Risk Attitude

Readings:

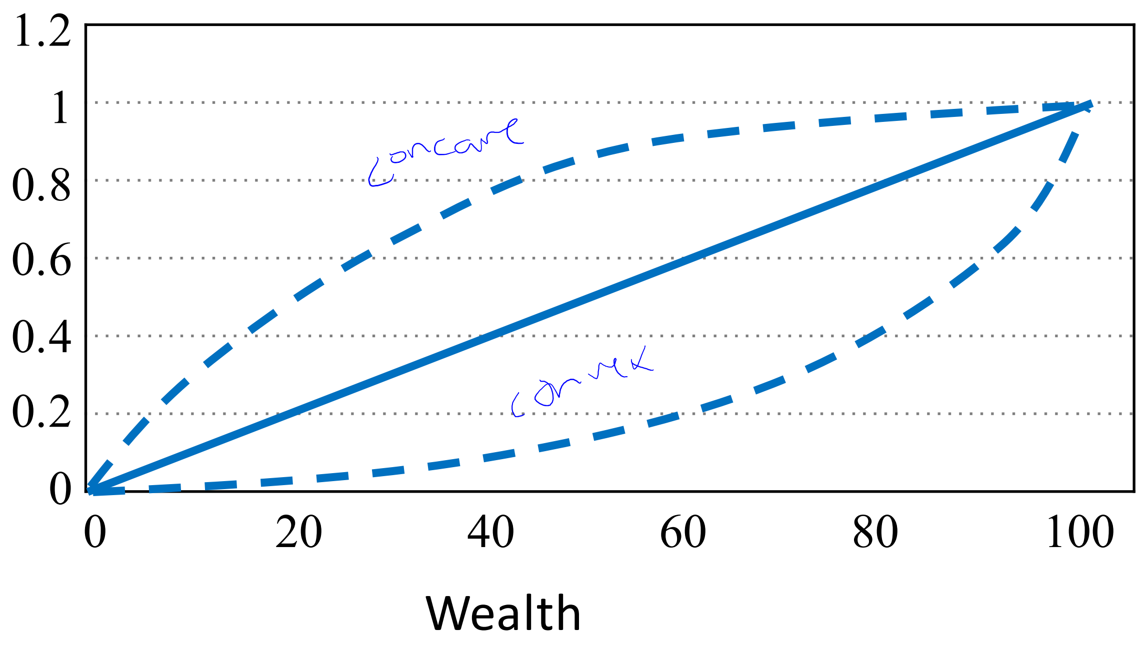
- Making Hard Decisions with decision tools by Clemen and Reilly, Chapter 14, 15

Learning Objectives:

- Be able to explain the concept of risk aversion.
- Be able to explain and use utility mathematical functions.
- Be able to explain risk aversion.
- Be able to explain the difference between constant and variable risk aversion.
- Be able to assess utility functions with relatively simple procedures.

Three types of risk attitude

1. Risk Averse: Utility function is …. concave
2. Risk Neutral: Utility function is ….
3. Risk Seeking: Utility function is …. convex

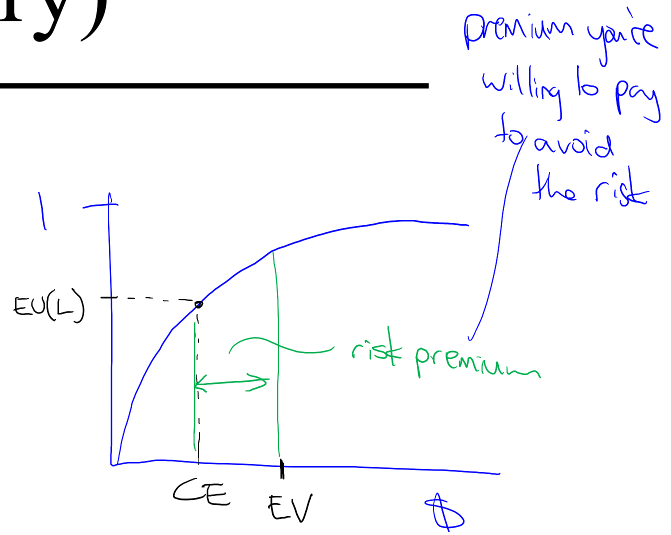


Relation between EU(lottery) and CE(lottery)

$$EU(L) = U(CE)$$

A more formal definition of certain equivalent (CE) is:

