ハミルトニアン $H = \dfrac{p_1^2 + p_2^2}{2} - \left(\dfrac{c}{\sqrt{x_1^2+x_2^2}} + \dfrac{\delta}{x_1^2+x_2^2} \right)$ (3.41)

$K(r) = \dfrac{c}{r} + r^2$ とおくと、例 3.11 の極座標変換より

$$H(q,p) = -K(q_1) + \frac{p_1^2}{2} + \frac{p_2^2}{2q_1^2} \qquad (3.9)$$

$$= \frac{p_1^2}{2} + \frac{p_2^2}{2q_1^2} - \frac{c}{q_1} - \frac{\delta}{q_1^2} \qquad (3.42)$$

。

H は q_2 を含まないから p_2 は第1積分。$p_2 = \alpha$ とおくと、

$$H_0 = \frac{p_1^2}{2} + \frac{\alpha^2}{2q_1^2} - \frac{c}{q_1} - \frac{\delta}{q_1^2} = \frac{p_1^2}{2} + \left(\frac{\alpha^2}{2} - \delta\right)\frac{1}{q_1^2} - \frac{c}{q_1}$$

$$= \frac{p_1^2}{2} + \left(\frac{\alpha^2}{2} - \delta\right)\left(\frac{1}{q_1^2} - \frac{2}{\alpha^2 - 2\delta}\frac{c}{q_1}\right)$$

$$= \frac{p_1^2}{2} + \left(\frac{\alpha^2}{2} - \delta\right)\left(\frac{1}{q_1} - \frac{c}{\alpha^2 - 2\delta}\right)^2 - \left(\frac{\alpha^2}{2} - \delta\right)\left(\frac{c}{\alpha^2 - 2\delta}\right)^2$$

$$= \frac{p_1^2}{2} + \left(\frac{\alpha^2}{2} - \delta\right)\left(\frac{1}{q_1} - \frac{c}{\alpha^2 - 2\delta}\right)^2 - \frac{c^2}{2(\alpha^2 - 2\delta)} \qquad (3.43)$$

は積分曲線上で定数，$\alpha^2 > 2\delta$ のとき，

$$\sqrt{2H_0 + \frac{c^2}{\alpha^2 - 2\delta}} = \beta, \quad \sqrt{1 - \frac{2\delta}{\alpha^2}} = \frac{1}{\rho}$$

とおくと

(3.43) $\Rightarrow$ $p_1^2 + \dfrac{\alpha^2}{\rho^2}\left(\dfrac{1}{q_1} - \dfrac{c}{\alpha^2 - 2\delta}\right)^2 = \beta^2$

$$\Rightarrow \begin{cases} p_1 = \beta \sin\theta \\ \dfrac{1}{q_1} = \dfrac{\beta\alpha}{\rho}\cos\theta + \dfrac{c}{\alpha^2 - 2\delta} \end{cases} \qquad (3.44)$$

となる $\theta = \theta(t)$ が存在する。