SOC542 Statistical Methods in Sociology II

Introduction and Review

Dr. Thomas Davidson

Rutgers University

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Plan

- Introductions
- Course outline
- Statistics review
  - Notation
  - Statistical inference
- Tools
  - R
  - RStudio
  - Github
  - AI

Part I: Course outline

Learning goals

- The use of statistical analysis in empirical social science research
- Multiple regression
  - Conceptual and statistical background
  - Implementation in R
  - Interpretation
  - Violations assumptions and robustness
  - Frequentist and Bayesian approaches to estimation
- Proficiency in data handling, analysis, and visualization using R

Course outline

Structure

1. OLS regression (Weeks 1-4)
2. Non-linear variables and interactions (5-6)
3. Model checking and missing data (7)
4. Generalized linear models (8-11)
5. Clustered data (12)
6. Causal inference with observational data (13)
7. Presentations (14)

Course outline

Key themes

- Theory-driven statistical modeling
  - Causal, scientific models to guide implementation of statistical analyses

Course outline

Key themes

- Multiple approaches to estimation and inference
  - Quantification of uncertainty via Bayesian inference
  - Reduced emphasis on p-values

Course outline

Key themes

- Data-driven pedagogy
  - Preference for code and simulations over mathematical formalism

Course outline

Key themes

- Robust inferences
  - Appropriately specified statistical models
  - Model checking and comparison

Course outline

Key themes

- Reproducibility and transparency
  - Emphasis on reproducible and transparent analytic procedures

Course outline

Key themes

- Scientific communication
  - Clear, precise summaries of statistical models using tables and visualizations

Course outline

Key themes

- Theory-driven statistical modeling
- Multiple approaches to estimation and inference
- Data-driven pedagogy
- Robust inferences
- Reproducibility and transparency
- Scientific communication

Course outline

Assessment

- Homework assignments (40%)
  - Four assignments, each worth 10%
- Final paper (50%)
  - Phase 1: Research question, data, and methodology
  - Phase 2: Preliminary results
  - Phase 3: Submit final paper
- Presentations (10%)
  - Present results from final paper

Course outline

Required readings

- *Regression and Other Stories* by Andrew Gelman, Jennifer Hill, and Aki Vehtari
  - Main textbook, covers applied OLS regression and generalized models in R
- *Bayes Rules! An Introduction to Applied Bayesian Modeling* by Johnson, Alicia A., Miles Q. Ott, Mine Dogucu. 2021.
  - More detailed introduction Bayesian inference with examples in R

Course outline

Recommended readings

- *Causal Inference: The Mixtape* by Scott Cunningham
  - More formal econometric background and overview of causal approaches
- *Statistical Rethinking*, 2nd ed., by Richard McElreath
  - Supplementary textbook, provides additional material and deepens understanding of Bayesian inference and modern statistics
  - Recommended: McElreath's YouTube lecture series (2023 series currently underway)
- *R for Data Science* by Hadley Wickham and Garrett Grolemund.
  - A useful reference for data manipulation in R via the tidyverse
- *Data Visualization: A Practical Introduction* by Kieran Healy.
  - Great introduction to data visualization using R and ggplot

Part II: Statistics review

Notation review

Vectors

- A vector is a sequence of numbers
  - e.g. We take the heights of everyone in the class and record them in vector v

$$v = \{6.1, 5.9, 5.7, 6.0, 6.2, ..., 5.9\}$$

- We typically arrange these vertically as columns in a data table.
- We can use *indexing* to reference specific elements of the vector
  - v_i refers to arbitrary element i
  - v_1 indexes the first element, 6.1, and so on

Notation review

Summation

- The uppercase letter Σ is used to denote a summation. We can use it here to take the sum of all the values in vector v, where k is the length of the vector.

$$\sum v_i = \sum_{i=1}^{k} v_i = v_1 + v_2 + ... + v_k$$

- We can compute sums in R using the `sum()` function, where the thing we are summing over is included in the parentheses, e.g. `sum(v)`.

Notation review

Products

- The uppercase letter Π is used to denote the product operation.
  We will encounter it far less frequently than summation.

$$\prod v_i = \prod_{i=1}^{k} v_i = v_1 * v_2 * ... * v_k$$

Notation review

Matrices

- We often want to represent multiple vectors as a matrix.
- Let's say we also collected each students' age, we could represent the ages as a vector u.
  - v and u can be combined together in a matrix M (typically we use lowercase for vectors and constants and uppercase for matrices):

$$M = \begin{Bmatrix} 6.1 & 24 \\ 5.9 & 22 \\ 5.7 & 27 \\ 6.0 & 30 \\ 6.2 & 25 \\ \dots & \dots \\ 5.9 & 26 \end{Bmatrix}$$

Notation review

Matrices

- We can index elements of a matrix in the following way
  - $M_{i,j}$ refers to the i^{th} row of column j
  - e.g. $M_{4,2}$ indexes the age of the 4^{th} student

Notation review

Vectors and matrices in R

```
v <- c(1,2,3)
print(sum(v))
```

```
## [1] 6
```

```
print(prod(v))
```

```
## [1] 6
```

Notation review

Vectors and matrices in R

We can use `cbind` to combine vectors columnwise into a matrix.

```
u <- c(1,1,1)
M <- cbind(v,u)
print(M[3,1]) # M[row, column]
```

```
## v
## 3
```

```
print(M)
```

```
##      v u
## [1,] 1 1
## [2,] 2 1
## [3,] 3 1
```

Notation review

Vectors and matrices in R

We can *transpose* this matrix using `t` if we want to treat these columns as rows.

```
print(t(M))
```

```
##   [,1] [,2] [,3]
## v    1    2    3
## u    1    1    1
```

Statistics review

Random variables

- The value of a random variable depends on the outcome of random events
  - e.g. x is a random variable representing the outcome of a series of coin tosses

Statistics review

Probability distributions

- Random variables are drawn from probability distributions
  - The probability of tossing a coin and getting a head is a defined by the Bernoulli distribution
  - The number of heads in a sequence of coin tosses is defined by the binomial distribution
  - The height of a randomly chosen adult male is drawn from a normal distribution

Statistics review

Probability distributions

- Distributions are defined by parameters that modify their shape and scale.
  - A Bernoulli distribution has a single parameter, p
  - A binomial distribution has two parameters, n and p
  - A normal distribution has two parameters, a mean μ and a standard deviation of σ
    - Often we refer to this using the shorthand $N(\mu, \sigma)$

Statistics review

Probability distributions in R

A random variable drawn from a binomial distribution can take a value of 0 or 1, where p is the probability of a 1. $p = 0.5$ is equivalent to a fair coin toss. The Bernoulli distribution is a special case where $n = 1$.

```
x <- rbinom(n=1, 1, 0.5)
print(x)
```

```
## [1] 0
```

Statistics review

Probability distributions in R

In this case, I make 10 draws from a binomial distribution, simulating ten tosses of a fair coin.

```
x <- rbinom(10, 1, 0.5)
print(x)
```

```
##  [1] 0 1 0 0 1 1 0 0 1 0
```

Statistics review

Probability distributions in R

A Normal, or Gaussian, distribution is a continuous distribution with two parameters. The standard normal distribution has a mean $\mu = 0$ and standard deviation $\sigma = 1$. Here is a single random draw:

