Spherical Polarization: A measurement approach for compositional data

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December 30, 2022

Abstract

The concept of polarization has been a topic with growing interest across multiple disciplines (labor economics, political science, conflict studies, etc.). While there are many ways in which polarization can be quantified, there are notable shortcomings, particularly in cases of comparison of concentration across arranged groups as a proportion of the population of interest. In this article, I develop a new measurement of polarization for compositional data with ordered categories. This new measurement, "spherical polarization" accounts for the constraint that the sum of group concentration must always be equal to one while also avoiding issues of dependence and collinearity endemic to the application of current polarization approaches to compositional data. Further, by providing a tractable mapping from $\mathbb{R}^N \to \mathbb{R}$, spherical polarization provides a straightforward function by which changes in polarization can be measured with limited dependence on initial distributions, unlike contemporary measures. Applying this measure to past work of labor market polarization, I show that current quantification strategies are generally inappropriate for ordered compositional data, difficult to interpret, and routinely misestimate changes in polarization.

Word Count: 8056 words (including abstract, notes, and references)

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1 Introduction

The evolution of measures of polarization in the social sciences to apply to unique topics and situations has been key to understanding political party systems, conflict, urban development, and inequality. Early 2000's research into an apparent hollowing-out of particular occupational concentrations in the middle of the skill/earning spectrum relative to low- and high-skill occupations has accelerated, particularly post the 2008 Global Financial Crisis.

There is, however, a dearth of interpretable empirical analysis dedicated to this topic, given the inherent nature of distributional changes. Compositional data, if not addressed properly, can lead to erroneous or nonsensical results, and contemporary methods of accounting for the properties of such data can be opaque and computationally intensive. These difficulties in interpretation and quantification of polarization can have far-reaching effects, given that polarization is a topic that is not constrained to a specific sub-field of economics; rather it has broad coverage across the social sciences. Further, such research is commonly associated with policy creation and recommendations, making precise and understandable measurements of polarization all the more crucial.

In this paper, I create a new measure of polarization particularly applicable to the analysis of ordered compositional data. By exploiting an underlying feature of geodesic distance in Euclidean space, I am able to produce a function which maps an n-categorical ordered quantification of polarization into a one dimensional output, providing a tractable measure without having to rely on multiple interwoven components. By accounting for the underlying nature of these data, we are able to produce meaningful inference in a burgeoning labor field.

The major findings of this paper are as follows: First, I show that common measures of polarization are not appropriate for compositional data, through a number of comparisons. Second, I find that most contemporary literature does not provide empirical analysis insofar as polarization is concerned, and, when it does, such analyses are often misspecified, generating estimates that are largely unjustifiable and/or nonsensical. The remainder of the paper is as follows. Section 2 describes the evolution of polarization analysis, across a number of research fields, a brief introduction to issues with inference, and a background of compositional data measurement approaches. Section 3 elaborates upon polarization measurements, the statistics underpinning their quantification, and a discussion of their applicability and distributional properties. Section 4 describes the properties of spherical polarization and its relation to ordered compositional data. Section 5 reviews the distributional properties of contemporary polarization through comparisons of visualizations and replicates an empirical analysis from a recent research article, comparing spherical polarization to the original measures. Section 6 provides a brief summary and discussion of future research areas.

2 Literature

Although polarization, in the broad sense, is a concept that is easily understood, given its varying applications in the natural sciences, its usefulness in the scope of the social sciences has been developed only since the mid-1970's. Initially, this research was largely confined to political science, starting with Sartori (1976). Sartori's approach to political polarization was two-dimensional, consisting of measures of the degrees of atomization and dispersion. Atomization analyzes the number of distinct categories (*e.g,* political parties) in a political system, while dispersion looks at the ideological distance between the categories. This multidimensionality is essential to quantifications of polarization, as it sets the foundation for analysis of distributions of the categorized features. To elaborate, the expansion into a second dimension allows the axioms of polarization (Esteban and Ray 1994) to be satisfied. Subsequently, Sigelman and Yough (1978) improved upon Sartori's work by introducing a concept of an ideological center of gravity into their measurement. Adding a more tractable scale, this index mapped polarization in the party system as a distribution across the political spectrum, broadly from left to right on a seven-point scale, accounting for distribution between extremes on the spectrum and the total number of unique categories. While their center of gravity improvement is largely inapplicable to the labor applications of this paper, Sigelman and Yough (1978) establish an explicit baseline by which polarization can be understood, namely, "...the highest possible polarization value would obtain for a system in which 50% of the support is for parties at the extreme left pole and 50% is for parties at the extreme right pole. The least polarized system would be one in which all support is for parties occupying a single pole anywhere on the left-right continuum." The establishment of inherent bounds to such an index proves to be key in distinguishing polarization from a related, yet distinct research field, inequality.

Following this, Esteban and Ray (1994) is the canonical work insofar as empirical measurement of polarization in the social sciences is generally concerned.[1] Formally, they introduced a set of features by which polarization should appropriately be measured, and axioms of polarization by which a measure can be defined. Branching from the initial scope of polarization through political parties and party systems, the authors approached polarization as applied to conflict, that is, quantifying

[1]Subsequent research led by Esteban and Ray has added additional nuance into the measurement of polarization and comparisons to other measures and distinctions (see Duclos, Esteban and Ray (2004), Esteban and Ray (1999, 2011, 2012))

social divisions in association with unrest as well as economic distributions of income and wealth. Reynal-Querol (2002) and Montalvo and Reynal-Querol (2005) extend the discussion of polarization's link to conflict from an ethnic standpoint, distinguishing polarization from fractionalization in the analysis of civil wars and the impact of ethnic and religious identification grouping.

A growing field in the polarization literature is that of labor economics. Given that polarization measures concentrations and distributions, it is particularly apt to be incorporated into labor economics. The labor market polarization literature holds strong to a few publications, which were the initial foray into the distributional changes of employment. Autor, Katz and Kearney (2006) examine the structural distributions of occupational evolution for the United States from 1980 - 2000, arguing that acceleration in technological change is associated with a skill-biased occupational effect on the underlying distribution of tasks in the labor market. Separating these into distinct, ranked categories, the authors delineate job distribution by task, namely, abstract tasks, routine tasks, and manual tasks. Extending from Autor, Levy and Murnane (2003), the authors hypothesize that this is a function of computerization. More directly, it is posited that computerization is associated with increased demand for abstract tasks (those occupations which require high levels of technical, interpersonal, or cognitive ability), decreased demand for routine tasks (*e.g.* white collar and/or manufacturing jobs with some education, clerical jobs, etc), and an ambiguous effect on the demand for nonroutine task-associated jobs (service workers, for example). With that, these uneven demand shifts are associated with a "hollowing-out" of the occupational distribution for those with average education and technical skills. Given that the jobs associated with the more technical positions requiring interpersonal or leadership skills are associated generally with higher pay, those in the middle of the spectrum with average pay, and those in manual non-routine jobs with low pay, Autor, Katz and Kearney (2006) suggest that this shift to the tails of the distribution (or, at a minimum, shift away from the center) is associated with wage inequality. Additional influential work in the area of labor market polarization includes Goos and Manning (2007), applying a similar analysis to the UK from 1979 - 1999, Goos, Manning and Salomons (2009), who look at a broad set of 16 European countries in a cross-national analysis, and Goos, Manning and Salomons (2014), who compare this routine-task-associated job decline from technological change with offshoring, finding that this change has a larger effect than offshoring, and affects polarization both in the broad sense as well as within industries. Jaimovich and Siu (2020) link slow recoveries in aggregate employment following contemporary recessions to diminishing concentration of middle-skill, routine occupations, categorizing occupations into non-routine cognitive, routine, and non-routine manual bins. Other ventures into the job polarization sphere include firm-level analysis of the impact of changes in technology on administrative office jobs (Dillender and Forsythe 2022), the channeling of routine-biased technological change by wage-setting institutions to account for heterogeneity across countries (Oesch and Rodriguez-Menes 2011), and attempts to address the empirical concerns regarding the link between job polarization and inequality, through effect decomposition of cross-national panel data in Europe (Bussolo, Torre and Winkler 2018).

Comparison of labor-related data in levels can lead to ambiguities in interpretability. This issue is magnified when looking at cross-national data or other panel data in which heterogeneous cross-sections are observed over time. With that, employment data is often either log-transformed or converted into compositional data, the latter of which is particularly suitable for polarization analysis. A function of compositional data, inherently, is that the components are not independent.

A common misapplication of these properties is shown in attempts to account for endogeneity via inclusion of instruments, for example. Let there be a system of equations,

$$y_{im} = x'_{im}\delta_m + \epsilon_{im}$$

where $i = 1, ..., n$ (sample size), m is the number of equations (in this case, the number of components of labor polarization), x is a $L_m \times 1$ vector of endogenous explanatory variables, and there exists a $K_m \times 1$ vector of instruments, denoted z_{im}, where $\mathbb{E}[z_{im}\epsilon_{im}] = 0$. Estimating single-equation-GMM (*i.e.* running each estimation separately) requires the assumption that the optimal weight matrix for the system of equations is block diagonal. Underlying this assumption, this states that $\mathbb{E}[\epsilon_{im}\epsilon_{ih}z_{im}z'_{ih}] = 0 \ \ \forall\, m \neq h$, this being the (m, h) block of the variance of the limiting distribution of orthogonality conditions, implying that the equations are unrelated. Contrary to this assumption, however, is a fundamental property of compositional data, that the components are functions of each other, thereby violating this assumption. Potential violations of this assumption can be found across the economic literature, including analyses of firm behavior (Shim and Yang 2018), price studies of information technologies (Jerbashian 2019), housing economics (Parkhomenko 2022) , and computerization (Dillender and Forsythe 2022).

