$$= \begin{pmatrix} B_{11} & B_{21} & B_{31} \\ B_{12} & B_{22} & B_{32} \\ B_{13} & B_{23} & B_{33} \end{pmatrix}^{-1} (x \times y) = ({}^tB)^{-1}(x \times y) = B(x \times y)$$

□

補題 2.61 $B \in SO(3)$, $\widehat{Bx} = B\hat{x}B^{-1}$

[証明] $\widehat{Bx}\, y = Bx \times y = B(x \times B^{-1}y)$

$$= B\hat{x}B^{-1}y$$

□

補題 2.62 U: 反対称行列

$$\widehat{Ux} = U\hat{x} - \hat{x}U = [U, \hat{x}] .$$

[証明]

$$\widehat{Ux} = \widehat{\begin{pmatrix} 0 & -u_3 & u_2 \\ u_3 & 0 & -u_1 \\ -u_2 & u_1 & 0 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}}$$

$$= \widehat{\begin{pmatrix} u_2x_3 - u_3x_2 \\ u_3x_1 - u_1x_3 \\ u_1x_2 - u_2x_1 \end{pmatrix}}$$

$$= \begin{pmatrix} 0 & -u_1x_2 + u_2x_1 & u_3x_1 - u_1x_3 \\ u_1x_2 - u_2x_1 & 0 & -u_2x_3 + u_3x_2 \\ -u_3x_1 + u_1x_3 & u_2x_3 - u_3x_2 & 0 \end{pmatrix} \quad \Delta$$

$$[U, \hat{x}] = \begin{pmatrix} 0 & -u_3 & u_2 \\ u_3 & 0 & -u_1 \\ -u_2 & u_1 & 0 \end{pmatrix}\begin{pmatrix} 0 & -x_3 & x_2 \\ x_3 & 0 & -x_1 \\ -x_2 & x_1 & 0 \end{pmatrix} - \begin{pmatrix} 0 & -x_3 & x_2 \\ x_3 & 0 & -x_1 \\ -x_2 & x_1 & 0 \end{pmatrix}\begin{pmatrix} 0 & -u_3 & u_2 \\ u_3 & 0 & -u_1 \\ -u_2 & u_1 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} -u_3x_3 - u_2x_2 & u_2x_1 & u_3x_1 \\ u_1x_2 & -u_3x_3 - u_1x_1 & u_3x_2 \\ u_1x_3 & u_2x_3 & -u_2x_2 - u_1x_1 \end{pmatrix}$$

$$- \begin{pmatrix} -u_3x_3 - u_2x_2 & u_1x_2 & u_1x_3 \\ u_2x_1 & -u_3x_3 - u_1x_1 & u_2x_3 \\ u_3x_1 & u_3x_2 & -u_2x_2 - u_1x_1 \end{pmatrix}$$