定義 2.16

$$df = \sum_i \frac{\partial f}{\partial x^i} dx^i,$$ 微分0形式の外微分

$$d\left(\sum_{1\leq i_1<\cdots<i_k\leq n} f^{i_1\cdots i_k} dx^{i_1}\wedge\cdots\wedge dx^{i_k}\right)$$

$$= \sum_{1\leq i_1<\cdots<i_k\leq n} df^{i_1\cdots i_k}\wedge dx^{i_1}\wedge\cdots\wedge dx^{i_k}$$ □

補題 2.17　u:微分k形式，v:微分l形式　について

$$d(u\wedge v) = du\wedge v + (-1)^k u\wedge dv$$

[証明]　$d(u_1+u_2) = du_1+du_2$ と (iii) 分配法則により，

$$u = f\,dx^{i_1}\wedge\cdots\wedge dx^{i_k},\quad v = g\,x^{j_1}\wedge\cdots\wedge x^{j_l}$$

の場合を示せばよい。

$^\exists a, ^\exists b$ s.t. $i_a = j_b$ のとき　$d(u\wedge v) = 0$
$= du\wedge v + (-1)^k u\wedge dv$。

$^\forall a, ^\forall b$　$i_a \neq j_b$ のとき，

$$d(u\wedge v) = d(fg\,dx^{i_1}\wedge\cdots\wedge dx^{i_k}\wedge dx^{j_1}\wedge\cdots\wedge dx^{j_l})$$

$$= \sum_i \frac{\partial(fg)}{\partial x^i} dx^i\wedge dx^{i_1}\wedge\cdots\wedge dx^{i_k}\wedge dx^{j_1}\wedge\cdots\wedge dx^{j_l}$$

$$= \sum_i \left(f\frac{\partial g}{\partial x^i} + g\frac{\partial f}{\partial x^i}\right) dx^i\wedge dx^{i_1}\wedge\cdots\wedge dx^{i_k}\wedge dx^{j_1}\wedge\cdots\wedge dx^{j_l}$$

一方，$du\wedge v = d(f\,dx^{i_1}\wedge\cdots\wedge dx^{i_k})\wedge(g\,dx^{j_1}\wedge\cdots\wedge dx^{j_l})$

$$= \left(\sum_i \frac{\partial f}{\partial x^i} dx^i\wedge dx^{i_1}\wedge\cdots\wedge dx^{i_k}\right)\wedge(g\,dx^{j_1}\wedge\cdots\wedge dx^{j_l})$$

$$= \sum_i \frac{\partial f}{\partial x^i} g\,dx^i\wedge dx^{i_1}\wedge\cdots\wedge dx^{i_k}\wedge dx^{j_1}\wedge\cdots\wedge dx^{j_l}$$