(iii) $i_m \neq j$, $j_n = j$ のとき,

$$i_{\mathbb{X}}(u\wedge v) = (-1)^{k+n-1} dx^{i_1}\wedge\cdots\wedge dx^{i_k}\wedge dx^{j_1}\wedge\cdots\wedge dx^{n-1}\wedge dx^{n+1}\wedge\cdots\wedge dx^{j_l}$$

$$= 0 + (-1)^k u\wedge i_{\mathbb{X}}(v)$$

$$= i_{\mathbb{X}}(u)\wedge v + (-1)^k u\wedge i_{\mathbb{X}}(v) .$$

(iv) $i_m = j$, $j_n = j$ のとき,

$$i_{\mathbb{X}}(u\wedge v) = i_{\mathbb{X}}(0) = 0 .$$

$$i_{\mathbb{X}}(u)\wedge v + (-1)^k u\wedge i_{\mathbb{X}}(v)$$

$$= (-1)^{m-1} dx^{i_1}\wedge\cdots\wedge dx^{i_{m-1}}\wedge dx^{i_{m+1}}\wedge\cdots\wedge dx^{i_k}\wedge dx^{j_1}\wedge\cdots\wedge dx^{j_n}\wedge\cdots\wedge dx^{j_l}$$

$$+ (-1)^{k+n-1} dx^{i_1}\wedge\cdots\wedge dx^{i_m}\wedge\cdots\wedge dx^{i_k}\wedge dx^{j_1}\wedge\cdots\wedge dx^{j_{n-1}}\wedge dx^{j_{n+1}}\wedge\cdots\wedge dx^{j_l}$$

$$= (-1)^{m-1+k-m+n-1} dx^{i_1}\wedge\cdots\wedge dx^{i_{m-1}}\wedge dx^{j_n}\wedge dx^{i_{m+1}}\wedge\cdots\wedge dx^{i_k}\wedge dx^{j_1}\wedge\cdots\wedge dx^{j_{n-1}}\wedge dx^{j_{n+1}}\wedge\cdots\wedge dx^{j_l}$$

$$+ (-1)^{k+n-1} dx^{i_1}\wedge\cdots\wedge dx^{i_m}\wedge\cdots\wedge dx^{i_k}\wedge dx^{j_1}\wedge\cdots\wedge dx^{j_{n-1}}\wedge dx^{j_{n+1}}\wedge\cdots\wedge dx^{j_l}$$

$$= 0 .$$

一般の場合は 計算規則 3.3 (ii), (iii) より 明らか。 ∎

補題 3.6　内部積の座標変換不変性

$\Phi : U \to V$: 可微分同相写像 , $\mathbb{X}$: U 上のベクトル場

u : V 上の微分形式

のとき,

$$\Phi^* i_{\Phi_* \mathbb{X}}(u) = i_{\mathbb{X}}(\Phi^* u)$$