[証明] 練習問題

$$\begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = \begin{pmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$ とする。

i) $$\Phi^* i_1 \Phi_*(V) = \Phi^* i_1 \Phi_* \left(\sum_{i=1}^{3} V^i \frac{\partial}{\partial x^i} \right)$$

$$= \Phi^* i_1 \left(\sum_{i=1}^{3} V^i \sum_{j=1}^{3} \frac{\partial y^j}{\partial x^i} \frac{\partial}{\partial y^j} \right)$$

$$= \Phi^* i_1 \left(\sum_{i=1}^{3} V^i \sum_{j=1}^{3} A_{ji} \frac{\partial}{\partial y^j} \right)$$

$$= \Phi^* i_1 \left(\sum_{j=1}^{3} \sum_{i=1}^{3} V^i A_{ji} \frac{\partial}{\partial y^j} \right)$$

$$= \Phi^* \left(\sum_{j=1}^{3} \sum_{i=1}^{3} V^i A_{ji} \, dy^j \right)$$

$$= \sum_{j=1}^{3} \left(\sum_{i=1}^{3} V^i A_{ji} \right) \left(\sum_{k=1}^{3} \frac{\partial y^j}{\partial x^k} dx^k \right)$$

$$= \sum_{j=1}^{3} \left(\sum_{i=1}^{3} V^i A_{ji} \right) \left(\sum_{k=1}^{3} A_{jk} \, dx^k \right)$$

$$= \sum_{j=1}^{3} \sum_{k,i=1}^{3} V^i A_{ji} A_{jk} \, dx^i$$

$$= \sum_{i=1}^{3} \sum_{j=1}^{3} V^i A_{ji} A_{ji} \, dx^i$$

$$= \sum_{i=1}^{3} V^i \, dx^i$$

$$= i_1 \left(\sum_{i=1}^{3} V^i \frac{\partial}{\partial x^i} \right). \qquad \square$$

$i = k \Rightarrow \sum_{k,i=1}^{3} A_{ji} A_{jk} = 1$

$\therefore AA^t = A^t A = I$

ii) $$\Phi^* i_2 \Phi^*(V) = \Phi^* i_2 \Phi_* \left(V^1 \frac{\partial}{\partial x^1} + V^2 \frac{\partial}{\partial x^2} + V^3 \frac{\partial}{\partial x^3} \right)$$

$$= \Phi i_2 \quad V^1 \left(\frac{\partial y^1}{\partial x^1} \frac{\partial}{\partial y^1} + \quad + \frac{\partial y^3}{\partial x^1} \frac{\partial}{\partial y^3} \right)$$

$$+ V^2 \left(\qquad \qquad \right)$$

$$+ V^3 \left(\frac{\partial y^1}{\partial x^3} \frac{\partial}{\partial y^1} + \cdots + \frac{\partial y^3}{\partial x^3} \frac{\partial}{\partial y^3} \right)$$