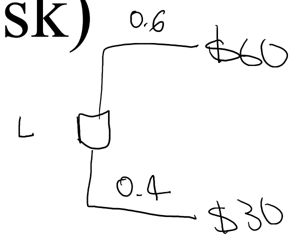
$$CE(L) = U^{-1}(EU(L))$$



EV – CE: Risk premium (the premium you pay to avoid the risk)

If CE < EV (Risk premium > 0) → DM is Risk Averse

If CE > EV (Risk premium < 0) → DM is Risk Seeking



$U(60) = 0.9 \quad U(35) = 0.65$

$EU(L) = 0.6 \times 0.9 + 0.4 \times 0.65 = 0.8$

$EV = 0.6 \times 60 + 0.4 \times 35 = 50$

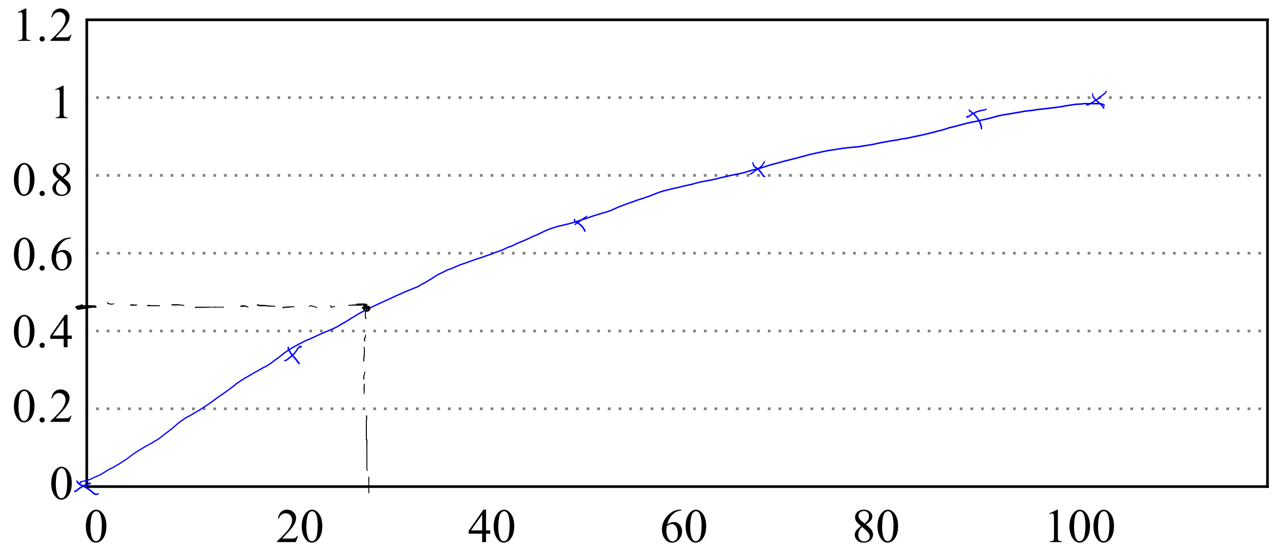
- Most people are:
  - ❑Risk averse
  - ❑Risk neutral
  - ❑Risk seeking
- Characteristics of utility functions:
  1. U(x) is continuous
  2. U(x) is monotonically increasing
  3. U(x) is defined only up to a linear transformation. a+bU(x) is a valid representation of the same preferences as U(x)

Risk attitude

- Let's assess a utility function for the $0-$100 range.

Approximately:

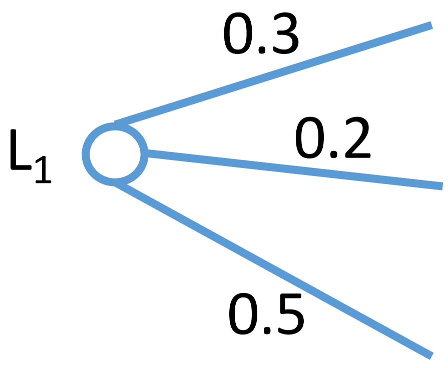
Value	Utility
0	0
20	0.35
50	0.67
70	0.83
80	0.9
90	0.95
100	1



What does this tell us? → risk averse

1. Is u(x) greater than, equal to, or less than x (x is dollar outcome)?
2. Does this relationship hold everywhere?
3. Can you guess at a formula that would fit the shape of this curve pretty well?

