SOC542 Statistical Methods in Sociology II

Interactions

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February 27, 2023

Plan

- Introducing interactions
- Types of interactions and their interpretations
- Marginal effects

Introducing interactions

What is an statistical interaction?

- Consider the following population model:

$$y = \beta_0 + \beta_1 x + \beta_2 z + u$$

- The coefficients β_1 and β_2 measure the relationship between x and y and z and y, respectively.
  - The interpretation of either coefficient requires that we hold the other constant. $>$- What if we expect the effect of x to vary as a function of z?

Introducing interactions

What is an statistical interaction?

- If we expect there to be an **interaction** between x and z, such that the effect of x on y varies according to the level of z, we can add an **interaction term** into our model formula.

$$y = \beta_0 + \beta_1 x + \beta_2 z + \beta_3 xz + u$$

- β_0 and β_1 are now considered as the **main effects**.
- β_3 is the coefficient for the interaction term, representing the effect of x *times* z.

Introducing interactions

A simple population model

```
N <- 1000
x <- rnorm(N)
z <- rnorm(N)
y <- 3*x + 2*z + -5*(x*z) + rnorm(N, 10)
```

Introducing interactions

Comparing models

	(1)	(2)
(Intercept)	10.029***	10.010***
	(0.153)	(0.032)
x	2.935***	2.981***
	(0.157)	(0.033)
z	2.099***	2.016***
	(0.151)	(0.031)
x × z		-4.980***
		(0.034)
Num.Obs.	1000	1000
R2	0.351	0.972
R2 Adj.	0.350	0.972
F	269.689	11455.353
RMSE	4.82	1.00

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Introducing interactions

Example: intersectional inequalities

- We can use interaction terms as a way to encode theoretical knowledge about the relationship between variables.
- For example, if we expect there to be differences in income related to the interaction between sex and race, we can add an interaction term to a model:

$$Income = \beta_0 + \beta_1 Sex + \beta_2 Race + \beta_3 Age + \beta_4 Sex * Race + u$$

Introducing interactions

Main effects and interactions

- In general, it is recommended to *include the main effects in any model with interactions*.
  - Type II errors are more likely when interpreting interaction terms with main effects omitted.
  - The interpretation of the model can change substantially if main effects are excluded.[1]

[1] See this Stata blog for further discussion: https://stats.oarc.ucla.edu/stata/faq/what-happens-if-you-omit-the-main-effect-in-a-regression-model-with-an-interaction/

Types of interactions

Dummy-dummy

$$y = \beta_0 + \beta_1 Male + \beta_2 Union + \beta_3 Male * Union + u$$

Types of interactions

Dummy-dummy

	(1)	(2)	(3)	(4)
(Intercept)	20.308***	33.095***	26.552***	33.181***
	(1.323)	(3.614)	(3.788)	(5.808)
sex	10.104***		9.756***	-0.933
	(1.902)		(1.910)	(7.355)
union		-2.204*	-1.694	-3.492*
		(0.971)	(0.963)	(1.534)
sex $\times$ union				2.965
				(1.970)
Num.Obs.	900	900	900	900
R2	0.030	0.006	0.034	0.036
R2 Adj.	0.029	0.005	0.032	0.033
F	28.211			
RMSE	28.49	28.85	28.44	28.40

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Types of interactions

Dummy-dummy

$$y = \beta_0 + \beta_1 Male + \beta_2 Union + \beta_3 Male * Union + u$$

- Female and non-unionized are the reference categories.
- β_1 and β_2 represent the main effects of sex and union membership on the outcome.
- The coefficient β_3 represents the expected difference in the effect of union membership for men versus women.[2]
- The expected income for a male unionized worker is $\beta_0 + \beta_1 + \beta_2 + \beta_3$. The same quantity for a female unionized worker is $\beta_0 + \beta_2$.

[2] Note the symmetrical interpretation here: the difference in the effect of sex for union members versus non-members. See McElreath 8.2 for further discussion.

