Computing Theory HW set 2

Context Free Grammar - A grammar G = (V, T, S, P) is said to be context-free if all productions in P have the form "**A→x,**" where $A \in V$ and $x \in (V \cup T)^*$.

Context Free Language - A language L is said to be context-free if and only if there is a context-free grammar G such that L = L (G).

Non-Regular Language - A language that cannot be defined by a regular expression
A NRL can also not be accepted by any Finite Automata or Transition Graph

Pumping Lemma - Let L be an infinite regular language. Then there exists some positive integer m such that any $w \in L$ $|w| \geq m$ can be decomposed as ***W = XYZ*** with $|XY| \leq m$ and $|Y| \geq 1$ such that $W_i = XY^iZ$

Chapter 4

3. Show that the language $L = \{w : n_a(w) = n_b(w)\}$ is not regular. Is L* regular?

Assume L is a regular language
$X = a^{m-j}$
$Y = a^j$ where $J \geq 1$
$Z = b^m$

Assume $i = 2 \rightarrow XY^2Z = a^{m-j} (a^j)^2 b^m$
$= a^{m-j}a^{2j} b^m$
$= a^{m+j} b^m \notin L$ **Therefore L is not regular**

4.a Prove that the following languages are not regular. (a) $L = \{a^n b^l a^k : k \geq n + l\}$.

***Note: 1 vs l (L)**

Assume L is regular
String 1: n = 1, l=2, k ≥ 1+2
abbaaa
String 2: n=1, l = 3, k ≥ 1+3
abbbaaaa

X= aa
Y = bb
Z = aaa

$W = XY^iZ$

W = aa bbbb aaa

This string does not belong to the given language therefore the language is not regular.

(d) $L = \{a^n b^l : n \leq l\}$.

Assume L is regular

String 1: n = 1, l = 2

abb

String 2: n = 2, l=2

aabb

Let:

X = a

Y = b

Z = b

Case 3: $W = XY^iZ$

$= XY^2Z$

W = abbb

This string does not belong to the given language therefore the language is not regular.

15. Consider the languages below. For each, make a conjecture whether or not it is regular. Then prove your conjecture

a) **$L = \{a^n b^l a^k : n + l + k > 5\}$**

L is a regular language

Let n= 1, l = 2, k = 3

abbaaa

- One can easily construct an FA for this
- The arrangement if a & b can be in any order imaginable as long as the sum of n, l, & k is greater than 5.

b) **$L = \{a^n b^l a^k : n > 5, l > 3, k \leq l\}$**

The language is not regular

Assume the L is regular

Let $n = 6, l = 4, k = 3$

aaaaaa bbbb aaa

Let $W=XY^iZ$

X = aaa

Y = aaa bbbb

Z = aaa

$W = XY^2Z$

= aaa aaabbbb aaabbbb aaa

This string does not belong to the given language therefore the language is not regular.

f) $L = \{a^n b^l : n \geq 100, l \leq 100\}$

The language is regular

The language has a regular expression which is $a^{100}a^* (\lambda + b + b^2 + \dots + b^{100})$.

g) $L = \{a^n b^l : |n-l| = 2\}$

Assume L is regular

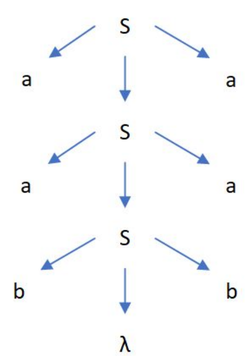
$W = XY^iZ$

<u>Chapter 5</u>

<u>5.1</u>

2. Draw the derivation tree corresponding to the derivation in Example 5.1.

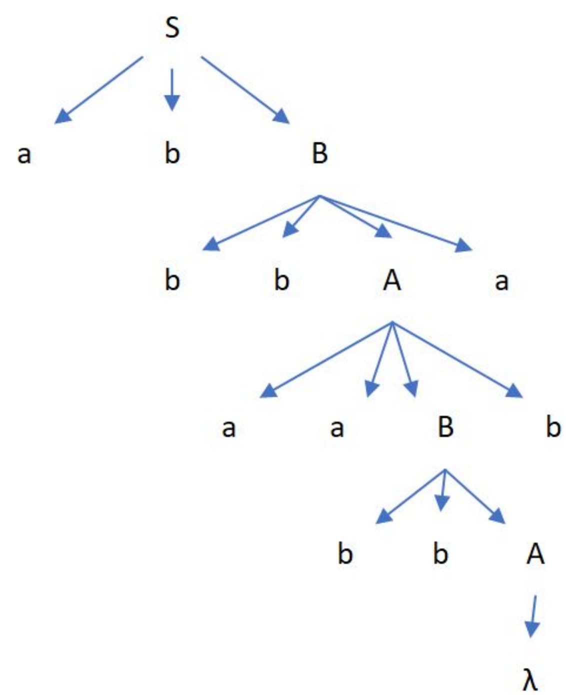
$S \rightarrow aSa \rightarrow aaSaa \rightarrow aabSaa \rightarrow aabbaa$



3. Give a derivation tree for w = abbbaabbaba for the grammar in Example 5.2. Use the derivation tree to find a leftmost derivation.

S → abB
A → aaBb
B → bbAa
A → λ

S → abB
→ abbAa
→ abbaaBba
→ abbbaabbAaba
→ abbbaabbaba



7. Find context-free grammars for the following languages (with n ≥ 0, m ≥ 0).

(a) L = {$a^n b^m$: n ≤ m + 3}.

L = {aaa, abbb, aabbb, …}
S → aSb | X | Y
X → a | aa | aaa | λ
Y → bY | b

(d) L = {$a^n b^m$: 2n ≤ m ≤ 3n}.

