

Step-up Gear Train for Electric Wind Turbine

Mechanical Design Project, MECH 325

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1.0 - Project Background and Introduction

Team 9 developed a step-up gear train for a wind turbine which will be used by Casa Congo, an NGO operating a school and a farm in El Astillero, Nicaragua. The turbine is designed to be replicable and have the ability to accept a range of gear ratios based on the generator. Team 9 was presented with a generator, and therefore only developed one unique gear train for this project. Team 9 was responsible for designing and setting specifications for gears, shafts, bearings and other components in order to reach a desired RPM for the turbine.

This project was chosen through a democratic selection process. After explaining and ranking each of the potential projects, such as a mechanical foosball table, a winch for SailBot, and a volleyball pitcher. The wind turbine project was ultimately chosen, firstly because of the scope of the project. As the project was already well-defined, it lent itself well for analysis in the context of the Mech 325 project. Another reason for the choice of this project was because of the potential impact it could have on the community it was helping.

The wind turbine gear train is part of a larger project currently undertaken by UBC Sustainingengineering in partnership with Casa Congo, an NGO operating in El Astillero, Nicaragua. The wind turbine is already being developed by the Sustainingengineering Mechanical Team, and the generator was already chosen. This wind turbine's ultimate purpose is to power an irrigation system currently used by the Casa Congo facility, while maintaining a replicable, repairable and modular design.

In this case, the client would be UBC Sustainingengineering, who is a proxy for Casa Congo. As UBC Sustainingengineering is the organization developing the wind turbine, they are the client for this project.

2 - Purpose of Project

The purpose of this project was to create a gear train which would translate the rotational movement of a wind turbine, increase the RPM, and transmit it to a generator. The gear train will primarily use spur gears, based on the materials available to our client. The client expressed interest in primarily recycling scrap materials from a vehicle scrapyard, where spur gears are highly prominent, given that motorcycles are the most prominent vehicle in El Astillero.

2.1 - Design Requirements

The design requirements as required by the client of this project are outlined as follows:

- It must be within a nacelle or box of size 30*15*15 in
- It must only use spur gears
- Gear train must step up turbine 230 RPM to 1500-2000 RPM at motor
- Design for 5 year life cycle at 12 hours per day 365 days per year

2.2 - Evaluation Criteria

The key evaluation criteria come from a few factors, which mainly stem from the needs of the client.

- The RPM and torque delivered to the motor
- The amount of overhead machining required by the client to fit the gears into the box (30in*15in*15in).
- Rated life of the system
- The complexity of chosen components (OTS components vs custom-made) for our system

2.3 - Key Components to Analyze

There are key components to analyze. Given the needs and the restraints from the client, we were already limited in the scope of the project. We are primarily focused on analysing:

- Spur Gears
- Bearings and bearing Housings
- Shafts
- Couplings
- Retaining Rings
- Keys

3 - Appropriateness of the Detailed Analysis

3.1 - Project Assumptions and Simplifications

There are some assumptions made for this project. First, the shaft analysis for the shaft which is attached to the turbine had already been chosen by the client, and was deemed out of scope.

However, the bearings for this shaft are not, and so, the bearings were specified in this project.

Another thing that was deemed out of scope is the actual housing for the gears, the only focus were the gears, shafts and its accessories. Bolts and other fasteners are also out of scope for this project. Those components will be chosen by the client, as the client is designing the housing.

The braking/stopping system is currently being developed independently by the client, so that is also out of scope for the sake of this project.

3.2 - Inputs for system

For this system, input parameters were provided by the client. In table 1 below, the given inputs for the system are given, and the calculations have stemmed from these input parameters.

Moreover, a targeted output target is also listed given the purpose of this project is for power generation.

Table 1: Inputs for the System

Input	Value
RPM	230 RPM
Torque	12.5 NM
Power	0.4037 HP
Thrust force on Turbine Shaft	10 lbf
Weight of Turbine Blade and Hub	5 lbs
Output	Value
RPM	Between 1500-2000 RPM

3.3 - Safety Considerations

Given that this project is aimed to be used by a community and will be near people, there are a few safety considerations which must be taken into account. The most devastating concern would be if the turbine were to break apart and fling parts of it everywhere. However, given that this project's scope only deals with the gear box itself, one of the larger concerns is that if the gears were to fail, no energy would be generated from the turbine. Another related safety issue, although out of scope, would be the potential for the drive to run at such a speed at which the generator would be unable to keep up, causing fire or other mechanical damage. However, given that the breaking/stopping mechanism is also out of our scope, these are considerations we have noted but have not taken any real action towards at this time.

3.4 Major Assumptions and Simplifications

3.4.1 Gears

The gear material is specified to be 0.2% carbon steel. Research showed that the Brinell hardness for this material ranged from 119 to 235. For analysis, a Brinell hardness of 150 was assumed. The quality number for the gears was assumed to be seven as it fits within most commercial quality gears. The rim thickness factor was assumed to be one because the gears appeared uniform thickness from the Boston gear catalogue. The surface condition factor was assumed to be one as no detrimental surface finish is expected if gears are bought from a manufacturer. For

lubrication, the client will design an oil bath for the intermediate shaft gears to provide adequate lubrication.

3.4.2 - Shaft

The shaft was assumed to be SAE 1020 CD steel due to it being relatively cheap and midrange ultimate tensile and yield strengths. In reality, the shaft could be a different material since the stakeholders may have a different shaft material on hand. The detailed shaft analysis was conducted on the intermediate shaft. Since the motor shaft has less torque and for consistency, it is the same diameter as the intermediate shaft. The turbine shaft calculations are out of scope.

3.4.3 - Intermediate Shaft Bearings

To specify the deep groove ball bearings fitted for the intermediate shaft, axial loading was assumed to be negligible. This was assumed due to spur gears applying only radial and tangential force onto the shaft. This simplifies the uncertainty of including axial forces in bearing load calculations, and allows for locking methods such as set screws (included on SKF's bearing catalogue selection) to be used for securing the shaft.

3.4.4 - Tapered Roller Bearings

In analyzing the tapered roller bearings, the Weibull model, as used by Timken, was chosen. This model includes: $x_o = 0$; $\theta = 4.48$ and $b = 3/2$. It was assumed that the only thrust force to be carried by the bearings is an external force of 10 lb exerted by the wind onto the turbine shaft (this value was provided to us by the client). A life of 300 million cycles is desired, this yields a desired life of 21739 hours. The diameter of the turbine shaft was assumed to be 1in, as provided by the client. In addition, the application factor was assumed to be 1.4 for light impact.

3.4.5 - Motor Coupled Shaft Bearings

In analyzing the reaction forces necessary to specify bearings for the motor coupled shaft, the end of the shaft connected to the coupling is assumed to have negligible radial forces. Therefore, the only radial forces contributing to the shaft analysis are the reaction forces at both bearings, and the gear connected near the end of the shaft. In addition, axial loading was assumed to be negligible. This was assumed due to the spur gear applying only radial and tangential force onto the shaft. This simplifies the uncertainty of including axial forces in bearing load calculations, and allows for locking methods such as set screws (included on SKF's bearing catalogue selection) to be used for securing the shaft.

3.4.6 - Retaining Rings

To keep the gears on the shaft, a shoulder will be created on one side of the gear, and a retaining ring will be used on the other. Since spur gears do not produce axial thrust forces, it is assumed

that the thrust force applied on the retaining rings will be negligible. Therefore when analyzing the retaining rings, any thrust load capacity should be sufficient.

3.4.7 - Couplings

The largest assumption which was made for this part was that it would have a key for the gear shaft, and not use a keyless coupling on that end. All torques and RPMs were taken directly from other analyses.

3.4.8 - Keys

For the keys, a few assumptions were made. First the design factor chosen was $N=3$, as outlined from Mott's Machine Elements in Mechanical Design. The material chosen was 1018 Carbon Steel, also based from Mott's Machine Elements in Mechanical Design. All torques and diameters for the gear shafts were taken from previous sections.

3.5 Analysis Factors

3.5.1 Gears

Gears were analyzed using the roadmap described by the textbook, Shigley's Mechanical Engineering Design 9th edition, which is based on AGMA standards (ANSI/AGMA 2001-D04). Analysis factors used are listed below:

- Overload factor
- Dynamic factor
- Size factor
- Load-distribution factor
- Rim thickness factor
- Geometry factor
- Quality factor
- Reliability factor
- Surface condition factor
- Stress cycle factor
- Hardness ratio factor
- Temperature factor

3.5.2 - Shafts

The intermediate shaft was analyzed using the same textbook as mentioned in the previous section. Analysis factors used are listed below:

- Surface condition modification factor
- Size modification factor

- Load modification factor
- Temperature modification factor
- Reliability factor
- Miscellaneous modification factor
- Notch sensitivity
- Shear notch sensitivity
- Bending stress concentration factor
- Torsional stress concentration factor
- Design factor

3.5.3 - Bearings

There were various types of bearings analyzed, along with their housing components. The pillow block and flange gears were analyzed using the process outlined in Machine Elements in Mechanical Design (Mott, 6th edition). The tapered roller bearings were analyzed using the method outlined in Shigley's Mechanical Engineering Design (Budynas & Nisbett, 10th Edition). The factors used in our analyses are listed below:

- Reliability factor
- Life adjustment factor for reliability
- Dynamic load rating
- Application factor
- Bearing type factor (ball, roller)
- Weibull distribution parameters

3.5.4 - Retaining Rings

The retaining rings were designed with the assumption that the axial thrust on the gears are negligible. Due to this assumption, there were no analysis factors used when analyzing the retaining rings.

3.5.5 - Couplings

The single coupling was analyzed using the procedure outlined by LoveJoy in their Power Transmission Products 2016 Catalogue. Analysis factors used are listed below:

- Design Factor

3.5.6 - Keys

The keys were analyzed by combining methods from Mott's Machine Elements in Mechanical Design and the Boston Gear Catalogue. Analysis factors used are listed below:

- Design Factor

4.0 - Final Component Specifications

The final design of the compound reverted gear train is displayed in figure 1. With the associated specs of the components below in Table 2.

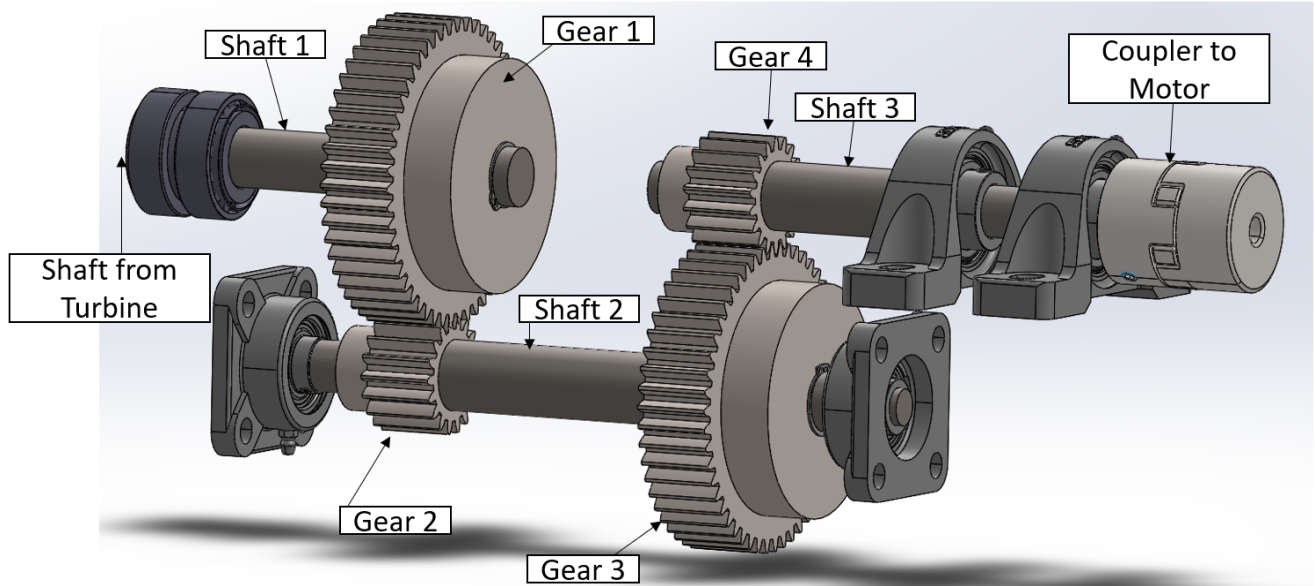


Figure 1: Completed and Labeled Gear Train Assembly for Wind Turbine

Table 2: Final Assembly Input and Output Specifications

Input Parameters from client	Value
Horsepower in from turbine	0.4 Horsepower
Axial Load from turbine	10 Pounds
Average Turbine Speed	230 Rpm
Turbine Life Cycle	5 Years at 12 hours per day
Output Parameters	Value
Final Gear Ratio	7.72
Final Output Speed	1775 Rpm
Diametral Pitch	10 teeth/inch
Gear 1 & 3 Teeth	50 Teeth
Gear 2 & 4 Teeth	18 Teeth
Shaft Center to Center Distance	3.4 Inches

4.1 - Gears

Product info for the gears were obtained from the Boston Gear Catalogue. Gear 1 and 3 are 50 tooth steel spur gears with a diametral pitch of 10 and a total length of 2.13 in per gear. Gear 2 and 4 are 18 tooth steel spur gears with the same diametral pitch and a total length of 1.87 in per gear. All gears have a face width of 1.25 in and a 20° pressure angle. The catalogue number for the larger gears is Y550A and YF18 for the smaller gears. Bores are dimensioned to fit the shafts. Stresses on the gears are shown in Table 4. Full analysis is shown in [Appendix A](#).

4.1.1 - Safety Factors

Factors of safety were based on a lifetime of 20 years at 12 hour days. A summary of safety factors for each gear is listed in Table 3.

Table 3: Gear Safety Factors

Gear #	Bending Stress Safety Factor	Contact Stress Safety Factor
1	12.06	1.71
2	8.88	1.64
3	29.44	2.65
4	21.68	2.55

4.1.2 - Gear Stress

Stress analysis showed that the limiting factor for the gears is through contact stress. Bending stress was not deemed to be an issue. The gears will have a lifetime of 10^8 cycles with a safety factor greater than one. It is expected that the order of lifetime between the gears will be Gear 2, Gear 1, Gear 4, Gear 3.

Table 4: Gear Stresses and Safety Factors

Gear	Bending Stress (psi)	Safety Factor	Contact Stress (psi)	Safety Factor
1	1763.85	12.06	41 890.75	1.71
2	2317.58	8.88	42 611.79	1.64
3	699.97	29.44	26 370.48	2.65

4	918.40	21.68	26 824.37	2.55
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See Section 6.1 [CAD Renders and Drawings](#) for the 2D shaft drawings. For load, shear, and bending moment diagrams of the intermediate shaft, see [Appendix B: Shaft Analysis](#). As described in Table 4, the limiting safety factor is 3.7 at the shoulder of Gear 2.

Numerical specifications of the intermediate shaft (Shaft 2):

- Diameter for bearings: 0.625"
- Diameter for Gear 2: 0.875"
- Diameter for Gear 3: 1"
- Diameter between Gears 2 and 3: 1.2"
- Center distance between Gears 2 and 3: 6"
- Center distance between Gear 2 to Bearing A and Gear 3 to Bearing B: 2.5"
- Maximum torque: 79 lb-in
- Rotational speed: 639 rpm
- Maximum bending moment: 178 lb-in (center of Gear 2)
- Max shear: 65 lb (between Bearing A and Gear 2)

Numerical specifications of the turbine shaft (Shaft 1 also out of scope):

- Rotational speed: 230 rpm
- Maximum torque: 110.6 lb-in

Numerical specifications of the motor shaft (Shaft 3):

- Rotational speed: 1775 rpm
- Maximum torque: 14.3 lb-in
- Diameter for Gear 4: 0.875"
- Pillow Block Center-to-center distance: 2.5"
- Center distance between Gear 4 and first pillow block bearing: 4"

4.2 - Shafts

Shaft person please add here the final chosen component and where it came from

4.2.1 - Shaft Safety Factors

The points along the shaft with the highest expected stress concentrations were identified from the torque and bending moment diagrams in [Appendix B: Shaft Analysis](#). Table 5 shows the safety factors of the points of interest. From the analysis the point with the smallest resultant safety factor was the shoulder of Gear 2 at 3.7.

Table 5: Shaft Safety Factors

Safety Factor Type	Value
Design, n_d	2.5
Gear 2 Shoulder	3.7
Gear 2 Key	4.0 (near shoulder), 6.0 (opposite to shoulder)
Gear 2 Retaining Ring Groove	4.0
Bearing “A” Shoulder	5.6

4.3 - Bearings

4.3.1 - Safety Factors

An application factor of 1.4 was used in analyzing the tapered roller bearings. Although the turbine shaft will likely not encounter impacts requiring a factor of 1.4, this value was chosen to allow for additional safety. An application factor of 1.4 is also chosen for the bearings on the intermediate and motor-coupled shafts, for the same reason.

4.3.2 - Flanged Housing Ball Bearings for Intermediate Shaft (Shaft 2)

The bearings chosen for the intermediate shaft will be F4B 010-TF bearings with 4-bolt flange housing, with a $\frac{5}{8}$ ” shaft diameter. Refer to [Appendix C](#) for the details and bearing analysis. Both ends of the intermediate shaft contain the same flanged ball bearings to satisfy our evaluation criteria of reducing the number of unique components, and were calculated using the higher radial load of the two determined reaction forces.

