SOC542 Statistical Methods in Sociology II

Interactions

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Plan

- Introducing interactions
- Types of interactions and their interpretations
- Marginal effects

Introducing interactions

What is an statistical interaction?

- Consider the following population model:

$$y = \beta_0 + \beta_1 x + \beta_2 z + u$$

- The coefficients β_1 and β_2 measure the relationship between x and y and z and y, respectively.
  - The interpretation of either coefficient requires that we hold the other constant.
- What if we expect the effect of x to vary as a function of z?

Introducing interactions

What is an statistical interaction?

- If we expect there to be an **interaction** between x and z, such that the effect of x on y varies according to the level of z, we can add an **interaction term** into our model formula.

$$y = \beta_0 + \beta_1 x + \beta_2 z + \beta_3 xz + u$$

- β_0 and β_1 are now considered as the **main effects**.
- β_3 is the coefficient for the interaction term, representing the effect of x times z.

Introducing interactions

A simple population model

```r
N <- 1000
x <- rnorm(N)
z <- rnorm(N)
y <- 3*x + 2*z + -5*(x*z) + rnorm(N, 10)
```

Introducing interactions

Comparing models

	Model 1	Model 2
(Intercept)	10.029***	10.010***
	(0.153)	(0.032)
x	2.935***	2.981***
	(0.157)	(0.033)
z	2.099***	2.016***
	(0.151)	(0.031)
x $\times$ z		-4.980***
		(0.034)
Num.Obs.	1000	1000
R2	0.351	0.972
R2 Adj.	0.350	0.972
F	269.689	11455.353

+ $p < 0.1$, * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Introducing interactions

Example: intersectional inequalities

- We can use interaction terms as a way to encode theoretical knowledge about the relationship between variables.
- For example, if we expect there to be differences in income related to the interaction between sex and race, we can add an interaction term to a model:

$$Income = \beta_0 + \beta_1 Sex + \beta_2 Race + \beta_3 Age + \beta_4 Sex * Race + u$$

Introducing interactions

Main effects and interactions

- In general, it is recommended to *include the main effects in any model with interactions*.
  - Type II errors are more likely when interpreting interaction terms with main effects omitted.
  - The interpretation of the model can change substantially if main effects are excluded.[1]

[1] See this Stata blog for further discussion: https://stats.oarc.ucla.edu/stata/faq/what-happens-if-you-omit-the-main-effect-in-a-regression-model-with-an-interaction/

Types of interactions

Dummy-dummy

$$y = \beta_0 + \beta_1 Male + \beta_2 Union + \beta_3 Male * Union + u$$

Types of interactions

Dummy-dummy

	Model 1	Model 2
(Intercept)	26551.509***	33180.553***
	(3787.960)	(5807.909)
sex	9755.789***	-933.337
	(1910.401)	(7354.886)
union	-1693.873+	-3492.301*
	(963.140)	(1534.419)
sex × union		2964.964
		(1970.186)
Num.Obs.	900	900
R2	0.034	0.036
R2 Adj.	0.032	0.033
F	15.685	11.226

+ $p < 0.1$, * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Types of interactions

Dummy-dummy

$$y = \beta_0 + \beta_1 Male + \beta_2 Union + \beta_3 Male * Union + u$$

- Female and non-unionized are the reference categories.
- β_1 and β_2 represent the main effects of sex and union membership on the outcome.
- The coefficient β_3 represents the expected difference in the effect of union membership for men versus women.[2]
- The expected income for a male unionized worker is $\beta_0 + \beta_1 + \beta_2 + \beta_3$. The same quantity for a female unionized worker is $\beta_0 + \beta_2$.

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[2] Note the symmetrical interpretation here: the difference in the effect of sex for union members versus non-members. See McElreath 8.2 for further discussion.

Types of interactions

Continuous-dummy

$$y = \beta_0 + \beta_1 Age + \beta_2 Sex + \beta_3 Age * Sex + u$$

Types of interactions

Continuous-dummy

	Model 1	Model 2
(Intercept)	4489.024+	7430.716*
	(2553.184)	(3394.090)
age	352.706***	285.651***
	(53.071)	(73.589)
sex	10158.427***	3941.453
	(1523.239)	(4967.082)
age × sex		139.644
		(106.196)
Num.Obs.	1358	1358
R2	0.064	0.065
R2 Adj.	0.063	0.063
F	46.342	31.488

+ $p < 0.1$, * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Types of interactions

Continuous-dummy

$$y = \beta_0 + \beta_1 Age + \beta_2 Sex + \beta_3 Age * Sex + u$$

- The coefficients β_1 and β_2 represent the main effects of age and sex on income.
- For females, β_1 represents the relationship between age and income. For males, the relationship is $\beta_1 + \beta_3$.
  - Thus, the interaction term allows the *slope* to vary according to sex.