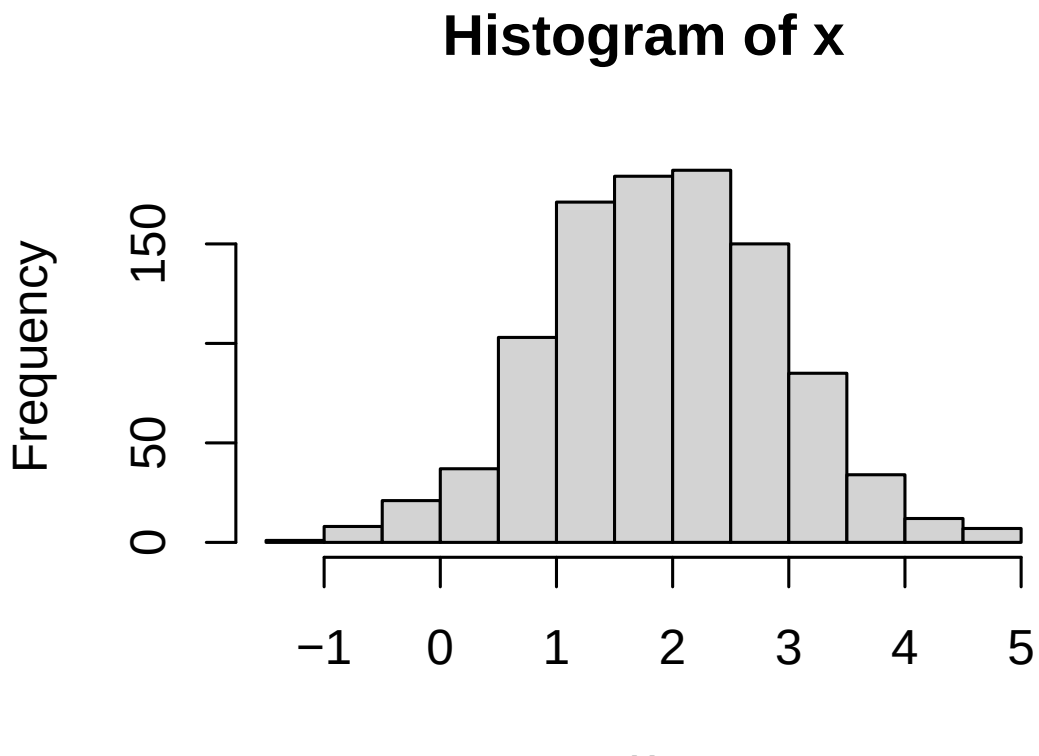
```
x <- rnorm(1, mean = 0, sd = 1)
print(x)
```

```
## [1] 1.477438
```

Statistics review

Probability distributions in R

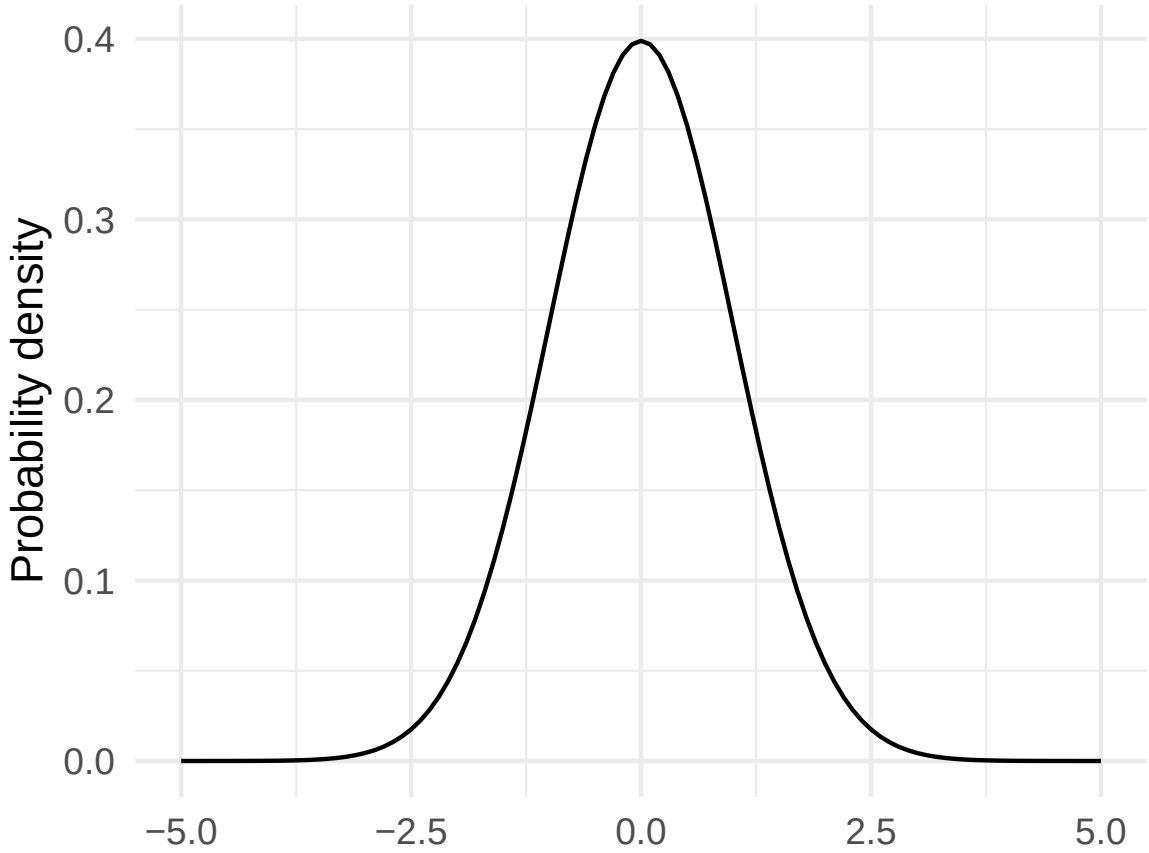
In this case, we can make 1000 draws from a normal distribution with a mean of 2 and a standard deviation of 1. Since there are a lot of values it is best to plot them using a histogram.