Other features of these ball bearings include:

- A wide inner ring, to minimize shaft misalignment due to the load being distributed over a greater shaft area
- Set screws for securing the shaft axially
- Flingers to splash away foreign material (oil, dirt, rain) to provide clean operation

Numerical Specifications

- Dynamic capacity = 2150 lbf
- Mass = 1.0 lb
- Largest dimension sizes = 3” x 3” x 1.283”

- [Price \(source\)](#): \$75.88 each

Results of Calculations

- Application factor: 1.4 for machinery with light impact
- Minimum dynamic capacity = 1550.42 lbf
- Design radial load = 71.15 lbf
- Resulting Reliability = 0.99
- Shaft nominal diameter = 0.625 in
- Rotational Speed = 639 rpm
- Shaft tolerance class = js5

4.3.3 - Tapered Roller Bearings for Turbine Shaft (Shaft 1)

The tapered roller bearings selected for the turbine shaft are Timken Single-Row Type TS with a bore diameter of 1" (Inner Part No. 1780, Outer Part No. 1730). The analysis and calculations involved in selecting this bearing can be found in [Appendix C](#). Based on our analysis, the combined reliability for the bearing pair is 0.992, which satisfies our 0.99 goal.

The bearings are secured by the gearbox housing, with a flange for the inner bearing.

Numerical Specifications

- Bore diameter = 1"
- Outer diameter = 2.25"
- Width = 0.6875"
- Dynamic load rating = 3180 lbf
- $K = 1.69$
- Weight = 0.5 lb
- Price = \$46.50 ([Source](#))

Calculation Results

- Dynamic equivalent load, $F_{eA} = 756.06$ lbf
- Dynamic equivalent load, $F_{eB} = 871.27$ lbf
- Dimensionless design life = 3.33
- Minimum dynamic capacity, Bearing A = 2256.3 lbf
- Minimum dynamic capacity, Bearing B = 2592.9 lbf
- Combined reliability = 0.992

4.3.4 - Pillow Block Housing Ball Bearings for Motor Shaft (Shaft 3)

The bearings chosen for the motor shaft will be P2B 014-RM bearings with pillow block housing, with a $\frac{7}{8}$ " shaft diameter. Refer to [Appendix C](#) for the details and bearing analysis.

Other features of these ball bearings include:

- A narrow inner ring, to account for the limited space between the gear and bearing
- Set screws for securing the shaft axially

Numerical Specifications

- Dynamic capacity = 3150 lbf
- Mass = 1.7 lb
- Largest dimension sizes = 5 1/2" x 1 1/2" x 2.781"
- [Price \(source\)](#): \$56.61 each

Results of Calculations

- Application factor: 1.4 for machinery with light impact
- Minimum dynamic capacity = 1355.46 lbf
- Design radial load = 44.25 lbf
- Resulting Reliability = 0.99
- Shaft nominal diameter = 0.875 in
- Rotational Speed = 1775 rpm
- Shaft tolerance class = j6

4.4 Retaining Rings

4.4.1 - Safety Factors

There were no safety factors directly related to retaining ring selection, but was dependent on the shaft diameter which had multiple factors as shown in [Section 4.2](#).

4.4.2 - Retaining Ring Selection

The shaft diameter for the gears will be 0.875 and 1 inch for the smaller(gear 2 and 4) and larger(gear 1 and 3) gear respectively. Two different 15-7 stainless steel retaining rings have been specified to be used in the design. For the 0.875 inch shaft diameter, a 0.821 inch diameter groove should be made, with a width of 0.046 inches to hold a 0.81 inch ID retaining ring. For the 1.000 inch shaft diameter, a 0.94 inch diameter groove should be made, with a width of 0.046 inches to hold a 0.925 inch ID retaining ring. The maximum thrust capacity for each ring is 4,360 and 5,020 lbs respectively. There will be five retaining rings in total.

4.5 - Couplings

4.5.1 - Safety Factors

While there are no safety factors directly related with these couplings, a design factor was added, as mentioned earlier in the previous section, Based on the LoveJoy Power Transmission Products 2016 document, the design factor was 1.

4.5.2 - Coupling Selection

The coupling chosen will be bored by ourselves, choosing a Lovejoy Jaw Type Coupling: SOX Rubber 11070-35747 and 11070-41911. This is based on the LoveJoy Catalogue provided by the MECH 325 teaching team. The sizes were chosen in order to conform to the gear shaft, with a key, and a keyless motor shaft. Another factor which was chosen was the elastomer material in order to accommodate for the temperatures and type of load that the system undergoes.

4.6 - Keys

4.6.1 - Safety Factors

There are no safety factors directly related to keys, a design factor was added, as mentioned earlier in the previous section, this was chosen to be 3. There was also an adjustment to the length of our key, by extending it beyond LMin, which can contribute towards increasing the safety of the key.

4.6.2 - Key Selection

There were keys chosen for the gears and for the coupling. The following table shows the keys chosen for each component.

Table 6: Key Selections

Component	Key (SAE 1018 Carbon Steel) (W*H*L) in ³
Gear One	0.25*0.25*1.75
Gear Two	0.25*0.25*1.5
Gear Three	0.25*0.25*1.75
Gear Four	0.25*0.25*1.5
Coupling	0.125*0.125*0.25

The gear couplings were based on the procedure outlined in Section 11 of Mott's Machine Elements in Mechanical Design. The key for the coupling was based out of the LoveJoy Catalogue provided by the MECH 325 teaching team. The lengths for all of these keys were extended beyond L Min in order to ensure that these keys would have a longer life before failure, as well as act as a form of safety for the system.

5.0 - Conclusion and Recommendation

Following all of the analyses performed for this project, we are confident in the components we have specified, given our consideration of safety factors to ensure the security of the device. Each section of the components has written a brief explanation outlining specifically the confidence in the analysis as well as any recommendations for that specific part.

Team 9 recommends that the design system be implemented into the client's workpiece. Given the work and contingencies Team 9 worked with to reduce complexity of the design and maintain a relatively reasonable safety factor for the design. A prototype based on these catalogue parts should be built, before attempting to build a chassis based on client-provided parts.

For further implementation, FEA analysis should be performed, as well as collecting more robust testing data from the client to ensure that the components are able to withstand some extreme forces before total failure. We also recommend further analysis into lubrication methods, in the system as a whole.

5.3.1 - Gears

Following the gear stress analysis, it is concluded that the specified gears outlined in section 4.1 confidently meets our design requirements and evaluation criteria. Analysing all gears at a power input of 0.4 hp with a lifetime of 5 years (12 hours/day) resulted in a minimum factor of safety of 1.64 for Gear 2. No further analysis is required but because the keyways for all the gears are not provided by the manufacturer, we recommend contacting a machinist to complete the gears.

5.3.2 - Shaft

Based on the analysis of the intermediate shaft, the chosen shaft dimensions are sufficient for the design requirements. The shaft diameter was sized to be larger than the minimum shaft diameter from the conservative DE-Goodman criterion. Furthermore, points of the highest stress concentrations were analyzed such as retaining rings, shoulders, and keyways and their respective safety factors calculated. The point on the shaft with the lowest safety factor was the

shoulder of Gear 2 of 3.7 which is reasonable. To verify these calculations, a prototype model should be assembled and extensively tested.

5.3.3 - Flanged Housing Ball Bearings for Intermediate Shaft

Following the flanged bearing analysis, it is concluded that the specified SKF bearings outlined in [Section 4.3.2](#) confidently meets our design requirements and evaluation criteria. Using the higher radial load of both bearings on the intermediate shaft, and a rated lifetime of 5 years (12 hours/day), the chosen bearing meets the minimum dynamic load capacity for 99% reliability by a factor of 1.39. As the catalogue already provided details on mounting and locking mechanisms for restraining axial movement, no shoulder diameter information was provided. We recommend building a physical prototype to test the assumed-negligible axial forces on the intermediate shaft, and doing further analysis on whether the integrated set screws will prevent the axial forces.

5.3.4 - Tapered Roller Bearings for Turbine Shaft

Following the tapered roller bearings analysis, it can be seen that the selected Timken tapered roller bearings, outlined in Section 4.3.3, satisfy our design requirements and evaluation criteria. Although the combined reliability for the two bearings is 99.2%, just exceeding our 99% desired reliability. In addition to this, the analysis was done with additional safety considerations in mind. For example, the turbine blades will likely not encounter impacts that require an application factor as large as 1.4. Furthermore, the thrust force due to the wind was taken based on worst-case data. The shaft will most likely experience smaller thrust forces than the 10lbf used in our bearing analysis. With these factors in mind, we are confident in our selected bearings. Further detailed design is required to determine how the bearings will be secured in the gearbox housing; this is highly dependent on the client and what material they select for the housing.

5.3.5 - Pillow Block Housing Ball Bearings for Motor Shaft

Following the pillow block bearing analysis, it is concluded that the specified SKF bearings outlined in [Section 5.3.4](#) confidently meets our design requirements and evaluation criteria. Using the higher radial load of both bearings on the motor-coupled shaft, and a rated lifetime of 5 years (12 hours/day), the chosen bearing meets the minimum dynamic load capacity for 99% reliability by a factor of 2.32. Similar to the flanged bearings, the same SKF catalogue was used to select appropriate pillow block bearings, which already included a set screw solution to constrain the shaft axially. We recommend building a physical prototype to test the assumed-negligible axial forces on the motor-coupled shaft, as well as determining the radial forces applied at the coupling end. This can be used to reduce the uncertainties of the bearing reaction forces, and analyze whether the integrated set screws will prevent axial forces.

5.3.6 - Retaining Rings

Following the analysis of the retaining rings, it is concluded that the specified retaining rings should meet the design requirements and evaluation criteria. The analysis of the spur gears showed that there should be negligible thrust on the gears due to them being spur gears, therefore a maximum thrust capacity of 4360 lbs and 5020 lbs should be more than sufficient. Aside from editing the shaft to accommodate for the necessary grooves, the retaining rings can be implemented as designed.

5.3.7 - Coupling

Based on the analysis of the coupling, it's concluded that the coupling chosen is sufficient for the mechanism it is attached to. Although there it is necessary for the actual component to be tested, using the design factor has already shown that the coupling's rated RPM and Torque are higher than the estimated load, which can be referred to in [Appendix D2](#). Further work would include testing to see if the coupling will be able to withstand and function in the designed capacity.

5.3.8 - Keys

Based on the analyses of the keys seen in [Appendix D3](#), and from there we are able to see that the keys will be sufficient given that they have exceeded the L_{min} , using a factor of safety of three. Given the extension of the key lengths, this applies some sort of safety factor, lending confidence to our analysis. Moreover, the material used for keys is stronger than that of the gears, alleviating some fears with regards to material failure. Finally, for further work, the keys should be machined and tested in a prototype setting.

6.0 - Other Considerations

6.1 CAD Renders and Drawings

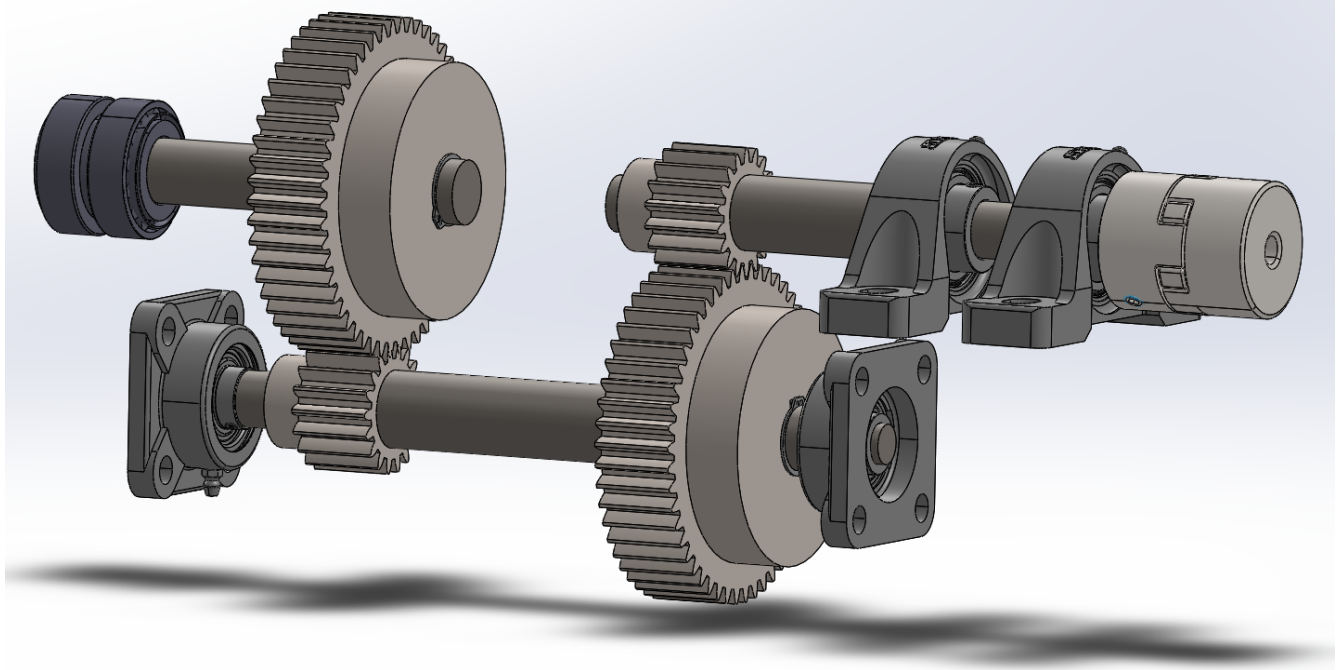


Figure 2: Completed Assembly Render of Gear Train

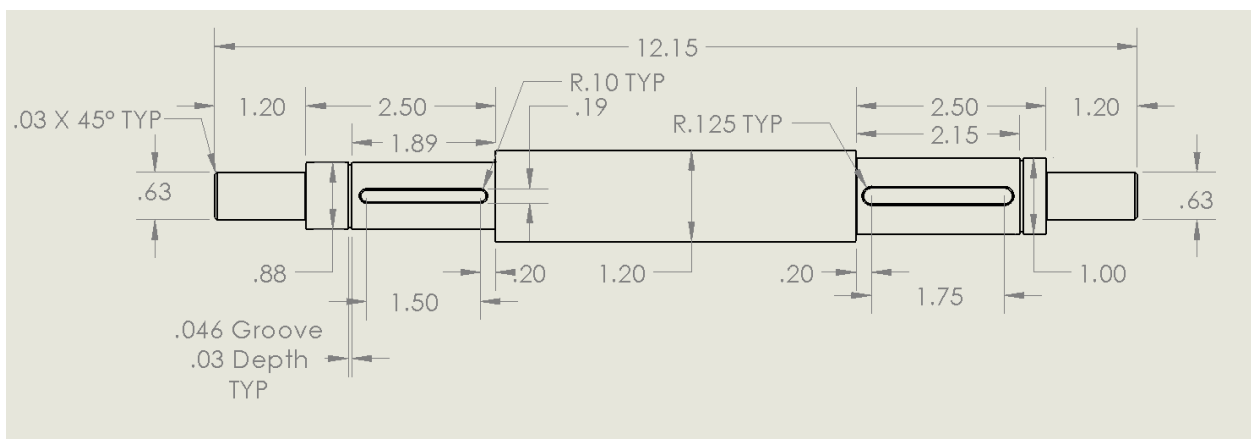


Figure 3: Intermediate Shaft, Shaft Two Engineering Drawing in Inches

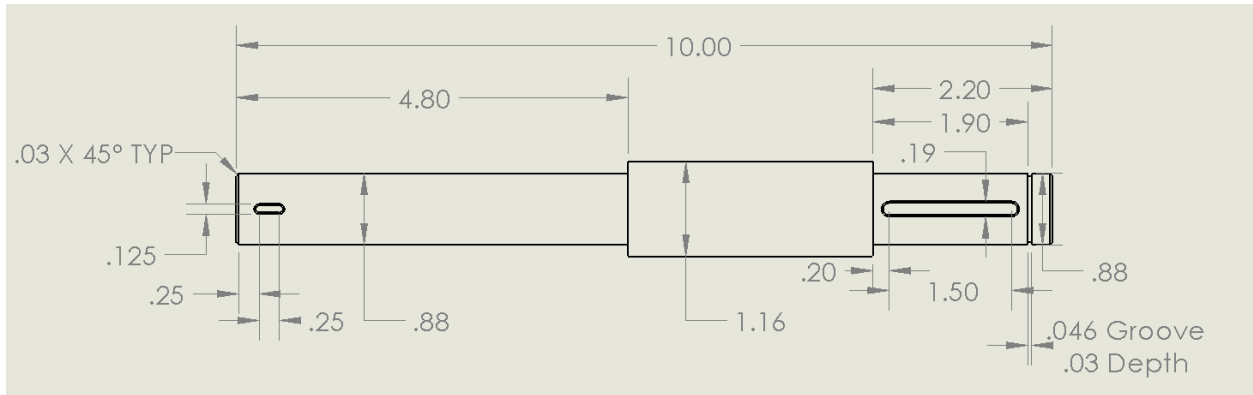


Figure 4: Shaft to Motor, Shaft Three engineering drawing in Inches

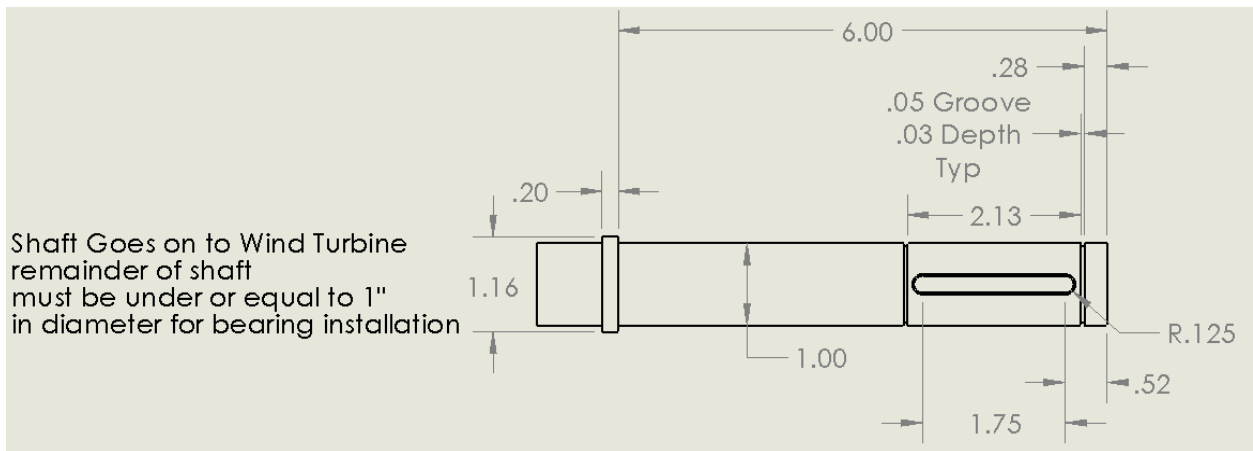


Figure 5: Gear Train End of Turbine Shaft, Shaft One Engineering Drawing in Inches

Appendix A: Gear Analysis

Tables and figures are in reference to Shigley's Mechanical Engineering Design 9th edition unless otherwise specified.