Example:



Outcome	Utility
\$80	0.9
\$70	0.83
\$0	0

$EU(L_1) = 0.3 \times 0.9 + 0.2 \times 0.83 = 0.44$

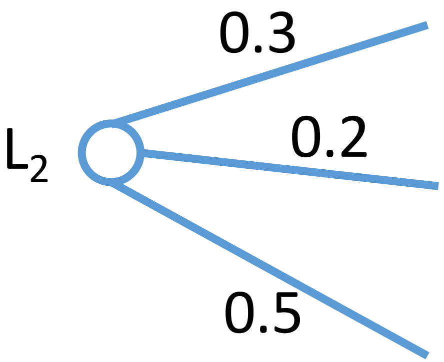
$CE(L_1) = \sim \$28$

$EV(L_1) = 0.3 \times 80 + 0.2 \times 70$

$= 24 + 14 = \$38$

- What does this imply about a risk aversion person's willingness to sell a lottery?

- Now compare lottery 1 to lottery 2 bellow. Which would the decision maker prefer?



Outcome	Utility
\$90	0.95
\$50	0.67
\$20	0.35

$EU(L_2) =$ $0.3\times0.95+0.2\times0.67+0.5\times0.35$ $=$

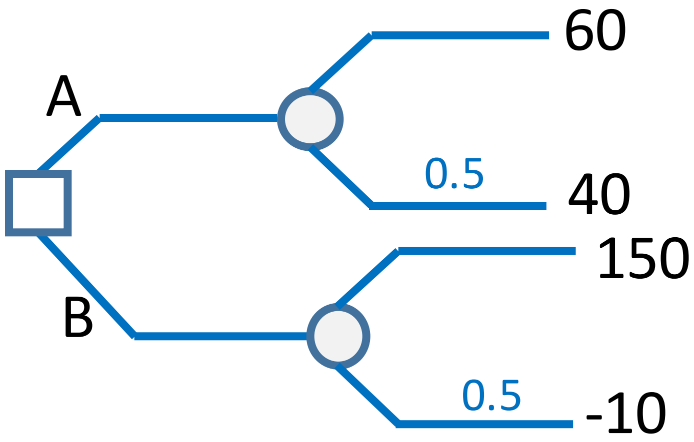
$CE(L_2) =$

$EV(L_2) =$ $0.3\times90+0.2\times50 + 0.5\times20$

If $EU[L_2] > EU[L_1]$ then $CE(L_2) > CE(L_1)$

Example 2

- Let's assume the DM has utility function of $U(x) = \ln(x + 100)$



EU =

CE =

EV=

risk averse

$EU_A = 0.5 \times \ln(60+100) + 0.5 \times \ln(40+100) = 5.008$

$5.008 = \ln(CE_A + 100)$

$e^{5.008} - 100 = CE_A = \cancel{49.6}\ 48.4$

