$$\frac{dL}{dt} = \begin{pmatrix} \frac{db_1}{dt} & \frac{da_1}{dt} & \frac{da_3}{dt} \\ \frac{da_1}{dt} & \frac{db_2}{dt} & \frac{da_2}{dt} \\ \frac{da_3}{dt} & \frac{da_2}{dt} & \frac{db_3}{dt} \end{pmatrix}$$

$$= \begin{pmatrix} -2a_1^2 + 2a_3^2 & a_1b_1 - a_1b_2 & a_3b_3 - a_3b_1 \\ a_1b_1 - a_1b_2 & -2a_2^2 + 2a_1^2 & a_2b_2 - a_2b_3 \\ a_3b_3 - a_3b_3 & a_2b_2 - a_2b_3 & -2a_3^2 + 2a_2^2 \end{pmatrix}$$

$$= \begin{pmatrix} -a_1^2 + a_3^2 & -a_1b_2 + a_2a_3 & -a_1a_2 + a_3b_3 \\ a_1b_1 - a_2a_3 & a_1^2 - a_2^2 & a_1a_3 - a_2b_3 \\ -a_3b_1 + a_1a_2 & -a_3a_1 + a_2b_2 & -a_3^2 + a_2^2 \end{pmatrix}$$

$$- \begin{pmatrix} a_1^2 - a_3^2 & -a_1b_1 + a_3a_2 & a_3b_1 - a_1a_2 \\ a_1b_2 - a_2a_3 & -a_1^2 + a_2^2 & a_1a_3 - a_2b_2 \\ a_2a_1 - a_3b_3 & -a_1a_3 + a_2b_3 & a_3^2 - a_2^2 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & -a_1 & a_3 \\ a_1 & 0 & -a_2 \\ -a_3 & a_2 & 0 \end{pmatrix} \begin{pmatrix} b_1 & a_1 & a_3 \\ a_1 & b_2 & a_2 \\ a_3 & a_2 & b_3 \end{pmatrix} - \begin{pmatrix} b_1 & a_1 & a_3 \\ a_1 & b_2 & a_2 \\ a_3 & a_2 & b_3 \end{pmatrix} \begin{pmatrix} 0 & -a_1 & a_3 \\ a_1 & 0 & -a_2 \\ -a_3 & a_2 & 0 \end{pmatrix}$$

$$= B(t)\,L(t) - L(t)\,B(t)$$