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$$= \frac{d}{d\delta}\int_0^1 \Big((\mathbb{p}+\delta\Delta\mathbb{p})\cdot(\dot{\mathbb{q}}+\delta\Delta\dot{\mathbb{q}}) - H(t, \mathbb{q}+\delta\Delta\mathbb{q}, \mathbb{p}+\delta\Delta\mathbb{p})\Big)dt \Big|_{\delta=0}$$

$$= \int_0^1 \Big(\Delta\mathbb{p}(\dot{\mathbb{q}}+\delta\Delta\dot{\mathbb{q}}) + (\mathbb{p}+\delta\Delta\mathbb{p})\Delta\dot{\mathbb{q}}\Big) - \Delta\mathbb{q}\frac{\partial H}{\partial \mathbb{q}_\delta} - \Delta\mathbb{p}\frac{\partial H}{\partial \mathbb{p}_\delta}\Big) dt \Big|_{\delta=0}$$

$$= \int_0^1 \Big(\Delta\mathbb{p}\cdot\dot{\mathbb{q}} + \underline{\mathbb{p}\cdot\Delta\dot{\mathbb{q}}} - \Delta\mathbb{q}\frac{\partial H}{\partial \mathbb{q}} - \Delta\mathbb{p}\frac{\partial H}{\partial \mathbb{p}}\Big)dt$$

$$= \underbrace{\Big[\mathbb{p}\cdot\Delta\mathbb{q}\Big]_0^1}_{\substack{\| \\ 0 \ \because\ \Delta\mathbb{q}(0)=\Delta\mathbb{q}(1)=0}} - \int_0^1 \dot{\mathbb{p}}\Delta\mathbb{q}\,dt + \int_0^1\Big(\Delta\mathbb{p}\cdot\dot{\mathbb{q}} - \Delta\mathbb{q}\frac{\partial H}{\partial \mathbb{q}} - \Delta\mathbb{p}\frac{\partial H}{\partial \mathbb{p}}\Big)dt$$

$$= \int_0^1\Big(\Big(\underline{\frac{d\mathbb{q}}{dt} - \frac{\partial H}{\partial \mathbb{p}}}\Big)\Delta\mathbb{p} - \Big(\underline{\frac{d\mathbb{p}}{dt} + \frac{\partial H}{\partial \mathbb{q}}}\Big)\Delta\mathbb{q}\Big)dt \qquad (1.38)$$

$$\therefore\ \frac{d}{d\delta}\mathcal{H}(\mathbb{q}+\delta\Delta\mathbb{q}, \mathbb{p}+\delta\Delta\mathbb{p})\Big|_{\delta=0} \Leftrightarrow \begin{cases} \dfrac{dq_i}{dt} = \dfrac{\partial H}{\partial p_i} \\ \dfrac{dp_i}{dt} = -\dfrac{\partial H}{\partial q_i}\end{cases}$$

□

$$\Omega(\mathbb{q}_0, \mathbb{q}_1, \mathbb{p}_0, \mathbb{p}_1 ; U) = \{(\mathbb{q}(t), \mathbb{p}(t)) : [0,1] \to U \subseteq \mathbb{R}^{2n}$$
$$\mid \mathbb{q}(0)=\mathbb{q}_0\ \ \mathbb{q}(1)=\mathbb{q}_1\ \ \mathbb{p}(0)=\mathbb{p}_0,\ \mathbb{p}(1)=\mathbb{p}_1\}$$

定義 1.28. $(\mathbb{q}(t), \mathbb{p}(t)) \in \Omega(\mathbb{q}_0, \mathbb{q}_1, \mathbb{p}_0, \mathbb{p}_1 : U)$

で $\mathcal{H}(\mathbb{q}, \mathbb{p})$ が極値をとる

$\overset{\text{def}}{\Longleftrightarrow}$ $(\Delta\mathbb{q}(t), \Delta\mathbb{p}(t))$: 任意の変分.

$$\Delta\mathbb{q}(0) = \Delta\mathbb{q}(1) = \Delta\mathbb{p}(0) = \Delta\mathbb{p}(1) = 0$$

に対して. $\dfrac{d}{d\delta}\mathcal{H}(\mathbb{q}+\delta\mathbb{q}, \mathbb{p}+\delta\Delta\mathbb{p})\Big|_{\delta=0} = 0$ □