A list of variables is shown below:

Table A1: Gear Analysis Variables

Allowable bending stress number	S_t	Quality factor	Q
Allowable contact stress number	S_c	Reliability factor	K_R
Diametral pitch	P	Rim thickness factor	K_B
Dynamic factor	K_v	Rotational speed (rpm)	n
Elastic coefficient	C_p	Size factor	K_s
Face width	F	Stress cycle factor	Y_N
Geometry factor	J	Stress cycle factor	Z_N
Geometry factor	I	Surface condition factor	C_f
Hardness ratio factor	C_H	Temperature factor	K_T
Load-distribution factor	K_m	Torque	T
Overload factor	K_o	Transmitted load	W_t
Pinion pitch diameter	d	Pitch line velocity	V

Initial gear sizing estimates were based on the requirement of a two-step speed reduction from 230 rpm to ~1800 rpm. Gear pairs with one sized 50 teeth, and the other sized 18 teeth, satisfied this condition. The torque input is 110.7 lb*in. Therefore, 0.40 hp is transmitted through the system.

Gear 1

$$d = N/P = 50/10 = 5 \text{ in}$$

$$V = \frac{\pi d n}{12} = \frac{\pi(5)230}{12} = 301 \text{ fpm}$$

$$W_t = 33000 H/V = 2(0.40)/301 = 44.26 \text{ lb}$$

Assuming light shock power source + light shock driver, $K_o = 1$ according to the textbook, Machine Elements in Mechanical Design 6th edition Table 9-1. To evaluate K_v from Eq. (14-28) with a quality number $Q = 7$,

$$B = 0.25(12 - Q)^{2/3} = 0.25(12 - 7)^{2/3} = 0.73$$

$$A = 50 + 56(1 - B) = 50 + 56(1 - 0.73) = 65.06$$

Then from Eq. (14-27) the dynamic factor is

$$K_v = \left(\frac{A + \sqrt{V}}{A}\right)^B = \left(\frac{65.06 + \sqrt{301}}{65.06}\right)^{0.73} = 1.19$$

To determine the size factor, K_s , the Lewis form factor is needed. From Table 14-2, with $N = 50$ teeth, $Y = 0.409$. Thus from Eq. (a) of Sec. 14-10, with $F = 1.25$ in,

$$K_s = 1.192 \left(\frac{F\sqrt{Y}}{P}\right)^{0.0535} = 1.192 \left(\frac{1.25\sqrt{0.409}}{10}\right)^{0.0535} = 1.04$$

The load distribution factor, K_m , is determined from Eq. (14-30), where five terms are needed. They are, with $F = 1.25$ in,

Uncrowned, Eq. (14-30): $C_{mc} = 1$,

Eq. (14-32):

$$C_{pf} = \frac{F}{10d} - 0.0375 + 0.0125F = \frac{1.25}{10(5)} - 0.0375 + 0.0125(1.25) = 0.00313$$

,

Straddle-mounted pinion with bias to one side, Eq. (14-33): $C_{pm} = 1.1$,

Commercial enclosed gear units (Fig. 14-11): $C_{ma} = 0.147$,

Eq. (14-35): $C_e = 1$

Thus,

$$K_m = 1 + C_{mc}(C_{pf}C_{pm} + C_{ma}C_e) = 1 + (1)[0.00313(1.1) + 0.147(1)] = 1.15$$

Assuming constant thickness gears, the rim thickness factor $K_B = 1$.

The stress cycle factor with 3×10^8 cycles is,

$$Y_N = 1.6831N^{-0.0323} = 1.6831(3 * 10^8)^{-0.0323} = 0.896$$

From Table 14.10, with a reliability of 0.99, $K_R = 1$. From Fig. 14-18, the temperature and surface condition factors are $K_T = 1$ and $C_f = 1$.

The geometry factor is given by Fig. 14-6 so $J = 0.4$. The geometry factor for contact stress with $m_n = 1$ for external spur gears from Eq. (14-32), a gear ratio $m_G = 2.78$, and a pressure angle $\phi = 20^\circ$ is,

$$I = \frac{\cos(\phi)\sin(\phi)}{2m_n} \left(\frac{m_G}{m_G + 1} \right) = \frac{\cos(20^\circ)\sin(20^\circ)}{2(1)} \left(\frac{(2.78)}{(2.78)+1} \right) = 0.118$$

From Table 14-8, $C_p = 2300\sqrt{psi}$

The allowable bending stress number is given by using Fig. 14-2 with Grade 1 steel such that,

$$S_t = 77.3HB + 12150 = 77.3(150) + 12150 = 23745 \text{ psi}$$

Similarly, for the allowable contact stress number using Fig. 14-5,

$$S_c = 322HB + 29100 = 322(150) + 29100 = 77400 \text{ psi}$$

From Fig. 14-15,

$$Z_N = 1.4488N^{-0.023} = 1.4488(3 * 10^8)^{-0.023} = 0.92$$

Because all gears are the same material, the hardness ratio $C_H = 1$.

Substituting the appropriate terms for tooth bending Eq. (14-15) gives,

$$\sigma_B = W_t K_o K_v K_s \frac{P}{F} \frac{K_m K_B}{J} = 44.26(1.4)1.19(1.04) \frac{10}{1.25} \frac{1.15(1)}{0.4} = 1763.8 \text{ psi}$$

Substituting the appropriate terms for bending stress safety factor Eq. (14-41) gives,

$$n_{sf_B} = \frac{S_t}{\sigma_B} \frac{Y_N}{K_T K_R} = \frac{23745}{1763.8} \frac{0.896}{1(1)} = 12.06$$

Substituting the appropriate terms for tooth wear Eq. (14-15) gives,

$$\sigma_C = C_p \left(W_t K_o K_v K_s \frac{K_m C_f}{dF} \right)^{1/2} = 2300 \left[44.26 (1.4) 1.19(1.04) \frac{1.15}{5(1.25)} \frac{1}{0.118} \right]^{1/2} = 41890.63 \text{ psi}$$

Substituting the appropriate terms for contact stress safety factor Eq. (14-16) gives,

$$n_{sf_C} = \frac{S_c}{\sigma_C} \frac{Z_N C_H}{K_T K_R} = \frac{77400}{41890.63} \frac{0.92(1)}{1(1)} = 1.71$$

Gear 2

$$d = N/P = 18/10 = 1.8 \text{ in}$$

$$V = \frac{\pi d n}{12} = \frac{\pi(1.8)639}{12} = 301 \text{ fpm}$$

$$W_t = 33000 H/V = 2(0.40)/301 = 44.26 \text{ lb}$$

Assuming light shock power source + light shock driver, $K_o = 1$ according to the textbook, Machine Elements in Mechanical Design 6th edition Table 9-1. To evaluate K_v from Eq. (14-28) with a quality number $Q = 7$,

$$B = 0.25(12 - Q)^{2/3} = 0.25(12 - 7)^{2/3} = 0.73$$

$$A = 50 + 56(1 - B) = 50 + 56(1 - 0.73) = 65.06$$

Then from Eq. (14-27) the dynamic factor is

$$K_v = \left(\frac{A + \sqrt{V}}{A}\right)^B = \left(\frac{65.06 + \sqrt{301}}{65.06}\right)^{0.73} = 1.19$$

To determine the size factor, K_s , the Lewis form factor is needed. From Table 14-2, with $N = 50$ teeth, $Y = 0.309$. Thus from Eq. (a) of Sec. 14-10, with $F = 1.25$ in,

$$K_s = 1.192\left(\frac{F\sqrt{Y}}{P}\right)^{0.0535} = 1.192\left(\frac{1.25\sqrt{0.309}}{10}\right)^{0.0535} = 1.03$$

The load distribution factor, K_m , is determined from Eq. (14-30), where five terms are needed. They are, with $F = 1.25$ in,

Uncrowned, Eq. (14-30): $C_{mc} = 1$,

Eq. (14-32):

$$C_{pf} = \frac{F}{10d} - 0.0375 + 0.0125F = \frac{1.25}{10(1.8)} - 0.0375 + 0.0125(1.25) = 0.047$$

Straddle-mounted pinion with bias to one side, Eq. (14-33): $C_{pm} = 1.1$,

Commercial enclosed gear units (Fig. 14-11): $C_{ma} = 0.147$,

Eq. (14-35): $C_e = 1$

Thus,

$$K_m = 1 + C_{mc}(C_{pf}C_{pm} + C_{ma}C_e) = 1 + (1)[0.047(1.1) + 0.147(1)] = 1.20$$

Assuming constant thickness gears, the rim thickness factor $K_B = 1$.

The stress cycle factor with $8.4 * 10^8$ cycles is,

$$Y_N = 1.6831N^{-0.0323} = 1.6831(8.4 * 10^8)^{-0.0323} = 0.867$$

From Table 14.10, with a reliability of 0.99, $K_R = 1$. From Fig. 14-18, the temperature and surface condition factors are $K_T = 1$ and $C_f = 1$.

The geometry factor is given by Fig. 14-6 so $J = 0.315$. The geometry factor for contact stress with $m_n = 1$ for external spur gears from Eq. (14-32), a gear ratio $m_G = 2.78$, and a pressure angle $\phi = 20^\circ$ is,

$$I = \frac{\cos(\phi)\sin(\phi)}{2m_n} \left(\frac{m_G}{m_G + 1} \right) = \frac{\cos(20^\circ)\sin(20^\circ)}{2(1)} \left(\frac{(2.78)}{(2.78)+1} \right) = 0.118$$

From Table 14-8, $C_p = 2300\sqrt{psi}$

The allowable bending stress number is given by using Fig. 14-2 with Grade 1 steel such that,

$$S_t = 77.3HB + 12150 = 77.3(150) + 12150 = 23\,745\,psi$$

Similarly, for the allowable contact stress number using Fig. 14-5,

$$S_c = 322HB + 29100 = 322(150) + 29100 = 77\,400\,psi$$

From Fig. 14-15,

$$Z_N = 1.4488N^{-0.023} = 1.4488(8.4 * 10^8)^{-0.023} = 0.90$$

Because all gears are the same material, the hardness ratio $C_H = 1$.

Substituting the appropriate terms for tooth bending Eq. (14-15) gives,

$$\sigma_B = W_t K_o K_v K_s \frac{P}{F} \frac{K_m K_B}{J} = 44.26(1.4)1.19(1.03) \frac{10}{1.25} \frac{1.20(1)}{0.315} = 2317.6\,psi$$

Substituting the appropriate terms for bending stress safety factor Eq. (14-41) gives,

$$n_{sf_B} = \frac{S_t}{\sigma_B} \frac{Y_N}{K_T K_R} = \frac{23745}{2317.6} \frac{0.867}{1(1)} = 8.88$$

Substituting the appropriate terms for tooth wear Eq. (14-15) gives,

$$\sigma_C = C_p \left(W_t K_o K_v K_s \frac{K_m C_f}{dF} \frac{1}{I} \right)^{1/2} = 2300 \left[44.26 (1.4)1.19(1.03) \frac{1.20}{1.8(1.25)} \frac{1}{0.118} \right]^{1/2} = 42\,611.67\,psi$$

Substituting the appropriate terms for contact stress safety factor Eq. (14-16) gives,

$$n_{sf_C} = \frac{S_c}{\sigma_C} \frac{Z_N C_H}{K_T K_R} = \frac{77400}{42611.67} \frac{0.90(1)}{1(1)} = 1.64$$

Gear 3

$$d = N/P = 50/10 = 5 \text{ in}$$

$$V = \frac{\pi d n}{12} = \frac{\pi(5)639}{12} = 836 \text{ fpm}$$

$$W_t = 33000 H/V = 2(0.40)/836 = 15.93 \text{ lb}$$

Assuming light shock power source + light shock driver, $K_o = 1$ according to the textbook, Machine Elements in Mechanical Design 6th edition Table 9-1. To evaluate K_v from Eq. (14-28) with a quality number $Q = 7$,

$$B = 0.25(12 - Q)^{2/3} = 0.25(12 - 7)^{2/3} = 0.73$$

$$A = 50 + 56(1 - B) = 50 + 56(1 - 0.73) = 65.06$$

Then from Eq. (14-27) the dynamic factor is

$$K_v = \left(\frac{A + \sqrt{V}}{A} \right)^B = \left(\frac{65.06 + \sqrt{836}}{65.06} \right)^{0.73} = 1.31$$

To determine the size factor, K_s , the Lewis form factor is needed. From Table 14-2, with $N = 50$ teeth, $Y = 0.409$. Thus from Eq. (a) of Sec. 14-10, with $F = 1.25$ in,

$$K_s = 1.192 \left(\frac{F\sqrt{Y}}{P} \right)^{0.0535} = 1.192 \left(\frac{1.25\sqrt{0.409}}{10} \right)^{0.0535} = 1.04$$

The load distribution factor, K_m , is determined from Eq. (14-30), where five terms are needed. They are, with $F = 1.25$ in,

$$\text{Uncrowned, Eq. (14-30): } C_{mc} = 1,$$

$$\text{Eq. (14-32):}$$

$$C_{pf} = \frac{F}{10d} - 0.0375 + 0.0125F = \frac{1.25}{10(5)} - 0.0375 + 0.0125(1.25) = 0.00313$$

$$\text{, Straddle-mounted pinion with bias to one side, Eq. (14-33): } C_{pm} = 1.1,$$

$$\text{Commercial enclosed gear units (Fig. 14-11): } C_{ma} = 0.147,$$

$$\text{Eq. (14-35): } C_e = 1$$

Thus,

$$K_m = 1 + C_{mc}(C_{pf}C_{pm} + C_{ma}C_e) = 1 + (1)[0.00313(1.1) + 0.147(1)] = 1.15$$

Assuming constant thickness gears, the rim thickness factor $K_B = 1$.

The stress cycle factor with 8.4×10^8 cycles is,

$$Y_N = 1.6831N^{-0.0323} = 1.6831(2.8 * 10^8)^{-0.0323} = 0.867$$

From Table 14.10, with a reliability of 0.99, $K_R = 1$. From Fig. 14-18, the temperature and surface condition factors are $K_T = 1$ and $C_f = 1$.

The geometry factor is given by Fig. 14-6 so $J = 0.4$. The geometry factor for contact stress with $m_n = 1$ for external spur gears from Eq. (14-32), a gear ratio $m_G = 2.78$, and a pressure angle $\phi = 20^\circ$ is,

$$I = \frac{\cos(\phi)\sin(\phi)}{2m_n} \left(\frac{m_G}{m_G+1} \right) = \frac{\cos(20^\circ)\sin(20^\circ)}{2(1)} \left(\frac{(2.78)}{(2.78)+1} \right) = 0.118$$

From Table 14-8, $C_p = 2300\sqrt{psi}$

The allowable bending stress number is given by using Fig. 14-2 with Grade 1 steel such that,

$$S_t = 77.3HB + 12150 = 77.3(150) + 12150 = 23745 \text{ psi}$$

Similarly, for the allowable contact stress number using Fig. 14-5,

$$S_c = 322HB + 29100 = 322(150) + 29100 = 77400 \text{ psi}$$

From Fig. 14-15,

$$Z_N = 1.4488N^{-0.023} = 1.4488(8.4 * 10^8)^{-0.023} = 0.90$$

Because all gears are the same material, the hardness ratio $C_H = 1$.