Types of interactions

Continuous-dummy

$$y = \beta_0 + \beta_1 Age + \beta_2 Sex + \beta_3 Age * Sex + u$$

Types of interactions

Continuous-dummy

	(1)	(2)
(Intercept)	4.489	7.431*
	(2.553)	(3.394)
age	0.353***	0.286***
	(0.053)	(0.074)
sex	10.158***	3.941
	(1.523)	(4.967)
age × sex		0.140
		(0.106)
Num.Obs.	1358	1358
R2	0.064	0.065
R2 Adj.	0.063	0.063
F	46.342	31.488
RMSE	27.97	27.95

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Types of interactions

Continuous-dummy

$$y = \beta_0 + \beta_1 Age + \beta_2 Sex + \beta_3 Age * Sex + u$$

- The coefficients β_1 and β_2 represent the main effects of age and sex on income.
- For females, β_1 represents the relationship between age and income. For males, the relationship is $\beta_1 + \beta_3$.
  - Thus, the interaction term allows the *slope* to vary according to sex.

Types of interactions

Continuous-continuous

$$y = \beta_0 + \beta_1 Age + \beta_2 Educ + \beta_3 Age * Educ + u$$

Types of interactions

Continuous-continuous

	(1)	(2)
(Intercept)	-32.587***	-2.246
	(4.263)	(12.926)
age	0.333***	-0.340
	(0.051)	(0.275)
educ	3.026***	0.850
	(0.258)	(0.913)
age × educ		0.048*
		(0.019)
Num.Obs.	1357	1357
R2	0.122	0.126
R2 Adj.	0.121	0.124
F	94.136	65.057
RMSE	27.10	27.04

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Types of interactions

Continuous-continuous

$$y = \beta_0 + \beta_1 Age + \beta_2 Educ + \beta_3 Age * Educ + u$$

- The intercept no longer has a meaningful education (income when age and education equal zero).
  - GHV 12.2 discuss standardization to make intercepts more interpretable in such contexts.
- β_1 and β_2 represent the main effects of age and education.
- The interaction term β_3 captures how the effect of education on income varies as a function of age.

Types of interactions

Continuous-continuous

- The effect of education on income is now also a function of age:

$$\frac{\Delta y}{\Delta_{Educ}} = \beta_2 + \beta_3 Age$$

- Similarly,

$$\frac{\Delta y}{\Delta_{Age}} = \beta_1 + \beta_3 Educ$$

Types of interactions

Continuous-continuous

- If Age changes by ΔAge and Educ by ΔEduc, the expected change in y is:

$$\Delta y = (\beta_1 + \beta_3 Educ)\Delta Age + (\beta_2 + \beta_3 Age)\Delta Educ + \beta_3 \Delta Age \Delta Educ$$

- The coefficient β_3 represents the effect of a unit increase in age *and* education, beyond the sum of the individual effects of unit increases alone.

Types of interactions

Dummy-categorical

	(1)	(2)
(Intercept)	22.657***	21.686***
sex	10.354***	12.357***
raceBlack	-8.753***	-4.062
raceOther	-9.069***	-8.545*
sex × raceBlack		-11.600**
sex × raceOther		-1.164

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Types of interactions

Dummy-categorical

$$y = \beta_0 + \beta_1 Male + \beta_2 Black + \beta_3 Other + \beta_4 Black\,Male + \beta_5 Other\,Male + u$$

- There is a separate coefficient for the interaction between the dummy variable and each of the categories, with the exception of the reference group.
- The interpretation is the same as the dummy-dummy model.

Types of interactions

Continuous-categorical

	(1)	(2)
(Intercept)	12.391***	11.405***
age	0.334***	0.356***
raceBlack	-8.403***	-1.744
raceOther	-6.901**	-8.009
age $\times$ raceBlack		-0.158
age $\times$ raceOther		0.030

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Types of interactions

Categorical-categorical

	(1)	(2)
(Intercept)	23.151***	22.096***
raceBlack	-8.627***	-4.916
raceOther	-8.231***	-7.528
bibleInspired Word	4.485*	
bibleAncient Book	8.583***	
raceWhite × bibleInspired Word		5.815*
raceBlack × bibleInspired Word		2.259
raceOther × bibleInspired Word		2.473
raceWhite × bibleAncient Book		10.015***
raceBlack × bibleAncient Book		-3.627
raceOther × bibleAncient Book		14.067*

