

$$\begin{aligned}
\rightarrow d\mathcal{V} &= \left( \frac{\partial A_1}{\partial x^1} dx^1 + \frac{\partial A_1}{\partial x^2} dx^2 + \frac{\partial A_1}{\partial x^3} dx^3 + \frac{\partial A_1}{\partial t} dt \right) \wedge dx^1 \\
&+ \left( \frac{\partial A_2}{\partial x^1} dx^1 + \frac{\partial A_2}{\partial x^2} dx^2 + \frac{\partial A_2}{\partial x^3} dx^3 + \frac{\partial A_2}{\partial t} dt \right) \wedge dx^2 \\
&+ \left( \frac{\partial A_3}{\partial x^1} dx^1 + \frac{\partial A_3}{\partial x^2} dx^2 + \frac{\partial A_3}{\partial x^3} dx^3 + \frac{\partial A_3}{\partial t} dt \right) \wedge dx^3 \\
&- \left( \frac{\partial \varphi}{\partial x^1} dx^1 + \frac{\partial \varphi}{\partial x^2} dx^2 + \frac{\partial \varphi}{\partial x^3} dx^3 + \frac{\partial \varphi}{\partial t} dt \right) \wedge dt \\
&= \left( \frac{\partial A_3}{\partial x^2} - \frac{\partial A_2}{\partial x^3} \right) dx^2 \wedge dx^3 + \left( \frac{\partial A_1}{\partial x^3} - \frac{\partial A_3}{\partial x^1} \right) dx^3 \wedge dx^1 + \left( \frac{\partial A_2}{\partial x^1} - \frac{\partial A_1}{\partial x^2} \right) dx^1 \wedge dx^2 \\
&- \left( \frac{\partial \varphi}{\partial x^1} + \frac{\partial A_1}{\partial t} \right) dx^1 \wedge dt - \left( \frac{\partial \varphi}{\partial x^2} + \frac{\partial A_2}{\partial t} \right) dx^2 \wedge dt - \left( \frac{\partial \varphi}{\partial x^3} + \frac{\partial A_3}{\partial t} \right) dx^3 \wedge dt
\end{aligned}$$

$$= B_1 dx^2 \wedge dx^3 + B_2 dx^3 \wedge dx^1 + B_3 dx^1 \wedge dx^2$$

$$+ E_1 dx^1 \wedge dt + E_2 dx^2 \wedge dt + E_3 dx^3 \wedge dt$$

$$= \mathcal{U}$$

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$$\begin{aligned}
\rightarrow &= \left( \frac{\partial}{\partial x^2} \left( \frac{\partial A_2}{\partial x^1} - \frac{\partial A_1}{\partial x^2} \right) - \frac{\partial}{\partial x^3} \left( \frac{\partial A_1}{\partial x^3} - \frac{\partial A_3}{\partial x^1} \right) + \frac{\partial}{\partial t} \left( \frac{\partial \varphi}{\partial x^1} + \frac{\partial A_1}{\partial t} \right) \right) dx^1 \\
&+ \left( \frac{\partial}{\partial x^3} \left( \frac{\partial A_3}{\partial x^2} - \frac{\partial A_2}{\partial x^3} \right) - \frac{\partial}{\partial x^1} \left( \frac{\partial A_2}{\partial x^1} - \frac{\partial A_1}{\partial x^2} \right) + \frac{\partial}{\partial t} \left( \frac{\partial \varphi}{\partial x^2} + \frac{\partial A_2}{\partial t} \right) \right) dx^2 \\
&+ \left( \frac{\partial}{\partial x^1} \left( \frac{\partial A_1}{\partial x^3} - \frac{\partial A_3}{\partial x^1} \right) - \frac{\partial}{\partial x^2} \left( \frac{\partial A_3}{\partial x^2} - \frac{\partial A_2}{\partial x^3} \right) + \frac{\partial}{\partial t} \left( \frac{\partial \varphi}{\partial x^3} + \frac{\partial A_3}{\partial t} \right) \right) dx^3 \\
&+ \left( \frac{\partial}{\partial x^1} \left( -\frac{\partial \varphi}{\partial x^1} - \frac{\partial A_1}{\partial t} \right) + \frac{\partial}{\partial x^2} \left( -\frac{\partial \varphi}{\partial x^2} - \frac{\partial A_2}{\partial t} \right) + \frac{\partial}{\partial x^3} \left( -\frac{\partial \varphi}{\partial x^3} - \frac{\partial A_3}{\partial t} \right) \right) dt
\end{aligned}$$

$$\begin{aligned}
&= \left( \frac{\partial^2}{\partial t^2} - \left( \frac{\partial}{\partial x^1} \right)^2 - \left( \frac{\partial}{\partial x^2} \right)^2 - \left( \frac{\partial}{\partial x^3} \right)^2 \right) A_1 dx^1 + \left( \frac{\partial^2}{\partial x^2 \partial x^1} A_2 + \frac{\partial^2}{\partial x^3 \partial x^1} A_3 + \frac{\partial^2}{\partial t \partial x^1} \varphi \right) dx^1 \\
&+ \left( \frac{\partial^2}{\partial t^2} - \left( \frac{\partial}{\partial x^1} \right)^2 - \left( \frac{\partial}{\partial x^3} \right)^2 \right) A_2 dx^2 + \left( \frac{\partial^2}{\partial x^3 \partial x^2} A_3 + \frac{\partial^2}{\partial x^1 \partial x^2} A_1 + \frac{\partial^2}{\partial t \partial x^2} \varphi \right) dx^2 \\
&+ \left( \frac{\partial^2}{\partial t^2} - \left( \frac{\partial}{\partial x^1} \right)^2 - \left( \frac{\partial}{\partial x^2} \right)^2 \right) A_3 dx^3 + \left( \frac{\partial^2}{\partial x^1 \partial x^3} A_1 + \frac{\partial^2}{\partial x^2 \partial x^3} A_2 + \frac{\partial^2}{\partial t \partial x^3} \varphi \right) dx^3 \\
&+ \left( -\left( \frac{\partial}{\partial x^1} \right)^2 - \left( \frac{\partial}{\partial x^2} \right)^2 - \left( \frac{\partial}{\partial x^3} \right)^2 \right) \varphi dt + \left( -\frac{\partial^2}{\partial x^1 \partial t} A_1 - \frac{\partial^2}{\partial x^2 \partial t} A_2 - \frac{\partial^2}{\partial x^3 \partial t} A_3 \right) dt
\end{aligned}$$

$$= (\star)$$