$$\Rightarrow \left| \ell(\varepsilon+t) - \ell(t) - \varepsilon \mathbb{V}(\ell(t)) \right| < \varepsilon^2 C$$

$$\Rightarrow \left| \varphi_{\varepsilon+t}(\mathbb{p}) - \varphi_t(\mathbb{p}) - \varepsilon \mathbb{V}(\varphi_t(\mathbb{p})) \right| < \varepsilon^2 C$$

$t=0$ として

$$\Rightarrow \left| \varphi_\varepsilon(\mathbb{p}) - (\mathbb{p} + \varepsilon \mathbb{V}(\mathbb{p})) \right| < \varepsilon^2 C 。$$

ε が十分小のとき $\varphi_\varepsilon(\mathbb{p}) \fallingdotseq (p + \varepsilon \mathbb{V}(p))$

$$[\mathbb{V}, \mathbb{W}](\mathbb{p}) = \lim_{\varepsilon \to 0} \frac{D\varphi_{-\varepsilon} \mathbb{W}(p + \varepsilon \mathbb{V}(p)) - \mathbb{W}(p)}{\varepsilon}$$

$$= \lim_{\varepsilon \to 0} \frac{D\varphi_{-\varepsilon} \frac{d(p + \varepsilon \mathbb{V}(p))}{dt} - \mathbb{W}(p)}{\varepsilon}$$

$$= \lim_{\varepsilon \to 0} \frac{D\varphi_{-\varepsilon}\left(\mathbb{W}(p) + \varepsilon \frac{d\mathbb{V}(p)}{dt} - \mathbb{W}(p)\right)}{\varepsilon}$$

$$= D\varphi_0 \frac{d\mathbb{V}}{dt}(p) + \lim_{\varepsilon \to 0} \frac{D\varphi_{-\varepsilon} \mathbb{W}(p) - \mathbb{W}(p)}{\varepsilon}$$

$$= D\varphi_0 \frac{d^2 p}{dt} + \lim_{\varepsilon \to 0} \frac{D\varphi_{-\varepsilon} \mathbb{W}(p) - \mathbb{W}(p)}{\varepsilon}$$

$$= D\varphi_0 \frac{d\mathbb{W}}{dt}(p) + \lim_{\varepsilon \to 0} \frac{D\varphi_{-\varepsilon} \mathbb{W}(p) - \mathbb{W}(p)}{\varepsilon}$$

$$= \frac{d\mathbb{W}}{dt}(p) + \lim_{\varepsilon \to 0} \frac{D\varphi_{-\varepsilon} \mathbb{W}(p) - \mathbb{W}(p)}{\varepsilon}$$

$$= \sum_{i=1} \frac{d\mathbb{W}}{dx_i} \frac{dp^i}{dt} + \lim_{\varepsilon \to 0} \frac{D\varphi_{-\varepsilon} \mathbb{W}(p) - \mathbb{W}(p)}{\varepsilon}$$

$$= \sum_{i=1} \frac{d\mathbb{W}}{dx_i} V^i(p) + \lim_{\varepsilon \to 0} \frac{D\varphi_{-\varepsilon} \mathbb{W}(p) - \mathbb{W}(p)}{\varepsilon} \quad (2.30)$$

Δ

$$-\frac{d\mathfrak{z}}{dl} = \lim_{\varepsilon \to 0} \frac{\mathfrak{g}(0-\varepsilon) - \mathfrak{g}(0)}{\varepsilon} = \lim_{\varepsilon \to 0} \frac{\varphi_{-\varepsilon}(\mathfrak{g}) - \mathfrak{g}}{\varepsilon} = -\mathbb{V}(\mathfrak{g})$$

両辺を χ^i で偏微分。