

$$\begin{aligned}
[\text{解}] \quad \Phi^* u &= f(r \cos \theta, r \sin \theta) d(r \cos \theta) + g(r \cos \theta, r \sin \theta) d(r \sin \theta) \\
&= f(r \cos \theta, r \sin \theta) \cos \theta dr + f(r \cos \theta, r \sin \theta) r \frac{\partial \cos \theta}{\partial \theta} d\theta \\
&\quad + g(r \cos \theta, r \sin \theta) \sin \theta dr + g(r \cos \theta, r \sin \theta) r \frac{\partial \sin \theta}{\partial \theta} d\theta \\
&= (f(r \cos \theta, r \sin \theta) \cos \theta + g(r \cos \theta, r \sin \theta) \sin \theta) dr \\
&\quad + (-r f(r \cos \theta, r \sin \theta) \sin \theta + r g(r \cos \theta, r \sin \theta) \cos \theta) d\theta.
\end{aligned}$$

補題 2.24 $u = \sum f_{i_1 \dots i_k} dx^{i_1} \wedge \dots \wedge dx^{i_k}, \quad v = \sum g_{j_1 \dots j_k} dx^{j_1} \wedge \dots \wedge dx^{j_k}$ □

$$(i) \quad \Phi^*(u+v) = \Phi^*u + \Phi^*v$$

$$(ii) \quad \Phi^*(u \wedge v) = \Phi^*u \wedge \Phi^*v$$

$$(iii) \quad \Phi^*(du) = d(\Phi^*u),$$

$$(iv) \quad \Phi^*\Phi^*(u) = (\Phi\Phi)^*(u).$$

注意 2.25 (iii) の類似がベクトル場では成り立たない。

[証明] (i)

$$\Phi^*(u+v) = \sum_{1 \leq i_1 < \dots < i_k \leq m} (f_{i_1 \dots i_k} + g_{i_1 \dots i_k}) \Phi d\varphi^{i_1} \wedge \dots \wedge d\varphi^{i_k}$$

$$\rightarrow \text{杉浦光夫} = \sum (f(\varphi) + g(\varphi)) d\varphi^{i_1} \wedge \dots \wedge d\varphi^{i_k}$$

「解析入門I」

$$P57 \text{ 定義6} = \sum f(\varphi) d\varphi^{i_1} \wedge \dots \wedge d\varphi^{i_k} + \sum g(\varphi) d\varphi^{i_1} \wedge \dots \wedge d\varphi^{i_k}$$

$$= \sum f \circ \Phi d\varphi^{i_1} \wedge \dots \wedge d\varphi^{i_k} + \sum g \circ \Phi d\varphi^{i_1} \wedge \dots \wedge d\varphi^{i_k}$$

$$= \Phi^*u + \Phi^*v$$