CSC 135 Lu

Test 2

Last Name Davis First Name Trevor Grade ______/100

1. (10%) Give a formal definition of Context Free Grammar G = (V, T, S, P). Define each of the 4 major components in a sentence, and form of P in set notation.

A grammar $G=(V,T,S,P)$ is said to be Context-free if all Productions in P have the form $A \to x$, where $A \in V$ and $x \in (V \cup T)$

V is the set of non-terminal symbols

T is a set of terminals and $V \cup T$ is null. They don't appear on left side of Production

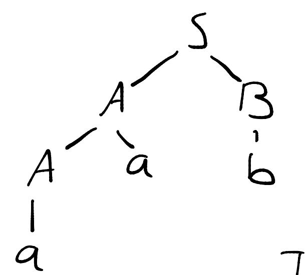
S is the start symbol. It appears in the initial string

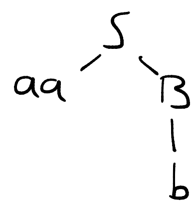
P is the set of rules. They are rules for replacing non terminal symbols.

2. (40 %)
   (a) Show that the following grammar is ambiguous by using parsing trees.

   $S \to AB \mid aaB$
   $A \to a \mid Aa$
   $B \to b$

String : aab





The grammar is ambiguous because we are able to draw 2 different parse trees for the same String

   (b) Find a CFG for the following language: $L = \{a^n b^m : n \neq m\}$

$n \neq m = n > m \cup n < m$

$n > m$
$S_1 \to aS_1b \mid a$
$S_1 \to aS_1$

$n < m$
$S_2 \to aS_2b \mid b$
$S_2 \to S_2b$

$S \to S_1 \mid S_2$
$S_1 \to aS_1b \mid a \mid aS_1$
$S_2 \to aS_2b \mid b \mid S_2b$

3. (40%) Prove the following language is non-regular using Pumping Lemma step by step and lead to a contradiction at the end: $L = \{a^n b^{n+3} : n \geq 0\}$

Assume that L is regular

String: {abbbb, aabbbbb, aaabbbbbb

$w = xy^iz$

Case 1: $y = a^r$

$xyz = \underset{x}{a}\ \underset{y}{aa}\ \underset{z}{bbbbbb}$

$xy^2z = aaaaabbbbbb$ $w \notin L$

case 2: $y = a^r b^m$

$xyz = \underset{x}{aa}\ \underset{y}{abb}\ \underset{z}{bbbb}$

$xy^2z = aa\ abbabb\ bbb$ $w \notin L$

case 3: $y = b^m$

$xyz = \underset{x}{aaa}\ \underset{y}{bbb}\ \underset{z}{bbb}$

$xy^2z = aaa\ bbb\ bbb\ bbb$ $w \notin L$

Hence because of the Pumping lemma, L is not a regular language.

4. (10 %) Prove $L = \{a^n b^k : n \geq 2, k \leq 14\}$ is regular.

This language is regular because there exists a regular expression which defines it as well as an FA

$aaa^*(\lambda + b + bb + \ldots b^{14})$