$$\frac{d\theta}{dt} = \frac{\partial\theta}{\partial x} V^x(r\cos\theta, r\sin\theta) + \frac{\partial\theta}{\partial y} V^y(r\cos\theta, r\sin\theta)$$

$$= -\frac{\sin\theta}{r} V^x(r\cos\theta, r\sin\theta) + \frac{\cos\theta}{r} V^y(r\cos\theta, r\sin\theta) \qquad \square$$

問1　$\frac{d\mathbb{x}}{dt} = \|\mathbb{x}\|^4 \mathbb{x}$,　$\mathbb{x} = (x(t), y(t))$ を極座標に変換して解け。

解)　$\|\mathbb{x}\|^4 \mathbb{x} = r^4 (r\cos\theta, r\sin\theta)$

$$\begin{cases} \frac{dr}{dt} = \cos\theta \cdot r^5 \cos\theta + \sin\theta \cdot r^5 \sin\theta = r^5 \\ \frac{d\theta}{dt} = -\frac{\sin\theta}{r} r^5 \cos\theta + \frac{\cos\theta}{r} r^5 \sin\theta = 0 \end{cases}$$

$$\begin{cases} t = -\frac{1}{4} r^{-4} + r_0 \\ \theta = \theta_0 \end{cases} \Rightarrow \begin{cases} r = \{4(t - r_0)\}^{-\frac{1}{4}} \\ \theta = \theta_0 \end{cases}$$

(r_0, θ_0 は定数)

(b) ベクトル場の座標変換

U, V : open set on $\mathbb{R}^2$

$\Phi : U \to V$: 可微分同相写像

$: \mathbb{x} = (x^1, \cdots, x^n) \mapsto \mathbb{y}(\mathbb{x}) = (y^1(\mathbb{x}), \cdots, y^n(\mathbb{x}))$

$$y^i(\mathbb{x}) = y^i(x^1, \cdots, x^n)$$

$$\mathbb{y} = (y^1, \cdots, y^n) = \Phi(\mathbb{x}) = \Phi(x_1, \cdots, x_n)$$

$\mathbb{W} = (W^1, \cdots, W^n)$: U 上のベクトル場 : $\mathbb{x} \to \mathbb{W}(\mathbb{x})$

$\Phi_* \mathbb{W} = \left(\cdots, \sum_{j=1}^{n} \frac{\partial y^i}{\partial x^j}(\mathbb{x}) W^j(\mathbb{x}), \cdots\right)$: V 上のベクトル場

$\mathbb{y} \mapsto \Phi_* \mathbb{W}(\mathbb{y})$

$D\Phi_{\mathbb{x}} = {}_n\left(\frac{\partial y^i}{\partial x^j}\right)^n$: ヤコビ行列