$EU_B = 0.5 \times \ln(250) + 0.5 \ln(90) = 4.998$

$e^{4.9998} - 100 = 48.1 = CE_B$

$EV_A = 0.5 \times 60 + 0.5 \times 40 = 50 \quad EV_B = 70$

The exponential utility function

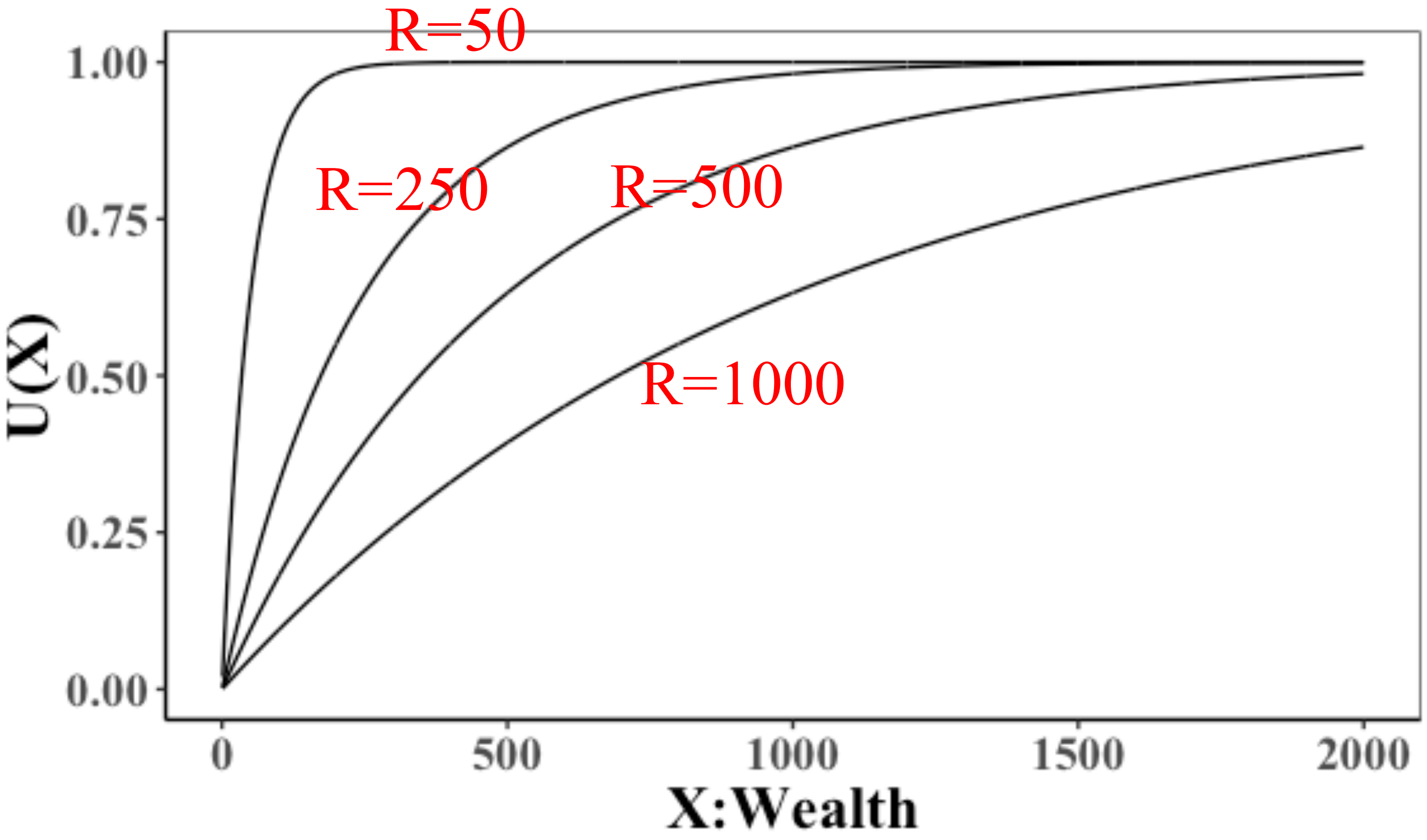
$U(X) = 1 - e^{-\frac{x}{R}}$;   R: risk tolerance

e.g. if indiff at

0.5 $8

0.5 -$4

→ R = 8



Assessing the risk tolerance for exponential utility

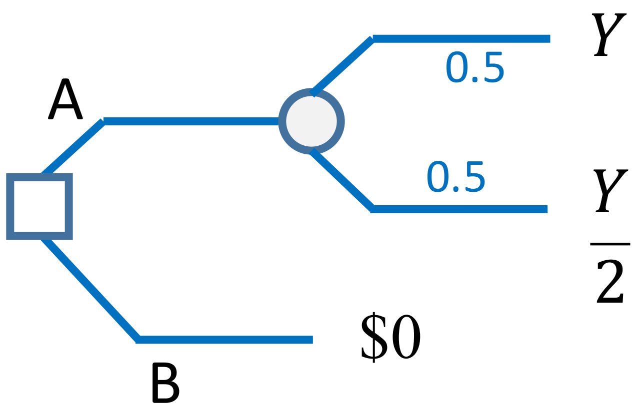
- Consider the following:

Ask the decision maker what value of Y would make him or her indifferent between a lottery with the following two outcomes and receiving $0 for certain:

a) Win $Y with probability 0.5

b) Lose $Y/2 with probability 0.5

The largest Y for which you would prefer the gamble to $0 for certain is approximately equal to your risk tolerance.



In-class Exercise:

- Work in pairs. Assess your partner's risk tolerance. Write it down. Then switch and have your risk tolerance assessed. Write it down.

