$$= \left(\frac{\partial V^x}{\partial x} dx + \frac{\partial V^x}{\partial y} dy + \frac{\partial V^x}{\partial z} dz \right) \wedge dy \wedge dz$$

$$+ \left(\frac{\partial V^y}{\partial x} dx + \frac{\partial V^y}{\partial y} dy + \frac{\partial V^y}{\partial z} dz \right) \wedge dz \wedge dx$$

$$+ \left(\frac{\partial V^z}{\partial x} dx + \frac{\partial V^z}{\partial y} dy + \frac{\partial V^z}{\partial z} dz \right) \wedge dx \wedge dy$$

$$= \left(\frac{\partial V^x}{\partial x} + \frac{\partial V^y}{\partial y} + \frac{\partial V^z}{\partial z} \right) dx \wedge dy \wedge dz$$

$$= * \left(\frac{\partial V^x}{\partial x} + \frac{\partial V^y}{\partial y} + \frac{\partial V^z}{\partial z} \right)$$

$$= * (\mathrm{div}\, \mathbb{V})$$ □

問9　ベクトル場 $\mathbb{V}$, $\mathbb{W}$ に対して.

$$i_1(\mathbb{V}) \wedge i_1(\mathbb{W}) = i_2(\mathbb{V} \times \mathbb{W}).$$

(証)　$i_1(\mathbb{V}) \wedge i_2(\mathbb{W})$

$$= (V^x dx + V^y dy + V^z dz) \wedge (W^x dx + W^y dy + W^z dz)$$

$$= V^x W^y dx \wedge dy + V^x W^z dx \wedge dz$$

$$+ V^y W^x dy \wedge dx + V^y W^z dy \wedge dz$$

$$+ V^z W^x dz \wedge dx + V^z W^y dz \wedge dy$$

$$= (V^x W^y - V^y W^x) dx \wedge dy + (V^z W^x - V^x W^z) dz \wedge dx$$

$$+ (V^y W^z - V^z W^y) dy \wedge dz$$

$$= * \big((V^y W^z - V^z W^y) dx + (V^z W^x - V^x W^z) dy$$

$$+ (V^x W^y - V^y W^x) dz \big)$$