Substituting the appropriate terms for tooth bending Eq. (14-15) gives,

$$\sigma_B = W_t K_o K_v K_s \frac{P}{F} \frac{K_m K_B}{J} = 15.93(1.4)1.31(1.04) \frac{10}{1.25} \frac{1.15(1)}{0.4} = 699.0 \text{ psi}$$

Substituting the appropriate terms for bending stress safety factor Eq. (14-41) gives,

$$n_{sf_B} = \frac{S_t}{\sigma_B} \frac{Y_N}{K_T K_R} = \frac{23745}{699.0} \frac{0.867}{1(1)} = 29.44$$

Substituting the appropriate terms for tooth wear Eq. (14-15) gives,

$$\sigma_C = C_p (W_t K_o K_v K_s \frac{K_m C_f}{dF I})^{1/2} = 2300[15.93 (1.4)1.31(1.04) \frac{1.15}{5(1.25)} \frac{1}{0.118}]^{1/2} = 26270.48 \text{ psi}$$

Substituting the appropriate terms for contact stress safety factor Eq. (14-16) gives,

$$n_{sf_C} = \frac{S_c}{\sigma_C} \frac{Z_N C_H}{K_T K_R} = \frac{77400}{26270.48} \frac{0.90(1)}{1(1)} = 2.65$$

Gear 4

$$d = N/P = 18/10 = 1.8 \text{ in}$$

$$V = \frac{\pi d n}{12} = \frac{\pi(1.8)1775}{12} = 836 \text{ fpm}$$

$$W_t = 33000 H/V = 2(0.40)/836 = 15.93 \text{ lb}$$

Assuming light shock power source + light shock driver, $K_o = 1$ according to the textbook, Machine Elements in Mechanical Design 6th edition Table 9-1. To evaluate K_v from Eq. (14-28) with a quality number $Q = 7$,

$$B = 0.25(12 - Q)^{2/3} = 0.25(12 - 7)^{2/3} = 0.73$$

$$A = 50 + 56(1 - B) = 50 + 56(1 - 0.73) = 65.06$$

Then from Eq. (14-27) the dynamic factor is

$$K_v = \left(\frac{A + \sqrt{V}}{A} \right)^B = \left(\frac{65.06 + \sqrt{836}}{65.06} \right)^{0.73} = 1.31$$

To determine the size factor, K_s , the Lewis form factor is needed. From Table 14-2, with $N = 50$ teeth, $Y = 0.309$. Thus from Eq. (a) of Sec. 14-10, with $F = 1.25$ in,

$$K_s = 1.192 \left(\frac{F\sqrt{Y}}{P} \right)^{0.0535} = 1.192 \left(\frac{1.25\sqrt{0.309}}{10} \right)^{0.0535} = 1.03$$

The load distribution factor, K_m , is determined from Eq. (14-30), where five terms are needed. They are, with $F = 1.25$ in,

$$\text{Uncrowned, Eq. (14-30): } C_{mc} = 1,$$

$$\text{Eq. (14-32):}$$

$$C_{pf} = \frac{F}{10d} - 0.0375 + 0.0125F = \frac{1.25}{10(1.8)} - 0.0375 + 0.0125(1.25) = 0.047,$$

$$\text{Straddle-mounted pinion with bias to one side, Eq. (14-33): } C_{pm} = 1.1,$$

$$\text{Commercial enclosed gear units (Fig. 14-11): } C_{ma} = 0.147,$$

$$\text{Eq. (14-35): } C_e = 1$$

Thus,

$$K_m = 1 + C_{mc}(C_{pf}C_{pm} + C_{ma}C_e) = 1 + (1)[0.047(1.1) + 0.147(1)] = 1.20$$

Assuming constant thickness gears, the rim thickness factor $K_B = 1$.

The stress cycle factor with $2.3 * 10^9$ cycles is,

$$Y_N = 1.6831N^{-0.0323} = 1.6831(2.3 * 10^9)^{-0.0323} = 0.839$$

From Table 14.10, with a reliability of 0.99, $K_R = 1$. From Fig. 14-18, the temperature and surface condition factors are $K_T = 1$ and $C_f = 1$.

The geometry factor is given by Fig. 14-6 so $J = 0.315$. The geometry factor for contact stress with $m_n = 1$ for external spur gears from Eq. (14-32), a gear ratio $m_G = 2.78$, and a pressure angle $\phi = 20^\circ$ is,

$$I = \frac{\cos(\phi)\sin(\phi)}{2m_n} \left(\frac{m_G}{m_G + 1} \right) = \frac{\cos(20^\circ)\sin(20^\circ)}{2(1)} \left(\frac{(2.78)}{(2.78)+1} \right) = 0.118$$

From Table 14-8, $C_p = 2300\sqrt{psi}$

The allowable bending stress number is given by using Fig. 14-2 with Grade 1 steel such that,

$$S_t = 77.3HB + 12150 = 77.3(150) + 12150 = 23745 \text{ psi}$$

Similarly, for the allowable contact stress number using Fig. 14-5,

$$S_c = 322HB + 29100 = 322(150) + 29100 = 77400 \text{ psi}$$

From Fig. 14-15,

$$Z_N = 1.4488N^{-0.023} = 1.4488(2.3 * 10^9)^{-0.023} = 0.88$$

Because all gears are the same material, the hardness ratio $C_H = 1$.

Substituting the appropriate terms for tooth bending Eq. (14-15) gives,

$$\sigma_B = W_t K_o K_v K_s \frac{P}{F} \frac{K_m K_B}{J} = 15.93(1.4)1.31(1.03) \frac{10}{1.25} \frac{1.20(1)}{0.315} = 918.4 \text{ psi}$$

Substituting the appropriate terms for bending stress safety factor Eq. (14-41) gives,

$$n_{sf_B} = \frac{S_t}{\sigma_B} \frac{Y_N}{K_T K_R} = \frac{23745}{918.4} \frac{0.839}{1(1)} = 21.68$$

Substituting the appropriate terms for tooth wear Eq. (14-15) gives,

$$\sigma_c = C_p \left(W_t K_o K_v K_s \frac{K_m C_f}{dF} \frac{1}{I} \right)^{1/2} = 2300 \left[15.93 (1.4)1.31(1.03) \frac{1.20}{1.8(1.25)} \frac{1}{0.118} \right]^{1/2} = 26824.37 \text{ psi}$$

Substituting the appropriate terms for contact stress safety factor Eq. (14-16) gives,

$$n_{sf_c} = \frac{S_c}{\sigma_c} \frac{Z_N C_H}{K_T K_R} = \frac{77400}{26824.37} \frac{0.88(1)}{1(1)} = 2.55$$

Appendix B: Shaft Analysis

Table B1: Shaft Analysis Variables

Adjusted endurance strength	S_e	Alternating moment	M_a
Endurance strength ($S'_e = 0.5S_{ut}$)	S'_e	Midrange torque	T_m
Surface condition modification factor	k_a	First iteration bending stress concentration factor	K_t
Size modification factor	k_b	First iteration torsional stress concentration factor	K_{ts}
Load modification factor	k_c	Bending stress concentration factor	K_f
Temperature modification factor	k_d	Torsional stress concentration factor	K_{fs}
Reliability factor	k_e	Ultimate strength	S_{ut}
Miscellaneous modification factor	k_f	Yield strength	S_y
Notch sensitivity	q	Shear notch sensitivity	q_s

B1. Load, Shear, and Bending Moment Analysis

To determine shaft size, the shaft length dimensions and gear reaction forces from [Appendix A: Gear Analysis](#) were used as inputs to the shaft analysis. A free body diagram of the intermediate shaft was drawn, seen at the top of Fig. [B1.1](#). Using the contact forces, the sum of forces and moments in both the x-y and x-z planes to find the bearing reaction forces. Using this information, load, shear, bending moment, and torque diagrams were created as seen in Figures [B1.1](#), [B1.2](#), [B1.3](#). The maximum moment and torque was found to be $M_{max} = 177.8 \text{ lb} \cdot \text{in}$, $T_{max} = 78.9 \text{ lb} \cdot \text{in}$ at Gear 2. Figure [B1.3](#) also shows the identified points of greatest stress concentrations shown with orange arrows.

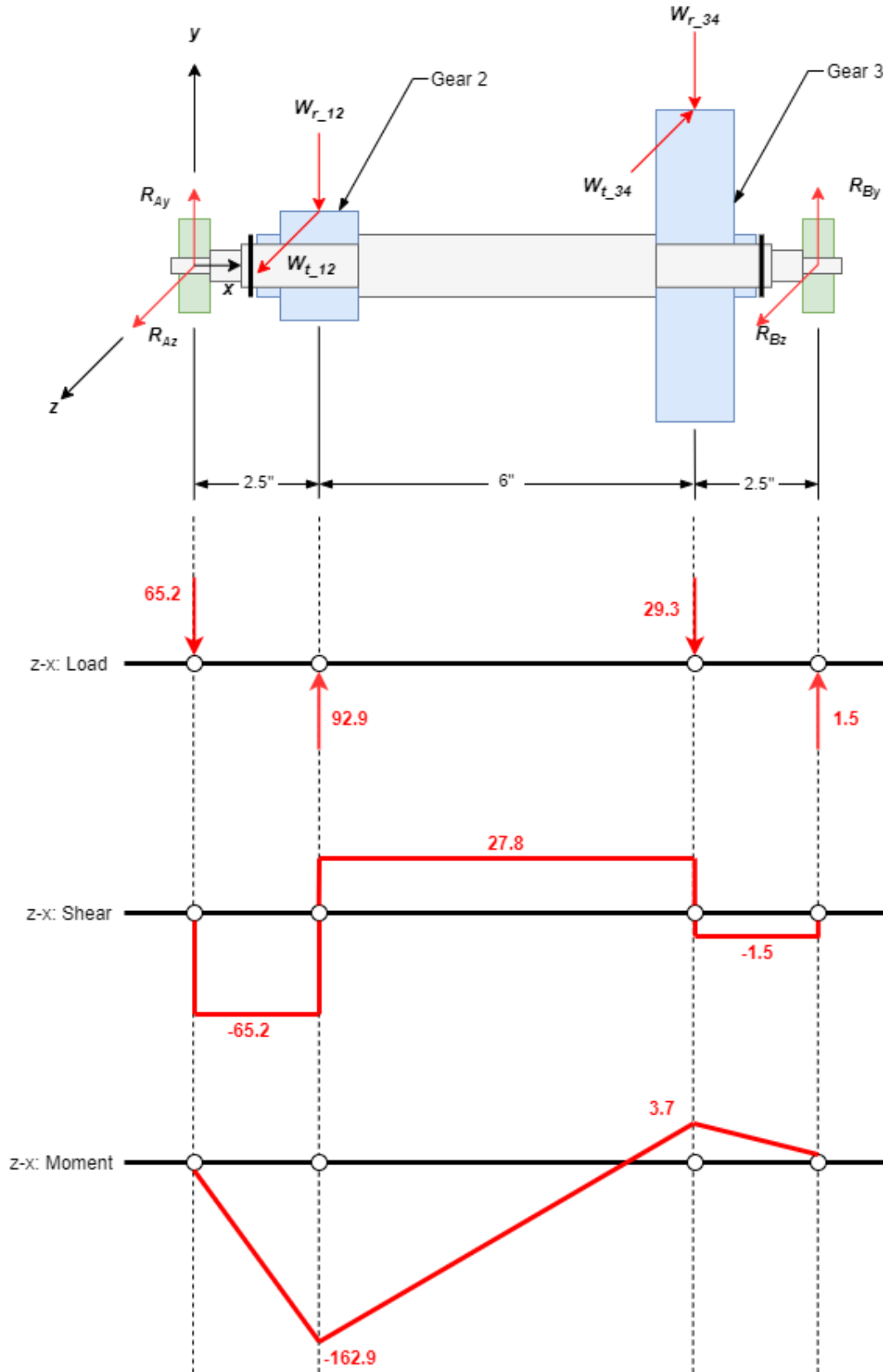


Fig. B1.1: z-x Plane Load, Shear, and Moment Diagrams

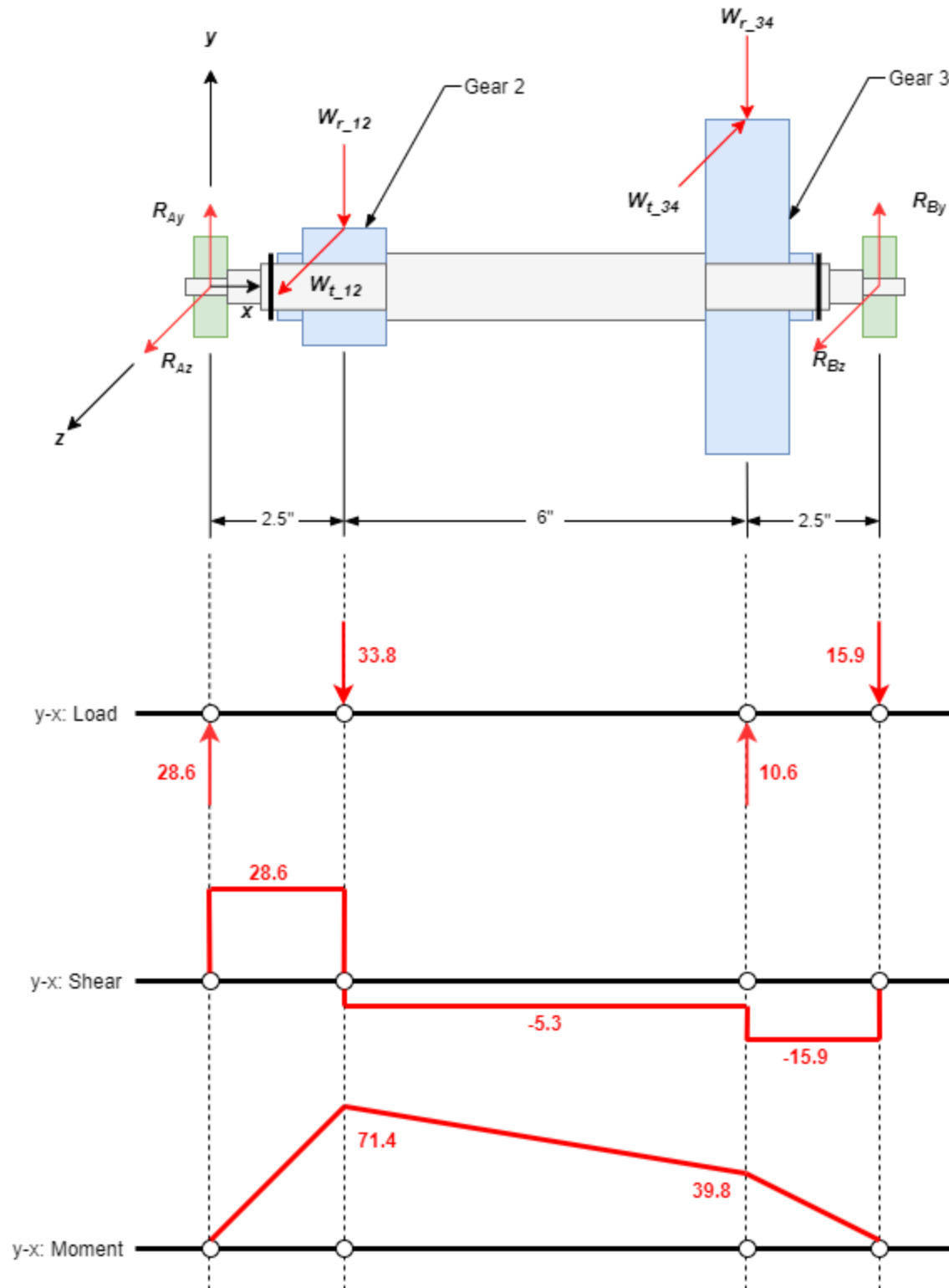


Fig. B1.2: y-x Plane Load, Shear, and Moment Diagrams

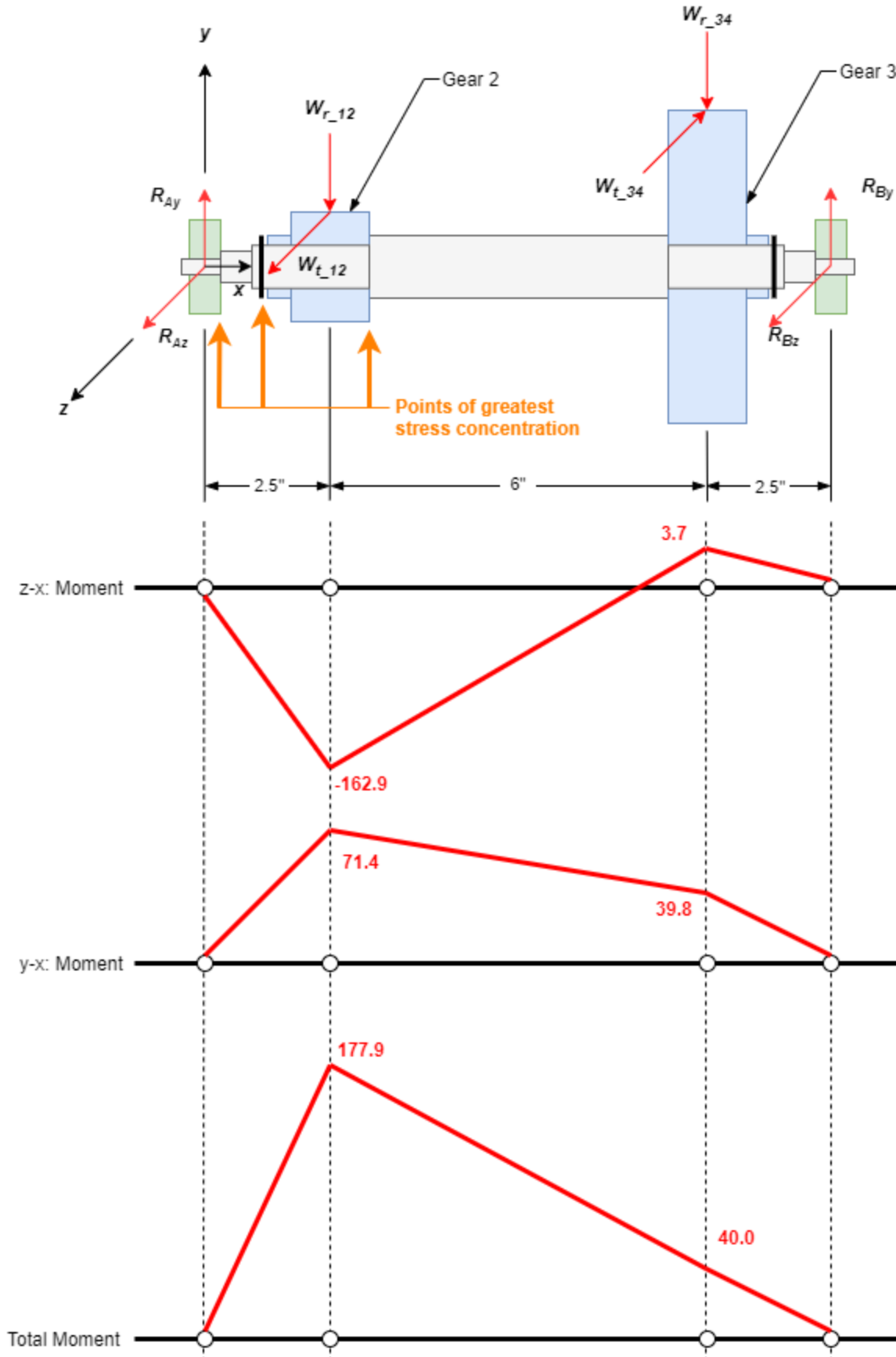


Fig. B1.3: Total Moment Diagrams and Points of Largest Stress Concentrations

B2. Shaft Failure Analysis

B2.1. Minimum Shaft Diameter

SAE 1020 CD steel ($s_{ut} = 68$ ksi, $s_y = 57$ ksi) was chosen for it being relatively inexpensive. To count for various conditions, the Marin equation was used to modify the design endurance limit. From the shafts shown in Section [5.1 CAD Renders and Drawings](#), this analysis covers the intermediate shaft. Since the motor shaft has less torque acting on it, it was assumed that the intermediate shaft would experience greater stresses and therefore it should be appropriate to select the same diameter shaft for the motor. The input shaft from the turbine rotors is assumed to be out of scope. **Note:** the equations below all refer to Shigley's *Mechanical Engineering Design 9th Edition*.

