

Test 3

Last Name Davis First Name Trevor Grade /100

1. (20%) Give a description of Chomsky hierarchy of grammar in a table.

type	Language generated	production restrictions $X \rightarrow Y$	accepter
0	unrestricted	unrestricted	Turing machine
1	Context - Sensitive grammar	$\alpha \rightarrow \beta$ where $ \alpha \leq \beta $ $\alpha \in V^* N V^*$ $\beta \in V^*$	Linear bounded Automata
2	Context free grammar	$A \rightarrow \alpha$ $A \in N$ $\alpha \in V^*$	Pushdown automata
3	Regular	$A \rightarrow xB$ $A \rightarrow \epsilon$ $A, B \in N$ $x \in T^*$	Finite automata

2. (30 %) For each of the following languages, **construct a grammar** and **indicate** its Chomsky hierarchy type (s) (pick all integer type [0, 1, 2, 3] that apply)

(a) $L(a^n b^n a^n, n \geq 1)$

(b) $L(\{a^{2^n} b^n, n \geq 0\})$

a) $S \rightarrow aba \mid aAba$
 $Ab \rightarrow bA$
 $Aa \rightarrow Bbaa$
 $bB \rightarrow Bb$
 $aB \rightarrow aa \mid aAa$

This is a context Sensitive grammar. It is a type 1 grammar

b) $S \rightarrow aaSb \mid aab \mid \lambda$

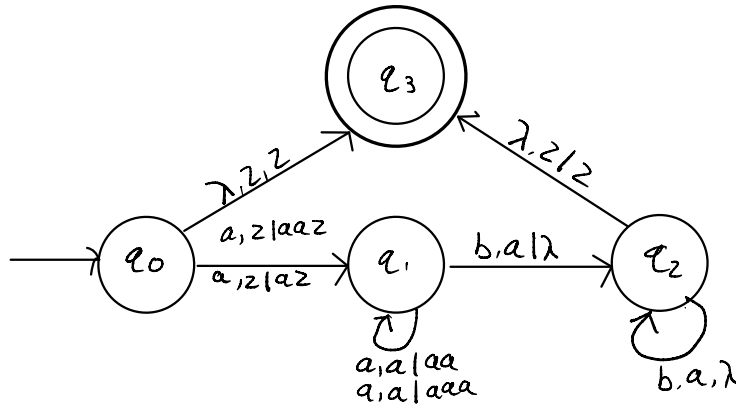
This is a context free grammar and is a type 2 grammar

3. (30%) Construct a PDA that accept the following language on $\{a, b\}$ in two steps

$$L = \{a^n b^m : n \leq m \leq 2n\}$$

(a) Give a brief algorithm design in English.

(b) Give the PDA transition graph based on your algorithm in (a).



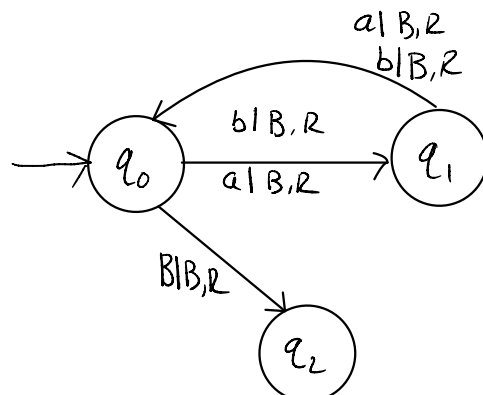
$$\begin{aligned} (q_0, \lambda, z) &= (q_3, z) \\ (q_0, a, z) &= (q_1, aaz) \\ (q_0, a, z) &= (q_1, aaz) \\ (q_1, a, a) &= (q_1, aaaa) \\ (q_1, a, a) &= (q_1, aaaa) \\ (q_1, b, a) &= (q_2, \lambda) \\ (q_2, b, a) &= (q_2, \lambda) \\ (q_2, \lambda, z) &= (q_3, z) \end{aligned}$$

4. (20%) For the following languages on $\{a, b\}$, $L = \{w : |w| \geq 0 \text{ and is multiple of } 2\}$

(a) Write a grammar that generates L ; (b) Construct a TM in TG that will accept L .

$$\begin{aligned} a) \quad S &\rightarrow AA \mid \lambda \\ A &\rightarrow aAa \mid aAb \mid bAa \mid bAb \mid \lambda \end{aligned}$$

$$L = \{\lambda, aa, ab, ba, bb, aabb, aaaa, \dots\}$$



$B = \text{blank symbol}$