$$\begin{tikzcd} x \arrow[r, "W"] \arrow[d, "\Phi"'] & W(x) \arrow[d] \\ y \arrow[r, "\Phi_* W"'] & \Phi_* W(y) \end{tikzcd}$$

定義 2.2 $\quad \Phi_* W \overset{\text{def}}{=} D\Phi_{\Phi^{-1}(y)} W(\Phi^{-1}(y))$

$$= D\Phi_x W(x)$$

$$= \begin{pmatrix} \frac{\partial y^1}{\partial x^1} & \cdots & \frac{\partial y^1}{\partial x^n} \\ \vdots & \frac{\partial y^i}{\partial x^j} & \vdots \\ \frac{\partial y^n}{\partial x^1} & \cdots & \frac{\partial y^n}{\partial x^n} \end{pmatrix} \begin{pmatrix} W^1(x) \\ \vdots \\ W^n(x) \end{pmatrix} \qquad \square$$

補題 2.3 $\quad \ell(t) : (a, b) \to U$: ベクトル場 W の解曲線

$\Rightarrow \Phi(\ell(t)) : (a, b) \to V$: ベクトル場 $\Phi_* W$ の解曲線.

[証明] $\quad \ell(t) = (\ell^1(t), \cdots, \ell^n(t))$ とおくと,

$$\frac{d\Phi(\ell(t))}{dt} = \sum_i \frac{\partial \Phi}{\partial x^i}(\ell(t)) \frac{d\ell^i}{dt}(t) = \sum_i \frac{\partial \Phi}{\partial x^i}(\ell(t)) W^i(\ell(t))$$
$$= \Phi_* W(\ell(t)) .$$

$\square$

補題 2.4 $\quad \Phi : U \to V, \quad \Psi : V \to W$: 可微分同相写像

W, X : U 上のベクトル場, $f : U$ 上の関数

のとき,

$$\begin{cases} \Phi_*(W + X) = \Phi_*(W) + \Phi(X) \\ \Phi_*(fW)(y) = f(\Phi^{-1}(y)) \Phi_*(W)(y) \\ (\Psi\Phi)_*(W) = \Psi_*(\Phi_*(W)) \end{cases}$$

[証明] $\quad \Phi_*(W + X) = D\Phi_x (W + X)$

$$= \left(\sum_j \frac{\partial y^i}{\partial x^j} (W^j + X^j) \right)$$

$$= D\Phi_x W + D\Phi_x X = \Phi_*(W) + \Phi_*(X) \quad \square$$