$$= -4\left(\frac{\partial x}{\partial \sigma_1}\frac{\partial}{\partial x}+\frac{\partial y}{\partial \sigma_1}\frac{\partial}{\partial y}+\frac{\partial z}{\partial \sigma_1}\frac{\partial}{\partial z}\right)\left(\frac{\partial x}{\partial \sigma_1}\frac{\partial}{\partial x}+\frac{\partial y}{\partial \sigma_1}\frac{\partial}{\partial y}+\frac{\partial z}{\partial \sigma_1}\frac{\partial}{\partial z}\right)$$

$$= -4\,\varphi_*\left(\frac{\partial}{\partial \sigma_1}\right)\cdot\varphi_*\left(\frac{\partial}{\partial \sigma_1}\right) \qquad (3.76)$$

$$\left.\frac{\partial \psi}{\partial \sigma_1}\right|_{u=\sigma_1} = -(\sigma_1-\sigma_2)(\sigma_1-\sigma_3)$$

$$= -\psi'(\sigma_1) \quad (u\text{で微分して } u=\sigma_1) \text{だから}$$

(3.75)の右辺をσ_1で偏微分して $u=\sigma_1$ とおくと

$$\frac{\partial}{\partial \sigma_1}\frac{2\psi(u)}{f(u)} = 2\frac{-\psi'(\sigma_1)}{f(\sigma_1)}$$

$$-4\,\varphi_*\left(\frac{\partial}{\partial \sigma_1}\right)\cdot\varphi_*\left(\frac{\partial}{\partial \sigma_1}\right) = -2\frac{\psi'(\sigma_1)}{f(\sigma_1)}$$

$$\| \qquad\qquad (3.77)$$

$$g_{11} = \frac{\psi'(\sigma_1)}{2f(\sigma_1)}$$

σ_2についても同様。

$$H=\frac{1}{2}\sum_{ij} g^{ij}p_i p_j = \frac{1}{2}\frac{2f(\sigma_1)}{\psi'(\sigma_1)}p_1^2 + \frac{1}{2}\frac{2f(\sigma_2)}{-\psi'(\sigma_2)}p_2^2$$

$$\begin{pmatrix} g_{11} & 0 \\ 0 & g_{22}\end{pmatrix}^{-1} = \frac{-f(\sigma_1)}{(\sigma_1-\sigma_2)(\sigma_1-\sigma_3)}p_1^2 + \frac{f(\sigma_2)}{(\sigma_2-\sigma_1)(\sigma_2-\sigma_3)}p_2^2$$

$$= \begin{pmatrix} \frac{g_{22}}{g_{11}g_{22}} & 0 \\ 0 & \frac{g_{11}}{g_{11}g_{22}}\end{pmatrix} = \frac{1}{(\sigma_2-\sigma_1)}\left(\frac{f(\sigma_1)}{(\sigma_3-\sigma_1)}p_1^2 - \frac{f(\sigma_2)}{(\sigma_3-\sigma_2)}p_2^2\right).$$

(3.78)

ハミルトン・ヤコビの方程式は

$$H = \frac{1}{(\sigma_2-\sigma_1)}\left(\frac{f(\sigma_1)\left(\frac{\partial S}{\partial \sigma_1}\right)^2}{(\sigma_3-\sigma_1)} - \frac{f(\sigma_2)\left(\frac{\partial S}{\partial \sigma_2}\right)^2}{(\sigma_3-\sigma_2)}\right) \qquad (3.79)$$

リウビル型