$$\Phi_*\left(\frac{\partial}{\partial\theta}\right)=\frac{\partial x}{\partial\theta}\frac{\partial}{\partial x}+\frac{\partial y}{\partial\theta}\frac{\partial}{\partial y}+\frac{\partial z}{\partial\theta}\frac{\partial}{\partial z}$$
$$=-r\sin\theta\cos\varphi\frac{\partial}{\partial x}+r\cos\theta\cos\varphi\frac{\partial}{\partial y}\left(+\ 0\frac{\partial}{\partial z}\right)$$
$$\Phi_*\left(\frac{\partial}{\partial\varphi}\right)=-r\cos\theta\sin\varphi\frac{\partial}{\partial x}-r\sin\theta\sin\varphi\frac{\partial}{\partial y}+r\cos\varphi\frac{\partial}{\partial z}$$

2.2 u: 奇数次の微分形式 $u\wedge u=0$

[証明] $u\wedge u=(-1)^{2n+1}u\wedge u=-u\wedge u$

$2u\wedge u=0 \quad \therefore\ u\wedge u=0$ □

2.3 $f_1,\cdots,f_m\in C^\infty(\mathbb{R}^n)$，次の二つは同値

(1) $\mathrm{grad}\, f_1(p),\cdots,\mathrm{grad}\, f_m(p)$ は1次従属

(2) $df_1\wedge\cdots\wedge df_m$ は p で 0

[証明] (1) ⇒ (2)

$$df_1\wedge\cdots\wedge df_m \overset{\text{問5}}{=} \sum_{1\le j_1<\cdots<j_m\le n}\underbrace{\det\left(\frac{\partial f_i}{\partial f_{j_k}}\right)}_{(1)\text{より}\ 0}dx^{j_1}\wedge\cdots\wedge dx^{j_m}$$
□

2.4 $\Omega=\mathbb{R}^3/\{0\}$ $\Phi:\Omega\to\mathbb{R}:\Phi(x)=\|x\|$ のとき

$\alpha\in\mathbb{R}$ $\Phi^*(r^\alpha dr)$?

(解)
$$\Phi^*(r^\alpha dr)=(x^2+y^2+z^2)^{\frac{\alpha}{2}}\left(\frac{\partial r}{\partial x}dx+\frac{\partial r}{\partial y}dy+\frac{\partial r}{\partial z}dz\right)$$
$$=(x^2+y^2+z^2)^{\frac{\alpha}{2}}\frac{1}{\sqrt{x^2+y^2+z^2}}(x\,dx+y\,dy+z\,dz)$$
$$=(x^2+y^2+z^2)^{\frac{\alpha-1}{2}}(x\,dx+y\,dy+z\,dz)$$
□