$$= -\Phi^*(dH(\mathbb{Q},\mathbb{P}))$$

$$\therefore \quad i_{\Phi_* X_{H(q,p)}}\Omega = -dH(\mathbb{Q},\mathbb{P}) \quad (\because \ \Phi^* \text{は } 1\text{-}1\ (?))$$

$$\therefore \quad \Phi_* X_{H(q,p)} = X_{H(\mathbb{Q},\mathbb{P})} \quad (\because \text{ 補題 } 3.7)$$ ■

定義 3.8

$$\Phi \text{が正準変換} \quad \overset{\text{def}}{\Longleftrightarrow} \quad \Phi^*\Omega = \omega$$ □

(b) 正準変換の作り方 (1) … 点変換

$(\mathbb{Q},\mathbb{P}) = \Phi(q,p) = (\Phi(q), \Phi(q,p))$ のとき,

$\Phi^*\Omega = \omega$ とすると, ($\Leftrightarrow$ Φが正準変換)

$$\Phi^*\Omega = \Phi^*\sum_{i=1}^n dP^i\wedge dQ^i = \sum_{i=1}^n\left(\sum_{j=1}^n\left(\frac{\partial P^i}{\partial q^j}dq^j + \frac{\partial P^i}{\partial p^j}dp^j\right)\wedge\sum_{k=1}^n\left(\frac{\partial Q^i}{\partial q^k}dq^k + \frac{\partial Q^i}{\partial p^k}dp^k\right)\right)$$

$$=\sum_{i=1}^n\sum_{j=1}^n\sum_{k=1}^n\left\{\begin{aligned}&\left(\frac{\partial P^i}{\partial q^j}\frac{\partial Q^i}{\partial q^k} - \frac{\partial P^i}{\partial q^k}\frac{\partial Q^i}{\partial q^j}\right)dq^j\wedge dq^k\\ &+\left(\frac{\partial P^i}{\partial q^j}\frac{\partial Q^i}{\partial p^k} - \frac{\partial P^i}{\partial p^k}\frac{\partial Q^i}{\partial q^j}\right)dq^j\wedge dp^k\\ &+\left(\frac{\partial P^i}{\partial p^j}\frac{\partial Q^i}{\partial p^k} - \frac{\partial P^i}{\partial p^k}\frac{\partial Q^i}{\partial p^j}\right)dp^j\wedge dp^k\end{aligned}\right\}$$

$$= \sum_{i=1}^n dp^i\wedge dq^i$$

よって,

$$\sum_{i=1}^n\left(\frac{\partial P^i}{\partial p^j}\frac{\partial Q^i}{\partial p^k} - \frac{\partial P^i}{\partial p^k}\frac{\partial Q^i}{\partial p^j}\right) = 0 \qquad (3.3) \qquad (3番目)$$

$$\sum_{i=1}^n\left(\frac{\partial P^i}{\partial q^j}\frac{\partial Q^i}{\partial q^k} - \frac{\partial P^i}{\partial q^k}\frac{\partial Q^i}{\partial q^j}\right) = 0 \qquad (3.4) \qquad (1番目)$$

$$\sum_{i=1}^n\left(\frac{\partial P^i}{\partial p^j}\frac{\partial Q^i}{\partial q^k} - \frac{\partial P^i}{\partial q^k}\underbrace{\frac{\partial Q^i}{\partial p^j}}_{0\ (\because\ \mathbb{Q}=\mathbb{Q}(q))}\right) = \delta_{jk} \qquad (3.5) \qquad (2番目)$$ △

$$(D\Phi)_{11} = \left(\frac{\partial Q^i}{\partial q^j}\right), \qquad (D\Phi)_{12} = \left(\frac{\partial Q^i}{\partial p^j}\right),$$

$$(D\Phi)_{21} = \left(\frac{\partial P^i}{\partial q^j}\right), \qquad (D\Phi)_{22} = \left(\frac{\partial P^i}{\partial p^j}\right), \qquad \text{とする。}$$