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Types of interactions

Three-way interactions

	(1)	(2)
(Intercept)	29.210***	34.756***
sex	9.774***	-0.909
raceBlack	-8.485**	-4.235
raceOther	-9.443**	-9.296*
union	-1.740	-3.413*
sex × raceWhite × union		3.353
sex × raceBlack × union		0.182
sex × raceOther × union		3.275

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Types of interactions

Interpreting interactions

- Interactions terms make models more challenging to interpret.
  - Like polynomial regression, the effect of a single predictor is represented by more than one coefficient (e.g. $y = \beta_0 + \beta_1 x + \beta_2 z + \beta_3 xz + u$).
- Three-way and more complex interactions are even more difficult to interpret and should be avoided unless there are strong theoretical reasons to use them.

Marginal effects

Definitions

- A **marginal effect** is the relationship between change in single predictor and the dependent variable while *holding other variables constant*.
- The **average marginal effect (AME)** is the *average* change in the outcome y as a function of a unit change in x_i over all observations.
  - Coefficients in a standard OLS model represent average marginal effects.
- This quantity becomes more complicated to calculate when interaction terms are included, since the effect of a change in x_i now depends on multiple parameters.

Marginal effects

Computing marginal effects

- Frequentist marginal effects computed by calculating *partial derivatives* and variance approximations are used to construct confidence intervals.
  - e.g. $ME(x_i) = \frac{\delta y}{\delta x_i}$.
  - We can use the `margins` package in R to do this.[3]
- Bayesian marginal effects can be calculated by sampling from the posterior distribution.

[3] See documentation for the margins package for further details.

Marginal effects

Marginal effects and OLS regression

	(1)
(Intercept)	-38.952***
	(4.250)
sex	11.317***
	(1.447)
age	0.315***
	(0.050)
educ	3.154***
	(0.253)

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Marginal effects

Marginal effects and OLS regression

Note how the average marginal effects are equal to the OLS coefficients.

```
library(margins)
me <- margins(m)
summary(me)
```

```
##   factor     AME     SE       z      p  lower   upper
##      age  0.3148 0.0504  6.2424 0.0000 0.2160  0.4137
##     educ  3.1538 0.2534 12.4477 0.0000 2.6572  3.6504
##      sex 11.3172 1.4475  7.8185 0.0000 8.4802 14.1542
```

Marginal effects

Marginal effects with non-linear variables

	(1)
(Intercept)	-77.887***
sex	11.300***
age	2.239***
	-0.021***
educ	3.063***

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Marginal effects

Marginal effects with non-linear variables

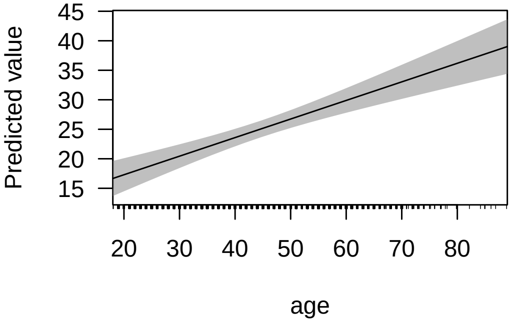
The margins commands are the same as above. Note how the AME now represents the total effect of age across the two parameters. There is no separate marginal effect for age squared.

```
##  factor     AME     SE       z      p  lower   upper
##     age  0.3915 0.0510  7.6752 0.0000 0.2915  0.4914
##    educ  3.0625 0.2499 12.2569 0.0000 2.5728  3.5523
##     sex 11.3001 1.4255  7.9271 0.0000 8.5061 14.0940
```

Marginal effects

Marginal effects with non-linear variables

We can also visualize the marginal effect of age in a continuous space, highlighting how it incorporates the squared term.