$$S_e = k_a k_b k_c k_d k_e k_f S'_e, S'_e = 0.5 S_{ut} \quad (\text{Shigley, eq. 6-18})$$

The size modification factor, $k_a = 0.883$, was determined by the following equation:

$$k_a = a S_{ut}^b \quad (\text{Shigley, eq. 6-19})$$

Factors a and b were determined through Shigley Table 6-2. Assuming the worst case Machined shaft, $a = 2.70$ ksi and $b = -0.265$.

The surface condition modification factor, $k_b = 0.9$, was determined by the following equation:

$$k_b = 0.879 d^{-0.107}, 0.11" \leq d \leq 2" \quad (\text{Shigley, eq. 6-20})$$

This equation requires knowing the diameter of the shaft beforehand. A guessed diameter was initially chosen, and then corrected once the full calculation was complete.

The load modification factor was set to $k_c = 1$ as in this case, torsion is combined with bending.

The temperature modification factor, $k_d = 1$, was determined by the following equation:

$$k_b = 0.975 + 0.432 \cdot 10^{-3}T - 0.115 \cdot 10^{-5}T^2 + 0.104 \cdot 10^{-8}T^3 - 0.595 \cdot 10^{-12}T^4, \quad (\text{Shigley, eq. 6-27})$$

$$70 \leq T \leq 1000^\circ\text{F}$$

The reliability factor, k_e , was determined using Shigley Table 6-5. For a required reliability of 99.9%, $k_e = 0.753$.

The miscellaneous modification factor was set to $k_f = 1$ since specific details related to it are out of scope of this project. The resulting endurance strength was found to be $S_e = 20.4 \text{ ksi}$

The DE-Goodman shaft stress criterion (Shigley eq. 7-8) was used to determine the shaft diameter. Assuming $T_a = M_m = 0$ (since this application has constant bending and torsion and only rotates in one direction), Shigley eq. 7-8 reduces to:

$$d = \left\{ \frac{16n_d}{\pi} \left(\frac{2K_f M_a}{S_e} + \frac{\sqrt{3}K_{fs} T_m}{S_{ut}} \right) \right\}^{1/3} \quad (\text{Shigley, eq. 7-8 reduced})$$

The safety factor was chosen to be $n_d = 2.5$ as per recommendation by Shigley to have $2.5 \leq n_d \leq 3.0$. For a conservative initial calculation, it was assumed $K_f = K_t$, $K_{fs} = K_{ts}$ where $K_t = 2.7$, $K_{ts} = 2.2$ are found from Shigley Table 7-1 assuming a sharp shoulder fillet of $r/d = 0.02$. M_a and T_m are found from the bending moment and torque diagrams. As a result, $d = 0.82$ " (minimum) for the gear bore diameter.

From the [Appendix A: Gear Analysis](#) calculation, the optimal available gears were determined to have a bore diameter of 0.875" for Gear 2 and 1" for Gear 3. Both bore diameters are larger than the minimum specified previously. The Shigley textbook recommends a shaft shoulder ratio $D/d = 1.2$. Thus, the shaft diameter between the gears is 1.2" to accommodate Gear 3.

B2.2. Gear 2 Shoulder Safety Factor

For analysis of the Gear 2 shoulder, Gear 2 has a bore diameter of 0.875". At the shoulder the alternating moment and midrange torque are $M_a = 158.8 \text{ lb} \cdot \text{in}$, $T_m = 78.9 \text{ lb} \cdot \text{in}$. The diameter of the shoulder is 1.2". Therefore the shaft shoulder ratio is $D/d = 1.37$. Assuming a worst-case fillet ratio of $r/d = 0.02$ (Table 7-1 Shigley, p.373), the fillet radius is $r = 0.0175$ ".

To find K_f and K_{fs} , $K_t = 2.7$ and $K_{ts} = 2.1$, first found using Shigley Figures A-15-9 and A-15-8 (p.1028) using the $r/d = 0.02$ and $D/d = 1.4$ ratios from previously. q and q_s are found using Shigley Figures 6-20 (p.295) and 6-21 (p.296) using $r = 0.0175$ " and $S_{ut} = 68 \text{ ksi}$. K_f and K_{fs} are then calculated by the following equations:

$$K_f = 1 + q(K_t - 1), K_{fs} = 1 + q_s(K_{ts} - 1) \quad (\text{Shigley, eq. 6-32})$$

Thus, $K_f = 2.02$ and $K_{fs} = 1.72$.

Next S_e needs to be recalculated for the 0.875" shaft diameter. $k_b = 0.89$, recalculated using the same equation as before - Shigley eq. 6-20. $S_e = 20.3 \text{ ksi}$ using the Marin Equation (Shigley eq. 6-18).

To find the midrange and alternating stresses the following equations are used:

$$\sigma'_a = \frac{32K_f M_a}{\pi d^3} \quad (\text{Shigley, eq. 7-5})$$

$$\sigma'_m = \sqrt{3} \frac{16K_{fs} T_m}{\pi d^3} \quad (\text{Shigley, eq. 7-6})$$

Thus, $\sigma'_a = 4.9 \text{ ksi}$, $\sigma'_m = 1.8 \text{ ksi}$. To find the final safety factor, the Modified Goodman criterion was used due to it being the most conservative.

$$\frac{1}{n_f} = \frac{\sigma'_a}{S_e} + \frac{\sigma'_m}{S_{ut}} \quad (\text{Shigley, eq. 6-46})$$

Thus, $n_f = 3.7$.

B2.3. Gear 2 Key Groove Safety Factor

For analysis of the Gear 2 key at both sides of gear 2, Gear 2 has a bore diameter of 0.875". At the shoulder the alternating moment and midrange torque are $M_a = 158.8 \text{ lb} \cdot \text{in}$, $T_m = 78.9 \text{ lb} \cdot \text{in}$ and on the opposite end of the key the alternating moment and midrange torque are $M_a = 97.8 \text{ lb} \cdot \text{in}$, $T_m = 78.9 \text{ lb} \cdot \text{in}$. Assuming a worst-case fillet ratio of $r/d = 0.02$ (Table 7-1 Shigley, p.373), the fillet radius is $r = 0.0175"$.

To find K_f and K_{fs} , $K_t = 2.14$ and $K_{ts} = 3$, first found using Shigley Table 7-1 Shigley (p.373). q and q_s are found using Shigley Figures 6-20 (p.295) and 6-21 (p.296) using $r = 0.0175"$ and $S_{ut} = 68 \text{ ksi}$. K_f and K_{fs} are then calculated by the following equations:

$$K_f = 1 + q(K_t - 1), K_{fs} = 1 + q_s(K_{ts} - 1) \quad (\text{Shigley, eq. 6-32})$$

Thus, $K_f = 1.7$ and $K_{fs} = 2.3$. S_e is the same as Section B2.2.

To find the midrange and alternating stresses the following equations are used:

$$\sigma'_a = \frac{32K_f M_a}{\pi d^3} \quad (\text{Shigley, eq. 7-5})$$

$$\sigma'_m = \sqrt{3} \frac{16K_{fs} T_m}{\pi d^3} \quad (\text{Shigley, eq. 7-6})$$

Thus, $\sigma'_a = 4.1 \text{ ksi}$, $\sigma'_m = 2.4 \text{ ksi}$ for the shoulder side and $\sigma'_a = 2.5 \text{ ksi}$, $\sigma'_m = 2.4 \text{ ksi}$ for the opposite side. To find the final safety factor, the Modified Goodman criterion was used due to it being the most conservative.

$$\frac{1}{n_f} = \frac{\sigma'_a}{S_e} + \frac{\sigma'_m}{S_{ut}} \quad (\text{Shigley, eq. 6-46})$$

Thus, $n_f = 4.1$ for the shoulder side and $n_f = 6.0$ for the opposite side.

B2.4. Gear 2 Retaining Ring Safety Factor

For analysis of the Gear 2 retaining ring, Gear 2 has a bore diameter of 0.875". At the retaining ring, the alternating moment and midrange torque are $M_a = 97.75 \text{ lb} \cdot \text{in}$, $T_m = 0 \text{ lb} \cdot \text{in}$.

To find K_f and K_{fs} , $K_t = 5$ and $K_{ts} = 3$, first found using Shigley Table 7-1 Shigley (p.373). q and q_s are found using Shigley Figures 6-20 (p.295) and 6-21 (p.296) using $r = 0.0175"$ and $S_{ut} = 68 \text{ ksi}$. K_f and K_{fs} are then calculated by the following equations:

$$K_f = 1 + q(K_t - 1), K_{fs} = 1 + q_s(K_{ts} - 1) \quad (\text{Shigley, eq. 6-32})$$

Thus, $K_f = 3.4$ and $K_{fs} = 2.3$. S_e is the same as Section B2.2.

To find the midrange and alternating stresses the following equations are used:

$$\sigma'_a = \frac{32K_f M_a}{\pi d^3} \quad (\text{Shigley, eq. 7-5})$$

$$\sigma'_m = \sqrt{3} \frac{16K_{fs} T_m}{\pi d^3} \quad (\text{Shigley, eq. 7-6})$$

Thus, $\sigma'_a = 5.1 \text{ ksi}$, $\sigma'_m = 0 \text{ ksi}$. To find the final safety factor, the Modified Goodman criterion was used due to it being the most conservative.

$$\frac{1}{n_f} = \frac{\sigma'_a}{S_e} + \frac{\sigma'_m}{S_{ut}} \quad (\text{Shigley, eq. 6-46})$$

Thus, $n_f = 4.0$.

B2.5. Bearing A Shoulder Safety Factor

For analysis of the bearing A shoulder, bearing A has a bore diameter of $0.625"$. At the shoulder the alternating moment and midrange torque are $M_a = 44.5 \text{ lb} \cdot \text{in}$, $T_m = 0 \text{ lb} \cdot \text{in}$. The diameter of the shoulder is $1.4"$ since Gear 2 bore diameter is $0.875"$. Assuming a worst-case fillet ratio of $r/d = 0.02$ (Table 7-1 Shigley, p.373), the fillet radius is $r = 0.0175"$.

To find K_f and K_{fs} , $K_t = 2.7$ and $K_{ts} = 2.1$, first found using Shigley Figures A-15-9 and A-15-8 (p.1028) using the $r/d = 0.02$ and $D/d = 1.4$ ratios from previously. q and q_s are

found using Shigley Figures 6-20 (p.295) and 6-21 (p.296) using $r = 0.0175"$ and $S_{ut} = 68 \text{ ksi}$. K_f and K_{fs} are then calculated by the following equations:

$$K_f = 1 + q(K_t - 1), K_{fs} = 1 + q_s(K_{ts} - 1) \quad (\text{Shigley, eq. 6-32})$$

Thus, $K_f = 2.02$ and $K_{fs} = 1.72$.

Next S_e needs to be recalculated for the 0.625" shaft diameter. $k_b = 0.92$, recalculated using the same equation as before - Shigley eq. 6-20. $S_e = 20.9 \text{ ksi}$ using the Marin Equation (Shigley eq. 6-18).

To find the midrange and alternating stresses the following equations are used:

$$\sigma'_a = \frac{32K_f M_a}{\pi d^3} \quad (\text{Shigley, eq. 7-5})$$

$$\sigma'_m = \sqrt{3} \frac{16K_{fs} T_m}{\pi d^3} \quad (\text{Shigley, eq. 7-6})$$

Thus, $\sigma'_a = 3.8 \text{ ksi}$, $\sigma'_m = 0 \text{ ksi}$. To find the final safety factor, the Modified Goodman criterion was used due to it being the most conservative.

$$\frac{1}{n_f} = \frac{\sigma'_a}{S_e} + \frac{\sigma'_m}{S_{ut}} \quad (\text{Shigley, eq. 6-46})$$

Thus, $n_f = 5.6$.

Appendix C: Bearing Analysis

Intermediate Shaft Bearing Analysis

Table C1: Shaft Bearing Variables

Radial load at Bearing A	R_A	Weibull parameter	x_0
Radial load at Bearing B	R_B	Weibull parameter	θ
Application factor	a_f	Weibull parameter	b
Design life in revolutions	L_{design}	Constant for ball bearings	a
Rating life for bearings in revolutions	L_{10}	Reliability	R
Design life, non-dimensionalized	x_{design}	Design radial load	F_{design}
Rated dynamic capacity	C_{10}		

To determine bearings fitted for the intermediate shaft, the reaction forces from the previous intermediate shaft analysis (See Appendix B) must be used. The reaction force components on bearings A and B are combined to get a radial load for bearing specification.

$$\begin{aligned}
 R_{A,y} &= 28.558 \text{ lb} \\
 R_{A,x} &= -65.164 \text{ lb} \\
 R_{B,y} &= 15.916 \text{ lb} \\
 R_{B,x} &= 1.486 \text{ lb}
 \end{aligned}$$

These radial loads are:

$$\begin{aligned}
 R_A &= \sqrt{R_{A,y}^2 + R_{A,x}^2} = 71.15 \text{ lb} \\
 R_B &= \sqrt{R_{B,y}^2 + R_{B,x}^2} = 15.99 \text{ lb}
 \end{aligned}$$

The larger radial load is chosen for bearing specification, and both sides of the shaft will use the same bearing for simplification. This simplification helps follow the evaluation criteria of using simple components and requires fewer custom parts.

$$F_{design} = R_A = 71.15 \text{ lb}$$

For an initial estimation of bearing bore diameter, the gear diameter on the shaft must be accounted for, to ensure that our bearings will have a shoulder. Using the results from Appendix B, the minimum shaft diameter at the gear is 0.823 in. We estimated that the ratio of bore diameter for the bearings with respect to the gear shaft diameter is:

$$\frac{\text{Gear shaft diameter}}{\text{Bearing bore diameter}} = 1.625$$

Therefore, the bearing bore diameter is in the vicinity of 0.52 in. With an approximate bore diameter set, the minimum rated dynamic capacity $C_{10,min}$ is calculated, using the equation:

$$C_{10,min} = F_{design} * a_f * \left[\frac{x_{design}}{x_0 + (\theta - x_0) \ln(\frac{1}{R})^{1/b}} \right]^{1/a} \quad (\text{Eqn. 11-9, Shigley 10}^{th})$$

where:

$a = 3$ for ball bearings

$x_0 = 0.02$, default from Shigley's

$b = 1.483$, default from Shigley's

$\theta = 4.459$, default from Shigley's

$a_f = 1.4$, application factor for machinery with light impact

x_{design} is calculated using the number of revolutions specified from our client throughout the lifetime of the component (5 years, 12 hours/day = 21600 hours)

$$x_{design} = \frac{L_{design}}{L_{10}} = 828.1$$

where:

$$L_{10} = 10^6 \text{ cycles}$$

$$L_{design} = (21600 \text{ hours}) * (639 \text{ rpm}) * (60 \text{ min/hr}) = 828.1 * 10^6 \text{ cycles}$$

Using these values,

$$C_{10,min} = 1550.4 \text{ lb}$$

Using [this SKF catalogue](#), we can specify an appropriate bearing based on the minimum dynamic capacity $C_{10,min}$ and an approximate bearing bore diameter of 0.62 in.

