$$\int_S du = \int_{\partial S} u .$$

[証明] $\mathbb{W}$: $i_1(\mathbb{W}) = u$ なるベクトル場とする。($i_1(\mathbb{W}) = \mathbb{W}^*$)

$$\int_S du = \int_S d(\mathbb{W}^*) \underset{(2.21)}{=} \int_S i_2(\operatorname{rot}\mathbb{W})$$

$$\underset{\text{補題 } 2.36}{=} \int_S \operatorname{rot}\mathbb{W}\, dS$$

$$\underset{\text{「電磁場～」定理 } 2.36}{=} \int_a^b \mathbb{W}\cdot d\boldsymbol{\ell} = (*)$$

$\boldsymbol{\ell} : [a, b] \to \mathbb{R}^3$: Lの向きを保つパラメータ

$\overset{\text{def}}{\Longleftrightarrow}$ $\boldsymbol{\ell} \in [m] = \{x : [a, b] \to L \mid x(t) = m(s) \Rightarrow {}^{\exists}C > 0,\ \frac{dx}{dt} = C\frac{dm}{dt}$

$u = u^x dx + u^y dy + u^z dz$ とおくと

$$\mathbb{W} = u^x\frac{\partial}{\partial x} + u^y\frac{\partial}{\partial y} + u^z\frac{\partial}{\partial z}$$

$$\therefore\ (*) = \int_a^b \left(W^x\frac{d\ell^1}{dt} + W^y\frac{d\ell^2}{dt} + W^z\frac{d\ell^3}{dt}\right)dt$$

$$= \int_a^b \boldsymbol{\ell}^*(W^x dx + W^y dy + W^z dz) = \int_L u .$$