Types of interactions

Continuous-continuous

$$y = \beta_0 + \beta_1 Age + \beta_2 Educ + \beta_3 Age * Educ + u$$

Types of interactions

Continuous-continuous

	Model 1	Model 2
(Intercept)	-32586.610***	-2246.022
	(4262.602)	(12926.170)
age	333.107***	-339.621
	(51.486)	(275.473)
educ	3025.906***	850.117
	(258.389)	(912.517)
age $\times$ educ		48.073*
		(19.340)
Num.Obs.	1357	1357
R2	0.122	0.126
R2 Adj.	0.121	0.124
F	94.136	65.057

+ $p < 0.1$, * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Types of interactions

Continuous-continuous

$$y = \beta_0 + \beta_1 Age + \beta_2 Educ + \beta_3 Age * Educ + u$$

- The intercept no longer has a meaningful education (income when age and education equal zero).
  - GHV 12.2 discuss standardization as an approach to make intercepts more interpretable in such contexts.
- β_1 and β_2 represent the main effects of age and education.
- The interaction term β_3 captures how the effect of education on income varies as a function of age.

Types of interactions

Continuous-continuous

- The effect of education on income is now also a function of age:

$$\frac{\Delta y}{\Delta_{Educ}} = \beta_2 + \beta_3 Age$$

- Similarly,

$$\frac{\Delta y}{\Delta_{Age}} = \beta_1 + \beta_3 Educ$$

Types of interactions

Continuous-continuous

- If Age changes by ΔAge and Educ by ΔEduc, the expected change in y is:

$$\Delta y = (\beta_1 + \beta_3 Educ)\Delta Age + (\beta_2 + \beta_3 Age)\Delta Educ + \beta_3 \Delta Age \Delta Educ$$

- The coefficient β_3 represents the effect of a unit increase in age *and* education, beyond the sum of the individual effects of unit increases alone.

Types of interactions

Dummy-categorical

	Model 1	Model 2
(Intercept)	22656.825***	21686.485***
sex	10354.422***	12357.436***
raceBlack	-8752.952***	-4062.267
raceOther	-9068.523***	-8545.222*
sex × raceBlack		-11599.813**
sex × raceOther		-1164.189

$+$ p $<$ 0.1, * p $<$ 0.05, ** p $<$ 0.01, *** p $<$ 0.001

Types of interactions

Dummy-categorical

$$y = \beta_0 + \beta_1 Male + \beta_2 Black + \beta_3 Other + \beta_4 Black\,Male + \beta_5 Other\,Male + u$$

- There is a separate coefficient for the interaction between the dummy variable and each of the categories, with the exception of the reference group.
- The interpretation is the same as the dummy-dummy model.

Types of interactions

Continuous-categorical

	Model 1	Model 2
(Intercept)	12391.315***	11405.129***
age	334.001***	355.556***
raceBlack	-8402.871***	-1744.355
raceOther	-6900.846**	-8009.313
age $\times$ raceBlack		-157.566
age $\times$ raceOther		29.827

$+$ $p < 0.1$, * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Types of interactions

Categorical-categorical

	Model 1	Model 2
(Intercept)	23151.207***	22095.578***
raceBlack	-8626.737***	-4916.366
raceOther	-8231.254***	-7528.201+
bible2	4485.160*	
bible3	8582.685***	
raceWhite × bible2		5814.876*
raceBlack × bible2		2258.917
raceOther × bible2		2473.170
raceWhite × bible3		10014.844***
raceBlack × bible3		-3627.312
raceOther × bible3		14067.405*

+ $p < 0.1$, * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Types of interactions

Three-way interactions

	Model 1	Model 2
(Intercept)	29210.035***	34756.000***
sex	9773.802***	-909.086
raceBlack	-8484.756**	-4235.477
raceOther	-9443.001**	-9295.560*
union	-1739.915+	-3412.829*
sex × raceWhite × union		3353.493+
sex × raceBlack × union		182.054
sex × raceOther × union		3275.209

+ $p < 0.1$, * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Types of interactions

Interpreting interactions

- Interactions terms make models more challenging to interpret.
  - Like polynomial regression, the effect of a single predictor is represented by more than one coefficient (e.g. $y = \beta_0 + \beta_1 x + \beta_2 z + \beta_3 xz + u$).
- Three-way and more complex interactions are even more difficult to interpret and should be avoided unless there are strong theoretical reasons to use them.

Marginal effects

Definitions

- A **marginal effect** is the relationship between change in single predictor and the dependent variable while *holding other variables constant*.
- The **average marginal effect (AME)** is the *average* change in the outcome y as a function of a unit change in x_i over all observations.
  - Coefficients in a standard OLS model represent average marginal effects.
- This quanity becomes more complicated when interaction terms are included, since the effect of a change in x_i now depends on multiple parameters.

Marginal effects

Computing marginal effects

- Frequentist marginal effects computed by calculating *partial derivatives* and approximating variance.
  - e.g. $ME(x_i) = \frac{\delta y}{\delta x_i}$.
  - We can use the `margins` package in R to do this.[3]
- Bayesian marginal effects can be calculated by sampling from the posterior distribution.