Statistics review

Probability distributions in R

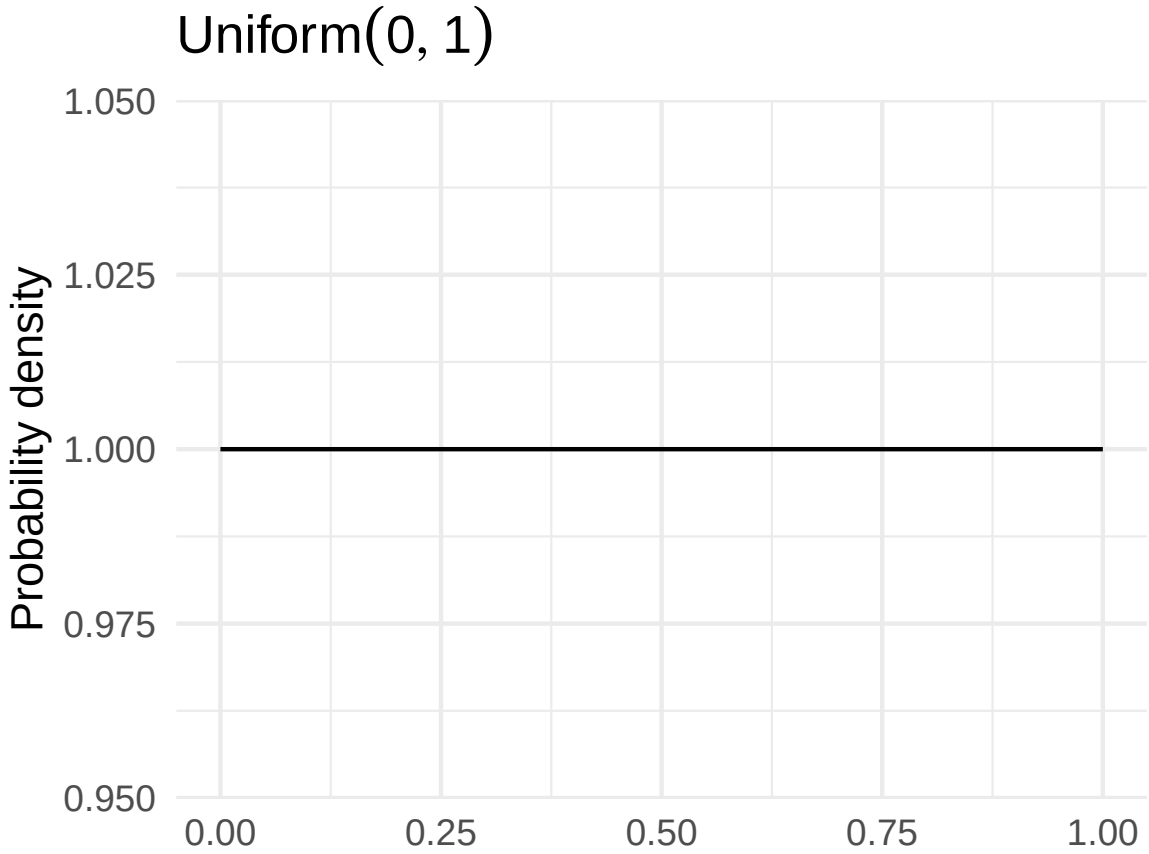
A *probability density function* shows the probability of observing particular random draws from a given distribution.



Statistics review

Probability distributions in R

In a uniform distribution, every value in the interval $[a, b]$ is equally likely.



Statistics review

Expected values

- The expectation of a random variable is denoted by $\mathbb{E}[x]$. It is the long-run average of the random variable over many repeated trials.
- The expected value of a constant $x = c$ is c. E.g. $\mathbb{E}[2] = 2$
- The expected value of a random variable is its mean.
  - e.g. If x represents a vector of values drawn from a normal distribution, our best guess as to the value of any one realization x_i is μ.
- We can use expectations notation to represent relationships between variables, e.g. $\mathbb{E}[y|x]$

Statistics review

Expectations as weighted averages

- Discrete case, where p_i is the probability of observing a particular value of x

$$\mathbb{E}[x] = \Sigma_{i=1}^{k} x_i p_i$$

- In the continuous case, where $f(x)$ is a probability density function, the expectation is an integral over the possible values of x.

$$\mathbb{E}[x] = \int_{-\infty}^{\infty} x f(x) dx$$

Statistics review

Populations and samples

- Classical statistics is based upon the assumption that our observations x are drawn from an underlying population.
- In a simple random sample, we draw n instances of a random variable from the population
  - These draws are assumed to be *independent* and *identically distributed* (IID)
- For example, a hypothetical population might be adults residing in the United States and a sample would be a randomly sampled subset of these adults.

Statistics review

Means

- The population mean value of a random variable is defined in the following manner

$$\mu_x = \frac{1}{N}\sum_{i=1}^{N} x_i = \frac{\sum_{i=1}^{N} x_i}{N}$$

- The sample mean $\bar{x}$ is defined by the following equation. n is lowercase to denote $n \subseteq N$

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i = \frac{\sum_{i=1}^{n} x_i}{n}$$

Statistics review

Variance

- Variance is the average of the squared deviations from the mean. For the population it is defined as

$$\sigma_x^2 = \frac{1}{N}\Sigma(x_i - \mu)^2$$

- The sample variance includes a degrees of freedom correction.

$$\sigma_{\bar{x}}^2 = \frac{1}{n-1}\Sigma(x_i - \bar{x})^2$$

Statistics review

Standard deviation

- Variance is difficult to interpret. The standard deviation is a scaled measure of variance, typically denoted using σ. In the population, it is equal to

$$\sigma_x = \sqrt{\sigma_x^2} = \sqrt{\frac{1}{N}\Sigma(x_i - \mu)^2}$$

- The sample standard deviation is thus

$$\sigma_{\bar{x}} = \sqrt{\sigma_{\bar{x}}^2} = \sqrt{\frac{1}{n-1}\Sigma(x_i - \bar{x})^2}$$

Statistics review

The sampling distribution of the mean

- The mean of an IID random sample is distributed according to a *sampling distribution*
- It has the following properties:

$$\mathbb{E}[\bar{x}] = \mu_x$$

$$var(\bar{x}) = \sigma^2_{\bar{x}} = \frac{\sigma^2_x}{N}$$

Statistics review

Simulating the sampling distribution

We can simulate the sample mean of 1000 normal distributions with the same population parameters and compare the sample means to the expectations.

```
mu <- 100; sigma <- 10; n <- 100 # population parameters
sims <- replicate(1000, mean(rnorm(n, mu, sigma)))
print(round(mean(sims),2)) # E[mu] = 100
```

```
## [1] 99.99
```

```
print(round(var(sims),2)) # E[sigma^2/n] = (10^2)/100 = 1
```

```
## [1] 0.9
```

Statistics review

The Law of Large Numbers

- When sample size is large, $\bar{x}$ is close to μ_x with a high probability
- A large IID sample can therefore approximate the sampling distribution
  - Such samples are *asymptotic* because approximations become exact in the limit as $n \to \infty$
- Under such conditions, $\bar{x}$ is considered a *consistent* estimator for μ_x

Statistics review

The Law of Large Numbers

The sample mean from a large IID sample closely approximates the population mean.

```
large.sample <- rnorm(1e6, mu, sigma)
print(mean(large.sample))
```

