(c) ベクトル場の記号

$\mathbb{W}$: 開集合 U 上のベクトル場

$x^1, \cdots, x^n$: U の座標　　　　のとき

$$\mathbb{W} = (W^1, \cdots, W^n)$$

$$\doteq W^1 \frac{\partial}{\partial x^1} + \cdots + W^n \frac{\partial}{\partial x^n} = \sum_{i=1}^n W^i \frac{\partial}{\partial x^i} \qquad (2.4)$$

$$0 \cdot \frac{\partial}{\partial x^i} = 0, \qquad 1 \cdot \frac{\partial}{\partial x^i} = \frac{\partial}{\partial x^i} \qquad \Delta$$

例

$$\mathbb{W} = \frac{\partial}{\partial x^2} - \frac{\partial}{\partial x^3} = (0, 1, -1, 0, \cdots, 0)$$

$$= 0\frac{\partial}{\partial x^1} + 1\frac{\partial}{\partial x^2} + (-1)\frac{\partial}{\partial x^3} + 0\frac{\partial}{\partial x^4} + \cdots + 0\frac{\partial}{\partial x^n} \qquad \Delta$$

定義 2.2 より,

$$\Phi_*\left(\sum_{i=1}^n W^i \frac{\partial}{\partial x^i}\right) \overset{\text{記号の意味}}{=} \Phi_* \mathbb{W}$$

$$\underset{\text{Def 2.2}}{=} \begin{pmatrix} \frac{\partial y^1}{\partial x^1} & \frac{\partial y^1}{\partial x^i} & \frac{\partial y^1}{\partial x^n} \\ \frac{\partial y^j}{\partial x^1} & \frac{\partial y^j}{\partial x^i} & \frac{\partial y^j}{\partial x^n} \\ \frac{\partial y^n}{\partial x^1} & \frac{\partial y^n}{\partial x^i} & \frac{\partial y^n}{\partial y^n} \end{pmatrix} \begin{pmatrix} W^1 \\ W^i \\ W^n \end{pmatrix} = \begin{pmatrix} \sum_{i=1}^n \frac{\partial y^1}{\partial x^i} W^i \\ \sum_{i=1}^n \frac{\partial y^j}{\partial x^i} W^i \\ \sum_{i=1}^n \frac{\partial y^n}{\partial x^i} W^i \end{pmatrix}$$

$$= \sum_{j=1}^n \sum_{i=1}^n \frac{\partial y^j}{\partial x^i} W^i \frac{\partial}{\partial y^j}$$

$$\mathbb{W} = (0, \cdots, \underset{i\text{ th}}{1}, \cdots, 0)$$ とすると