(2) $M_c = \{ (q_1, q_2, p_1, p_2) \mid H(q_1, q_2, p_1, p_2) = c \}$, $c > 0$

$$\lambda(c) = \inf \{ G(q_1, q_2, p_1, p_2) \mid (q_1, q_2, p_1, p_2) \in M_c \}$$

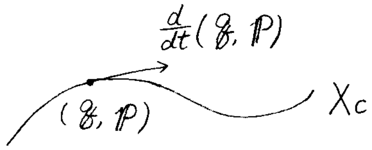
$$\mu(c) = \sup \{ G(q_1, q_2, p_1, p_2) \mid (q_1, q_2, p_1, p_2) \in M_c \}$$

$$X_c = \{ (q_1, q_2, p_1, p_2) \in M_c \mid G(q_1, q_2, p_1, p_2) = \lambda(c) \}$$

$$Y_c = \{ (q_1, q_2, p_1, p_2) \in M_c \mid G(q_1, q_2, p_1, p_2) = \mu(c) \}$$

とするとき，X_c, Y_c は閉曲線である。

[証明] $$\frac{d}{dt}(q_1, q_2, p_1, p_2) = \left(\frac{dq_1}{dt}, \frac{dq_2}{dt}, \frac{dp_1}{dt}, \frac{dp_2}{dt} \right) = \left(\frac{\partial H}{\partial p_1}, \frac{\partial H}{\partial p_2}, -\frac{\partial H}{\partial q_1}, -\frac{\partial H}{\partial q_2} \right)$$



$$\frac{dH}{dt} = \frac{\partial H}{\partial q_1}\frac{dq_1}{dt} + \frac{\partial H}{\partial q_2}\frac{dq_2}{dt} + \frac{\partial H}{\partial p_1}\frac{dp_1}{dt} + \frac{\partial H}{\partial p_2}\frac{dp_2}{dt}$$

$$= 0 \quad = \alpha \cdot 0$$

$$= \alpha \left(\frac{\partial G}{\partial q_1}\frac{dq_1}{dt} + \frac{\partial G}{\partial q_2}\frac{dq_2}{dt} + \frac{\partial G}{\partial p_1}\frac{dp_1}{dt} + \frac{\partial G}{\partial p_2}\frac{dp_2}{dt} \right)$$

$$\therefore \quad \frac{\partial H}{\partial q_i} = \alpha \frac{\partial G}{\partial q_i} \qquad \frac{\partial H}{\partial p_i} = \alpha \frac{\partial G}{\partial p_i} \qquad \left(\frac{dq}{dt}, \frac{dp}{dt} \right) \neq 0$$

$$\therefore \quad \alpha (p_2, -p_1, -q_2, q_1) = (2q_1 e^{q_1^2+q_2^2}, 2q_2 e^{q_1^2+q_2^2}, p_1, p_2)$$

$G(q_1, q_2, p_1, p_2) = \lambda(c)$ より