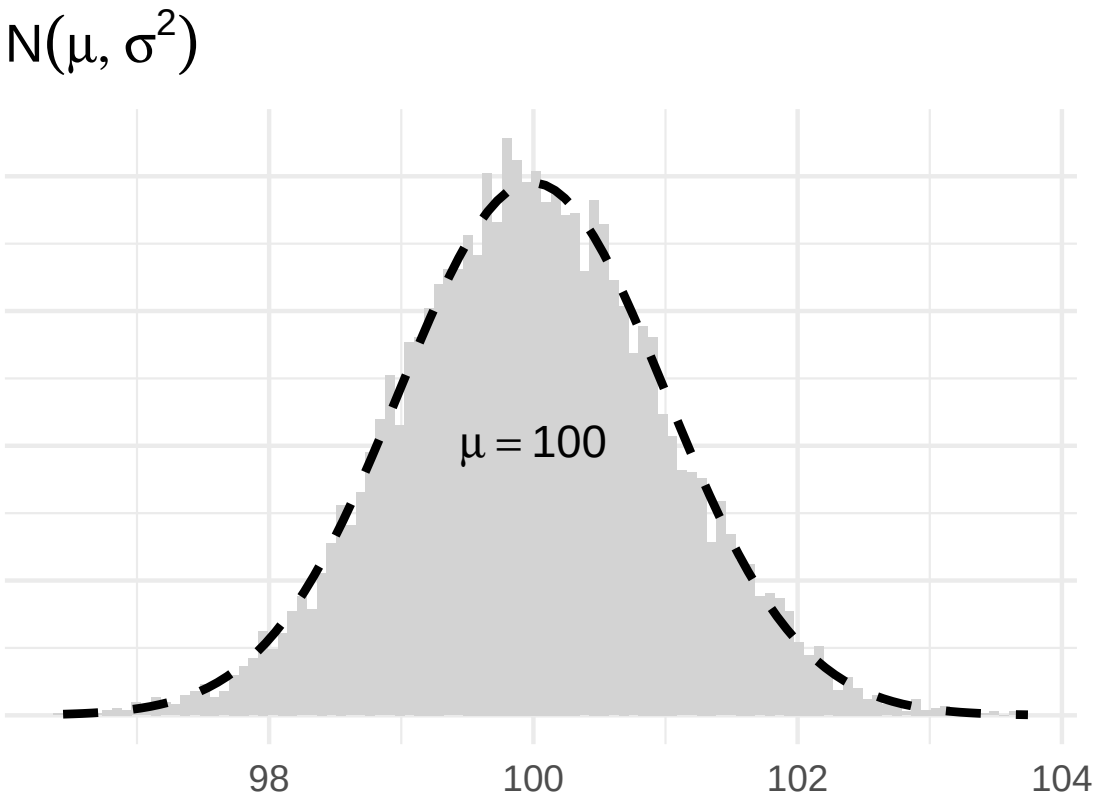
```
## [1] 99.99705
```

Statistics review

The Central Limit Theorem

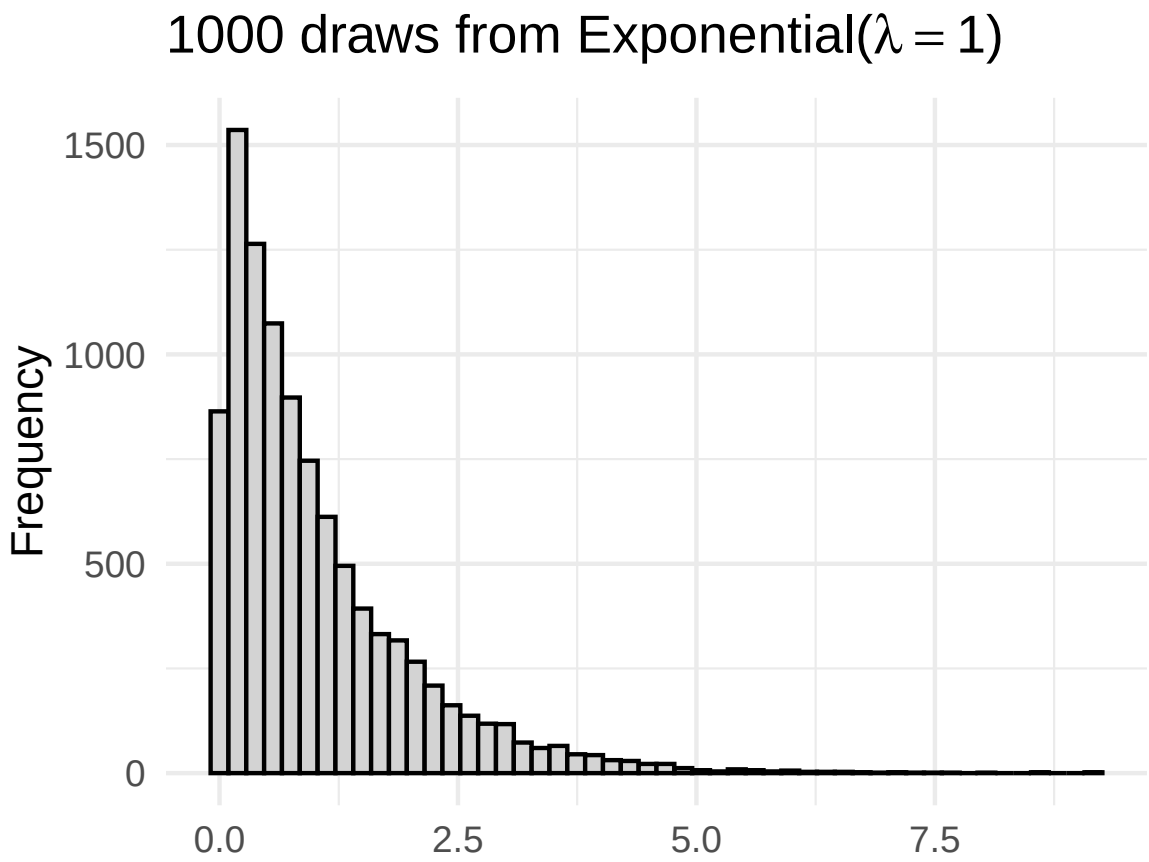
- CLT: The distribution of $\bar{x}$ is well approximated by a normal distribution when n is large
  - This is approximately true even if x are not normally distributed

Statistics review

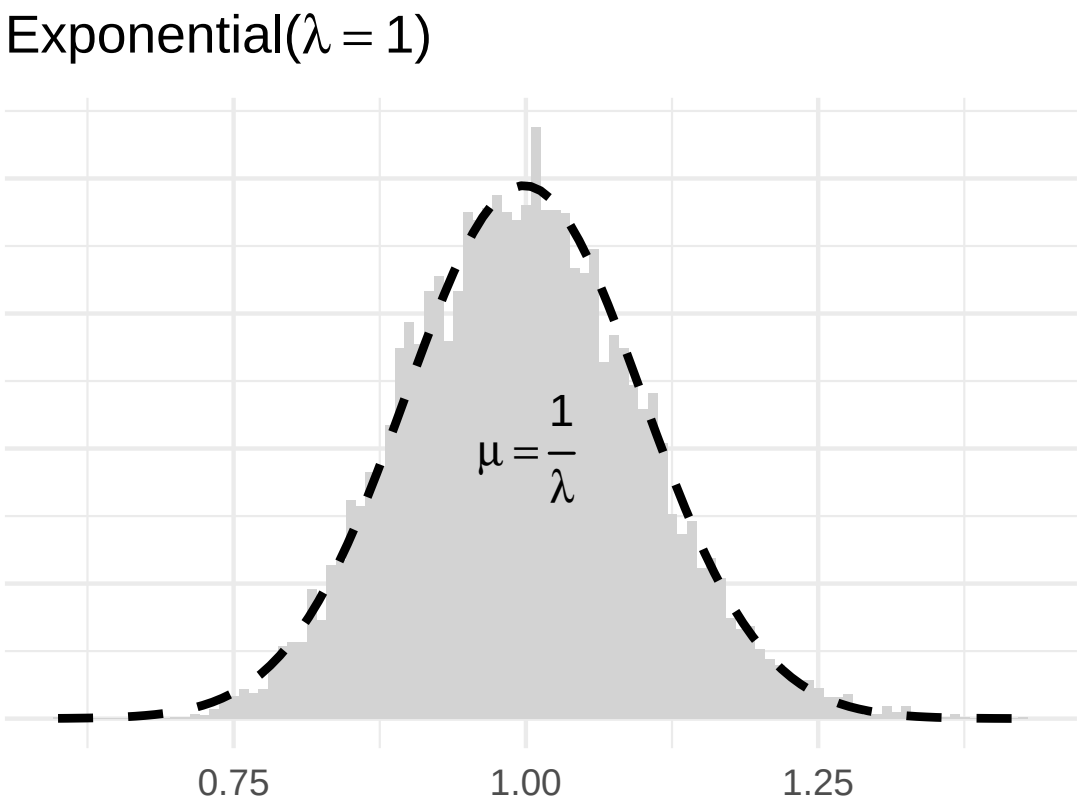


Distribution of 10,000 sample means, where n = 100

Statistics review



Statistics review



Distribution of 10,000 sample means, where n = 100.

