

問3 (2.13) について

[解] (2.13) 右边

$$\begin{aligned} &= \left(\frac{\partial f_{12}}{\partial x^1} dx^1 + \frac{\partial f_{12}}{\partial x^2} dx^2 + \frac{\partial f_{12}}{\partial x^3} dx^3 \right) \wedge dx^1 \wedge dx^2 \\ &+ \left(\frac{\partial f_{23}}{\partial x^1} dx^1 + \frac{\partial f_{23}}{\partial x^2} dx^2 + \frac{\partial f_{23}}{\partial x^3} dx^3 \right) \wedge dx^2 \wedge dx^3 \\ &+ \left(\frac{\partial f_{13}}{\partial x^1} dx^1 + \frac{\partial f_{13}}{\partial x^2} dx^2 + \frac{\partial f_{13}}{\partial x^3} dx^3 \right) \wedge dx^1 \wedge dx^3 \\ &= \frac{\partial f_{12}}{\partial x^3} dx^3 \wedge dx^1 \wedge dx^2 + \frac{\partial f_{23}}{\partial x^1} dx^1 \wedge dx^2 \wedge dx^3 \\ &\quad + \frac{\partial f_{13}}{\partial x^2} dx^2 \wedge dx^1 \wedge dx^3 \\ &= \left(\frac{\partial f_{12}}{\partial x^3} + \frac{\partial f_{23}}{\partial x^1} - \frac{\partial f_{13}}{\partial x^2} \right) dx^1 \wedge dx^2 \wedge dx^3 \end{aligned}$$

(C) 一般次元空間の中の微分形式

$$(x^1, \dots, x^n) \in \mathbb{R}^n$$

定義 2.13 \mathbb{R}^n の開集合 U 上の微分形式とは

$$\sum_{1 \leq i_1 < \dots < i_k \leq n} f_{i_1 \dots i_k} dx^{i_1} \wedge \dots \wedge dx^{i_k}$$

なる形式的な和のこと。

$f_{i_1 \dots i_k}$ は U 上の関数。

$n=4$ $k=3$ のとき

$$\begin{aligned} &f_{123} dx^1 \wedge dx^2 \wedge dx^3 + f_{124} dx^1 \wedge dx^2 \wedge dx^4 \\ &+ f_{134} dx^1 \wedge dx^3 \wedge dx^4 + f_{234} dx^2 \wedge dx^3 \wedge dx^4 \end{aligned}$$