From an alternative model specification standpoint, let's consider a model in which the explanatory variables are the proportional categorized groupings. Initially, one can anticipate that coefficient estimates on one of the categories will be dropped from the analyses, stemming from correlation.

Intuitively, this makes sense, as each of the categories is a function of the other which must fluctuate accordingly given a change in any component, as the sum of these must continue to sum to one. Beyond this, attempts at causal inference could be severely impacted by the incorporation of these variables in their proportional form, as shown by Young (2022), who finds that two-stage-least-square estimation in Stata is highly sensitive to seemingly inconsequential factors in the setup, such as variable order, in the presence of near collinearity among explanatory variables. Given the properties underlying compositional data, this collinearity is likely. From this, additional approaches are required to create meaningful analyses with compositional data.

Katz and King (1999), posit that such analyses of systems of linear equations (in the case of multiparty electoral data, in their case, but applicable to labor polarization as well) "usually generate nonsensical results." The critical facet of these analyses are the restrictions that the shares be positive and less than one and that the sum of these proportions to be equal to be one. Although straightforward, these conditions are often overlooked. Looking at the categorized bins as separate dependent variables using solely an OLS estimator implies that it is possible for observations to be less than zero or greater than one, which is impossible for such data. Katz and King (1999) has the disadvantage of being quite mathematically complex, however. Tomz, Tucker and Wittenberg (2002) simplify the Katz and King (1999) setup through an application of Seemingly Unrelated Regressions to Feasible Generalized Least Squares, while Jackson (2002) addresses this through an iterative Generalized Least Squares procedure. A central component of these methods is the application of the multivariate logistic transformation in order to formalize these bounds restrictions. A shortcoming in applying these measures directly to applications of labor market polarization, however, is that they are largely order invariant. To clarify, an X unit shift in concentration from Party A to Party B would be equal in change to an X unit shift in concentration from Party A to Party C, ceteris paribus, regardless of the relative ideological positions of the party. This is a discordant feature insofar as links to labor market polarization is concerned, as there is an ordered component to the occupational grouping. Further, while the Seemingly Unrelated Regressions approach provides useful analysis robust to correlation across categories, there is the additional drawback of a redundant degree of freedom issue, that is, if all components are included in a regression (even incorporating a Seemingly Unrelated Regression approach) one category will be forced to drop from the equation, as the remaining categories are jointly perfectly collinear with it (Kagalwala, Philips and Whitten 2021). Separate from the simplex planar application of restrictions from Katz and King (1999), there is a nascent field of computational analyses extending such approaches to the n-dimensional plane, incorporating a dimension-reduction hyperspherical transformation of the data (Wang et al. 2007), although the applications are limited to the realm of economic forecasting, and are such not directly applicable to distributional analyses such as labor market polarization.

A recurring theme in labor market polarization research is a reliance on descriptive statistics and/or visualizations in order to convey a theme of labor polarization. This can be shown in a variety of forms: summary statistics and descriptive tables reflecting the initial shares and percentage growth of categorized occupations (Dabla-Norris, Pizzinelli and Rappaport 2019, Dwyer and Wright 2003, 2019, Fernández-Macías 2012, Goos, Manning and Salomons 2009, Oesch and Rodriguez-Menes 2011), bar graphs (observed values) of concentration across these occupational categories at specific points in time (Abel and Deitz 2012, Dabla-Norris, Pizzinelli and Rappaport 2019, Kalleberg 2011), bar charts (in percentage change, both in relative and absolute terms) of occupational groupings at specific points in time (Abel and Deitz 2012, Autor 2010, Dwyer and Wright 2003, 2019, Fernández-Macías 2012, Jaimovich and Siu 2020, Oesch and Rodriguez-Menes 2011), line graphs reflecting the changes across a larger number of time periods for these occupational groups (Abel and Deitz 2012, Dabla-Norris, Pizzinelli and Rappaport 2019), and line graphs reflecting the smoothed changes across granular categorization of occupational bins between set time periods (Autor 2010, Autor, Katz and Kearney 2006), among others. The common thread among these articles is that these visualizations and statistics are solely of a descriptive nature, that is, while painting a compelling picture of the evolution of occupational distributions, much contemporary literature does not provide empirical analysis of such polarization (either its causes or effects). This is not to suggest that the aforementioned research is misleading or poor quality. It does, however, emphasize a quandary posed to researchers delving into labor polarization analysis. On the one hand, there is limited inference that can be justifiably gleaned from research entirely through descriptive statistics and visualizations; on the other, empirical analysis of these data are likely to be flawed, given the properties of compositional data previously discussed.

With that, the remaining sections will provide a measure of polarization stemming from the economic and political science foundations of polarization measurement, while providing for more accurate measurement through incorporating the restrictions underlying compositional data.

3 Quantifying Polarization: A Review

Initially applied through the lens of conflict, generally, the Esteban and Ray (1999, 1994) features and axioms for political polarization are fundamental to properly measuring polarization across a breadth of disciplines. More explicitly, data with which one would like to measure polarization should have the following properties:

1. There must be significant homogeneity within each group by which observations are categorized.

2. Conversely, there must be substantial heterogeneity across these categorizations.

3. There must be a relatively small number of groupings, and these groupings should be of significant size/concentration. That is, individual observations carry little informative effect

Given these properties of the data, Esteban and Ray assert that a polarization measurement should hold to the following axioms:

1. Pooling two relatively small masses that are very close to one another should increase polarization. This is because it effectively hollows out the center

2. An outward shift of an intermediate mass raises polarization

3. A shift of mass from a central concentration outwards, equally in both directions, increases polarization

They asserted that a polarization index should incorporate the group size and heterogeneity across groups, as well as the group homogeneity, and argue that the polarization index (applying through the political science lens, for clarity in interpretation) should increase when political parties with more extreme positions gain votes from the center, because it increases the group homogeneity, such that:

$$P(\pi, y) = K \sum_{i=1}^{n} \sum_{j=1}^{n} \pi_{it}^{1+\alpha} \pi_{jt} |y_{it} - y_{jt}|$$

Where i indicates parties, k represents countries, t is year, n is the number of parties, $y_{it} - y_{jt}$ represents the ideological distance between parties i and j, and α is a sensitivity parameter, such that $a \in [1, 1.6]$ For the analysis comparison in Section 5, we analyze the Esteban & Ray polarization function in the three-point distribution setting.

As previously discussed, Montalvo and Reynal-Querol (2005) and Reynal-Querol (2002) continued along this path, extending the Esteban and Ray polarization quantification to a cross-sectional ethnic conflict study. Rather than polarization being a hollowing of the middle, polarization here is based on concentrations of distinct masses where it is maximized when all masses are equal (greatest evidence of ideological conflict). This isn't as applicable to the labor polarization data, however, as it is order invariant.

$$P = 1 - \sum_{i=1}^{n}(0.5 - \pi_i)^2(\frac{\pi_i}{4})$$

where π_i denotes the proportion of the total that are within group i.

As mentioned previously, Sigelman and Yough (1978) quantified an early iteration of polarization measurement as a distance between poles in the spirit of Sartori (1976).

$$Polarization = \sum_{i=1}^{n} f_i(x_i - \overline{x})^2$$

where f_i is the concentration within group i (applied to voteshares in the original analysis), x_i is the political left-right placement for party i, and and $\overline{x}$ is the ideological center of the system as a whole.

As discussed in Schmitt (2016), Evans (2002) finds a potential flaw in Sigelman and Yough's measure, where two party systems may overestimate overall polarization relative to a party system with ideological alienation of extreme parties and a clustered center of masses.

Dalton (2008) looks at the weighted ideological distance among parties' left-right positions as well. The Dalton polarization index is relatively simple and straightforward, and includes the ideological heterogeneity across parties in addition to the size of each party, leading it to be a commonly used measure.

$$PI_{kt} = \sqrt{\sum_{i=1}^{n}[\pi_{it}\{\frac{y_{it} - y_{mt}}{5}\}^2]}$$

where n is the number of parties, π is the vote share, i is a party indicator, y is the party position, and m represents the average party.

Gunderson (2022) defines party polarization as the amount by which parties take positions far from the ideological center in an election. This is reflected via the standard deviation of parties' positions on a given dimension of features of disagreement, weighted by their voteshare.

$$P = \sqrt{\frac{1}{N}\sum_{i=1}^{N}((LR_i - \overline{LR}) * \pi_i)}$$

As shown in Section 5, the Sigelman & Yough, Dalton, and Gunderson comparisons are nearly identical, apart from minor differences in weighting applications.

Additionally, Lelkes (2016) interprets polarization as a bimodality coefficient. That is, it is a function of the moments of the distribution.

$$BC = \frac{m_3^2 + 1}{m_4 + 3(\frac{(n-1)^2}{(n-2)(n-3)})}$$

where m_3 denotes the skewness, m_4 is the excess kurtosis (kurtosis - 3), and n is the sample size. This bimodality coefficient, however, is limited to the two-mass case, where we are looking at the distance between concentrations in a binary categorized system (ideological positions of the democrat vs republican party, for example). As shown in Section 5, incorporating a bipolarization measure into a > 2 dimensional polarization measure renders outputs with little meaningful significance.

