$$\int_S W \cdot dS \underset{\text{def}}{=} \int_U W \cdot \left(\frac{\partial \varphi}{\partial x} \times \frac{\partial \varphi}{\partial y} \right) dx\, dy$$

$$= \int_U \det \begin{pmatrix} W^x & \frac{\partial \varphi^1}{\partial x} & \frac{\partial \varphi^1}{\partial y} \\ W^y & \frac{\partial \varphi^2}{\partial x} & \frac{\partial \varphi^2}{\partial y} \\ W^z & \frac{\partial \varphi^3}{\partial x} & \frac{\partial \varphi^3}{\partial y} \end{pmatrix} dx\, dy$$

$$= \int_U \left(W^x \frac{\partial \varphi^2}{\partial x} \frac{\partial \varphi^3}{\partial y} + W^y \frac{\partial \varphi^3}{\partial x} \frac{\partial \varphi^1}{\partial y} + W^z \frac{\partial \varphi^1}{\partial x} \frac{\partial \varphi^2}{\partial y} \right.$$
$$\left. - W^z \frac{\partial \varphi^2}{\partial x} \frac{\partial \varphi^1}{\partial y} - W^y \frac{\partial \varphi^1}{\partial x} \frac{\partial \varphi^3}{\partial y} - W^x \frac{\partial \varphi^3}{\partial x} \frac{\partial \varphi^2}{\partial y} \right) dx\, dy$$

$$= \int_U W^x \left(\frac{\partial \varphi^2}{\partial x} \frac{\partial \varphi^3}{\partial y} - \frac{\partial \varphi^3}{\partial x} \frac{\partial \varphi^2}{\partial y} \right) dx\, dy \quad (\wedge)$$
$$+ W^y \left(\frac{\partial \varphi^3}{\partial x} \frac{\partial \varphi^1}{\partial y} - \frac{\partial \varphi^1}{\partial x} \frac{\partial \varphi^3}{\partial y} \right) dx\, dy \quad (\wedge)$$
$$+ W^z \left(\frac{\partial \varphi^1}{\partial x} \frac{\partial \varphi^2}{\partial y} - \frac{\partial \varphi^2}{\partial x} \frac{\partial \varphi^1}{\partial y} \right) dx\, dy \quad (\wedge)$$

$$= \int_U W^x \left(\frac{\partial \varphi^2}{\partial x} dx + \frac{\partial \varphi^2}{\partial y} dy \right) \wedge \left(\frac{\partial \varphi^3}{\partial x} dx + \frac{\partial \varphi^3}{\partial y} \right)$$
$$+ W^y \left(\frac{\partial \varphi^3}{\partial x} dx + \frac{\partial \varphi^3}{\partial y} dy \right) \wedge \left(\frac{\partial \varphi^1}{\partial x} dx + \frac{\partial \varphi^1}{\partial y} dy \right)$$
$$+ W^z \left(\frac{\partial \varphi^1}{\partial x} dx + \frac{\partial \varphi^1}{\partial y} dy \right) \wedge \left(\frac{\partial \varphi^2}{\partial x} dx + \frac{\partial \varphi^2}{\partial y} dy \right)$$

$$= \int_S W^x\, d\varphi^2 \wedge d\varphi^3 + W^y\, d\varphi^3 \wedge d\varphi^1 + W^z\, d\varphi^1 \wedge d\varphi^2$$

$$= \int_S i_2(W) \qquad\qquad \square$$