

$$\{ \{f, g\}, h \} + \{ \{g, h\}, f \} + \{ \{h, f\}, g \}$$

$$= \left\{ \sum_i \left(\frac{\partial f}{\partial g_i} \frac{\partial g}{\partial p_i} - \frac{\partial f \partial g}{\partial p_i \partial g_i} \right), h \right\} + \left\{ \sum_i \left(\frac{\partial g \partial h}{\partial g_i \partial p_i} - \frac{\partial g \partial h}{\partial p_i \partial g_i} \right), f \right\} + \left\{ \sum_i \left(\frac{\partial h \partial f}{\partial g_i \partial p_i} - \frac{\partial h \partial f}{\partial p_i \partial g_i} \right), g \right\}$$

$$= \sum_{i,j} \left(\frac{\partial}{\partial g_j} \left(\frac{\partial f \partial g}{\partial g_i \partial p_i} - \frac{\partial f \partial g}{\partial p_i \partial g_i} \right) \frac{\partial h}{\partial p_j} - \frac{\partial}{\partial p_j} \left(\frac{\partial f \partial g}{\partial g_i \partial p_i} - \frac{\partial f \partial g}{\partial p_i \partial g_i} \right) \frac{\partial h}{\partial g_j} \right)$$

$$+ \sum_{i,j} \left(\frac{\partial}{\partial g_j} \left(\frac{\partial g \partial h}{\partial g_i \partial p_i} - \frac{\partial g \partial h}{\partial p_i \partial g_i} \right) \frac{\partial f}{\partial p_j} - \frac{\partial}{\partial p_j} \left(\frac{\partial g \partial h}{\partial g_i \partial p_i} - \frac{\partial g \partial h}{\partial p_i \partial g_i} \right) \frac{\partial f}{\partial g_j} \right)$$

$$+ \sum_{i,j} \left(\frac{\partial}{\partial g_j} \left(\frac{\partial h \partial f}{\partial g_i \partial p_i} - \frac{\partial h \partial f}{\partial p_i \partial g_i} \right) \frac{\partial g}{\partial p_j} - \frac{\partial}{\partial p_j} \left(\frac{\partial h \partial f}{\partial g_i \partial p_i} - \frac{\partial h \partial f}{\partial p_i \partial g_i} \right) \frac{\partial g}{\partial g_j} \right)$$

$$= \sum_{i,j} \left(\underbrace{\frac{\partial^2 f}{\partial g_j \partial g_i} \frac{\partial g}{\partial p_i} \frac{\partial h}{\partial p_j}}_{(1)} + \underbrace{\frac{\partial f}{\partial g_i} \frac{\partial^2 g}{\partial g_j \partial p_i} \frac{\partial h}{\partial p_j}}_{(2)} - \underbrace{\frac{\partial^2 f}{\partial g_i \partial p_i} \frac{\partial g}{\partial g_j} \frac{\partial h}{\partial p_j}}_{(3)} - \underbrace{\frac{\partial f}{\partial p_i} \frac{\partial^2 g}{\partial g_j \partial g_i} \frac{\partial h}{\partial p_j}}_{(4)} \right.$$

$$\left. - \underbrace{\frac{\partial^2 f}{\partial p_i \partial g_i} \frac{\partial g}{\partial p_j} \frac{\partial h}{\partial g_j}}_{(5)} - \underbrace{\frac{\partial f}{\partial g_i} \frac{\partial^2 g}{\partial p_i \partial p_j} \frac{\partial h}{\partial g_j}}_{(6)} + \underbrace{\frac{\partial^2 f}{\partial p_i \partial p_i} \frac{\partial g}{\partial g_j} \frac{\partial h}{\partial g_j}}_{(7)} + \underbrace{\frac{\partial f}{\partial p_i} \frac{\partial^2 g}{\partial p_i \partial g_j} \frac{\partial h}{\partial g_j}}_{(8)} \right)$$

$$+ \sum_{i,j} \left(\underbrace{\frac{\partial^2 g}{\partial g_j \partial g_i} \frac{\partial h}{\partial p_i} \frac{\partial f}{\partial p_j}}_{(9)} + \underbrace{\frac{\partial g}{\partial g_i} \frac{\partial^2 h}{\partial g_j \partial p_i} \frac{\partial f}{\partial p_j}}_{(10)} - \underbrace{\frac{\partial^2 g}{\partial g_i \partial p_i} \frac{\partial h}{\partial g_j} \frac{\partial f}{\partial p_j}}_{(11)} - \underbrace{\frac{\partial g}{\partial p_i} \frac{\partial^2 h}{\partial g_j \partial g_i} \frac{\partial f}{\partial p_j}}_{(12)} \right.$$

$$\left. - \underbrace{\frac{\partial^2 g}{\partial p_i \partial g_i} \frac{\partial h}{\partial p_j} \frac{\partial f}{\partial g_j}}_{(13)} - \underbrace{\frac{\partial g}{\partial g_i} \frac{\partial^2 h}{\partial p_i \partial p_j} \frac{\partial f}{\partial g_j}}_{(14)} + \underbrace{\frac{\partial^2 g}{\partial p_i \partial p_i} \frac{\partial h}{\partial g_j} \frac{\partial f}{\partial g_j}}_{(15)} + \underbrace{\frac{\partial g}{\partial p_i} \frac{\partial^2 h}{\partial p_i \partial g_j} \frac{\partial f}{\partial g_j}}_{(16)} \right)$$

$$+ \sum_{i,j} \left(\underbrace{\frac{\partial^2 h}{\partial g_j \partial g_i} \frac{\partial f}{\partial p_i} \frac{\partial g}{\partial p_j}}_{(17)} + \underbrace{\frac{\partial h}{\partial g_i} \frac{\partial^2 f}{\partial g_j \partial p_i} \frac{\partial g}{\partial p_j}}_{(18)} - \underbrace{\frac{\partial^2 h}{\partial g_i \partial p_i} \frac{\partial f}{\partial g_j} \frac{\partial g}{\partial p_j}}_{(19)} - \underbrace{\frac{\partial h}{\partial p_i} \frac{\partial^2 f}{\partial g_j \partial g_i} \frac{\partial g}{\partial p_j}}_{(20)} \right.$$

$$\left. - \underbrace{\frac{\partial^2 h}{\partial p_i \partial g_i} \frac{\partial f}{\partial p_j} \frac{\partial g}{\partial g_j}}_{(21)} - \underbrace{\frac{\partial h}{\partial g_i} \frac{\partial^2 f}{\partial p_i \partial p_j} \frac{\partial g}{\partial g_j}}_{(22)} + \underbrace{\frac{\partial^2 h}{\partial p_i \partial p_i} \frac{\partial f}{\partial g_j} \frac{\partial g}{\partial g_j}}_{(23)} + \underbrace{\frac{\partial h}{\partial p_i} \frac{\partial^2 f}{\partial p_i \partial g_j} \frac{\partial g}{\partial g_j}}_{(24)} \right)$$

$$= 0$$