微分形式

$$u = B_1\, dx^2\wedge dx^3 + B_2\, dx^3\wedge dx^1 + B_3\, dx^1\wedge dx^2 + E_1\, dx^1\wedge dt + E_2\, dx^2\wedge dt + E_3\, dx^3\wedge dt$$

について。

$$\begin{cases} \mathrm{rot}\, \boldsymbol{E} = -\dfrac{d\boldsymbol{B}}{dt} & \text{電磁誘導} \\ \mathrm{div}\, \boldsymbol{B} = 0 & \text{ガウス} \end{cases}$$

マクスウェルの方程式 ⟺

$$du = \left(\frac{\partial B_1}{\partial x^1}dx^1 + \frac{\partial B_1}{\partial x^2}dx^2 + \frac{\partial B_1}{\partial x^3}dx^3 + \frac{\partial B_1}{\partial t}dt\right)\wedge dx^2\wedge dx^3$$

$$+\left(\frac{\partial B_2}{\partial x^1}dx^1 + \frac{\partial B_2}{\partial x^2}dx^2 + \frac{\partial B_2}{\partial x^3}dx^3 + \frac{\partial B_2}{\partial t}dt\right)\wedge dx^3\wedge dx^1$$

$$+\left(\frac{\partial B_3}{\partial x^1}dx^1 + \frac{\partial B_3}{\partial x^2}dx^2 + \frac{\partial B_3}{\partial x^3}dx^3 + \frac{\partial B_3}{\partial t}dt\right)\wedge dx^1\wedge dx^2$$

$$+\left(\frac{\partial E_1}{\partial x^1}dx^1 + \frac{\partial E_1}{\partial x^2}dx^2 + \frac{\partial E_1}{\partial x^3}dx^3 + \frac{\partial E_1}{\partial t}dt\right)\wedge dx^1\wedge dt$$

$$+\left(\frac{\partial E_2}{\partial x^1}dx^1 + \frac{\partial E_2}{\partial x^2}dx^2 + \frac{\partial E_2}{\partial x^3}dx^3 + \frac{\partial E_2}{\partial t}dt\right)\wedge dx^2\wedge dt$$

$$+\left(\frac{\partial E_3}{\partial x^1}dx^1 + \frac{\partial E_3}{\partial x^2}dx^2 + \frac{\partial E_3}{\partial x^3}dx^3 + \frac{\partial E_3}{\partial t}dt\right)\wedge dx^3\wedge dt$$

$$= \left(\frac{\partial B_1}{\partial x^1} + \frac{\partial B_2}{\partial x^2} + \frac{\partial B_3}{\partial x^3}\right)dx^1\wedge dx^2\wedge dx^3$$

$$+\left(\frac{\partial B_1}{\partial t} + \frac{\partial E_3}{\partial x^2} - \frac{\partial E_2}{\partial x^3}\right)dx^2\wedge dx^3\wedge dt$$