].map(ontime_df['PLANE_AGE'].
        ⇔value_counts(normalize=True))
       delay_df['AGE_PCT'] = delay_df['PLANE_AGE'].map(delay_df['PLANE_AGE'].
        ⇔value counts(normalize=True))
       # Pair the age percentage with each unique plane age
       ontime_df = ontime_df.groupby(['PLANE_AGE']).head(n=1)[['PLANE_AGE', 'AGE_PCT']]
       delay_df = delay_df.groupby(['PLANE_AGE']).head(n=1)[['PLANE_AGE', 'AGE_PCT']]
       # Plot the percentage of planes in each age group for delayed and on-time_
        \hookrightarrow flights
       ax = plt.subplots()
       ax = sns.barplot(x=ontime_df.PLANE_AGE,y=ontime_df.
        GAGE_PCT,color='cadetblue',edgecolor='black',label='on-time',alpha=0.4)
       ax = sns.barplot(x=delay_df.PLANE_AGE,y=delay_df.
        AGE_PCT,color='lightcoral',edgecolor='black',label='delayed',alpha=0.4)
       ax.set title('Distribution of plane age for domestic departures in 2019',,,
        ⇔fontdict={'fontsize':18})
       ax.set_xlabel("Plane Age (Years)"), ax.set_ylabel("Plane frequency")
       plt.legend();
```



The difference between on-time and delayed departures appears to be quite small. The left hand side of the graph suggests that on-time flights use younger aircraft slightly more frequently. The middle of the graph suggests that delayed flights use older aircraft slightly more frequently.

Distribution of delays across airlines

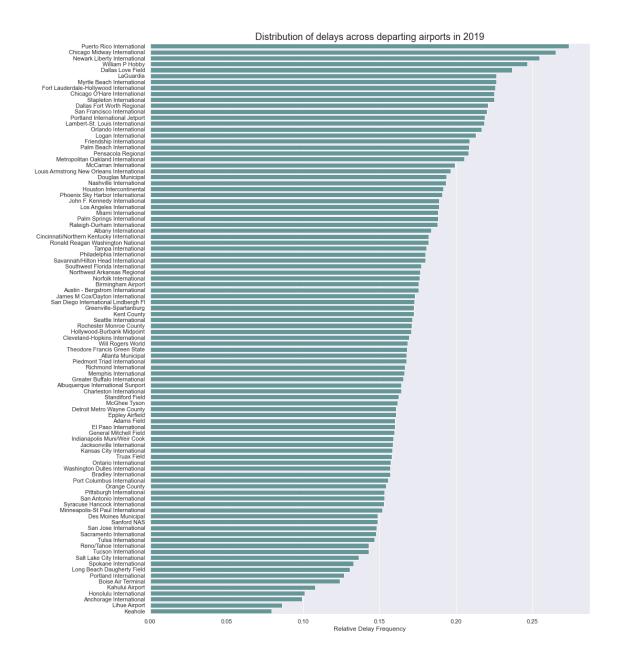
Next I will plot the distribution of delays across different airlines to see if there are any particular carriers that are better or worse at avoiding delays.



The clear winner here is Hawaiin Airlines which has the lowest relative delay frequency. The top two airlines are Frontier and Jetblue. I find this a little amusing because I often booked Frontier for flights to CU Boulder and I often encountered delays.

Distribution of delays across departing airports

This section is similar to the last, however instead of airline carriers I am plotting the relative frequency of delays over departing airports in the dataset. As with airline carrier, this should be interesting to see if there exist certain airports that are rife with delays.



Summary of EDA findings

- The dataset is biased towards on-time flights.
- The largest number of departure delays occur on June and on Thursday.
- Airlines that are delayed most frequently are: Frontier and Jetblue.
- Airports with the most frequent delays are: Puerto Rico Intl., Chicago Midway, and Newark Liberty
- SEGMENT_NUMBER and PRCP have the largest positive correlation with target class DEP_DEL15.
- The numeric features AVG_MONTHLY_PASS_AIRLINE, AVG_MONTHLY_PASS_AIRPORT, CONCURRENT_FLIGHTS were dropped.

Discussion of forseen difficulties

EDA has identified foreseen difficulties to consider when selecting a model for this classification task:

- 1. The target class DEP_DEL15 is imbalanced: 18.91% of the total flights in the dataset are delayed.
- 2. There remain an abundance of features after cleaning: 4 categorical, 18 numeric.
- 3. The dataset is large and may require algorithms with lower computational complexity.
- 4. Model input is heterogenous.

Each of these difficulties can be mitigated in different ways:

- 1. Class imbalance in DEP_DEL15 may be mitigated by sampling methods, such as oversampling delayed or undersampling on-time flights.
- 2. Heterogenous input may require encoding categorical features, or selecting a model that can handle categorical and numeric features.
- 3. Since the input size is quite large, it will be important to select models whose complexity has a lower polynomial k of the input size n, i.e. n^k .

1.5 Plans for Model Approach:

For this project, I intend to train two different models and then compare their accuracy. One will be relatively simple and the other will be more advanced. In both cases, I want to preserve interpretability because it could be interesting to see how each model weighs features differently.

Model 1: Logistic regression

The first classification algorithm that I plan to use is simple logistic regression. The reasons for this are:

- 1. Target class DEP DEL15 is categorical and binary which is well suited for logistic regression.
- 2. The algorithm is simple to run, resistant to overfitting, and can act as a starting point before attempting more advanced classification models.
- 3. The interpretability is high which allows us to better understand how the model predicts delays from the features.
- 4. It has favorable training time complexity $\mathcal{O}(np)$ and test time complexity $\mathcal{O}(p)$ for the large input data n and predictors p in this dataset.

One important caveat with logistic regression is that real-world data isn't often modeled very well with a linear decision boundary. It's unlikely that flight delays are bounded by some linear combination of weather conditions, airline traffic, or airports.

Furthermore, logistic models perform poorly with highly correlated features. The correlation matrix in EDA did identify and remove some highly correlated features, but there remain features that are moderately correlated with $|\rho| >= 0.50$.

In light of these difficulties, I will relegate logistic regression as a first pass attempt at classifying flight delays. After training the model, I will asses the accuracy of its predictions and then see if a more advanced algorithm can achieve greater model accuracy.

Model 2: Random Forest

The second classification algorithm that I plan to use is the random forest ensemble method. The reasons for this are:

- 1. It handles non-linear parameters and is immune to highly correlated features, which are both improvements from the logistic model.
- 2. It can leverage the size of the dataset by generating larger bootstrapped sampling distributions with lower standard error.
- 3. As with logistic regression, interpretability of each decision tree is high and features can be ranked in terms of importance.
- 4. The training time complexity for k trees, n data points, and p predictors is $\mathcal{O}(n\log(n)pk)$ which is favorable for the large n in this dataset.

The random forest model will require some parameter tuning and that could create some challenges due to the size of the dataset. For example, increasing the total number of trees and their depth might improve model accuracy, but it could create large increases in space and time complexity.

Handling class imbalance

Both random forest and logistic regression will be sensitive to feature imbalances in the training data. This imbalance does exist. The target class DEP_DEL15 labels 81.09% on-time flights and only 18.91% delayed flights, as was shown during EDA. If we were to train either model with a similarly imbalanced set of training data, the model could predict all flights as on-time and still be correct 81.09% of the time. Therefore, in both models, great care must be taken to balance the target class.

In the case of the logistic model, I will attempt to balance DEP_DEL15 by using re-sampling methods, such as oversampling delayed flights or undersampling on-time flights. In the case of random forest, I can balance by using these re-sampling methods during the bootstrap step.