Boxell, Gentzkow and Shapiro (2022) measure affective polarization, a separate form of polarization measurement, which is not addressed here. Other items which are outside the scope of this paper include polarization vs. fractionalization vs inequality[2] or bipolarization (polarization with only two distinct categories (see Foster and Wolfson (2010), Lelkes (2016), Wang and Tsui (2000), Wolfson (1994)).

4 Theory

This initial discussion will address polarization in the three-dimensional plane, holding to the surface of the unit sphere. This geometric restriction derives, in part from Katz and King (1999). Largely, the empirics surrounding polarization research, since the non-transformed data are three-dimensional (and thus difficult to interpret, jointly, in empirical analysis), are solely descriptive and/or visual, as previously shown. My motivation in the creation of this new method to measuring polarization is that most labor polarization research does not include a true quantification of polarization, and the ones that do often erroneously run individual sets of regression analyses for each category of occupational grouping separately. This neglects the fact that, by design, categories of compositional data are functions of each other. Restating the above, Katz and King (1999) hold that for such compositional data, two features must be present in the analysis to be properly specified:

1. $V_{ij} \in [0,1] \; \forall \; i,j$. That is, each component proportion is in the interval of [0,1]
2. $\sum_{j=1}^{J} V_{ij} = 1 \; \forall \; i$. That is, the sum of all components is equal to 1

[2] Kam, Indridason and Bianco (2017) discuss the nuanced, yet important differences between these at length

For the polarization measurement here, there are a few assumptions about the data that is to be transformed, starting with the Katz and King (1999) restrictions:

First, the sum of the components of the distribution must be equal to one, with each component being a nonnegative real.

$$\sum_{n=1}^{N} x_n = 1; x_n \in \mathbb{R}^+$$

The data can be separated into at least three ordinal bins, which can be associated with characteristics as listed below (in the 3 dimensional case):

- Two extreme bins (hereinafter referred to as low and high)
- One intermediate bin (hereinafter referred to as center)

This extends upon the foundation of Katz and King (1999) by allowing for a natural restriction of the data to have components which sum to 1 by incorporating the surface of the unit sphere as a constraint point, while also appropriately accounting for the ordered nature of the data. That is, the spherical model assumes that shifts between masses are likely to be between neighboring masses, as they share relatively more characteristic distinctions than with distributionally distant masses, and accounts for the full set of possible observations jointly for large shifts. These large shifts apply to both large shifts of masses between neighboring categories and shifts across multiple categories.

Polarization, in this case, is minimized when the entire proportion of the variable in question is concentrated in a single bin (ideological homogeneity), and is maximized when there is a 1:0:1 ratio of low : center : high concentration (hereinafter referred to as "perfect polarization").

With that, let the proportional split of low, center, and high be the squared components of the observed unit normal vector $\vec{n}_{obs} = < \hat{i}, \hat{j}, \hat{k} >$, respectively. Since $\hat{i}^2 + \hat{j}^2 + \hat{k}^2 = 1 \, \forall \, \hat{i}, \ \hat{j}, \ \hat{k}$, each vector endpoint is a point along the circumference of the unit sphere.

Given the above, let us define a new measure of polarization ("Spherical Polarization") as follows:

$$Spherical \ Polarization = (1 - |\hat{i}^2 - \hat{k}^2|)(\frac{\pi}{2} - arctan(\frac{||\vec{n}_{obs} \times \vec{n}_{pp}||}{\vec{n}_{obs} \cdot \vec{n}_{pp}}))$$

with i and k being the corresponding component values $\hat{i}, \ \hat{k}$ of $\vec{n}_{obs}$ (observed proportions), respectively, and $\vec{n}_{pp}$ being the unit normal vector of perfect polarization, $< \frac{\sqrt{2}}{2}, 0, \frac{\sqrt{2}}{2} >$ (*i.e.* a 50/50 concentration split between the extreme masses). This function consists of two general components, a measure of geodesic distance and a weighting factor. The right side of the equation measures the distance from the perfect polarization point to the observed vector endpoint across the edge of the unit sphere and inverts it, such that a high value of spherical polarization would imply relative proximity to the perfect polarization point. The arctangent function is included here, despite the simpler spherical distance formulas incorporating arcsine or arccosine, as arctangent is well-defined across all distances, whereas the arcsine and arccosine functions can have computational inaccuracies when the cosine of the central angle approaches 1. The weight is incorporated to account for the fact that polarization is minimized not only at the point farthest from the perfect polarization point (concentration solely within the center), but at single concentrations of low and high as well. This is not to imply that these are comparable insofar as their worth is concerned, only to say that it is a non-polarized system. To elaborate on this, let us look at this from the political science lens. If the goal were to measure political polarization as a function of voteshare within an election, 100% voteshare for an extreme left party, 100% voteshare for a moderate/center party, and 100% voteshare for an extreme right party would be equally non-polarized, as there is complete agreement by the voters. For a more concrete example, Milner (2021) finds that import and technology shocks are associated with strong shifts to the far-right, while other party families suggest little to no association. This would reflect a shift to an extreme, but not necessarily polarization, as there is no corresponding increase in the far-left nor hollowing out of the center. Polarization, to be clear, is not a measure focusing on shifts to a particular extreme, but a measure of weighted distributions with a spatial component.

As noted earlier, Evans (2002) finds that Sigelman and Yough (1978) err in the estimation of polarization depending on the fractionalization of the system. This mismeasurement stemming from degrees of fractionalization is avoided in the n-categorical quantification of spherical polarization, as it requires comparable ordered categorization of masses to be estimated. Spherical polarization, beyond the three mass case, however, is able to account for number of masses and their internal as well as overall distribution. The n-categorical spherical polarization formula is as follows:

$$Spherical\ Polarization = \frac{n-1}{2}\sum_{q}((1-|\hat{i}^2-\hat{k}^2|)(\frac{\pi}{2}-arctan(\frac{||\vec{n}_{obs}\times\vec{n}_{pp}||}{\vec{n}_{obs}\cdot\vec{n}_{pp}})))$$

where q is the unique symmetric opposing groupings of distributions, and n is the number of distinct masses. Thus, the number of pairings, q, will be equal to half of one fewer than the number of masses. As an example, let us imagine a system with nine distinct categories of masses (1-9, from low to high). Thus, the center mass would be associated with mass number 5. In order to account for internal distributions between neighboring masses, there would be an average of aggregations of distributional groupings:

- Aggregation 1: Mass 1,
  Aggregation 2: Mass 2-8,
  Aggregation 3: Mass 9
- Aggregation 1: Mass 1-2,
  Aggregation 2: Mass 3-7,
  Aggregation 3: Mass 8-9
- Aggregation 1: Mass 1-3,
  Aggregation 2: Mass 4-6,
  Aggregation 3: Mass 7-9
- Aggregation 1: Mass 1-4,
  Aggregation 2: Mass 5,
  Aggregation 3: Mass 6-9

From this, spherical polarization is thus quantified as a positive real number on the interval $[0, \frac{\pi}{2}]$. There are four key features of Spherical Polarization. First, it provides a mapping of the multidimensional compositional data in $\mathbb{R}^N$ to a one-dimensional measure with natural upper and lower bounds. This creates a clear mathematical tractability by which one can perform empirical analysis without violating the model properties discussed by Jackson (2002), Katz and King (1999), Tomz, Tucker and Wittenberg (2002). Second, it more precisely estimates changes in polarization, particularly for large shifts. Shifts from low polarization to high polarization can often be underestimated by contemporary polarization measures, as they are not explicitly constrained by the properties of compositional data, that is, the sum of the components need not be always equal to 1. Additionally, it operates as a function in which shifts between neighboring masses are nearly linear, provided that the shift does not cross the polarization pivot point (*i.e.* the point at which the low and high component are equal). Shifts between neighboring masses (holding other masses constant) are often nonlinear functions in contemporary measures. While this may be inconsequential (or even appropriate) for non-compositional data, a one unit transfer (1 percentage point) between neighboring masses should be anticipated to be a linear function, regardless of the initial concentrations (see Figure 1).

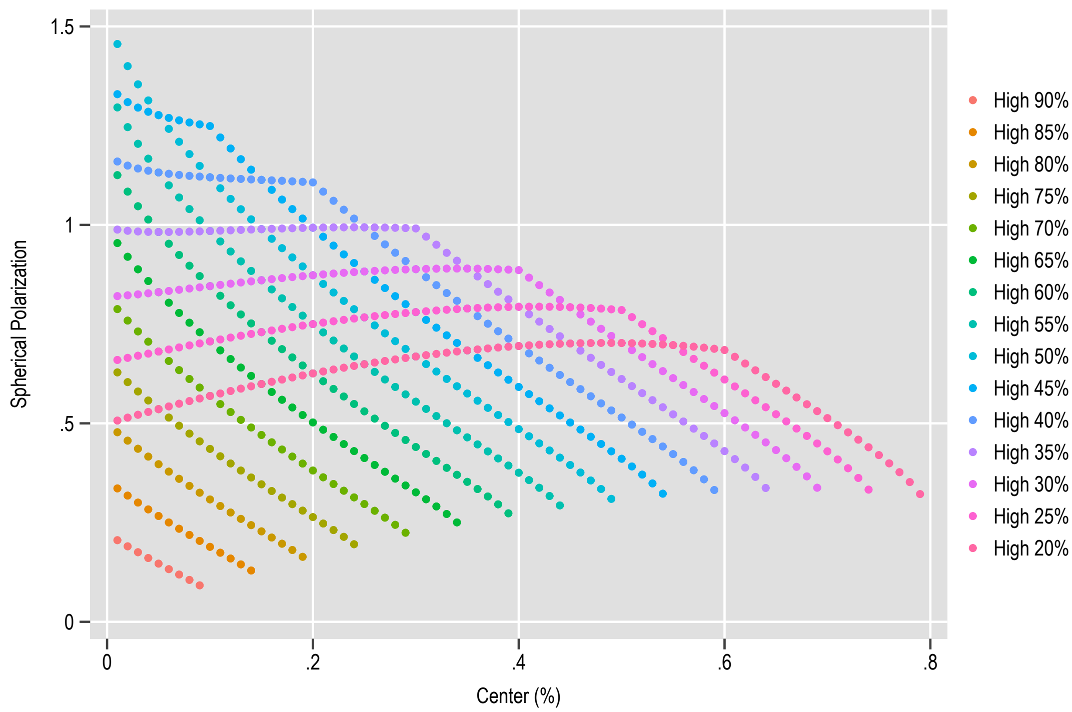


Figure 1: Scatterplot of spherical polarization given a one-unit shift from low to center, held constant at specified levels of high %. The pivot points indicate the point at which low is equal to high, thus the maximum polarization point, given the High concentration held constant.