Exercise:

- Using the exponential utility function once you know your risk tolerance:

  Assume you face the following lottery: $R=20$

  a) Win \$2000 with probability 0.4

  b) Win \$1000 with probability 0.4

  c) Win \$500 with probability 0.2

  How much should you be willing to sell this lottery?

$$EU(L) = 0.4\times\left[\left(1-\exp\left(-\frac{2000}{20}\right)\right)+\left(1-\exp\left(-\frac{1000}{20}\right)\right)\right]$$
$$+0.5\times\left(1-\exp\left(-\frac{500}{20}\right)\right) = \_\_\_\_$$

sell price = CE

$$EU(L) = 1-\exp\left(-\frac{CE_L}{20}\right)$$

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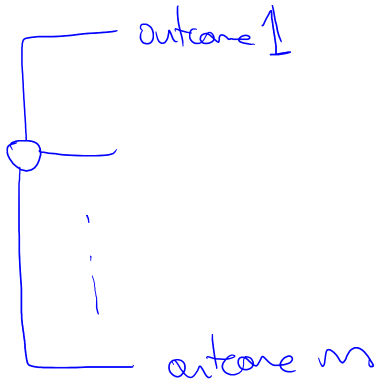
L :



$L = \min(\text{outcomes})$

$M = \max(\text{outcomes})$

We can say that:

$$U(L) < EU(L) < U(M)$$

$$L < CE(L) < M$$

Since $U(x)$ is an increasing then $CE(x)$ is also increasing

Now if $U(x)$ is concave:

$$U(L) < EU(L) < U(EV)$$

$$L < CE(L) < EV(L)$$

Now if $U(x)$ is convex:

$$U(EV) < EU(L) < U(M)$$

$$EV(L) < CE(L) < M$$

Risk aversion coefficient

- Let's $U(X)$ be the utility function, and $U'(X)$ and $U''(X)$ be the first and second derivatives of $U(X)$, respectively.

  Then $R(X) = -\frac{U''(X)}{U'(X)}$ is the **Arrow-Pratt Risk Aversion Coefficient.**

- Monotonically increasing property of $U(X)$ implies $U'(X) > 0$.

  So, $R(X)$ characterizes concavity of the utility function.

- As X increases, R …………… (so risk aversion depends on wealth level with this utility function)

- In General:

  $R(X) > 0$ implies Risk Aversion

  $R(X) < 0$ implies Risk Seeking

U(X) is increasing so first derivative is always +ve. if the 2nd derivative is +ve then it is convex (--> risk seeking) -- has negative risk aversion coefficient

if R(X) is +ve then the person is risk averse for this specific X

if R(x) is decreasing then this means that the person's risk aversion is decreasing in X -- people become less risk averse as they become more wealthy

$U(x)$ is increasing $\rightarrow$ $U'(x) > 0$

$R(x) = -\frac{U''(x)}{U'(x)}$ so if $U''(x) > 0$ then $R(x) < 0$

↳ convex

risk seeking $\rightarrow$ -ve $R(x)$

Example

- $U(X) = ln(X + A)$

- $U'(X) = \frac{1}{X+A}$
- $U''(X) = \frac{-1}{(X+A)^2}$
- $R(X) = -\frac{U''(x)}{U'(x)} = \frac{X+A}{(X+A)^2} = \frac{1}{X+A}$

$\rightarrow$ if $x > 0$ then $R(x) > 0$ $\rightarrow$ risk averse

$\rightarrow$ risk aversion is decreasing in X

Risk premium:
If $R(x)$ is decreasing then the ~~risk averse~~ risk premium is decreasing for a certain bet as the person becomes wealthier

Risk aversion coefficient

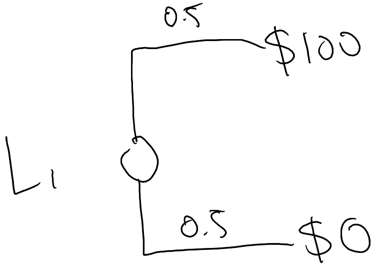
Is X the wealth or the amount that you're gambling?

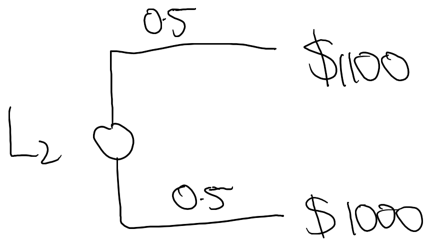
- $R(X)$: Decreasing function of X → Decreasing Risk Aversion

e.g. exponential function

- $R(X)$: Constant function of X → Constant Risk Aversion

Risk premium = the premium you're willing to pay to avoid the risk





If $CE(L_1) = 40$ then $CE(L_2) > \$1040$ if decreasing $R(x)$

$CE(L_1) = 40$ (for ex.)