Statistics review

Standard error of the sample mean

- The standard error of the sample mean is defined as the sample standard deviation divided by the square root of n.

$$SE_{\bar{x}} = \frac{\sigma_{\bar{x}}}{\sqrt{n}}$$

- The standard error is used to communicate *uncertainty* since we cannot observe the true population mean μ but only the sample mean $\bar{x}$.
- Theoretically, the standard error is the *standard deviation of the sampling distribution*.

Statistics review

Standard error as standard deviation of the sampling distribution

We can show that a standard error of a large random sample is a good approximate of the standard deviation of the sampling distribution.

```
N <- 10000
s.dist <- replicate(1000, mean(rnorm(N)))
print(round(sd(s.dist),3))
```

```
## [1] 0.01
```

```
x <- rnorm(N)
print(round(sd(x)/sqrt(N),3))
```

```
## [1] 0.01
```

Statistics review

Estimating the standard error of the mean in R

```
mu <- 10 # population mean
sigma2 <- 1 # population standard deviation
N <- 100
x <- rnorm(N, mu, sigma2)
print(mean(x)) # sample mean
```

```
## [1] 9.862993
```

```
print(sd(x)/sqrt(N)) # sample SE
```

```
## [1] 0.09972622
```

Statistics review

Confidence intervals

- The standard error is used to define a confidence interval. By convention, a 95% confidence interval has lower and upper bounds of

$$[\bar{x} - 1.96SE_{\bar{x}}, \bar{x} + 1.96SE_{\bar{x}}]$$

- When n is large, the 95% confidence interval around $\bar{x}$ contains the true value μ_x in 95% of all possible random samples.

Statistics review

Confidence intervals in R

We can use the results of the previous simulation to compute confidence intervals.

```
xbar <- mean(x)
SE <- sd(x)/sqrt(N)
lower <- xbar-1.96*SE
print(lower)
```

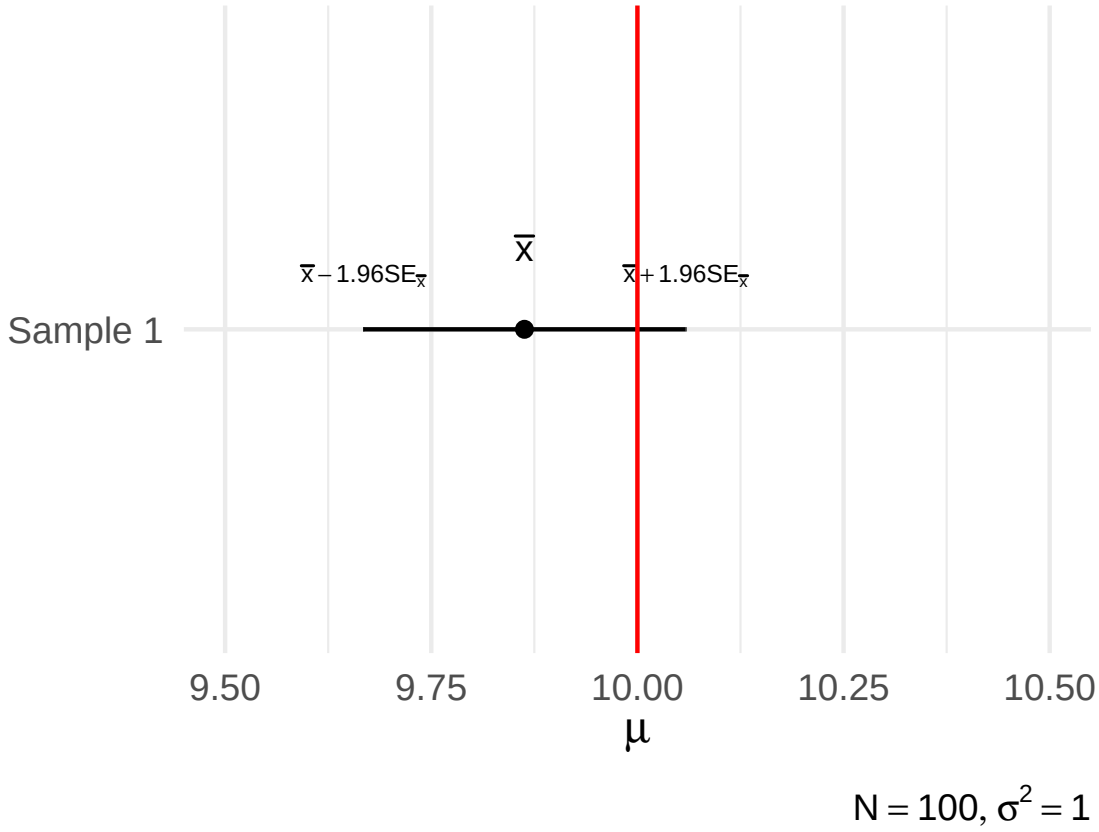
```
## [1] 9.667529
```

```
upper <- xbar+1.96*SE
print(upper)
```

```
## [1] 10.05846
```

Statistics review

Confidence intervals in R



Statistics review

One-sample t-tests

- We can use a one-sample t-test to assess whether there is a statistically significant difference between our sample mean and a null hypothesis.
- The *test statistic* t is obtained using the following equation:

$$t = \frac{\bar{x} - \mu}{\frac{\sigma_{\bar{x}}}{\sqrt{n}}}$$

Statistics review

p-values and statistical significance

- In classical statistics, we use a test statistic to calculate a *p-value*.
  - This represents the probability of drawing a test statistic at least as large as the observed test statistic, assuming the null hypothesis is correct
- Smaller p-values indicate that a result is less likely to be due to chance
  - By convention, $p < 0.05$ is considered the threshold for statistical significance.

Statistics review

One-sample t-tests in R

```
t.test(x, mu=10)
```

```
##
##  One Sample t-test
##
## data:  x
## t = -1.3738, df = 99, p-value = 0.1726
## alternative hypothesis: true mean is not equal to 10
## 95 percent confidence interval:
##   9.665114 10.060871
## sample estimates:
## mean of x
##  9.862993
```

Statistics review

One-sample t-tests in R

We can calculate the t-statistic and directly look up the corresponding p-value using a Student t distribution with N-1 degrees of freedom.[1]

```
t <- (xbar-mu)/(sd(x)/sqrt(N))
print(t)
```

```
## [1] -1.373835
```

