

2形式 $v = v^{12} dx \wedge dy + v^{23} dy \wedge dz + v^{31} dz \wedge dx$

3形式 $w = f dx \wedge dy \wedge dz$

に対して,

$$*f \stackrel{\text{def}}{=} f dx \wedge dy \wedge dz$$

$$*u \stackrel{\text{def}}{=} u^1 dy \wedge dz + u^2 dz \wedge dx + u^3 dx \wedge dy$$

$$*v \stackrel{\text{def}}{=} v^{23} dx + v^{31} dy + v^{12} dz$$

$$*w \stackrel{\text{def}}{=} f$$

$*$: Hodge のスター-作用素

$$\text{以後, } i_1(V) = V^*, \quad i_2(V) = *(V^*) \quad \text{と書く.}$$

□

補題 2.30 $f: U$ 上の関数

$$V = V^x \frac{\partial}{\partial x} + V^y \frac{\partial}{\partial y} + V^z \frac{\partial}{\partial z} : U \text{ 上のベクトル場 のとき}$$

$$i_1(\text{grad } f) = df \quad (2.20)$$

$$i_2(\text{rot } V) = di_1(V) \quad (2.21)$$

$$*(\text{div } V) = di_2(V) \quad (2.22)$$

$$\begin{aligned} [\text{証明}] \quad (2.20) \quad i_1(\text{grad } f) &= i_1\left(\frac{\partial f}{\partial x} \frac{\partial}{\partial x} + \frac{\partial f}{\partial y} \frac{\partial}{\partial y} + \frac{\partial f}{\partial z} \frac{\partial}{\partial z}\right) \\ &= \frac{\partial f}{\partial x} dx + \frac{\partial f}{\partial y} dy + \frac{\partial f}{\partial z} dz \\ &= df. \end{aligned}$$