$\Phi(y) = \Phi(y^1, \cdots, y^m) = (\varphi^1(y), \cdots, \varphi^n(y))$ とする。

$x = \Phi(y) = \Phi(\Psi(z)) = \Phi\Psi(z)$

$(\Phi\Psi)^* dx^i = d(\Phi\Psi)^* x^i$　　x^i は x^i の関数

$= d(\varphi^i \circ \Psi)$

$= \sum_{j=1}^{l} \frac{\partial \varphi^i(\Psi(z^1, \cdots, z^l))}{\partial z^j} dz^j$

$= \sum_{j=1}^{l} \sum_{k=1}^{m} \frac{\partial \varphi^i}{\partial y^k} \frac{\partial \psi^k}{\partial z^j} dz^j$　　（合成関数の微分）

一方　$\Psi^* \Phi^*(dx^i) = \Psi^*(d\varphi^i)$

$= \Psi^* \left(\sum_{k=1}^{m} \frac{\partial \varphi^i}{\partial y^k} dy^k \right)$

$= \sum_{k=1}^{m} \frac{\partial \varphi^i}{\partial y^k} d\psi^k$

$= \sum_{j=1}^{l} \sum_{k=1}^{m} \frac{\partial \varphi^i}{\partial y^k} \frac{\partial \psi^k}{\partial z^j} dz^j$

$u = dx^i$ のとき成立。一般の場合は上と (i) と (ii)　□

問6　$\Phi : \mathbb{R}^2 \to \mathbb{R}^3 : (s,t) \to (x, y, z) = (s^2, st, t^2)$

のとき　$\Phi^*(dx \wedge dy + x\,dy \wedge dz)$

解）$\Phi^*(dx \wedge dy + x\,dy \wedge dz)$

$= d(s^2) \wedge d(st) + s^2 d(st) \wedge d(t^2)$

$= (2s\,ds + 0\,dt) \wedge (t\,ds + s\,dt) + s^2(t\,ds + s\,dt) \wedge (0\,ds + 2t\,dt)$

$= 2st\,\underset{0}{\underset{\shortparallel}{ds \wedge ds}} + 2s^2\,ds \wedge dt + 2s^2t^2\,ds \wedge dt + s^3 2t\,dt \wedge dt$

$= 2s^2(1 + t^2)\,ds \wedge dt$　□