Slope Estimates of Shifts Across Masses				
Dependent Variable: Spherical Polarization				
	Pre Pivot Point		Post Pivot Point	
	Low → High	Center → High	Low → High	Center → High
	2.519	2.224	-2.519	-0.295
N	2550	2550	2550	2550
R^2	0.9734	0.9734	0.9734	0.9734

Table 1: Independent variable of interest is High percent. The first and third columns include Center concentration as a control, implying that changes in High will arise from shifts from Low. The second and fourth columns include Low concentration as a control, implying that changes in high arise from shifts from center. The data are subset to those combinations of concentrations on the left side vs the right side of the polarization pivot point.

Fourth, Spherical Polarization incorporates the spatial component of these distributions. To elaborate, as previously mentioned, Spherical Polarization requires that the data not only be compositional in nature, but that the categorization be ordinal, such that each non-center mass has a corresponding mass on the other side of the center, the same distance away. This is based upon the notion that masses in relatively close proximity with each other, while distinct enough to warrant separate categorization, are assumed to share more traits than masses with a larger separation. This serves to account for the fact that, given that this is compositional data, to move from a low extreme to a high extreme, you must account for the distance across the masses in between. From this, we would expect that, given the same starting concentration and assuming that a pivot point is not crossed, a one unit shift from low to center will have a smaller effect on polarization than a one unit shift from low to high, as the latter travels more distance. A straightforward comparison of OLS coefficients is in Table 1.

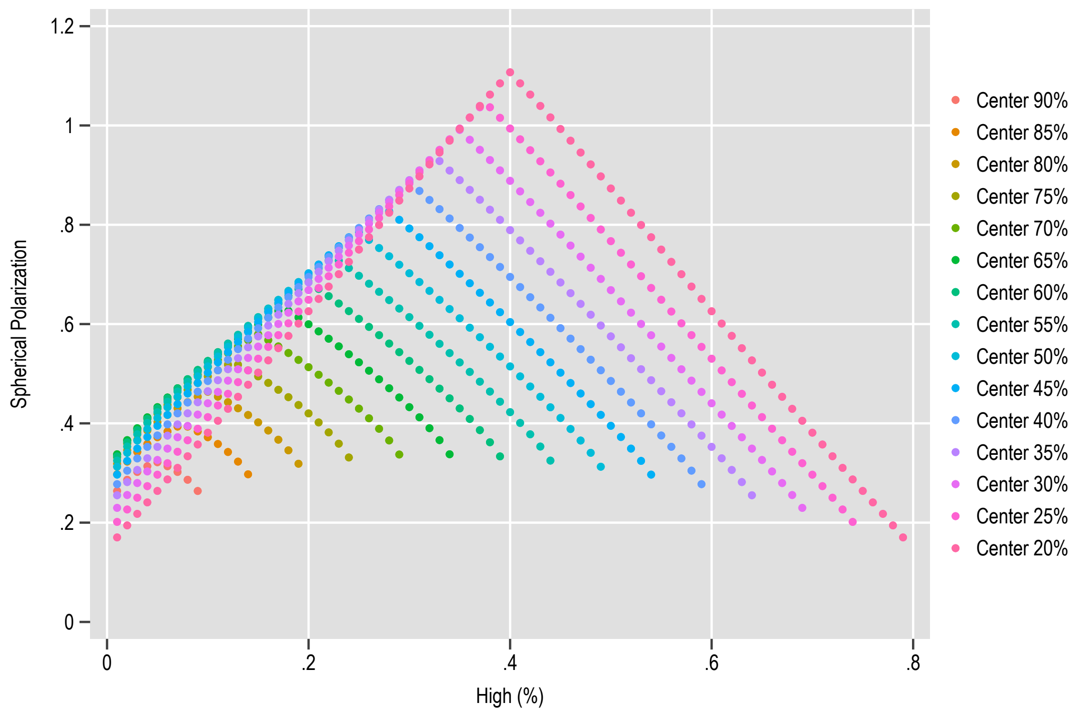


Figure 2: Scatterplot of spherical polarization given a one-unit shift from low to high at given levels of center concentration.

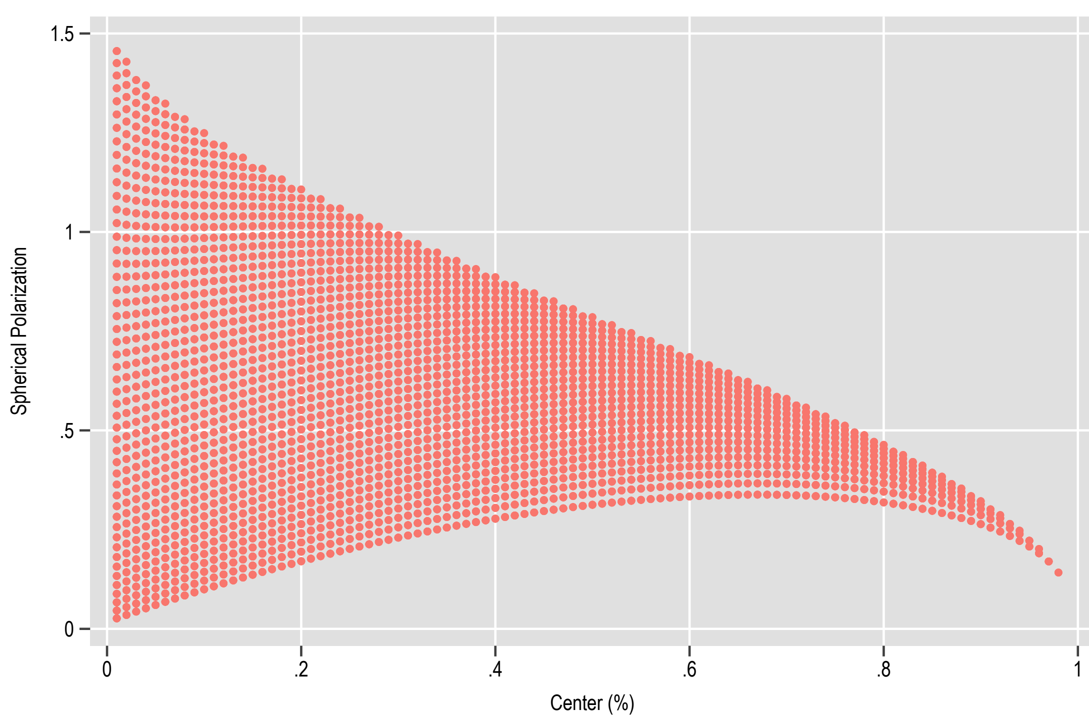


Figure 3: Scatterplot of all possible values of spherical polarization given across all concentrations of center

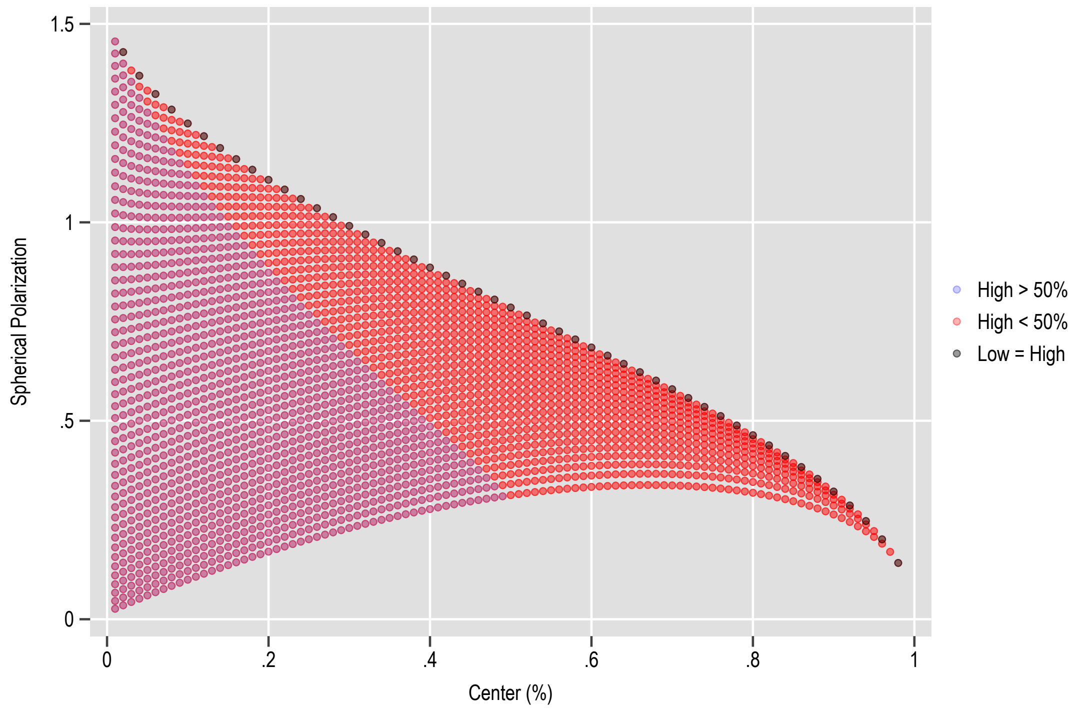


Figure 4: Scatterplot of all possible values of spherical polarization given across all concentrations of center. The overlapping blue region indicates the possible values of polarization across the concentration of center. To reinforce the claim that polarization is maximized, given concentration of center, where low concentration is equal to high concentration, the highest possible value for polarization at each observation of center percentage is marked in gray.