Other considerations in choosing a bearing:

- We chose bearings with flanged housing, to mount to the gearbox enclosure walls
- Set screws for securing the shaft axially
- Flingers to splash away foreign material (oil, dirt, rain) to provide clean operation

The SKF F4B 010-TF bearing was chosen, with a rated dynamic capacity $C_{10} = 2150$ lbf, and a bore diameter $D = 0.625$ in.

Source: [SKF Bearings and Mounted Products, pg 262](#)

The tolerances for the shaft and bore diameter are then specified. Referring to Table 1 on pg 52 of the SKF Bearing Installation Guide, the recommended inner ring fit is an **Interference fit**, based on:

- Rotating inner ring and stationary outer ring
- Constant load direction

Referring to Table 2 on pg 52 of the SKF Bearing Installation Guide, the recommended tolerance class is **js5**, based on:

- Light/variable loads
- Shaft diameter < 17mm ball bearing

[The SKF website](#) contains appropriate values for shaft seat tolerances. For tolerance class **js5**:

Table C2: Shaft Tolerances

Shaft nominal diameter - lower tolerance	-4 μm
Shaft nominal diameter - upper tolerance	+4 μm
Bearing bore diameter - lower tolerance	-8 μm
Bearing bore diameter - upper tolerance	0 μm

Tapered Roller Bearings for Turbine Shaft

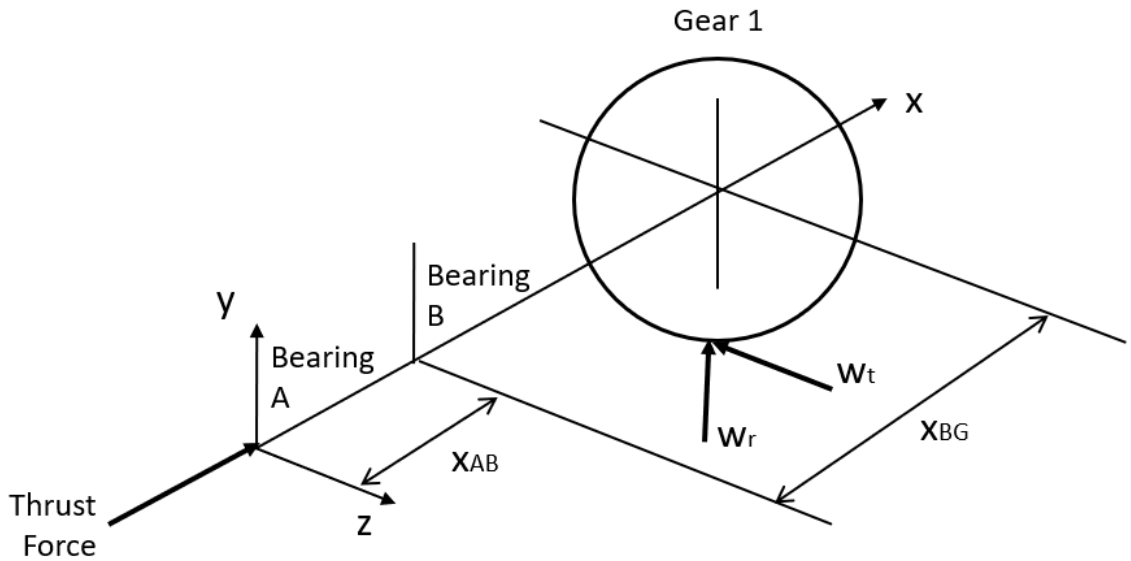


Fig. C1.1: Sketch of Forces on Turbine Shaft

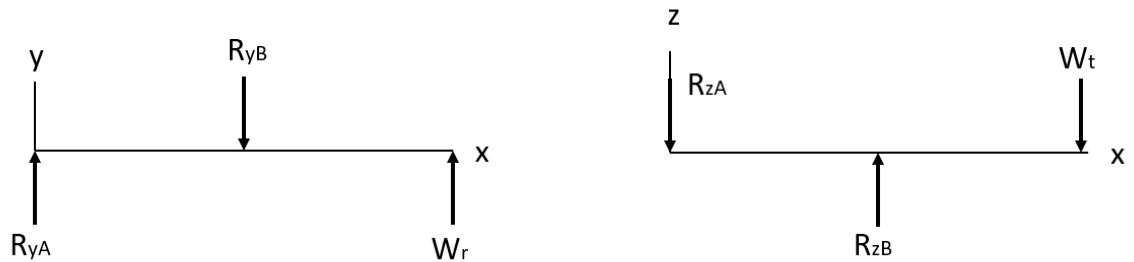


Fig. C1.2: Reaction Forces in xy and xz Planes

Table C3: Variables and Results

Variable	Value	Unit	Notes
W_t	44.26	lbf	Tangential force on Gear 1 (Appendix B)
W_r	16.11	lbf	Radial force on Gear 1 (Appendix B)
F_{ae}	10	lbf	External thrust force
X_{AB}	0.2	in	
X_{BG}	3.5	in	
a	10/3	-	Tapered Roller Bearing

x_0	0	-	Timken Weibell Model
θ	4.48	-	Timken Weibell Model
b	2/3	-	Timken Weibell Model
a_f	1.4	-	Light impact
\mathcal{L}_D	21739	hrs	
L_R	$90 \cdot 10^6$	-	
R_D	0.99	-	Desired Combined Reliability ($=R_A R_B$)

The forces acting on Gear 1, as calculated in Appendix B, were used to analyze the tapered rolling bearings for the turbine shaft. These forces were then used in force and moment balance equations to determine the reaction forces on bearings A and B. x_{AB} denotes the distance between bearings A and B, and x_{BG} denotes the distances between bearing B and Gear 1 on the shaft.

$$R_{zA} = \frac{W_t x_{BG}}{x_{AB}} = 774.47 \text{ lbf}$$

$$R_{zB} = \frac{W_t (x_{BG} + x_{AB})}{x_{AB}} = 818.72 \text{ lbf}$$

$$R_{yA} = \frac{W_r x_{BG}}{x_{AB}} = 281.11 \text{ lbf}$$

$$R_{yB} = \frac{W_r (x_{BG} + x_{AB})}{x_{AB}} = 297.99 \text{ lbf}$$

The radial forces on A and B are the vector sums of the z and y forces on each bearing.

$$F_{rA} = (R_{zA}^2 + R_{yA}^2)^{1/2} = 824.17 \text{ lbf}$$

$$F_{rB} = (R_{zB}^2 + R_{yB}^2)^{1/2} = 871.26 \text{ lbf}$$

Using initial guess of $K=1.5$ for both bearings, the induced loads on each bearing are found using:

$$F_i = \frac{0.47 F_r}{K} \quad (\text{Shigley, eq. 11-18, 10th ed.})$$

$$F_{iA} = 258.24 \text{ lbf}$$

$$F_{iB} = 273.00 \text{ lbf}$$

Because $F_{iA} \leq (F_{iB} + F_{ae})$, where F_{ae} is the external thrust force, equation 11-19 holds for the dynamic equivalent loads:

$$F_{eA} = 0.4F_{rA} + K_A(F_{iB} + F_{ae}) \quad (\text{Shigley, eq. 11-19a, 10th ed.})$$

$$F_{eB} = F_{rB} \quad (\text{Shigley, eq. 11-19b, 10th ed.})$$

$$F_{eA} = 754.16 \text{ lbf}$$

$$F_{eB} = 871.27 \text{ lbf}$$

Using a combined reliability of $R = 0.99$:

$$R_A = R_B = \sqrt{R} = 0.995$$

$$x_D = \frac{L_D n_D^{60}}{L_R} = 3.333$$

The catalogue rating for the bearings can be determined with equation 11-10:

$$C_{10} = a_f F_D \left[\frac{x_D}{x_0 + (\theta - x_0)(1 - R_D)^{1/b}} \right]^{1/a} \quad (\text{Shigley, eq. 11-10, 10th ed.})$$

$$C_{10A} = 2244.42 \text{ lbf}$$

$$C_{10B} = 2592.92 \text{ lbf}$$

From [Timken Tapered Roller Bearing Catalogue \(page 96\)](#), the following bearing was tentatively selected for both bearings:

Table C4: Tapered Bearing Data

Bore Diameter	1 in
Dynamic Load Rating	3180 lbf
K	1.69

Inner Part Number	15578
Outer Part Number	15520

The calculations were iterated using $K=1.69$. It was found that $F_{iA} \leq (F_{iB} + F_{ae})$ still holds, thus F_{eA} and F_{eB} are still calculated using equation 11-19.

Table C5: Variables and Values for Taper Bearings

Variable	Value	Unit
F_{iA}	242.30	lbf
F_{iB}	219.45	lbf
F_{eA}	756.06	lbf
F_{eB}	871.27	lbf
C_{10A}	2250.08	lbf
C_{10B}	2592.92	lbf
R_A	0.99738	-
R_B	0.99467	-

Thus the selected tapered roller bearings for both bearings A and B is sufficient. The combined reliability is calculated to be 0.99206.

Motor Coupled Shaft Bearing Analysis

Table C6: Variables for Motor Coupled Shaft Bearing Analysis

Radial load at Bearing A	R_A	Weibull parameter	x_0
Radial load at Bearing B	R_B	Weibull parameter	θ
Application factor	a_f	Weibull parameter	b
Design life in revolutions	L_{design}	Constant for ball bearings	a
Rating life for bearings in revolutions	L_{10}	Reliability	R
Design life, non-dimensionalized	x_{design}	Design radial load	F_{design}

Rated dynamic capacity	C_{10}		
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To determine bearings fitted for the motor-coupled shaft, the reaction forces must be determined.

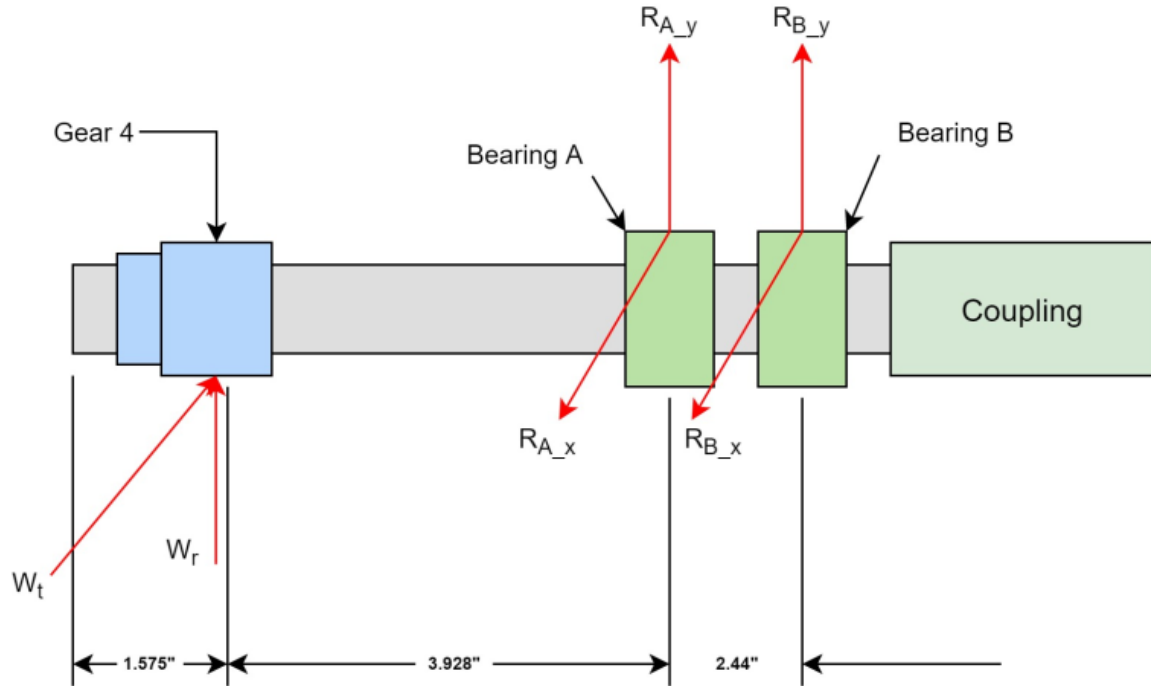


Fig. C1.3: x-y Reaction Diagrams

With the gear 4 forces $W_t = 15.93$ lb and $W_r = 5.799$ lb defined from the gear analysis section in Appendix A, the reaction forces at each bearing can be solved for through moment balance equations. Then, the reaction force components on both bearings are combined to get a radial load for bearing specification.

$$R_{A_y} = -15.134 \text{ lb}$$

$$R_{A_x} = -41.580 \text{ lb}$$

$$R_{B_y} = 9.335 \text{ lb}$$

$$R_{B_x} = 25.648 \text{ lb}$$

These radial loads are:

$$R_A = \sqrt{R_{A_y}^2 + R_{A_x}^2} = 44.25 \text{ lb}$$

$$R_B = \sqrt{R_{B_y}^2 + R_{B_x}^2} = 27.29 \text{ lb}$$

The larger radial load is chosen for bearing specification, and both sides of the shaft will use the same bearing for simplification. This simplification helps follow the evaluation criteria of using simple components and requires fewer custom parts.

$$F_{design} = R_A = 44.25 \text{ lb}$$

The bearing bore diameter is specified by the client to be 0.875 in. With this bore diameter set, the minimum rated dynamic capacity $C_{10,min}$ is calculated, using the equation:

$$C_{10,min} = F_{design} * a_f * \left[\frac{x_{design}}{x_0 + (\theta - x_0) \ln(\frac{1}{R})^{1/b}} \right]^{1/a} \quad (\text{Eqn. 11-9, Shigley } 10^{\text{th}})$$

where:

$a = 3$ for ball bearings

$x_0 = 0.02$, default from Shigley's

$b = 1.483$, default from Shigley's

$\theta = 4.459$, default from Shigley's

$a_f = 1.4$, application factor for machinery with light impact

x_{design} is calculated using the number of revolutions specified from our client throughout the lifetime of the component (5 years, 12 hours/day = 21600 hours)

$$x_{design} = \frac{L_{design}}{L_{10}} = 2300.4$$

where:

$$L_{10} = 10^6 \text{ cycles}$$

$$L_{design} = (21600 \text{ hours}) * (1775 \text{ rpm}) * (60 \text{ min/hr}) = 2300.4 * 10^6 \text{ cycles}$$

Using these values,

$$C_{10,min} = 1335.4 \text{ lb}$$

Using [this SKF catalogue](#), we can specify an appropriate bearing based on the minimum dynamic capacity $C_{10,min}$ and a bearing bore diameter of 0.825 in.

Other considerations in choosing a bearing:

- We chose bearings with pillow block housing, to mount to the gearbox enclosure
- A narrow inner ring, to account for the limited space between the gear and bearing

- Set screws for securing the shaft axially

The SKF P2B 014-RM bearing was chosen, with a rated dynamic capacity $C_{10} = 3150$ lbf, and a bore diameter $D = 0.875$ in.

Source: [SKF Bearings and Mounted Products, pg 259](#)

The tolerances for the shaft and bore diameter are then specified. Referring to Table 1 on pg 52 of the SKF Bearing Installation Guide, the recommended inner ring fit is an **Interference fit**, based on:

- Rotating inner ring and stationary outer ring
- Constant load direction

Referring to Table 2 on pg 52 of the SKF Bearing Installation Guide, the recommended tolerance class is **j6**, based on:

- Light/variable loads
- Shaft diameter between 18-100mm ball bearing

[The SKF website](#) contains appropriate values for shaft seat tolerances. For tolerance class **j6**:

Table C7: Shaft and Bearing Tolerances

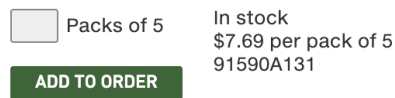
Shaft nominal diameter - lower tolerance	-4 μm
Shaft nominal diameter - upper tolerance	+9 μm
Bearing bore diameter - lower tolerance	-10 μm
Bearing bore diameter - upper tolerance	0 μm

Appendix D: Shaft Accessory Analysis

Appendix D1: Retaining Ring Analysis

With the assumptions of negligible axial force applied on the gears, retaining rings were specified using only the shaft diameter that it will be placed on. With 0.875 in and 1.000 in shaft diameters, 15-7 PH stainless steel rings were specified using a McMaster-Carr catalogue.

