$$D\varphi_t\cdot\varphi_\varepsilon=\begin{pmatrix}\frac{\partial\varphi_t^1(\varphi_\varepsilon)}{\partial x^1} & \frac{\partial\varphi_t^1(\varphi_\varepsilon)}{\partial x^i} & \frac{\partial\varphi_t^1(\varphi_\varepsilon)}{\partial x^n}\\ \frac{\partial\varphi_t^i(\varphi_\varepsilon)}{\partial x^1} & \frac{\partial\varphi_t^i(\varphi_\varepsilon)}{\partial x^i} & \frac{\partial\varphi_t^j(\varphi_\varepsilon)}{\partial x^n}\\ \frac{\partial\varphi_t^n(\varphi_\varepsilon)}{\partial x^1} & \frac{\partial\varphi_t^n(\varphi_\varepsilon)}{\partial x^i} & \frac{\partial\varphi_t^n(\varphi_\varepsilon)}{\partial x^n}\end{pmatrix}$$

$$=\begin{pmatrix} & & \\ & \frac{\partial\varphi_t^j}{\partial\varphi_\varepsilon^1}\frac{\partial\varphi_\varepsilon^1}{\partial x^i}+\frac{\partial\varphi_t^j}{\partial\varphi_\varepsilon^k}\frac{\partial\varphi_\varepsilon^k}{\partial x^i}+\frac{\partial\varphi_t^j}{\partial\varphi_\varepsilon^n}\frac{\partial\varphi_\varepsilon^n}{\partial x^i} & \\ & & \end{pmatrix}$$

$$=\left(\frac{\partial\varphi_t^j}{\partial\varphi_\varepsilon^k}\right)\left(\frac{\partial\varphi_\varepsilon^k}{\partial x^i}\right)=D\varphi_t\cdot D\varphi_\varepsilon$$

$$=D\varphi_{-t_0}\lim_{\varepsilon\to0}\left(\frac{D\varphi_{-\varepsilon}\,W(\varphi_{t_0+\varepsilon}(p))-W(\varphi_{t_0}(p))}{\varepsilon}\right)$$

$$=D\varphi_{-t_0}[V,W](\varphi_{t_0}(p))$$

$$=0.$$

$\Rightarrow D\varphi_{-t}W(\varphi_t(p))=W(p)$. tについて定値。

$\Rightarrow W(\varphi_t(p))=D\varphi_t W(p)$ (2.33)°°両辺に $D\varphi_t$ を掛ける。

$\ell(s)=\varphi_t(\psi_s(p))$ とおく.

$$\frac{d\ell(s)}{ds}=\frac{d}{ds}\varphi_t(\psi_s(p))=\left(\frac{\partial\varphi_t^i}{\partial\psi_s^k}\right)\left(\frac{d}{ds}\psi_s(p)\right)\leftarrow\text{合成関数の微分}$$

$$=D\varphi_t\,W(\psi_s(p))\leftarrow\text{1径数変換群}$$

$$=W(\varphi_t(\psi_s(p)))\leftarrow(2.33)$$

$$=W(\ell(s))$$

$\ell(s)$は Wの積分曲線。