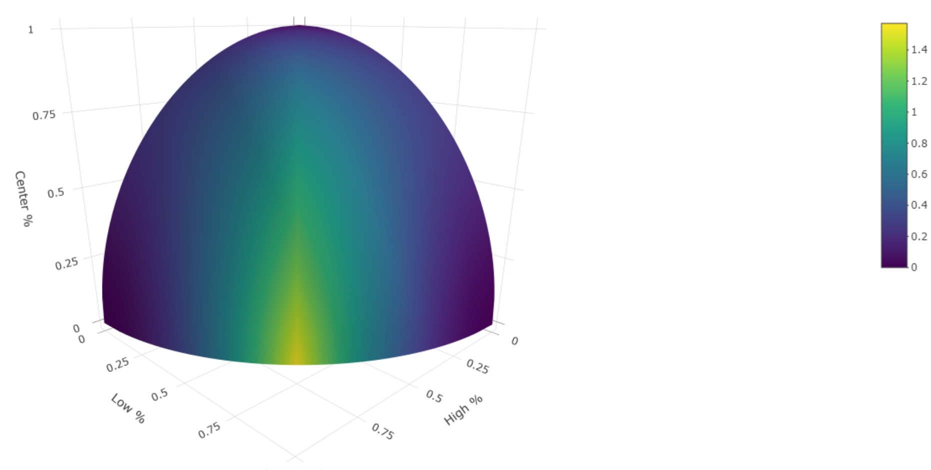


Figure 5: Three-dimensional rendering of spherical polarization. The dark concentrations at each maximum point for the individual vector components represent a minimal value of polarization, while the concentrated yellow represents perfect polarization, an equal split solely between low and high.

5 Comparisons: Test Data & Empirical Replications

Creating a dataset which encompasses all combinations of proportions to the hundredths digit (5151 observations), several interesting features become apparent comparing common measures with Spherical Polarization. For ease of interpretability, the analyses performed here are solely descriptive, and are not intended at this point to measure the degree to which Spherical Polarization is more accurate than the model it is being compared to. These visualizations and discussions serve to highlight the shortcomings of contemporary measures of polarization in their applications to ordered compositional data.

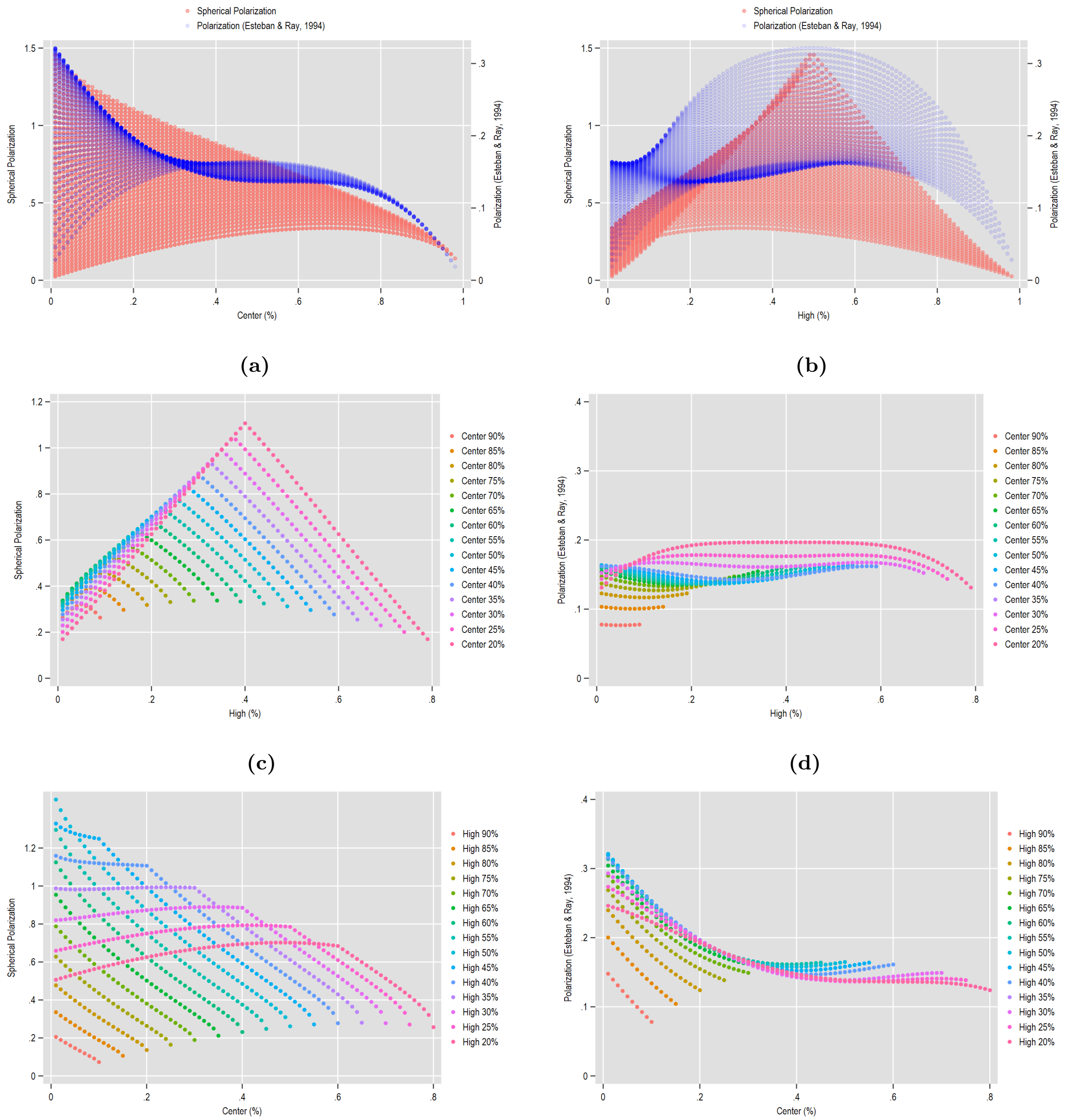
5.1 Esteban & Ray

Esteban and Ray (1994) quantify polarization for three-point distributions as follows:

$$Polarization \equiv [p^{1\,+\,\alpha}q \;+\; q^{1\,+\,\alpha}p]x \;+\; [p^{1\,+\,\alpha}r \;+\; r^{1\,+\,\alpha}p]y \;+\; [q^{1\,+\,\alpha}r \;+\; r^{1\,+\,\alpha}q](y \;-\; x)$$

where 0, x, and y (such that $0 \;<\; x \;<\; y$) are the groupings (in this case, low, center, and high, respectively). p, q, and r are the corresponding population weights. α can be chosen as desired, provided that $\alpha \;\geq\; 1$. The authors recommend a value of $\alpha = 1.6$.

As shown in Figure 6, there exist some issues in applying the Esteban and Ray quantification to compositional data. While the basic tenets of a polarization function are met, such as polarization's maxima when Center concentration is 0% in Panel 6a and maximization achieved when High is 50% in Panel 6b (we can reasonably assume that this is an even split between Low and High), Panel 6a reflects a "pinch-point" in the distribution at roughly one-third concentration in Center. For ordered compositional polarization appropriate for labor market analysis, one would expect that this distributional coverage would decrease in breadth monotonically in Center concentration. Intuitively, one can see why this pinch-point exists. Esteban and Ray (1994) was initially developed to measure polarization through the scope of conflict. As with Reynal-Querol (2002), conflict-associated polarization would also achieve a local maximum in the case of an even distribution of strength in the particular masses. Panels 6d and 6f further indicate the distributional issues encountered when comparing the behavior of this measure across a number of specifications, as they lack the straightforward comparisons possible under the Spherical Polarization framework.



(e)

(f)

Figure 6: Characteristics of polarization, Spherical Polarization vs Esteban and Ray (1994): (a) is a re-creation of Figure 3 for Spherical Polarization and Esteban & Ray's method; (b) compares these measures in a scatterplot for all possible values of polarization across all values of High.; (c) is a repeat of Figure 2, for reference; (d) reflects the same visualization of Figure 2 applied to Esteban and Ray; (e) is a repeat of Figure 1, for reference; (f) reflects the same visualization of Figure 2 applied to Esteban and Ray.

