補題 3.39 (i) φ_t: ベクトル場 $\mathbb{V}$ の生成する1径数変換群 が 正準変換

$\Longleftrightarrow$ (ii) $d\,i_1 J(\mathbb{V}) = 0$.

[証明] (i) $\Rightarrow$ $\varphi_t^* w = w$.

$$\Rightarrow \lim_{\varepsilon \to 0} \frac{\varphi_\varepsilon^* w - w}{\varepsilon} \underset{\substack{\uparrow \\ (3.34)}}{=} -d\,i_1 J(\mathbb{V}) = 0 \Rightarrow d\,i_1 J(\mathbb{V}) = 0$$

(ii) $\Rightarrow$

$$\frac{\partial}{\partial t} \varphi_t^* w \Big|_{t=t_0} = \lim_{\varepsilon \to 0} \frac{\varphi_{t_0+\varepsilon}^* w - \varphi_{t_0}^* w}{\varepsilon} = \lim_{\varepsilon \to 0} \frac{\varphi_{t_0}^* \varphi_\varepsilon^* w - \varphi_{t_0}^* w}{\varepsilon}$$

$$= \varphi_{t_0}^* \lim_{\varepsilon \to 0} \frac{\varphi_\varepsilon^* w - w}{\varepsilon} = -\varphi_{t_0}^* d\,i_1 J(\mathbb{V}) = 0$$

$$\Rightarrow \varphi_t^* w = \varphi_{t_0}^* w = \varphi_0^* w = w$$ ■

◎ 考えている領域を単連結とする.

$\Downarrow \longleftarrow$ 定理 2.31

$d\,i_1 J(\mathbb{V}) = 0 \Longleftrightarrow \exists G$, s.t $dG = i_1 J(\mathbb{V})$.

$$\mathbb{X}_G \overset{\text{def}}{=} \sum_{i=1}^{n} \left(\frac{\partial G}{\partial q_i} \frac{\partial}{\partial p_i} - \frac{\partial H}{\partial p_i} \frac{\partial}{\partial q_i} \right)$$ とする。

補題 3.30 (i) $dG = i_1 J(\mathbb{V}) \Longleftrightarrow$ (ii) $\mathbb{V} = -\mathbb{X}_G$

[証明] (i) $\Longleftrightarrow$ $\displaystyle\sum_i \frac{\partial G}{\partial p_i} dp_i + \frac{\partial G}{\partial q_i} dq_i = \sum_i V^i dp_i - V^{i+n} dq_i$

$$\Longleftrightarrow \begin{cases} \dfrac{\partial G}{\partial p_i} = V^i \\ \dfrac{\partial G}{\partial q_i} = -V^{i+n} \end{cases}$$

$$\Longleftrightarrow \mathbb{V} = \sum_i \frac{\partial G}{\partial p_i} \frac{\partial}{\partial q_i} - \frac{\partial G}{\partial q_i} \frac{\partial}{\partial p_i} = -\mathbb{X}_G \Longleftrightarrow \text{(ii)}$$ ■