ゆえに、 (1.37) $\Rightarrow$

$$\frac{\partial H}{\partial p_i} = \sum_{j=1}^{n}\left(\frac{\partial p_j}{\partial p_i} y_j + p_j \frac{\partial y_j}{\partial p_i}\right) - \frac{\partial L}{\partial t}\frac{\partial t}{\partial p_i} - \sum_{j=1}^{n}\left(\frac{\partial L}{\partial x_j}\frac{\partial x_j}{\partial p_i} + \frac{\partial L}{\partial y_j}\frac{\partial y_j}{\partial p_i}\right)$$

$$= y_i = \frac{dq_i}{dt} = \frac{dx_i}{dt}$$

このとき、 $$\frac{dp_i}{dt} = \frac{\partial^2 L}{\partial t \partial y_i}$$

$$-\frac{\partial H}{\partial q_i} = -\sum_{j=1}^{n}\left(\frac{\partial p_j}{\partial q_i} y_j + p_j \frac{\partial y_j}{\partial q_i}\right) + \frac{\partial L}{\partial t}\frac{\partial t}{\partial q_i} + \sum_{j=1}^{n}\left(\frac{\partial L}{\partial x_j}\frac{\partial x_j}{\partial q_i} + \frac{\partial L}{\partial y_j}\frac{\partial y_j}{\partial q_i}\right)$$

0 $\because$ $dp_j = \frac{\partial p_j}{\partial t} dt + \frac{\partial p_j}{\partial q_i} dq_i + \frac{\partial p_j}{\partial p_i} dp_i$

$$= \frac{\partial L}{\partial x_i}$$

$$\therefore \frac{\partial^2 L}{\partial t \partial y_i} = \frac{\partial L}{\partial x_i}$$

$$\therefore \frac{\partial^2 L}{\partial t \partial \dot{x}_i} = \frac{\partial L}{\partial x_i} \qquad \square$$

《まとめ》

1.1 勾配ベクトル場には周期解がない。

1.2 ハミルトン・ベクトル場ではエネルギー保存法則が成り立つ。