逆行列は
逆写像の
ヤコビ行列

$$
= {}^t\begin{pmatrix} \frac{\partial q^1}{\partial Q^1} & \cdots & \frac{\partial q^1}{\partial Q^j} & \cdots & \frac{\partial q^1}{\partial Q^n} \\ \vdots & & \vdots & & \vdots \\ \frac{\partial q^i}{\partial Q^1} & \cdots & \frac{\partial q^i}{\partial Q^j} & \cdots & \frac{\partial q^i}{\partial Q^n} \\ \vdots & & \vdots & & \vdots \\ \frac{\partial q^n}{\partial Q^1} & \cdots & \frac{\partial q^n}{\partial Q^j} & \cdots & \frac{\partial q^n}{\partial Q^n} \end{pmatrix} \begin{pmatrix} p^1 \\ \vdots \\ p^j \\ \vdots \\ p^n \end{pmatrix}
$$

i,jを交換

$$
= \begin{pmatrix} \frac{\partial q^1}{\partial Q^1} & \cdots & \frac{\partial q^j}{\partial Q^1} & \cdots & \frac{\partial q^n}{\partial Q^1} \\ \vdots & & \vdots & & \vdots \\ \frac{\partial q^1}{\partial Q^i} & \cdots & \frac{\partial q^j}{\partial Q^i} & \cdots & \frac{\partial q^n}{\partial Q^i} \\ \vdots & & \vdots & & \vdots \\ \frac{\partial q^1}{\partial Q^n} & \cdots & \frac{\partial q^j}{\partial Q^n} & \cdots & \frac{\partial q^n}{\partial Q^n} \end{pmatrix} \begin{pmatrix} p^1 \\ \vdots \\ p^j \\ \vdots \\ p^n \end{pmatrix} = \begin{pmatrix} \sum_j \frac{\partial q^j}{\partial Q^1} p^j \\ \vdots \\ \sum_j \frac{\partial q^j}{\partial Q^i} p^j \\ \vdots \\ \sum_j \frac{\partial q^j}{\partial Q^n} p^j \end{pmatrix}
$$

$$
(*) = \sum_i \left(\sum_j \frac{\partial q^j}{\partial Q^i} dp^j + \sum_j \sum_k \frac{\partial^2 q^j}{\partial q^k \partial Q^i} p^j dq^k \right) \wedge \left(\sum_j \frac{\partial Q^i}{\partial q^j} dq^j \right)
$$

$$
= \sum_{i,j,k} \frac{\partial q^j}{\partial Q^i} dp^j \wedge \frac{\partial Q^i}{\partial q^k} dq^k + \sum_{i,j,k,l} \frac{\partial^2 q^j}{\partial q^k \partial Q^i} p^j dq^k \wedge \frac{\partial Q^i}{\partial q^l} dq^l \quad (3.8)
$$

添え字の変換

$$
(\text{第1項}) = \sum_{j,k} \sum_i \frac{\partial q^j}{\partial Q^i} \frac{\partial Q^i}{\partial q^k} dp^j \wedge dq^k
$$

$$
= \sum_{j,k} \frac{\partial q^j}{\partial q^k} dp^j \wedge dq^k = \sum_j dp^j \wedge dq^k
$$

$$
(\text{第2項}) = \sum_{j,k,l} \left(\sum_i \frac{\partial^2 q^j}{\partial Q^i \partial q^k} \frac{\partial Q^i}{\partial q^l} \right) p^j dq^k \wedge dq^l
$$

$$
= \sum_{j,k,l} \frac{\partial^2 q^j}{\partial q^k \partial q^l} p^j dq^k \wedge dq^l = 0
$$

$$
\begin{cases} j \neq k \neq l \text{ のとき } 0 \\ j \neq k = l \text{ のとき } 0 \end{cases} \text{etc}
$$

$$
(3.8) = \sum_j dp^j \wedge dq^j
$$

問2 (i) $\Phi : (\mathbb{q}, \mathbb{p}) \mapsto (\mathbb{Q}, \mathbb{P})$ が点変換 $\Longleftrightarrow$ (ii) $\Phi^* \sum_{i=1}^n P^i dQ^i = \sum_i p^i dq^i$

証明 (i) $\Rightarrow$ $(\text{左辺}) = \sum_{i,j,k} \frac{\partial q^k}{\partial Q^i} p^k \frac{\partial Q^i}{\partial q^j} dq^j = \sum_j p^j dq^j$

iについての和をとると

$= \delta_{jk}$ ∵ 逆行列