L = {aabb, aabbb, aaaabbbb, aaaabbbbb, aaaabbbbbb, …}

$S \rightarrow aSbb \mid aSbbb \mid \lambda$

(f) $L = \{w \in \{a, b\}^* : n_a(v) \geq n_b(v)$, where v is any prefix of w$\}$.

L = {abaabb, aababb, ababab, …}
$S \rightarrow aSb \mid SS \mid \lambda$

8. Find context-free grammars for the following languages (with $n \geq 0$, $m \geq 0$, $k \geq 0$).

(a) $L = \{a^n b^m c^k : n = m \text{ or } m \leq k\}$.

L = {abc, abcc, aabbcc, …}

$S \rightarrow S_1 \mid S_2$

$S_1 \rightarrow AC$
$A \rightarrow aAb \mid \lambda$
$C \rightarrow Cc \mid \lambda$

$S_2 \rightarrow BD$
$B \rightarrow aB \mid \lambda$
$D \rightarrow bDc \mid E$
$E \rightarrow Ec \mid \lambda$

(b) $L = \{a^n b^m c^k : n = m \text{ or } m \neq k\}$.

L = {aabbc, abbc, aaabbc,...}
$S \rightarrow AB \mid CD$
$A \rightarrow aAb \mid \lambda$
$B \rightarrow cB \mid \lambda$
$C \rightarrow aC \mid \lambda$
$D \rightarrow ED_1 \mid D_1F$
$D_1 \rightarrow bD_1c \mid \lambda$
$E \rightarrow bE \mid b$
$F \rightarrow cF \mid c$

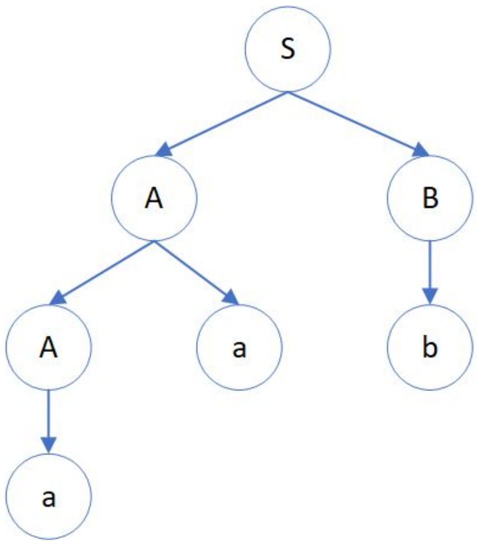
(d) $L = \{a^n b^m c^k : n + 2m = k\}$.

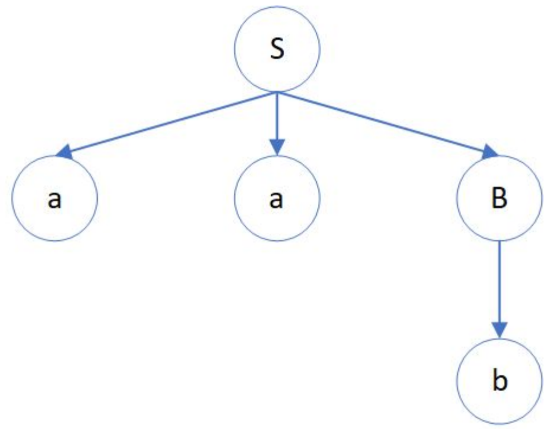
L = {abccc, abbccccc, aabcccc, …}
$S \rightarrow aSc \mid A$
$A \rightarrow bAcc \mid \lambda$

Chapter 5.2

6. Show that the following grammar is ambiguous

$$S \rightarrow AB \mid aaB$$
$$A \rightarrow a \mid Aa$$
$$B \rightarrow b$$





It is possible to derive aab using either $S \rightarrow AB$ or $S \rightarrow aaB$

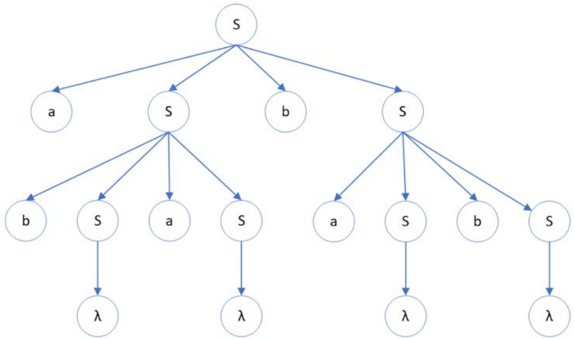
10. Give an unambiguous grammar that generates the set of all regular expressions on $\Sigma = \{a,b\}$.

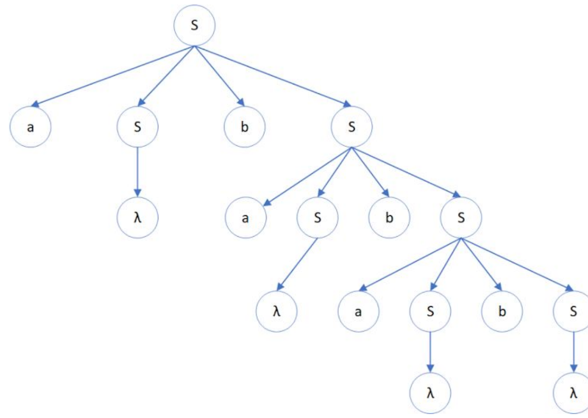
$S \rightarrow aSb \mid \lambda$
$S \rightarrow bSa \mid \lambda$

13. Show that the following grammar is ambiguous.

$S \rightarrow aSbS \mid bSaS \mid \lambda$

Considering the string ababab…





It produces more than one graph to get the same result. Therefore the grammar is ambiguous.