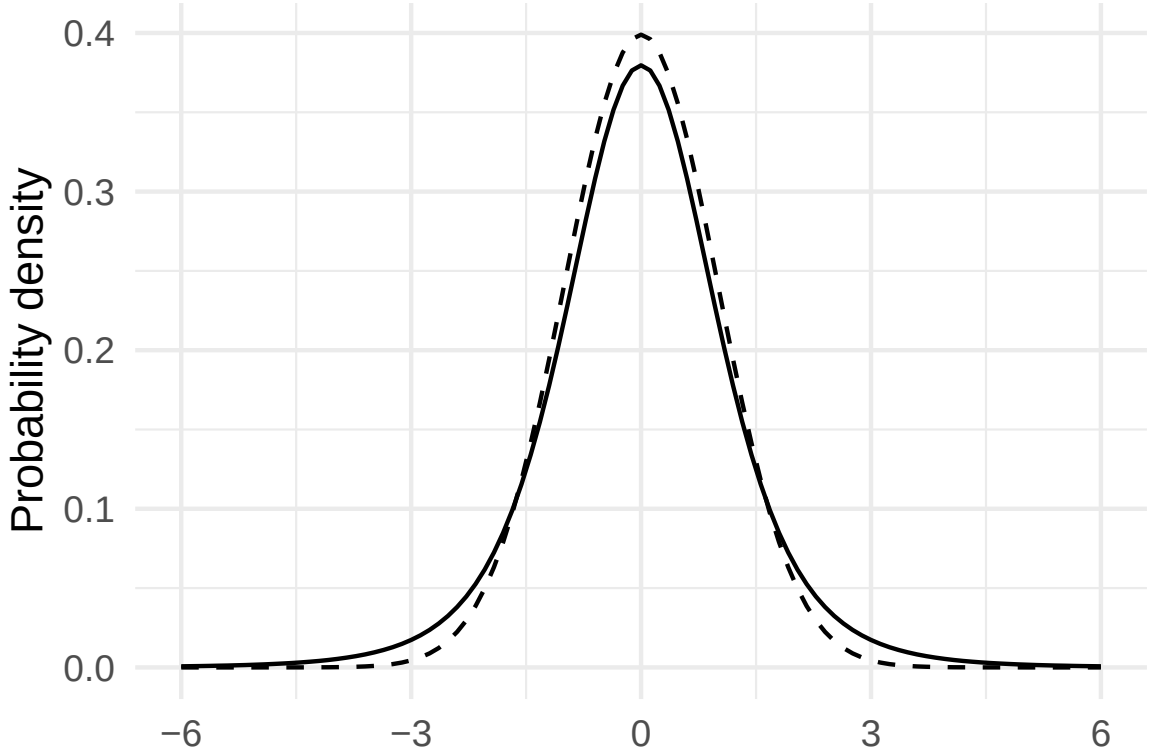
```
p <- pt(abs(round(t,5)), df = N-1, lower.tail = FALSE)
print(round(p*2,4)) # p*2 = two-tailed p-value
```

```
## [1] 0.1726
```

[1] Rounding ensures that the results are equivalent to the 't.test' function used above.

Statistics review

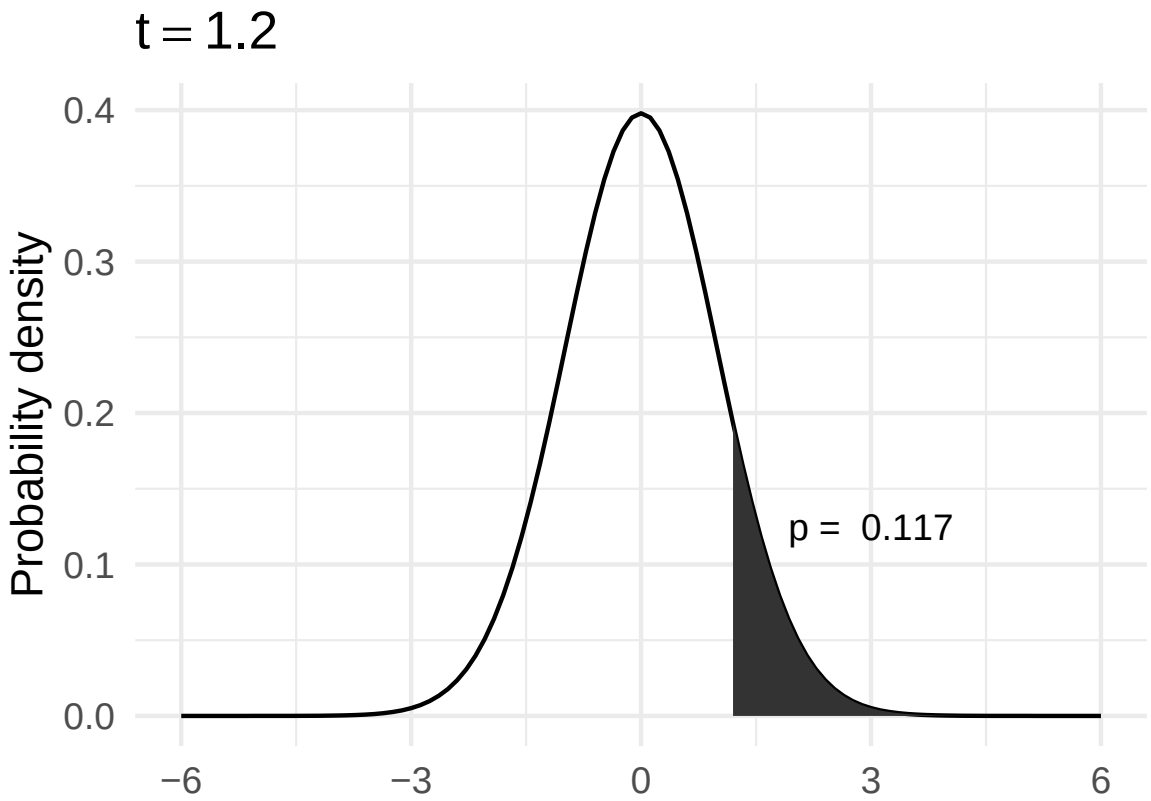
The t-test and the Student t distribution



Student t distribution with 5 d.f.
N(0,1) dashed line

Statistics review

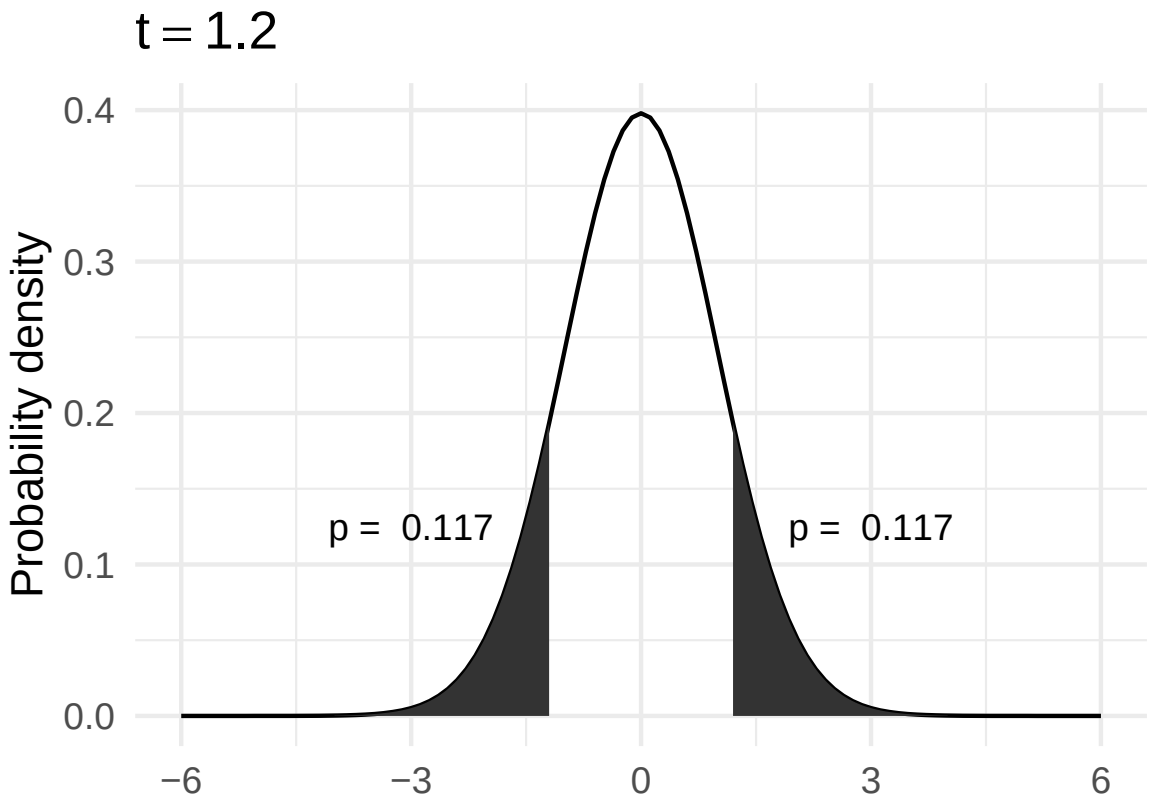
Area under one tail



Student t distribution with 99 degrees of freedom.

