問4　G_1, G_2：Hから決まるハミルトン方程式の第1積分

のとき、

$\{G_1, G_2\}$も第1積分

[証明]　$\{\{G_1, G_2\}, H\} = -\{\{G_2, H\}, G_1\} - \{\{H, G_1\}, G_2\}$

$= 0$　(ヤコビの恒等式、定理3.36) ■

問5　$\{f, gh\} = g\{f, h\} + h\{f, g\}$

[証明]　(左辺) $= \sum_i \left(\frac{\partial f}{\partial q^i} \frac{\partial (gh)}{\partial p_i} - \frac{\partial f}{\partial p_i} \frac{\partial (gh)}{\partial q^i} \right)$

$$= \sum_i \left(h \frac{\partial f}{\partial q^i} \frac{\partial g}{\partial p_i} - h \frac{\partial f}{\partial p_i} \frac{\partial g}{\partial q^i} \right) + \sum_i \left(g \frac{\partial f}{\partial q^i} \frac{\partial h}{\partial p_i} - g \frac{\partial f}{\partial p_i} \frac{\partial g}{\partial q^i} \right)$$

$= h\{f, g\} + g\{f, h\}$ ■

(d) ネーターの定理

$$\mathbb{V} = \sum_i \left(\frac{\partial H}{\partial q^i} \frac{\partial}{\partial p_i} - \frac{\partial H}{\partial p_i} \frac{\partial}{\partial q^i} \right) = \mathbb{X}_H = \sum_i \left(V^i \frac{\partial}{\partial p_i} + V^{n+i} \frac{\partial}{\partial q^i} \right)$$

φ_t：$\mathbb{V}$の生成する1径数変換群

(read aloud and write on the black load)

(a) 定理3.31　φ_tが正準変換 $\Leftrightarrow$ $^\exists G$ s.t. $\mathbb{V} = \mathbb{X}_G$

(b) 定理3.33　$H \circ \varphi_t = H$ $\Leftrightarrow$ $\mathbb{V}(H) = 0$

(?) $\Rightarrow$ $\mathbb{X}_G(H) = 0$

〜いつ成り立つであろうか。