[McMaster-Carr](#) for 0.875 inch shaft



Retaining Ring Type	External
Retaining Ring Style	Standard
System of Measurement	Inch
Material	15-7 PH Stainless Steel
Passivation	Passivated
For OD	7/8"
For Groove	
Diameter	0.821"
Diameter Tolerance	-0.003" to 0.003"
Width	0.046"
Width Tolerance	0" to 0.003"
Ring	
ID	0.81"
ID Tolerance	-0.01" to 0.005"
Thickness	0.042"
Thickness Tolerance	-0.002" to 0.002"
Min. Hardness	Rockwell C43
Thrust Load Capacity	4,360 lbs.
Magnetic Properties	Magnetic
Specifications Met	ASME B18.27.1
RoHS	RoHS 3 (2015/863/EU) Compliant
REACH	REACH (EC 1907/2006) (06/25/2020, 209 SVHC) Compliant
DFARS	Specialty Metals Compliant (252.225-7009)
Country of Origin	United States

Fig. D1.1: McMaster-Carr for 0.857 Inch Shaft

[McMaster-Carr](#) for 1.000 inch shaft

Packs of 5

In stock

\$8.88 per pack of 5

91590A133

ADD TO ORDER

Retaining Ring Type	External
Retaining Ring Style	Standard
System of Measurement	Inch
Material	15-7 PH Stainless Steel
Passivation	Passivated
For OD	1"
For Groove	
Diameter	0.94"
Diameter Tolerance	-0.003" to 0.003"
Width	0.046"
Width Tolerance	0" to 0.003"
Ring	
ID	0.925"
ID Tolerance	-0.01" to 0.005"
Thickness	0.042"
Thickness Tolerance	-0.002" to 0.002"
Min. Hardness	Rockwell C43
Thrust Load Capacity	5,020 lbs.
Magnetic Properties	Magnetic
Specifications Met	ASME B18.27.1
RoHS	RoHS 3 (2015/863/EU) Compliant
REACH	REACH (EC 1907/2006) (06/25/2020, 209 SVHC) Compliant
DFARS	Specialty Metals Compliant (252.225-7009)
Country of Origin	United States

Fig. D1.2: McMaster Carr for 1.000 Inch Shaft

Appendix D2: Coupling Analysis

The coupling calculations were based from the LoveJoy Power Transmission Products 2016 Catalogue, Page 18 “Steps in Selection a Jaw Coupling”. Based on existing designs of Wind turbines, an elastomer jaw coupling was used to account for variations between the gear shaft and the motor shaft.

For the gear shaft:

Power	4.845130427	RPM	1774.69
-------	-------------	-----	---------

These values were taken from the gear analysis section in Appendix A.

Now, the nominal torque is calculated from

$$Nominal Torque = \frac{HP * 63025}{RPM} \quad \text{(LoveJoy Power Transmission Products 2016 Catalogue, Page 18)}$$

By adding the values presented earlier, we can see that the nominal torque is:

$$172.066(in - lb) = \frac{4.875 * 63025}{1774.69}$$

From the Application Service Factors Chart (LoveJoy Power Transmission Products 2016 Catalogue, Page 20)(Figure D2.1), the service factor is chosen to be 1, as the application is a non-welding generator.

Application Service Factors

Chart 1

	Service Factors						Service Factors						Service Factors				
	Electric Motor w/ Standard Torque	Electric Motor w/ High Torque	Steam Turbines & En- gines w/ or more cyl*	Reciprocating Engines*			Electric Motor w/ Standard Torque	Electric Motor w/ High Torque	Steam Turbines & En- gines w/ or more cyl*	Reciprocating Engines*			Electric Motor w/ Standard Torque	Electric Motor w/ High Torque	Steam Turbines & En- gines w/ or more cyl*	Reciprocating Engines*	
				1-Cyl	2-Cyl					1-Cyl	2-Cyl					1-Cyl	2-Cyl
Agitators	1.00	1.25	1.00	1.7	1.3	Feeders						Beater, Pulper,					
Band Resaw (lumber).....	1.50	1.75	1.50	2.2	1.8	Belt, Screw.....	1.00	1.25	1.00	1.7	1.3	Jordans, Dresses.....	2.00	2.25	2.00	2.7	2.3
Barge Haul Puller	2.00	2.25	2.00	2.7	2.3	Reciprocating.....	2.50	2.75	2.50	3.2	2.8	Calenders, Dryers, Washers,					
Beaters	1.50	1.75	1.50	2.2	1.8	Filter, Press-oil	1.50	1.75	1.50	2.2	1.8	Thickener.....	1.50	1.75	1.50	2.2	1.8
Blowers						Generators						Converting Machines,					
Centrifugal.....	1.00	1.25	1.00	1.7	1.3	Not Welding.....	1.00	1.25	1.00	1.7	1.3	Conveyors.....	1.20	1.45	1.20	1.9	1.5
Lobe, Vane.....	1.25	1.50	1.25	2.0	1.6	Welding.....	2.00	2.25	2.00	2.7	2.3	Printing Presses	1.50	1.75	1.50	1.7	1.3
Bottling Machinery	1.25	1.50	1.25	2.0	1.6	Hoist.....	1.50	1.75	1.50	2.2	1.8	Pug Mill	1.75	2.00	1.75	2.0	1.6
Brew Kettles (distilling).....	1.25	1.50	1.25	2.0	1.6	Hammermills	2.00	2.25	2.00	2.7	2.3	Pumps					
Can Filling Machinery	1.00	1.25	1.00	1.7	1.3	Kilns	1.50	1.75	1.50	2.2	1.8	Centrifugal.....	1.00	1.25	1.00	1.7	1.3
Car Dumpers	2.50	2.75	2.50	3.2	2.8	Laundry Washers —						Gear, Rotary, Vane.....	1.25	1.50	1.25	2.0	1.6
Car Pullers	1.50	1.75	1.50	2.2	1.8	Reversing.....	2.00	2.25	2.00	2.7	2.3	Reciprocating:					
Card Machine	1.75	2.00	1.75	2.5	2.0	Lumber Machinery						1-Cyl. Single or					
Chiller (oil).....	1.50	2.00	1.25	2.0	2.0	Barkers, Edger Feeder,						Double Acting.....	2.00	2.25	2.00	2.7	2.3
Compressors						Live Roll.....	2.00	2.25	2.00	2.7	2.3	2-Cyl. Single Acting.....	2.00	2.25	2.00	2.7	2.3
Centrifugal.....	1.00	1.25	1.00	1.7	1.3	Planer, Slab Conveyor.....	2.00	2.25	2.00	2.7	2.3	2-Cyl. Double Acting.....	1.75	2.00	1.75	2.5	2.0
Screw, Lobe.....	1.25	1.50	1.25	2.0	1.6	Machine Tools						3 or more Cyl.....	1.50	1.75	1.50	2.2	1.8
Reciprocating.....	See Note					Punch Press-gear Driven,						Rubber Machinery					
Conveyors, Uniformly Fed						Plate Planer.....	2.00	2.25	2.00	2.7	2.3	Mixers.....	2.50	2.75	2.50	3.2	2.8
Assembly, Belt, Screw.....	1.00	1.25	1.00	1.7	1.3	Tapping Machinery,						Rubber Calender.....	2.00	2.25	2.00	2.7	2.3
Bucket, Sawdust.....	1.25	1.50	1.25	2.0	1.6	Bending Roll.....	2.00	2.25	2.00	2.7	2.3	Screens					
Live Roll, Shaker,						Main Drive.....	1.50	1.75	1.50	2.2	1.8	Air washing, Water.....	1.00	1.25	1.00	1.7	1.3
Reciprocating.....	3.00	3.25	3.00	3.7	3.3	Auxiliary Drives.....	1.00	1.25	1.00	1.7	1.3	Rotary—stone or gravel,					
Conveyors, Not Uniformly Fed						Metal Forming Machines						Dewatering.....	1.50	1.75	1.50	2.2	1.8
Assembly, Belt,						Draw Bench-carriage						Vibrating.....	2.50	2.75	2.50	3.2	2.8
Oven, Screw.....	1.20	1.45	1.20	1.9	1.5	& Main Drive.....	2.00	2.25	2.00	2.7	2.3	Grizzly.....	2.00	2.25	2.00	2.7	2.3
Reciprocating.....	2.50	2.75	2.50	3.2	2.8	Extruder, Forming Machine,						Shredders	1.50	1.75	1.50	2.2	1.8
Shaker.....	3.00	3.25	3.00	3.7	3.3	Wire Drawing.....	2.00	2.25	2.00	2.7	2.3	Steering Gears	1.00	1.25	1.00	1.7	1.3
Cookers —Brewing, Distilling,						Table Conveyors.....	2.50	2.75	2.50	3.2	2.8	Stokers	1.00	1.25	1.00	1.7	1.3
Food.....	1.25	1.50	1.25	2.0	1.6	Wire Winding, Coilers,						Suction Roll (paper).....	1.50	1.75	1.50	2.2	1.8
Cranes & Hoist ¹	2.00	2.25	2.00	2.7	2.3	Slitters.....	1.50	1.75	1.50	2.2	1.8	Textile Machinery					
Crushers —Cane (sugar), Stone, or Ore						Mills, Rotary Type						Dryers, Dyeing Machinery,					
.....	3.00	3.25	3.00	3.7	3.3	Ball, Kilns, Pebble,						Mangle.....	1.20	1.45	1.20	2.0	1.6
Dredges						Rolling, Tube.....	2.00	2.25	2.00	2.7	2.3	Loom, Spinner,					
Cable reels.....	2.00	2.25	2.00	2.7	2.3	Cement Kilns,						Tenter frames.....	1.50	1.75	1.50	2.2	1.8
Conveyors, Pumps,						Dryers, Coolers.....	2.00	2.25	2.00	2.7	2.3	Tumbling Barrels	1.75	2.00	1.75	2.5	2.0
Maneuvering Winches.....	1.50	1.75	1.50	2.2	1.8	Tumbling.....	1.50	1.75	1.50	2.2	1.8	Windlass	2.00	2.25	2.00	2.7	2.3
Cutter Head Drives.....	2.50	2.75	2.50	3.2	2.8	Mixers						Woodworking Mach.	1.00	1.25	1.00	1.7	1.3
Dynamometer	1.50	1.75	1.50	2.2	1.8	Concrete, continuous.....	1.75	2.00	1.75	2.5	2.0						
Evaporators	1.00	1.25	1.00	1.7	1.3	Muller.....	1.50	1.75	1.50	2.2	1.8						
Fans						Paper Mills											
Centrifugal.....	1.00	1.25	1.00	1.7	1.3	Agitator (mixers),											
Cooling Towers.....	2.00	2.25	2.00	2.7	2.3	Reel, Winder.....	1.20	1.45	1.20	1.9	1.5						
Forced Draft,						Winder.....	1.20	1.45	1.20	1.9	1.5						
Propeller.....	1.50	1.75	1.50	2.2	1.8	Barker (mechanical),											
Induced draft						Log Haul, Chipper.....	2.00	2.25	2.00	2.7	2.3						
w/damper control.....	2.00	2.25	2.00	2.7	2.3	Barking Drum											
Induced draft w/o						(spur gear).....	2.50	2.75	2.50	3.2	2.8						
damper control.....	1.25	1.50	1.25	2.0	1.6												

Caution: Applications involving reciprocating engines and reciprocating driven devices are subject to critical rotational speeds which may damage the coupling and/or connected equipment. Contact Lovejoy Engineering with specific requirements.

Note: ■ 1 indicates: If people are transported, Lovejoy does not recommend and will not warranty the use of the coupling.

Fig. D2.1: LoveJoy Power Transmission Products 2016 Catalogue Application Service Factors

Now, given the service factor and the nominal torque, the design torque can be calculated from

$$\text{Design Torque} = \text{Nominal Torque} * \text{Application Service Factor} \quad (\text{LoveJoy Power Transmission Products 2016 Catalogue, Page 18})$$

From the formula,

$$172.066(\text{in} - \text{lb}) = 172.066 * 1$$

Now, selecting from the Spider Performance Data (LoveJoy Power Transmission Products 2016 Catalogue, Page 21) (Figure D2.2), we can choose the material of the elastomer. Given the parameters outlined in the table, we choose SOX NBR Rubber.

Spider Performance Data

Chart 2

Characteristics	Temperature Range	Misalignment		Shore Hardness ¹	Dampening Capacity	Chemical Resistance ²	Color
		Angular Degree	Parallel Inch				
SOX (NBR) Rubber – Nitrile Butadiene (Buna N) Rubber is a flexible elastomer material that is oil resistant, resembles natural rubber in resilience and elasticity and operates effectively in temperature range of -40° to 212° F (-40° to 100° C). Good resistance to oil. Standard elastomer. (Also applies to SXB Cushions.)	-40° to 212° F -40° to 100° C	1°	.015	80A	HIGH	GOOD	BLACK
URETHANE – Urethane has greater torque capability than NBR (1.5 times), provides less dampening effect, and operates at a temperature range of -30° to 160° F (-34° to 71° C). Good resistance to oil and chemicals. Not recommended for cyclic or start/stop applications.	-30° to 160° F -34° to 71° C	1°	.015	55D L050-L110 90-95A L150-L225	LOW	VERY GOOD	BLUE
HYTREL® – Hytrel is a flexible elastomer designed for high torque and high temperature operations. Hytrel can operate in temperatures of -60° to 250° F (-51° to 121° C) and has an excellent resistance to oil and chemicals. Not recommended for cyclic or start/stop applications.	-60° to 250° F -51° to 121° C	1/2°	.015	55D	LOW	EXCELLENT	TAN
BRONZE – Bronze is a rigid, porous oil-impregnated metal insert exclusively for slow speed (maximum 250 RPM) applications requiring high torque capabilities. Bronze operations are not affected by extreme temperatures, water, oil, or dirt.	-40° to 450° F -40° to 232° C	1/2°	.010	—	NONE	EXCELLENT	BRONZE

Notes: ■ 1 indicates: NBR standard shore hardness is 80A ±5A – Except L035=60A. Other softer or harder designs are available in NBR material; consult Lovejoy.
■ 2 indicates: Chemical Resistance chart shown in Engineering Data Section (page ED-9).

Fig. D2.2: LoveJoy Power Transmission Products 2016 Catalogue Spider Performance Data

Now, selecting from the Jaw Nominal Rated Torque Chart 3 (LoveJoy Power Transmission Products 2016 Catalogue, Page 21) (Figure D2.3), we can choose the size of the coupling based on the chosen material, SOX (NBR), the design torque, and the max bore size in this circumstance. In this instance, we choose L/AL095.

In order to find the UPC number for the coupling, we refer to Figure D2.3, and choose a solid core for the coupling. This yields a UPC number of 11070.

L Type Spider UPC Number Selection Table

Spider Type	Coupling Size										
	L035	L050	L070	L075	L090/095	L099/100	L110	L150	L190	L225	L276
SOX (NBR) (Solid)	10118	10194	10406	10621	11070	11494	11724	12001	12274	12409	—
SOX (NBR) (open center)	—	—	10393	10620	10968	11492	11711	37880	37881	12406	12612
Urethane (Solid)	—	37786	10395	—	—	—	—	—	—	12417	—
Urethane (open center)	—	—	10411	10626	11075	11499	11729	12006	12280	—	—
Hytrel® (Solid)	—	25307	—	—	—	—	11717	11993	12265	12401	—
Hytrel® (open center)	—	—	25308	25309	25310	11486	38097	38098	38099	12400	—
Bronze (open center)	—	10198	10409	10624	11073	11497	11727	12004	12277	34517	25767
Snap Wrap (NBR) w/ring	—	—	—	—	24669	24670	24671	24672	24673	—	—
Snap Wrap (NBR) w/o ring	—	—	—	—	11071	11495	11725	12002	12275	—	—
SOX (NBR) Bulk - pk 25	50115	50116	50117	50118	50119	—	—	—	—	—	—
SOX (NBR) Bulk - pk 10	—	—	—	—	—	50120	50121	50122	—	—	—
Snap Wrap Urethane - solid ring	—	—	—	—	—	41170	41171	—	28284	26093	—
In-Shear Elastomer	—	—	—	—	71706	71707	71708	71709	71710	71711	71712
In-Shear Ring	—	—	—	—	71679	71680	71681	71682	71683	71684	71685

Fig. D2.3: LoveJoy Power Transmission Products 2016 Catalogue L Type Spider UPC Number Selection Table

Jaw Nominal Rated Torque Data

Chart 3

Size	Max Bore		Spider Material							
			SOX (NBR) Torque		Urethane Torque		Hytrel Torque		Bronze Torque	
	in	mm	in-lbs	Nm	in-lbs	Nm	in-lbs	Nm	in-lbs	Nm
L035	0.375	9	3.5	0.4	—	—	—	—	—	—
L/AL050	0.625	16	26.3	3.0	39	4.5	50	5.60	50	5.60
L/AL070	0.750	19	43.2	4.9	65	7.3	114	12.90	114	12.90
L/AL075	0.875	22	90.0	10.2	135	15.3	227	25.60	227	25.60
L/AL090	1.000	25	144.0	16.3	216	24.4	401	45.30	401	45.30
L/AL095	1.125	28	194.0	21.9	291	32.9	561	63.40	561	63.40
L/AL099	1.188	30	318.0	35.9	477	53.9	792	89.50	792	89.50
L/AL100	1.375	35	417.0	47.1	626	70.7	1,134	128.00	1,134	128.00
L/AL110	1.625	42	792.0	89.5	1,188	134.0	2,268	256.00	2,268	256.00
L150	1.875	48	1,240.0	140.0	1,860	210.0	3,708	419.00	3,706	419.00
AL150	1.875	48	1,450.0	163.8	—	—	—	—	—	—
L190	2.125	55	1,728.0	195.0	2,592	293.0	4,680	529.00	4,680	529.00
L225	2.625	65	2,340.0	264.0	3,510	397.0	6,228	704.00	6,228	704.00
L276	2.875	73	4,716.0	533.0	—	—	—	—	12,500	1 412.00
C226	2.500	64	2,988.0	338.0	—	—	5,940	671.00	5,940	671.00
C276	2.875	73	4,716.0	533.0	—	—	9,432	1 066.00	—	—
C280	3.000	76	7,560.0	854.0	—	—	13,866	1 567.00	—	—
C285	4.000	102	9,182.0	1 038.0	—	—	16,680	1 882.00	—	—
C295	3.500	89	11,340.0	1 281.0	—	—	22,680	2 563.00	22,680	2 563.00
C2955	4.000	102	18,900.0	2 136.0	—	—	37,800	4 271.00	37,800	4 271.00
H3067	4.500	114	33,395.0	3 774.0	—	—	47,196	5 333.00	47,196	5 333.00
H3567	5.000	127	46,632.0	5 269.0	—	—	63,000	7 119.00	63,000	7 119.00
H3667	5.629	143	64,812.0	7 323.0	—	—	88,200	9 966.00	88,200	9 966.00
H4067	6.250	159	88,224.0	9 969.0	—	—	126,000	14 237.00	126,000	14 237.00
H4567	7.000	178	119,700.0	13 525.0	—	—	170,000	19 209.00	170,000	19 209.00

Note: ■ Bronze has a maximum RPM capability of 250 RPM.