Statistics review

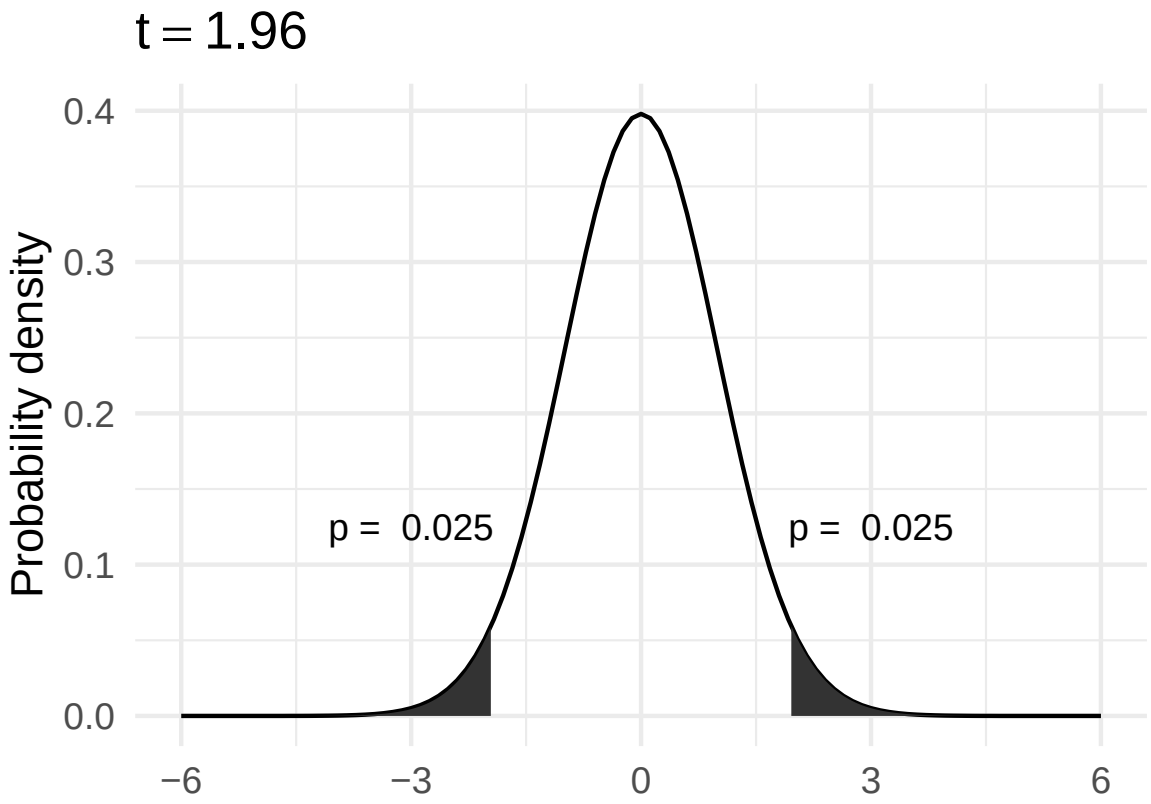
Area under both tails



Student t distribution with 99 degrees of freedom.

Statistics review

Area under both tails



Student t distribution with 999 degrees of freedom.

Statistics review

Random sampling and p-values

Another way to show the same result is to draw random variables from a Student t distribution and calculate the proportion that fall above the chosen significance threshold.

```
random.t <- rt(n = 10000, df = 1000-1)
round(length(random.t[abs(random.t) >= 1.96])/
          length(random.t),2)
```

```
## [1] 0.05
```

We can expect to see a t-statistic where $|t| \geq 1.96$ approximately 5% of the time purely due to chance.

Statistics review

Testing for differences in means

- Often we want to know whether the mean values of two random variables, μ_x and μ_y, are different from one another.
- We can test for this by computing calculating the difference between the sample means and the uncertainty about that difference
- The equation for the two-sample t-test is:

$$t = \frac{\mu_x - \mu_y}{SE(\mu_x - \mu_y)}$$

where

$$SE(\mu_x - \mu_y) = \sqrt{\frac{\sigma_x^2}{n_x} + \frac{\sigma_y^2}{n_y}}$$

Statistics review

Testing for differences in means

```
n <- 1000
x <- rnorm(n, mean = 12)
y <- rnorm(n, mean = 11)
xbar <- mean(x)
ybar <- mean(y)
varx <- (1/(n-1))*sum((x-xbar)^2)
vary <- (1/(n-1))*sum((y-ybar)^2)
SE <- sqrt((varx/n)+(vary/n))
t <- (xbar-ybar)/SE
print(round(t,3))
```

```
## [1] 20.831
```

Statistics review

Testing for differences in means

```
t.test(x, y)
```

```
##
##  Welch Two Sample t-test
##
## data:  x and y
## t = 20.831, df = 1994.7, p-value < 2.2e-16
## alternative hypothesis: true difference in means is not
## 95 percent confidence interval:
##  0.8753134 1.0572567
## sample estimates:
## mean of x mean of y
##  11.97403  11.00774
```

Statistics review

Type I and Type II errors

- A Type I (false positive) error occurs when we incorrectly reject the null hypothesis
  - e.g. In the one-sample test, we reject the hypothesis that $\bar{x} = 10$ given $\mu = 10$
- A Type II (false negative) error occurs when we incorrectly fail to reject the null hypothesis
  - e.g. If we did not have sufficient statistical power for the test above, we might fail to reject the null that $\bar{x} = \bar{y}$
- In classical statistics, if our chosen significance level is $p < 0.05$ then we expect to see such errors approximately 5% of the time

Statistics review

Sign (S) and magnitude (M) errors

- Gelman, Hill and Vehtari draw attention to two kinds of errors that are often overlooked but can lead to profoundly incorrect inferences:
  - Sign errors: the sign of a relationship is incorrect
    - e.g. We observe a positive relationship between x and y when the true relationship is negative
  - Magnitude errors: the observed effect is severely over- or under-estimated
    - e.g. We think $\bar{x} = 10$ but $\bar{x} = 0.1$

Statistics review

Expected mean and variance of two random variables

- The expected mean of the sum of two random variables is

$$E[x + y] = E[x] + E[y] = \mu_x + \mu_y$$

- The expected variance is the sum of the variances plus twice their covariance

$$var(x + y) = var(x) + var(y) + 2cov(x, y)$$

- If x and y are independent then $cov(x, y) = 0$ and $var(x + y) = var(x) + var(y)$