[3] See Thomas Leeper's documentation for the margins package for further details.

Marginal effects

Marginal effects and OLS regression

	Model 1
(Intercept)	-38952.254***
	(4249.685)
sex	11317.170***
	(1447.358)
age	314.828***
	(50.434)
educ	3153.801***
	(253.364)

+ $p < 0.1$, * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Marginal effects

Marginal effects and OLS regression

Note how the average marginal effects are equal to the OLS coefficients.

```
library(margins)
me <- margins(m)
summary(me)
```

```
##  factor        AME        SE       z      p     lower      upper
##     age   314.8284   50.4327  6.2425 0.0000  215.9821   413.6747
##    educ  3153.8010  253.3219 12.4498 0.0000 2657.2993  3650.3028
##     sex 11317.1702 1585.0688  7.1399 0.0000 8210.4925 14423.8479
```

Marginal effects

Marginal effects with non-linear variables

	Model 1
(Intercept)	-77886.516***
sex	11300.064***
age	2238.963***
I(age^2)	-20.769***
educ	3062.532***

+ $p < 0.1$, * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

[4]

[4] Note the use of the 'I' symbol when computing age-squared. This ensures that the margins command recognizes that this variable also relates to age.

Marginal effects

Marginal effects with non-linear variables

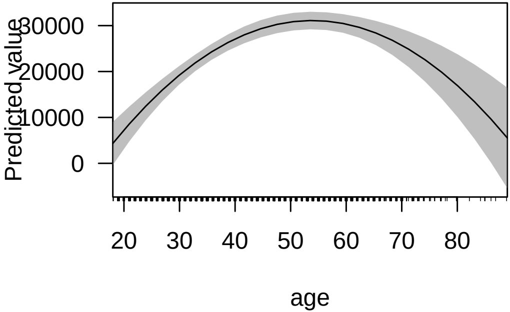
The margins commands are the same as above. Note how the AME now represents the total effect of age across the two parameters.

```
##  factor        AME        SE       z      p     lower      upper
##     age   391.4542   50.9703  7.6800 0.0000  291.5542   491.3541
##    educ  3062.5317  249.6433 12.2676 0.0000 2573.2398  3551.8235
##     sex 11300.0640 1467.4884  7.7003 0.0000 8423.8396 14176.2885
```

Marginal effects

Marginal effects with non-linear variables

We can also visualize the marginal effect of age in a continuous space.



Marginal effects

Marginal effects with interactions

	Model 1
(Intercept)	-71605.617***
sex	-2585.989
age	2211.447***
I(age^2)	-21.256***
educ	2780.345***
sex × educ	513.257
sex × age	148.844

$+$ p $<$ 0.1, * p $<$ 0.05, ** p $<$ 0.01, *** p $<$ 0.001

Marginal effects

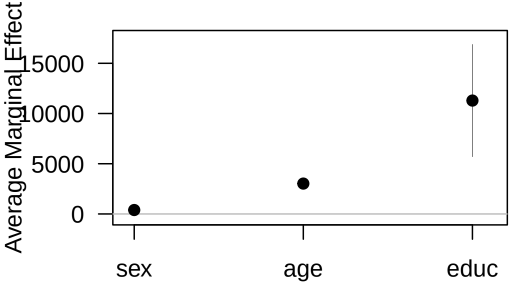
Marginal effects with interactions

```
##  factor        AME        SE       z      p     lower      upper
##     age   391.2588   50.9826  7.6744 0.0000  291.3347   491.1830
##    educ  3023.9249  251.5492 12.0212 0.0000 2530.8974  3516.9523
##     sex 11289.1676 2854.3856  3.9550 0.0001 5694.6746 16883.6605
```

Marginal effects

Plotting marginal effects

The `margins` package includes a `plot()` function to show the results of the table. The output can also be modified using `ggplot2`.

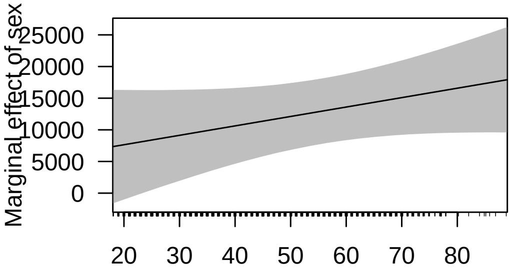


Marginal effects

Plotting conditional marginal effects

The `cplot` function can be used to plot the marginal effect while conditioning on another predictor. In this case, the marginal effect of sex on income over the range of age.

```
cplot(m, x = "age", dx = "sex", what = "effect")
```



Marginal effects

Marginal effects and generalized linear models

- Marginal effects are even more important when we consider generalized linear models (e.g. logistic regression) since coefficients often do not have clear interpretations on the outcome scale.[5]

[5] See the recommended reading, Mize 2019, for further discussion.

Next week

Topic

- Missing data
- Model specification, comparison, and robustness