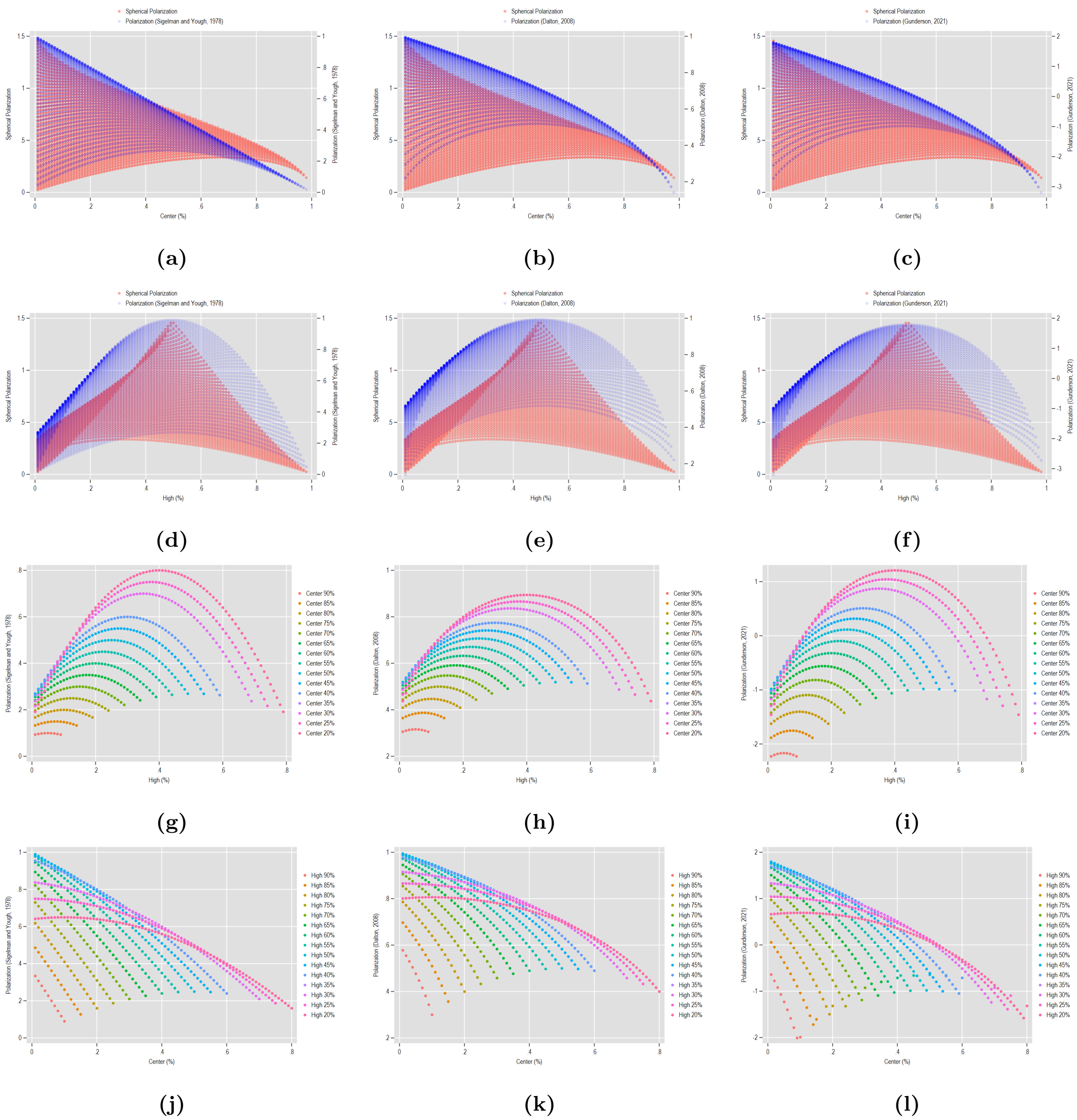


Figure 7: Characteristics of Polarization, Spherical Polarization vs Standard Deviation-Associated Functions: These comparisons mirror the same ordering from Figure 6, The first column shows Sigelman and Yough (1978), the second Dalton (2008), and the third Gunderson (2022).

5.2 Standard Deviation Approaches

As previously shown, there are a number of contemporary approaches which are derivations of system standard deviations (see Dalton (2008), Sigelman and Yough (1978), and Gunderson (2022)). While in the broad sense, these measures provide relatively reasonable estimates of polarization (as compared to other measures discussed here), there are some underlying potential issues in empirical applications. particularly two-mass systems (Evans 2002) and skewed distributions (Schmitt 2016).

Row 1 of Figure 7 suggests that the standard deviation supported measures underestimate the total number of possible values for shifts from low to high (or vice versa), given the more limited coverage compared to spherical polarization. Conversely, these measures appear to overrepresent the number of possible values for a shift between neighboring masses. These issues are aligned with the reasoning behind the ordinal nature of spherical polarization. Additionally, Rows 3 and 4 show that estimating shifts in polarization via a standard deviation-associated measure is more dependent upon initial distributions of masses compared to spherical polarization, regardless of whether a pivot point is crossed.

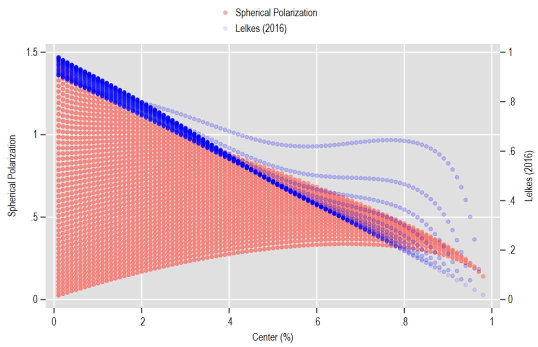
5.3 Other Approaches

Figure 8 reviews the visual distributional evolutions of a pair of other measures: Cirillo (2018) and Lelkes (2016). As discussed in Section 3, the Bimodality Coefficient from Lelkes (2016) does not smoothly apply to polarization measures beyond the two-mass system in which bipolarization measures are applied. The general applicability to the two-category approach is apparent in the relative concentration of paths and linearity as shown in Figure 8a and Figure 8g, the bimodality coefficient is overly sensitive in a highly skewed, leptokurtic system. The Cirillo (2018) analysis reflects the issue with contemporary measures of labor polarization via these distributional changes across specifications. While the Cirillo measurement (viewing this as a single-country, single industry system, grouping clerks and craft workers for straightforward application to the three-mass system) appears in the literature to be a relatively straightforward approach to occupation-based polarization, it neglects the behavior of such an index across changes in concentration. Apart from the initial information conveyed from the concentration of center (clerks & craft workers) jobs, shifts of workers from low (manual workers) to high (managers) are inconsequential to overall polarization of the system, as shown in Figure 8f. Figure 8h reflects this same concept, although it is not clear on its face, as each of the specified concentrations follows the same path (thus only reflecting a single specification in the figure), while also reflecting a near divide-by-zero concern as center occupational concentration approaches zero.

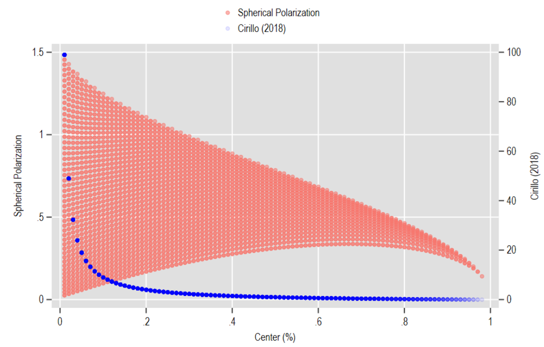
5.4 Empirical Replication: Bárány & Siegel (2018)

In addition to the overall evolution of Spherical Polarization relative to contemporary measures of polarization across feasible combinations of observations, I've applied the spherical polarization measure to existing research to show its tractability and interpretability in the applied empirical realm. Bárány and Siegel (2018) analyze the hollowing-out of middle-wage workers' earnings growth, relative to those low-wage and high-wage workers. Modifying the original analysis to incorporate the American Community Survey from 1976 - 2015 yields interesting results, particularly when running the same analyses using the Spherical Polarization approach. As with the original publication, the authors' quantification yields similar results across all decades for both occupational explanatory variables. It is difficult, however, to clearly map the authors' analysis to polarization, however. First, let us consider the explanatory variables here. Recall, the measures of occupational polarization discussed so far have been defined as a decline in the concentration of routine occupations with a relative increase of abstract and manual nonroutine occupational shares. However, this relative proportional diminishment is not explicitly included in the author's regression model. Given the nature of compositional data, inclusion of this measure would be effectively impossible, as it would cause another variable to drop from the analysis due to collinearity. If you are analyzing data in a situation where the aggregated shares sum to one, inclusion of all components would inherently cause a collinearity issue, as it would be a function of the other components. Further, given the lack of inclusion of the routine occupations, it leaves ambiguity as to the effect of polarization on wage growth. To clarify, we see that manual nonroutine occupations (the bottom tercile) are associated with a decrease in wage growth, while abstract professions (the top tercile) are associated with increased wage growth. While this may be a valid step in analyzing inequality in professions and the effect it has on earnings inequality, this provides no information insofar as labor market polarization is concerned.

