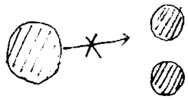
ρは無理数。　$n \neq m$ i.e $r \neq 0$



$$\Rightarrow \varphi_\rho^r(z_2) \neq z_2$$

$$\Rightarrow \varphi_\rho^r(I) \neq I \neq \varphi_\rho^{-r}(I)$$ （∵ φ_ρ は長さを変えない，同相写像だから）

$$\Rightarrow \inf\{|z - z_2| \mid z \notin \varphi_\rho^{-r}(I) \cup I \cup \varphi_\rho^r(I)\} > \varepsilon$$

(3.47) より　$I \cap \{\varphi_\rho^i(z_1) \mid i \in \mathbb{Z}\} = \phi$

$$\begin{aligned} &\varphi_\rho^n(I \cap \{\varphi_\rho^i(z_1) \mid i \in \mathbb{Z}\}) = \phi \\ &\qquad \| \\ &\varphi_\rho^n(I) \cap \varphi_\rho^n\{\varphi_\rho^i(z_1) \mid i \in \mathbb{Z}\}) \\ &\qquad \| \\ &\varphi_\rho^n(I) \cap \{\varphi_\rho^i(z_1) \mid i \in \mathbb{Z}\} \end{aligned}$$

（φ_ρ^n は 1 - 1）

$$\varphi_\rho^n(z_1) \notin \varphi_\rho^n(I)$$

$$\notin \varphi_\rho^n(I) \cup I \cup \varphi^{-n}(I). \quad (\forall n \in \mathbb{Z})$$

$$\Rightarrow |\varphi_\rho^n(z_1) - z_2| \in \{|z - z_2| \mid z \notin \varphi_\rho^{-r}(I) \cup I \cup \varphi_\rho^r(I)\}$$

$$\therefore \{|\varphi_\rho^n(z_1) - z_2| \mid n \in \mathbb{Z}\} \subseteqq \{|z - z_2| \mid z \notin \varphi_\rho^{-r}(I) \cup I \cup \varphi_\rho^r(I)\}.$$

したがって，

$$\varepsilon < \inf\{|z - z_2| \mid z \notin \varphi_\rho^{-r}(I) \cup I \cup \varphi_\rho^r(I)\} \overset{\text{部分集合}}{\leqq} \inf\{|\varphi_\rho^n(z_1) - z_2| \mid n \in \mathbb{Z}\}$$

(3.47) に矛盾. ■

$\widehat{\varphi_\rho^n(I)} = \hat{I}$，　補題 3.50　$\varphi_\rho^n(I) \cap \varphi_\rho^m = \phi$.

$$\sum_{i=1}^{n} \varphi_\rho^i(I) \leqq 2\pi, \quad \sum_{i=1}^{n+1} \varphi^i(I) > 2\pi \text{ のとき}$$

$\exists n, \exists m$ で　$\varphi_\rho^n(I) \cap \varphi_\rho^m \neq \phi$　矛盾.! ■