$EV(L_1) = \$50$

$CE(L_2) =$

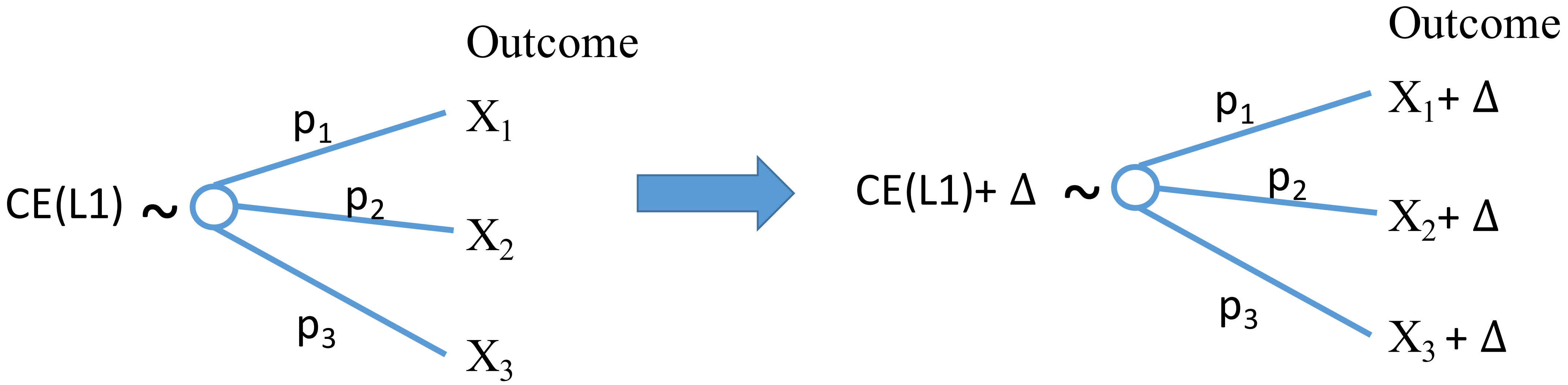
$EV(L_2) = \$1050$

→ $EV - CE < 10$ so $CE > \$1040$ if decreasing $R(x)$

Risk premium $= \$50 - CE_{L_1} = 10$

A special case: Constant Risk Attitude (a.k.a., the Delta Property)

- Consider a case in which adding Δ to each outcome of a lottery increases the CE by Δ. → This is a property of a constant risk aversion



- If this equivalence holds, the decision maker exhibits a constant risk attitude (the delta property)

Consequences of constant risk attitude

1. Risk attitude is same along utility curve (It does not depend on wealth).

2. Break-even payment for a lottery equals the CE

   Buying price = Selling price

   Graphically:

Examples

- Do the following utility functions exhibit constant risk attitude? For what range of parameters do they represent risk aversion? Risk seeking?

a) $U(X) = \ln(x + A)$

$U'(x) = \frac{1}{x+A}$ $U''(x) = \frac{-1}{(x+A)^2}$ $U''(x) < 0$ so $R > 0$

→ risk averse

→ concave

$R(x) = -\frac{\frac{-1}{(x+A)^2}}{\frac{1}{(x+A)}} = \frac{1}{x+A}$ → this is true

it is also decreasing

Examples

$b)\ U(X) = 1 - \exp(-x/R)$

$U'(x) = \frac{1}{R}\exp\left(-\frac{x}{R}\right)$

$U''(x) = \frac{-1}{R^2}\exp\left(-\frac{x}{R}\right)$

$R(x) = -\frac{\frac{-1}{R^2}}{\frac{1}{R}} = \frac{1}{R} \longrightarrow$ positive $\rightarrow$ risk averse

constant

Examples

c) $U(X) = x^{0.5}$ (positive roots only)

$U'(x) = 0.5x^{-0.5}$

$-1.5 - 0.5$

$U''(x) = -0.25x^{-1.5}$

$R(x) = -\dfrac{-0.25x^{-1.5}}{0.5x^{-0.5}} = \dfrac{1}{2x} > 0$ and decreasing

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