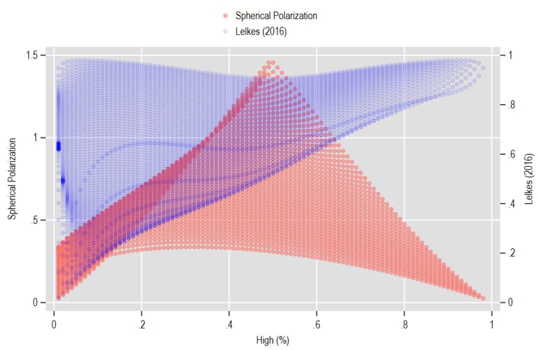
Conversely, running the same analysis with Spherical Polarization as an explanatory variable, we see that the mapping from a multidimensional combination of occupational concentrations to a one-dimensional measure allows clear interpretation of the association between labor market polarization and hourly wages. Across all years, it suggests a negative overall effect of occupational polarization on wage growth, generally, with all yearly bins post 1985 being strongly statistically significant at



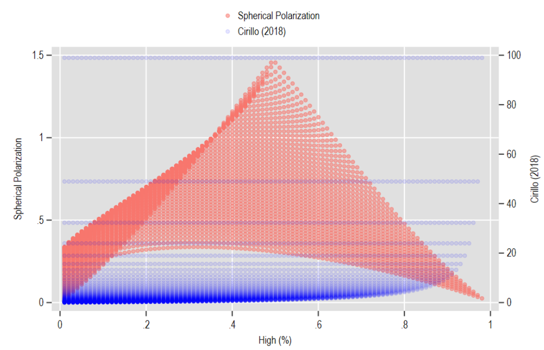
(a)



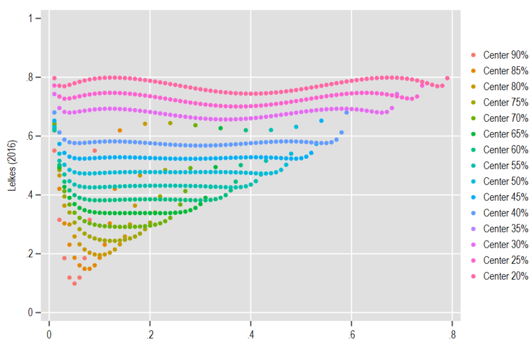
(b)



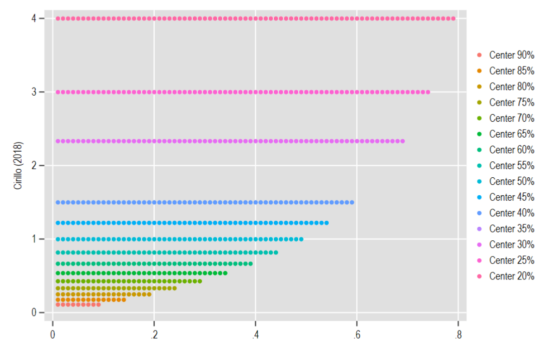
(c)



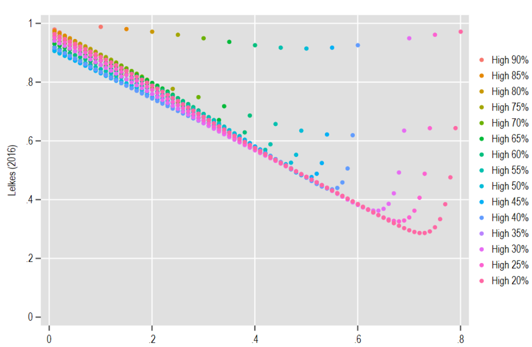
(d)



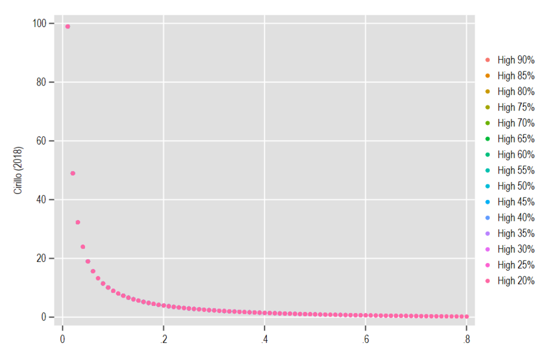
(e)



(f)



(g)



(h)

Figure 8: Characteristics of Polarization, Spherical Polarization vs Alternative Functions: These comparisons mirror the same ordering from Figure 6, The first column shows Lelkes (2016), the second Cirillo (2018).

Hourly Wages: Occupational Effects (Bárány and Siegel (2018) Replication) - Individual Level Results Dependent Variable: Log of hourly wages								
	1976-1985	1986-1995	1996-2005	2006-2015	1976-1985	1986-1995	1996-2005	2006-2015
Manual Nonroutine								
Coefficient	-0.186***	-0.202***	-0.185***	-0.164***				
Std. error	(0.006)	(0.007)	(0.005)	(0.007)				
Abstract								
Coefficient	0.264***	0.395***	0.471***	0.522***				
Std. error	(0.012)	(0.011)	(0.014)	(0.014)				
Spherical Polarization								
Coefficient					-0.262	-1.109***	-0.703***	-0.968***
Std. error					(0.187)	(0.191)	(0.134)	(0.087)
Number of observations	599870	592693	729670	782161	599870	592693	729670	782161
R-squared	0.214	0.231	0.237	0.237	0.168	0.146	0.125	0.109

Table 2: OLS, standard errors clustered by state. Models 1-4 explanatory variables are binary indicators of occupational class. Models 5-8 are annual state-level treatment of labor market polarization, measured through the aggregation of these indicators. Additional covariates not shown: Educational experience (in levels, a quadratic, cubic, and quartic), gender, and race.

conventional levels.

To ensure that these differences were not a function of the binned decades, I pool the Bárány and Siegel (2018) data and include state and year fixed effects in the OLS model. Promisingly, the estimate structure of the results aligned with those of Table 2. Again, however, the interpretation of job polarization excluding an integral input leaves comprehension of these results murky at best. Spherical Polarization, as with Table 2 has a negative and statistically significant association with hourly wage growth.

Further, as the results for spherical polarization could technically be a function of the differences in application of the employment variables (individual level indicators vs aggregated polarization), I aggregate the Bárány and Siegel (2018) data to the state level. With that, the inputs to spherical polarization are identical to the inputs included in the regression analysis. Again, the estimate structure of the results aligned with those of Table 2. As with the previous analyses, linking of the occupationally separated results to polarization seems difficult to justify. Spherical Polarization, as with Table 2 has a negative and statistically significant association with hourly wage growth.

Extending this again, one could argue that important detail was lost in the use of log wages in this analysis, as those with no reported hourly wage would be excluded from the analysis. With that, I applied the aggregated state-level analysis with a dependent variable of hourly wages in levels to polarization via a fixed effects Poisson regression estimator. To confirm, the Poisson fixed effects estimator is a valid application, as the dependent variable is non-negative with no natural

Hourly Wages: Occupational Effects - Pooled Results Dependent Variable: Log of hourly wages		
	Barany & Siegel	Polarization
Manual Nonroutine		
Coefficient	-0.184***	
Std. error	(0.006)	
Abstract		
Coefficient	0.435***	
Std. error	(0.012)	
Polarization		
Coefficient		-0.898***
Std. error		(0.116)
Number of observations	2704394	2704394
R-squared	0.243	0.148

Table 3: OLS, standard errors clustered by state. Year fixed effects. 1976 - 2015 American Community Survey. Additional covariates not shown: Educational experience (in levels, a quadratic, cubic, and quartic), gender, and race.

upper bound. Further, it has been proved that Poisson fixed effects regression is robust to arbitrary violation of the Poisson distribution and Poisson assumption (Wooldridge 1999). Interestingly, in the aggregated OLS models with state and year fixed effects as well as the Poisson fixed effects estimator, the Bárány and Siegel estimates lose their statistical significance for manual nonroutine work. Spherical Polarization maintains a negative and statistically significant relationship across the full set of specifications, albeit only at a $p < 0.1$ level of significance when incorporating the Poisson fixed effect estimator.

Hourly Wages: Occupational Effects - Aggregated Results Dependent Variable: Log of hourly wages / Hourly Wages						
	Barany & Siegel	Polarization	Barany & Siegel	Polarization	Barany & Siegel	Polarization
Manual Nonroutine						
Coefficient	-0.007**		-0.003		-0.009	
Std. error	(0.003)		(0.002)		(0.006)	
Abstract						
Coefficient	0.012***		0.007***		0.008*	
Std. error	(0.002)		(0.002)		(0.004)	
Polarization						
Coefficient		-0.817***		-0.290***		-0.451*
Std. error		(0.154)		(0.084)		(0.272)
Number of observations	2040	2040	2040	2040	2040	2040
R-squared	0.607	0.543	0.676	0.673		
Year FE	✓	✓	✓	✓	✓	✓
State FE			✓	✓	✓	✓
Estimator	OLS	OLS	OLS	OLS	Poisson FE	Poisson FE

Table 4: Models 1-4: OLS, standard errors clustered by state. Models 5-6: Poisson fixed effects estimator, bootstrap standard errors. Year fixed effects. 1976 - 2015 American Community Survey. Additional covariates not shown: Educational experience (in levels, a quadratic, cubic, and quartic), gender, and race.

