$$+\left(\frac{\partial B_2}{\partial t}+\frac{\partial E_1}{\partial x^3}-\frac{\partial E_3}{\partial x^1}\right)dx^3\wedge dx^1\wedge dt$$

$$+\left(\frac{\partial B_3}{\partial t}+\frac{\partial E_2}{\partial x^1}-\frac{\partial E_1}{\partial x^2}\right)dx^1\wedge dx^2\wedge dt$$

$$=0$$

このとき 定理2.32 ポアンカレの補題 より

$\exists\, v$: 1形式 s.t $dv = u$ 。

$\mathbb{A}$: 磁場のベクトルポテンシャル ($\mathrm{rot}\,\mathbb{A}=\mathbb{B}$)

φ : 電場のスカラーポテンシャル ($-\mathrm{grad}\,\varphi-\frac{d\mathbb{A}}{dt}=\mathbb{E}$)

とすると

$v = A_1dx^1+A_2dx^2+A_3dx^3-\varphi dt$ となる。

$$\begin{cases}\mathrm{div}\,\mathbb{E}=\rho & \rho:\text{電荷} \quad \text{ガウスの法則}\\ \mathrm{rot}\,\mathbb{B}-\frac{d\mathbb{E}}{dt}=j & j:\text{電流} \quad \text{アンペールの法則}\end{cases}$$

$$\Leftrightarrow\begin{cases}\frac{\partial E_1}{\partial x^1}+\frac{\partial E_2}{\partial x^2}+\frac{\partial E_3}{\partial x^3}=\rho\\ \left(\frac{\partial B_3}{\partial x^2}-\frac{\partial B_2}{\partial x^3}-\frac{\partial E_1}{\partial t},\ \frac{\partial B_1}{\partial x^3}-\frac{\partial B_3}{\partial x^1}-\frac{\partial E_2}{\partial t},\ \frac{\partial B_2}{\partial x_1}-\frac{\partial B_1}{\partial x^2}-\frac{\partial E_3}{\partial t}\right)\\ \quad=(j_1,j_2,j_3)\end{cases}$$

$$\Leftrightarrow\quad w = j_1dx^1+j_2dx^2+j_3dx^3+\rho\,dt$$

$$=\left(\frac{\partial B_3}{\partial x^2}-\frac{\partial B_2}{\partial x^3}-\frac{\partial E_1}{\partial t}\right)dx^1+\left(\frac{\partial B_1}{\partial x^3}-\frac{\partial B_3}{\partial x^1}-\frac{\partial E_2}{\partial t}\right)dx^2+\left(\frac{\partial B_2}{\partial x^1}-\frac{\partial B_1}{\partial x^2}-\frac{\partial E_3}{\partial t}\right)dx^3$$

$$+\left(\frac{\partial E_1}{\partial x^1}+\frac{\partial E_2}{\partial x^2}+\frac{\partial E_3}{\partial x^3}\right)dt$$