(1) $\Phi^*(U \wedge V)$

$$= \sum_{1 \le i_1 < \cdots < i_k \le m} \sum_{1 \le j_1 < \cdots < j_\ell \le n} f_{i_1 \cdots i_k} g_{i_1 \cdots i_k} \circ \Phi \, d\varphi^{i_1} \wedge \cdots \wedge d\varphi^{i_k} \wedge d\varphi^{j_1} \wedge \cdots \wedge d\varphi^{j_\ell}$$

$$= \sum_{1 \le i_1 < \cdots < i_k \le m} f_{i_1 \cdots i_k} \circ \Phi \, d\varphi^{i_1} \wedge \cdots \wedge d\varphi^{i_k} \sum_{1 \le j_1 < \cdots < j_\ell \le n} g_{j_1 < \cdots < j_\ell} \circ \Phi \, d\varphi^{j_1} \wedge \cdots \wedge d\varphi^{j_\ell}$$

$$= \Phi^* U \wedge \Phi^* V.$$

(2) (i), (ii) より $U = f\, dy^{i_1} \wedge \cdots \wedge dy^{i_k}$ について示せば十分。

$$\Phi^*(dU) = \Phi^*(df \wedge dy^{i_1} \wedge \cdots \wedge dy^{i_k})$$

$$= \Phi^*\left(\sum_i \frac{\partial f}{\partial y^i} dy^i \wedge dy^{i_1} \wedge \cdots \wedge dy^{i_k}\right)$$

$$= \sum_i \frac{\partial f}{\partial y^i} \circ \Phi \, d\varphi^i \wedge d\varphi^{i_1} \wedge \cdots \wedge d\varphi^{i_k}.$$

一方 $d(\Phi^* U) = d(f \circ \Phi \, d\varphi^{i_1} \wedge \cdots \wedge d\varphi^{i_k})$

$$= d(f \circ \Phi) \wedge d\varphi^{i_1} \wedge \cdots \wedge d\varphi^{i_k} + f \circ \Phi \, d(d\varphi^{i_1} \wedge \cdots \wedge d\varphi^{i_k})$$

$$= \sum_j \frac{\partial (f \circ \Phi)}{\partial x^j} dx^j \wedge d\varphi^{i_1} \wedge \cdots \wedge d\varphi^{i_k}$$

$$+ f \circ \Phi \; d(d\varphi^{i_1}) \wedge d\varphi^{i_2} \wedge \cdots \wedge d\varphi^{i_k}$$

$$+ f \circ \Phi \; d\varphi^{i_1} \wedge d(d\varphi^{i_2}) \wedge \cdots \wedge d\varphi^{i_k}$$

$$+ f \circ \Phi \; d\varphi^{i_1} \wedge d\varphi^{i_2} \wedge \cdots \wedge d(d\varphi^{i_k})$$

$$= \sum_{i,j} \frac{\partial f}{\partial y^i} \frac{\partial \varphi^i}{\partial x^j} \partial x^j \wedge d\varphi^{i_1} \wedge \cdots \wedge d\varphi^{i_k}$$

$$= \sum_i \frac{\partial f}{\partial y^i} \, d\varphi^i \wedge d\varphi^{i_1} \wedge \cdots \wedge d\varphi^{i_k}$$

(1) $\Psi(z) = \Psi(z^1, \cdots, z^\ell) = (\psi^1(z), \cdots, \psi^m(z))$