(ii) $\Rightarrow \quad \Phi^* \sum_i P^i dQ^i = \sum_{i,j} P^i \left(\frac{\partial Q^i}{\partial q^j} dq^j + \frac{\partial Q^i}{\partial p^j} dp^j\right) = \sum_j p^j dq^j$

$$\Rightarrow \begin{cases} \dfrac{\partial Q^i}{\partial p^j} = 0 \quad (\forall i, j) \\ p^j = \sum_i P^i \dfrac{\partial Q^i}{\partial q^j} \end{cases} \Rightarrow \begin{cases} \mathbb{Q} = \mathbb{Q}(q) \\ \mathbb{P} = {}^t D\mathbb{Q}(q)^{-1} \mathbb{p} \end{cases}$$

↑ 両辺に逆行列を掛ける。 ■

例3.11 2次元 極座標変換の場合

$(Q_1, Q_2) = (q_1 \cos q_2,\ q_1 \sin q_2)$ のとき

$$D\mathbb{Q} = \begin{pmatrix} \cos q_2 & -q_1 \sin q_2 \\ \sin q_2 & q_1 \cos q_2 \end{pmatrix}, \quad D\mathbb{Q}^{-1} = \begin{pmatrix} \cos q_2 & \sin q_2 \\ -\dfrac{\sin q_2}{q_1} & \dfrac{\cos q_2}{q_1} \end{pmatrix}$$

$${}^t D\mathbb{Q}^{-1} = \begin{pmatrix} \cos q_2 & -\dfrac{\sin q_2}{q_1} \\ \sin q_2 & \dfrac{\cos q_2}{q_1} \end{pmatrix},$$

(3.7)第2式 $\begin{cases} P_1 = p_1 \cos q_2 - p_2 \dfrac{\sin q_2}{q_1} \\ P_2 = p_1 \sin q_2 + p_2 \dfrac{\cos q_2}{q_1} \end{cases}$ Δ

中心力場のハミルトニアン

$$H(\mathbb{Q}, \mathbb{P}) = -K\left(\sqrt{Q_1^2 + Q_2^2}\right) + \frac{P_1^2 + P_2^2}{2}$$

に対して,

$$H(\Phi(q, \mathbb{p})) = H(q, \mathbb{p}) = -K_1(q_1) + \frac{p_1^2}{2} + \frac{p_2^2}{2q_1^2} \qquad (3.9).$$

$$\Rightarrow \quad \frac{dp_2}{dt} = -\frac{\partial H}{\partial q_2} = 0 \Rightarrow \begin{cases} p_2 = C_1 \\ -K(q_1) + \dfrac{p_1^2}{2} + \dfrac{C_1^2}{2q_1^2} = C_2 \end{cases}$$

(ハミルトニアンは時間について定数)

□