Marginal effects

Marginal effects with interactions

	(1)
(Intercept)	-71.606***
sex	-2.586
age	2.211***
	-0.021***
educ	2.780***
sex $\times$ educ	0.513
sex $\times$ age	0.149

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Marginal effects

Marginal effects with interactions

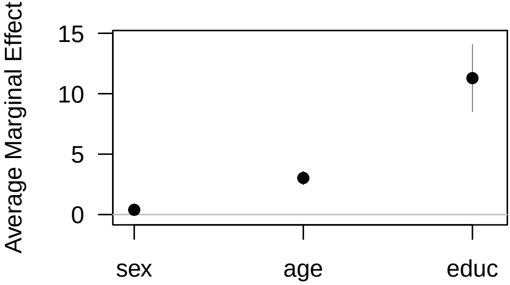
In this case, we can isolate the average marginal effect of each predictor.

```
##  factor     AME     SE       z      p  lower   upper
##     age  0.3913 0.0510  7.6747 0.0000 0.2913  0.4912
##    educ  3.0239 0.2511 12.0412 0.0000 2.5317  3.5161
##     sex 11.2892 1.4244  7.9255 0.0000 8.4974 14.0810
```

Marginal effects

Plotting marginal effects

The `margins` package includes a `plot()` function to show the results of the table. The output can also be modified using `ggplot2`.

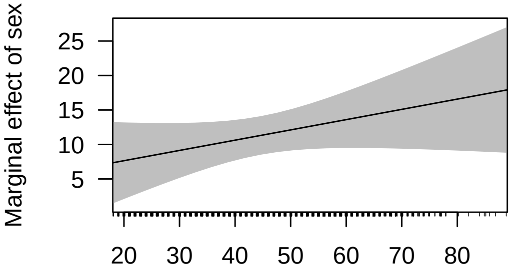


Marginal effects

Plotting conditional marginal effects

The `cplot` function can be used to plot the marginal effect while conditioning on another predictor. In this case, the marginal effect of sex on income over the range of age.

```
cplot(m, x = "age", dx = "sex", what = "effect")
```



Marginal effects

Bayesian estimation

	OLS	Bayesian
sex	-2.586	-1.865
	[-18.771, 13.599]	[-18.286, 13.949]
age	2.211	2.193
	[1.628, 2.795]	[1.649, 2.779]
	-0.021	-0.021
	[-0.027, -0.015]	[-0.027, -0.015]
educ	2.780	2.801
	[2.070, 3.491]	[2.092, 3.496]
sex $\times$ educ	0.513	0.473
	[-0.467, 1.494]	[-0.476, 1.473]
sex $\times$ age	0.149	0.144
	[-0.047, 0.344]	[-0.049, 0.331]

Marginal effects

Bayesian marginal effects

To get *average marginal effects*, we need to compute the expected value of the outcome at different levels of predictors.

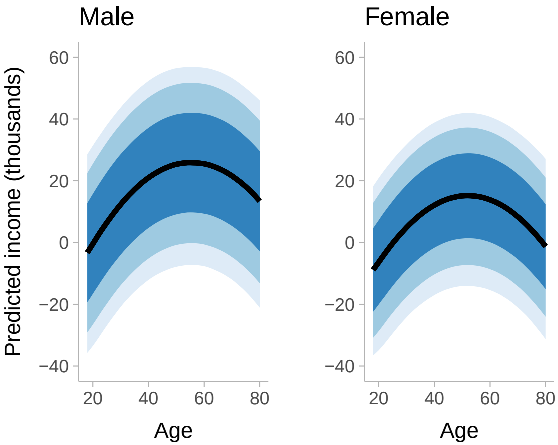
```
library(gridExtra)
library(tidybayes)

data.range <- expand_grid(sex = c(0,1),
                          educ = 1:20,
                          age = 18:80)

tidy_epred <- m.b %>% epred_draws(newdata = data.range)
```

Marginal effects

Bayesian marginal effects



Marginal effects

Marginal effects and generalized linear models

- In generalized linear models (GLMs), which will be out main focus after spring break, the coefficients often do not have clear interpretations on the outcome scale, making marginal effects evne more important for interpretation.[4]

[4] See the recommended reading, Mize 2019, for further discussion.

Next week

Topic

- Missing data
- Model specification, comparison, and robustness

Lab

- Specifying and interpreting interaction terms