Fig. D2.4: LoveJoy Power Transmission Products 2016 Catalogue Jaw Nominal Rated Torque Data

At this point in time, we use the Jaw Type Performance Ratings(LoveJoy Power Transmission Products 2016 Catalogue, Page 34) (Figure D2.4), to ensure that our chosen coupling is able to

Step-up Gear Train for Electric Wind Turbine

withstand the torque and RPM applied to it. Looking at the table, we can see that there is no column for our current rated RPM at 1774.69 RPM. Through extrapolation:

$$\frac{1800-1200}{5.5-3.72} = \frac{1800-1774.69}{5.5-x}$$

$$x=5.4\text{HP}$$

Given that 5.4HP>4.4HP, we can see that this coupling is suitable for our needs. The maximum torque rating for this is 194 in-lbs, which is larger than 172.066 in-lbs at which our system is going to run at.

Jaw Type Performance Ratings

Size	Elastomeric Member	Number of Jaws	Basic HP Ratings @ Varying RPM				Torque Rating		Max Bore in mm		Max x1000 RPM
			100	1200	1800	3600	in-lbs	Nm			
L, AL & LC Type											
L035	SOX (NBR)	2	0.006	0.07	0.10	0.22	3.5	0.40	0.375	9	31.0
L050/AL050	SOX (NBR)	2	0.042	0.50	0.75	1.51	26.3	2.97	0.625	16	18.0
L050/AL050	Hytrel®	2	0.080	0.96	1.43	2.88	50.0	5.65	0.625	16	18.0
L070/AL070	SOX (NBR)	2	0.070	0.84	1.23	2.52	43.2	4.88	0.750	19	14.0
L070/AL070	Hytrel	2	0.180	2.16	3.26	6.48	114.0	12.88	0.750	19	3.6
L075/AL075	SOX (NBR)	3	0.140	1.68	2.57	5.04	90.0	10.17	0.875	22	11.0
L075/AL075	Hytrel	3	0.360	4.32	6.48	12.96	227.0	25.65	0.875	22	3.6
L090/AL090/LC090	SOX (NBR)	3	0.230	2.76	4.11	8.28	144.0	16.27	1.000	25	9.0
L090/AL090	Hytrel	3	0.640	7.68	11.50	23.04	401.0	42.31	1.000	25	3.6
L095/AL095/LC095	SOX (NBR)	3	0.310	3.72	5.50	11.16	194.0	21.92	1.125	32	9.0
L095/AL095	Hytrel	3	0.890	10.68	16.00	32.04	561.0	63.38	1.125	32	3.6
L099/AL099/LC099	SOX (NBR)	3	0.500	6.00	9.10	18.00	318.0	35.93	1.180	30	7.0
L099/AL099	Hytrel	3	1.260	15.12	22.60	45.36	792.0	89.48	1.180	30	3.6
L100/AL100/LC100	SOX (NBR)	3	0.660	7.92	11.90	23.76	417.0	47.11	1.380	35	7.0
L100/AL100	Hytrel	3	1.800	21.60	32.40	64.80	1,134.0	128.12	1.380	35	3.6
L110/AL110/LC110	SOX (NBR)	3	1.260	15.12	23.00	45.36	792.0	89.48	1.620	42	5.0
L110/AL110	Hytrel	3	3.600	43.20	65.00	129.60	2,268.0	256.25	1.620	42	5.0
L150/LC150	SOX (NBR)	3	2.000	24.00	35.00	72.00	1,240.0	140.10	1.880	48	5.0
L150	Hytrel	3	5.900	70.80	106.00	212.40	3,708.0	418.95	1.880	48	5.0
AL-150	SOX (NBR)	4	2.300	27.60	41.40	82.80	1,450.0	163.83	1.880	48	5.0
L190/LC190	SOX (NBR)	3	2.700	32.40	49.00	97.20	1,728.0	195.24	2.120	55	5.0
L190	Hytrel	3	7.400	88.80	134.00	266.40	4,680.0	528.77	2.120	55	5.0
L225/LC225	SOX (NBR)	3	3.700	44.40	67.00	133.20	2,340.0	264.38	2.620	65	4.2
L225	Hytrel	3	9.900	118.80	178.00	356.40	6,228.0	703.67	2.620	65	4.2
L276	SOX (NBR)	3	7.500	90.00	135.00	+	4,716.0	532.84	2.880	73	1.8

Fig. D2.5: LoveJoy Power Transmission Products 2016 Catalogue Jaw Type Performance Ratings

From the Jaw Nominal Rated Torque Chart 3 (LoveJoy Power Transmission Products 2016 Catalogue, Page 21) (Figure D2.5), we can see that the maximum bore is 1.25in. This is greater than both the shaft of the motor as well as the shaft of the gear. At this point, the viability of this coupling is determined.

From the Jaw Nominal Rated Torque Chart 3 (LoveJoy Power Transmission Products 2016 Catalogue, Page 23-24) (Figure D2.5), we can choose the bore and keyway sizes required. For the motor shaft, which is 5/8 in in diameter, we can see that the 41911 is the most suitable. For the gear shaft, we have chosen 35747 in accordance with the size of the key.

Bore	Keyway	L035	L050	L070	L075	L090	L095	L099	L100	L110	L150	L190	L225	L276
1/8	No Keyway	10124	—	—	—	—	—	—	—	—	—	—	—	—
3/16	No Keyway	10126	—	—	—	—	—	—	—	—	—	—	—	—
1/4	No Keyway	10127	10206	10416	10680	10766	—	—	—	—	—	—	—	—
1/4	1/8 x 1/16	—	—	—	35744	—	—	—	—	—	—	—	—	—
5/16	No Keyway	10128	10207	10417	10681	26209	—	—	—	—	—	—	—	—
3/8	No Keyway	24687	10208	10418	10682	10767	—	—	—	—	—	—	—	—
3/8	3/32 x 3/64	—	46121	41985	37234	37235	—	—	—	—	—	—	—	—
3/8	1/8 x 1/16	—	44136	48829	35745	37236	—	—	—	—	—	—	—	—
7/16	No Keyway	—	10209	10419	10683	10768	11082	31297	11505	—	—	—	—	—
7/16	3/32 x 3/64	—	44713	44007	28089	28877	27613	38198	37237	—	—	—	—	—
7/16	1/8 x 1/16	—	—	44066	28875	28878	28879	38199	37238	—	—	—	—	—
1/2	No Keyway	—	10210	10420	10684	10769	11083	11333	11506	—	—	—	—	—
1/2	1/8 x 1/16	—	10211	10421	10685	26087	26088	11334	26089	—	—	—	—	—
9/16	No Keyway	—	10212	52338	10686	24976	37239	11335	11508	—	—	—	—	—
9/16	1/8 x 1/16	—	10213	10423	10687	28876	11084	38200	11509	—	—	—	—	—
5/8	No Keyway	—	10214	24771	44322	46052	41911	44174	44291	11733	12101	—	—	—
5/8	5/32 x 5/64	—	—	51104	37240	37241	37242	38201	37243	37244	37245	—	—	—
5/8	3/16 x 3/32	—	—	10424	10688	10771	11085	11336	11510	26211	26212	—	—	—
11/16	3/16 x 3/32	—	—	10425	10689	10772	11086	11337	11511	11734	12102	—	—	—
3/4	No Keyway	—	—	46116	56140	54282	56887	49705	45212	—	—	12285	12422	—
3/4	1/8 x 1/16	—	—	51719	35881	37246	37074	38202	37247	37248	37249	37250	—	—
3/4	3/16 x 3/32	—	—	10426	10690	10773	11087	11338	11512	11735	12103	38468	35882	—
13/16	3/16 x 3/32	—	—	—	10691	10774	11088	11339	11513	11736	12104	37252	37255	—
7/8	No Keyway	—	—	—	56941	—	—	59063	—	—	—	—	—	12582
7/8	3/16 x 3/32	—	—	—	10692	10775	11089	11340	11514	11737	12105	12286	12423	12585
7/8	1/4 x 1/8	—	—	—	—	38188	35747	38203	35686	35749	35750	37256	35753	54883
15/16	1/4 x 1/8	—	—	—	—	32332	11090	11341	11515	11738	12160	12287	12424	—
1	1/4 x 1/8	—	—	—	—	31296	11091	11342	11516	11739	12107	12288	12425	12586
1	3/16 x 3/32	—	—	—	—	37257	37258	38204	37259	37260	37261	37262	37263	—
1-1/16	1/4 x 1/8	—	—	—	—	—	11092	11343	11517	11740	12108	12289	12426	—
1-1/8	1/4 x 1/8	—	—	—	—	—	11093	11344	11518	11741	12109	12290	12427	12587
1-3/16	1/4 x 1/8	—	—	—	—	—	—	11345	11519	11742	12110	12291	12428	—
1-1/4	1/4 x 1/8	—	—	—	—	—	—	—	11520	11743	12111	12292	12429	12588
1-1/4	5/16 x 5/32	—	—	—	—	—	—	—	35748	35752	35751	37294	35754	12589
1-5/16	5/16 x 5/32	—	—	—	—	—	—	—	11521	11744	12112	12293	26090	—
1-3/8	5/16 x 5/32	—	—	—	—	—	—	—	11522	11745	12113	12294	12430	12590
1-3/8	3/8 x 3/16	—	—	—	—	—	—	—	44348	37265	37266	37267	37568	46758
1-7/16	3/8 x 3/16	—	—	—	—	—	—	—	—	11746	12114	12295	12431	12591
1-1/2	5/16 x 5/32	—	—	—	—	—	—	—	—	37269	37270	37271	37272	—
1-1/2	3/8 x 3/16	—	—	—	—	—	—	—	—	11747	12115	12296	12432	12592
1-9/16	3/8 x 3/16	—	—	—	—	—	—	—	—	11748	12116	37273	12433	45689
1-5/8	3/8 x 3/16	—	—	—	—	—	—	—	—	11749	12117	12297	12434	12593
1-11/16	3/8 x 3/16	—	—	—	—	—	—	—	—	—	12118	12298	12435	60057
1-3/4	3/8 x 3/16	—	—	—	—	—	—	—	—	—	12119	12299	12436	12594
1-3/4	7/16 x 7/32	—	—	—	—	—	—	—	—	—	37274	37275	37276	48250
1-13/16	1/2 x 1/4	—	—	—	—	—	—	—	—	—	12120	12300	26091	—
1-7/8	1/2 x 1/4	—	—	—	—	—	—	—	—	—	12121	12301	12437	12595
1-15/16	1/2 x 1/4	—	—	—	—	—	—	—	—	—	—	12302	12438	49762
2	1/2 x 1/4	—	—	—	—	—	—	—	—	—	—	12303	12439	12596
2-1/16	1/2 x 1/4	—	—	—	—	—	—	—	—	—	—	12304	26092	—
2-1/8	1/2 x 1/4	—	—	—	—	—	—	—	—	—	—	12305	12440	12597
2-3/16	1/2 x 1/4	—	—	—	—	—	—	—	—	—	—	—	12441	12598
2-1/4	1/2 x 1/4	—	—	—	—	—	—	—	—	—	—	—	12442	12599
2-3/8	5/8 x 5/16	—	—	—	—	—	—	—	—	—	—	—	12443	12602
2-5/8	5/8 x 5/16	—	—	—	—	—	—	—	—	—	—	—	41809	12605
2-7/8	3/4 x 3/8	—	—	—	—	—	—	—	—	—	—	—	—	12607

Notes: ■ Tolerances for bore and keyways are found in Engineering Data section (pages ED-10 and ED-11). All hubs supplied standard with one set screw.
 ■ Non-standard bores available – consult Lovejoy Engineering.
 ■ When referencing the Lovejoy UPC number in this table, include 685144 as a prefix to the number shown.

Fig. D2.6: LoveJoy Power Transmission Products 2016 Catalogue Jaw Nominal Rated Torque

At the end of the day, the coupling chosen is the Lovejoy Jaw Type Coupling Size L/AL095: SOX Rubber 11070-35747 and 11070-41911

Appendix D3: Keys

The keys chosen were dependent on the selection of the gears and couplings. For each type of part, the keys were specified. First, the gears were analyzed by using the torque and shaft

diameters from the previous sections. The following table shows the various values which were used.

Table D1: Table Parameters

Gear Torque			Shaft Diameters
Gear 1	110.64	Lb*in	1
Gear 2	39.83	Lb*in	0.875
Gear 3	39.83	Lb*in	1
Gear 4	14.34	Lb*in	0.875

Then, by using these shaft diameters and referring to Mott's Machine Elements in Mechanical Design Table 11-1, we found the shaft width and height.

TABLE 11-1 Key Size vs. Shaft Diameter							
U.S. inch sizes				SI metric sizes			
Nominal shaft diameter		Key dimensions		Nominal shaft diameter		Key dimensions	
Over (in)	to-including (in)	Width, <i>W</i> (in)	Height, <i>H</i> (in)	Over (mm)	to-including (mm)	Width, <i>W</i> (mm)	Height, <i>H</i> (mm)
0.3125	0.4375	0.09375	0.09375	6	8	2	2
0.4375	0.5625	0.1250	0.1250	8	10	3	3
0.5625	0.875	0.1875	0.1875	10	12	4	4
0.875	1.250	0.2500	0.2500	12	17	5	5
1.250	1.375	0.3125	0.3125	17	22	6	6
1.375	1.75	0.375	0.375	22	30	8	7
1.75	2.25	0.500	0.500	30	38	10	8
2.25	2.75	0.625	0.625	38	44	12	8
2.75	3.25	0.750	0.750	44	50	14	9
3.25	3.75	0.875	0.875	50	58	16	10
3.75	4.50	1.00	1.00	58	65	18	11
4.50	5.50	1.25	1.25	65	75	20	12
5.50	6.50	1.50	1.50	75	85	22	14
6.50	7.50	1.75	1.50	85	95	25	14
7.50	9.00	2.00	1.50	95	110	28	16
9.00	11.00	2.50	1.75	110	130	32	18
11.00	13.00	3.00	2.00	130	150	36	20
13.00	15.00	3.50	2.50	150	170	40	22
15.00	18.00	4.00	3.00	170	200	45	25
18.00	22.00	5.00	3.50	200	230	50	28
22.00	26.00	6.00	4.00	230	260	56	32
26.00	30.00	7.00	5.00	260	290	63	32
				290	330	70	36
				330	380	80	40
				380	440	90	45
				440	500	100	50

Note: Key sizes above the horizontal line are square; others are rectangular.

Fig. D3.1: Mott Table 11-1 Key Size VS Shaft Diameter

Based on the shaft diameters, we produce the table:

Table D2: Gears and Key Sizes

Gear Torque			Shaft Diameters	Key Width (in)	Key Height (in)
Gear 1	110.64	Lb*in	1	0.25	0.25
Gear 2	39.83	Lb*in	0.875	0.1875	0.1875
Gear 3	39.83	Lb*in	1	0.25	0.25
Gear 4	14.34	Lb*in	0.875	0.1875	0.1875

Now, because the 1018 steel which the keys are made of are stronger than the steel the gears are made of, we must use the following formulas from Mott's Machine Elements in Mechanical Design. The factors used:

Table D3: Key Parameters

Key Material	1018 Carbon Steel
Tensile Strength (ksi)	64
Yield Strength (ksi)	54
Design Factor (N)	3

For the minimum required key length for shear:

$$L_{min} = \frac{2T}{T_d * D * W} \quad (\text{Mott, Eq. 11-2})$$

Where T_d is:

$$T_d = 0.5 * s_y / N \quad (\text{Mott 11})$$

For the minimum required key length for compression:

$$L_{min} = \frac{4T}{\sigma_d * D * H} \quad (\text{Mott, Eq. 11-3})$$

Where σ_d is:

$$\sigma_d = \frac{s_y}{N} \quad (\text{Mott, Eq. 11-4})$$

Finally, from these calculations we find the following values:

Table D4: Key Selection

Gear 1 Analysis					
Lmin Compression	0.0983	Inches	Gear 2 Analysis		
Lmin Shear	0.0983	Inches	Lmin Compression	0.054	Inches
Shear Stress Theory	9000		Lmin Shear	0.054	
Hub Length	2.13	Inch	Hub Length	1.87	Inch
Spec Length	1.75	Inch	Spec Length	1.5	Inch
Gear 3 Analysis			Gear 4 Analysis		
Lmin Compression	0.0354	Inches	Lmin Compression	0.0194	Inches
Lmin Shear	0.0354	Inches	Lmin Shear	0.0194	Inches
Hub Length	2.13	Inch	Hub Length	1.87	Inch
Spec Length	1.75	Inch	Spec Length	1.5	Inch

Now, we looked at the hub length of each gear, and made sure that the spec length both did not exceed the hub length nor the minimum length for the key. We chose a spec length for each of the keys based on these criteria.