Statistics review

Covariance

- Covariance is the a measure of the joint variability of two random variables
- The expectation of the covariance between x and y is

$$cov(x, y) = E[xy] - E[x]E[y]$$

- For a population, the covariance is

$$cov(x, y) = \frac{1}{N}\Sigma(x_i - \mu_x)(y_i - \mu_y)$$

- Sample covariance is defined as

$$cov(x, y) = \frac{1}{n-1}\Sigma(x_i - \bar{x})(y_i - \bar{y})$$

Statistics review

Correlation

- Correlation is a scaled version of covariance. We divide the covariance by the product of the standard deviations.

$$\rho(x, y) = \frac{\frac{1}{n-1}\Sigma(x_i - \bar{x})(y_i - \bar{y})}{\sigma_x \sigma_y} = \frac{cov(x, y)}{\sigma_x \sigma_y}$$

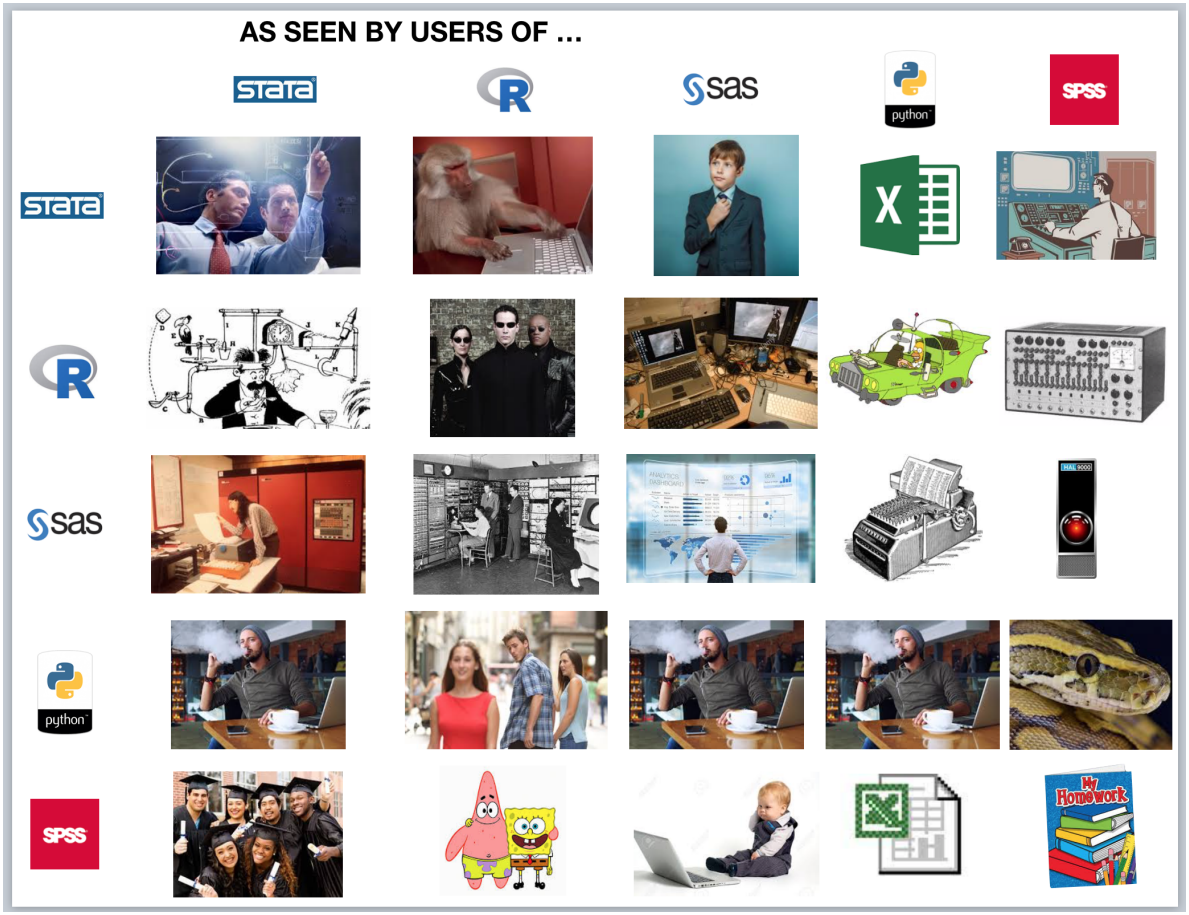
- The Greek letter ρ is typically used to refer to correlation. The correlation coefficient ranges from -1 to 1.

Part III: Tools

Why R?

- Free and open-source
- Versatile
  - Statistical computing (`lm`, `glm`, 'rstanarm")
  - Data manipulation (`tidyverse`)
  - Visualization (`ggplot`)
  - General purpose programming
- Active developer community
  - Lots of packages with regular updates
  - New statistical and econometric packages

Why R?



Source: Kieran Healey

RStudio

Overview

- RStudio is an Integrated Development Environment for programming in R
  - Run code in the console or in scripts
  - Easy to view data, objects in memory, plots
  - Easy to create output such as papers or slides
  - Integration with Github

RMarkdown

Overview

- RMarkdown is an interactive coding environment
  - RMarkdown documents can combine text, LaTeX code, R code, and any output.
  - These slides are rendered using RMarkdown
- You will be using RMarkdown for your homework assignments and hopefully your papers

Github

Overview

- Github is a platform for hosting code
  - Source code for open-source projects, such as R packages, is available
  - Version control keeps track of changes
  - Helps to ensure reproducible workflows
- The course website, homework assignments, and your final projects will be hosted on Github

Github

Set-up

- Follow the instructions on https://tinyurl.com/github542
- Step 1: Make an account
- Step 2: Install Github locally (depending on system)
- Step 3: Sync with RStudio
- Step 4: Clone the class repository

Artificial intelligence

Policy (from syllabus)

"I encourage you to use AI tools such as ChatGPT, Claude, and Gemini to learn about statistics, interrogate the readings, and improve your R code. You are permitted to use these tools to help edit your writing and code. However, you must solve the homework questions yourself and to write responses in your own words. You should also carefully check any output from AI to ensure its accuracy and validity. Please document any AI usage in your submissions."

Artificial intelligence

Common use cases

- Explanation
  - "I don't understand why we take the square root of a variance to get a standard deviation. Can you explaint this"
- Summarization
  - [Upload paper PDF] "Summarize the main argument of this paper"
- Question-answering
  - [Upload paper PDF] "Does this paper use logistic regression?"
- Debugging code
  - [Paste R code and error message] "Why is my code failing?"
- Enhancing code
  - [Paste R code and data snippet] "Write an R function to create a standardized version of each of these variables"

Artificial intelligence

Strengths

- AI systems "know" a lot about statistics as they have "read" much of the publically available information and many scientific papers
- Many models are trained to write code and can be helpful for debugging and assistance
- Latest models have capacity to analyze new information including PDFs, spreadsheets, and other information

Artificial intelligence

Weaknesses

- AI can "hallucinate" / "confabulate", including inaccurate information or incorrect code
  - Carefully verify and cross-reference information
- AI may not have syntax for more niche software, including some of the R packages we will use
  - Understanding the basics and how to do things yourself is critical
- AI can be biased, reproducing stereotypical representations and tropes
- AI often provide a "one-size-fits-all" solution to problems and will struggle with more creative tasks in research where we push boundaries of human knowledge

Next lecture

- Ordinary Least Squares regression with two variables

Lab 1

- RStudio and RMarkdown
- Manipulating random variables
- Intro to `tidyverse` and `ggplot` for data manipulation and visualization