6 Conclusion and Open Questions

In this paper, I create a new measure of polarization particularly applicable to the analysis of ordered compositional data. By accounting for the underlying nature of these data, this allows us to provide a tractable and justifiable application of these proportional data with a spatial component for analysis. Further, through comparison to other measures, we see that the incorporation of the restrictions involved in the Spherical Polarization measurement are justified, given the poor applicability performance of other non-compositional data-specific measures of polarization. There are extensions of this work which could be addressed in future research, namely applications beyond the largely 2-dimensional spectrum of categorization in most labor applications. That is, are there additional important characteristics of labor data orthogonal to the routine/cognitive aspects of current labor polarization research? Further, if this were to go beyond the standard 3-dimensional analysis incorporated here, how can results be structured in such a way that they are interpretable, particularly keeping in mind the issues associated with higher dimensional cross products, which lose generality once extended to analysis of octonions, such that there is not a unique result to the cross product as there is in the application to quaternions.

References

Abel, Jaison R and Richard Deitz. 2012. "Job Polarization and Rising Inequality in the Nation and the New York-Northern New Jersey Region." *Current Issues in Economics and Finance: Second District Highlights* 18(7).

Autor, David. 2010. "The Polarization of Job Opportunities in the U.S. Labor Market: Implications for Employment and Earnings." *The Hamilton Project Policy Proposal* . https://www.brookings.edu/wp-content/uploads/2016/06/04_jobs_autor.pdf.

Autor, David H., Frank Levy and Richard J. Murnane. 2003. "The Skill Content of Recent Technological Change: An Empirical Exploration*." *The Quarterly Journal of Economics* 118(4):1279–1333.

Autor, David H., Lawrence F. Katz and Melissa S. Kearney. 2006. "The Polarization of the U.S. Labor Market." *American Economic Review* 96(2):189–194.

Boxell, Levi, Matthew Gentzkow and Jesse M. Shapiro. 2022. "Cross-Country Trends in Affective Polarization." *The Review of Economics and Statistics* pp. 1–60.

Bussolo, Maurizio, Ivan Torre and Hernan Winkler. 2018. "Does Job Polarization Explain the Rise in Earnings Inequality?" *World Bank Group Policy Research Working Paper* 8652. https://openknowledge.worldbank.org/handle/10986/30879.

Bárány, Zsófia L. and Christian Siegel. 2018. "Job Polarization and Structural Change." *American Economic Journal: Macroeconomics* 10(1):57–89.

Cirillo, Valeria. 2018. "Job Polarization in European Industries." *International Labour Review* 157(1):39–63.

Dabla-Norris, Era, Carlo Pizzinelli and Jay Rappaport. 2019. "Job Polarization and the Declining Fortunes of the Young: Evidence from the United Kingdom." *IMF Working Papers Series* 2019(216). https://www.imf.org/en/Publications/WP/Issues/2019/10/11/Job-Polarization-and-the-Declining-Fortunes-of-the-Young-Evidence-from-the-United-Kingdom-48644.

Dalton, Russell J. 2008. "The Quantity and the Quality of Party Systems: Party System Polarization, Its Measurement, and Its Consequences." *Comparative Political Studies* 41(7):899–920.

Dillender, Marcus and Eliza Forsythe. 2022. "Computerization of White Collar Jobs." *NBER Working Paper Series* 29866.

Duclos, Jean-Yves, Joan Esteban and Debraj Ray. 2004. "Polarization: Concepts, Measurement, Estimation." *Econometrica* 72(6):1737–1772.

Dwyer, Rachel E. and Erik Olin Wright. 2003. "The Patterns of Job Expansions in the USA: A comparison of the 1960s and 1990s." *Socio-Economic Review* 1:289–325.

Dwyer, Rachel E. and Erik Olin Wright. 2019. "Low-Wage Job Growth, Polarization, and the Limits and Opportunities of the Service Economy." *RSF: The Russell Sage Foundation Journal of the Social Sciences* 5(4):56–76.

Esteban, Joan and Debraj Ray. 1999. "Conflict and Distribution." *Journal of Economic Theory* 87(2):379–415.

Esteban, Joan and Debraj Ray. 2011. “Linking Conflict to Inequality and Polarization.” *American Economic Review* 101(4):1345–1374.

Esteban, Joan and Debraj Ray. 2012. “Comparing Polarization Measures.” *The Oxford Handbook of the Economics of Peace and Conflict* pp. 127–151.

Esteban, Joan-María and Debraj Ray. 1994. “On the Measurement of Polarization.” *Econometrica* 62(4):819–851.

Evans, Jocelyn A.J. 2002. “In Defence of Sartori: Party System Change, Voter Preference Distributions, and Other Competitive Incentives.” *Party Politics* 8(2):155–175.

Fernández-Macías, Enrique. 2012. “Job Polarization in Europe? Changes in the Employment Structure and Job Quality, 1995-2007.” *Work and Occupations* 39(2):157–182.

Foster, James E and Michael C Wolfson. 2010. “Polarization and the decline of the middle class: Canada and the U.S.” *The Journal of Economic Inequality* 8(2):247–273.

Goos, Maarten and Alan Manning. 2007. “Lousy and Lovely Jobs: The Rising Polarization of Work in Britain.” *The Review of Economics and Statistics* 89(1):118–133.

Goos, Maarten, Alan Manning and Anna Salomons. 2009. “Job Polarization in Europe.” *American Economic Review: Papers & Proceedings* 99(2):58–63.

Goos, Maarten, Alan Manning and Anna Salomons. 2014. “Explaining Job Polarization: Routine-Biased Technological Change and Offshoring.” *American Economic Review* 104(8):2509–2526.

Gunderson, Jacob R. 2022. “When Does Income Inequality Cause Polarization?” *British Journal of Political Science* 52(3):1315–1332.

Jackson, John E. 2002. “A Seemingly Unrelated Regression Model for Analyzing Multiparty Elections.” *Political Analysis* 10(1):49–65.

Jaimovich, Nir and Henry E. Siu. 2020. “Job Polarization and Jobless Recoveries.” *The Review of Economics and Statistics* 102(1):129–147.

Jerbashian, Vahagn. 2019. “Automation and Job Polarization: On the Decline of Middling Occupations in Europe.” *Oxford Bulletin of Economics and Statistics* 81(5):1095–1116.

Kagalwala, Ali, Andrew Q. Philips and Guy D. Whitten. 2021. “What About the Rest of the Pie? A Dynamic Compositional Approach to Modeling Inequality.” *Social Science Quarterly* 102(4):1534–1552.

Kalleberg, Arne L. 2011. *Dimensions of Polarity.* Russell Sage Foundation chapter 4, pp. 61–81.

Kam, Christopher, Indridi Indridason and William Bianco. 2017. “Polarization in multiparty systems.” *Working Paper - Presented at Polarization, institutional design and the future of representative democracy* . https://www.democratic-anxieties.eu/wordpress/wp-content/uploads/2017/10/Indridason_Berlin2017.pdf.

Katz, Jonathan N. and Gary King. 1999. “A Statistical Model for Multiparty Electoral Data.” *American Political Science Review* 93(1):15–32.

Lelkes, Yphtach. 2016. “Mass Polarization: Manifestations and Measurements.” *Public Opinion Quarterly* 80(S1):392–410.

Milner, Helen V. 2021. "Voting for Populism in Europe: Globalization, Technological Change, and the Extreme Right." *Comparative Political Studies* 54(13):2286–2320.

Montalvo, José G and Marta Reynal-Querol. 2005. "Ethnic Polarization, Potential Conflict, and Civil Wars." *American Economic Review* 95(3):796–816.

Oesch, Daniel and Jorge Rodriguez-Menes. 2011. "Upgrading or Polarization? Occupational change in Britain, Germany, Spain, and Switzerland, 1990-2008." *Socio-Economic Review* 9:503–531.

Parkhomenko, Andrii. 2022. "Homeownership, Polarization, and Inequality." *SSRN Electronic Journal Working Paper* . http://dx.doi.org/10.2139/ssrn.3854352.

Reynal-Querol, Marta. 2002. "Ethnicity, Political Systems, and Civil Wars." *Journal of Conflict Resolution* 46(1):29–54.

Sartori, Giovanni. 1976. *Parties and Party Systems : A framework for analysis*. Cambridge: Cambridge University Press.

Schmitt, Johannes. 2016. "How To Measure Ideological Polarization in Party Systems." *Working Paper - Presented at the ECPR Graduate Student Conference, Tartu, 10–13 July* .

Shim, Myungkyu and Hee-Seung Yang. 2018. "Interindustry wage differentials, technology adoption, and job polarization." *Journal of Economic Behavior Organization* 146:141–160.

Sigelman, Lee and Syng Nam Yough. 1978. "Left-Right Polarization in National Party Systems: A Cross-National Analysis." *Comparative Political Studies* 11(3):355–379.

Tomz, Michael, Joshua A. Tucker and Jason Wittenberg. 2002. "An Easy and Accurate Regression Model for Multiparty Electoral Data." *Political Analysis* 10(1):66–83.

Wang, Huiwen, Qiang Liu, Henry M.K. Mok, Linghui Fu and Wai Man Tse. 2007. "A hyperspherical transformation forecasting model for compositional data." *European Journal of Operational Research* 179(2):459–468.

Wang, You-Qiang and Kai-Yuen Tsui. 2000. "Polarization Orderings and New Classes of Polarization Indices." *Journal of Public Economic Theory* 2(3):349–363.

Wolfson, Michael C. 1994. "When Inequalities Diverge." *The American Economic Review* 84(2):353–358.

Wooldridge, Jeffrey M. 1999. *Quasi-Likelihood Methods for Count Data*. John Wiley Sons, Ltd chapter 8, pp. 321–368.

Young, Alwyn. 2022. "Nearly Collinear Robust Procedures for 2SLS Estimation." *Working Paper* . https://personal.lse.ac.uk/YoungA/.