2 ベクトル場と微分形式

§2.1 ベクトル場の座標変換

(a) 常微分方程式の変数変換

$$\frac{dx}{dt} = \mathbb{V}(x) \qquad (2.1)$$

$$y^i = y^i(x) = y^i(x^1, \cdots, x^n).$$

x_i が (2.1) を満たすとして y^i を t で微分

$$\frac{dy^i}{dt}(t) = \sum_{j=1}^{n} \frac{\partial y^i}{\partial x^j}(x^1, \cdots, x^n)\, \frac{dx^j}{dt}(t)$$

$$= \sum_{j=1}^{n} \frac{\partial y^i}{\partial x^j}(x^1, \cdots, x^n)\, V^j(x^1, \cdots, x^n) \qquad (2.2)$$

(自励系)

$$\frac{dy}{dt} = \left(\sum_{j=1}^{n} \frac{\partial y^1}{\partial x^j} V^j, \ \cdots, \ \sum_{j=1}^{n} \frac{\partial y^m}{\partial x^j} V^j \right) \qquad (2.3)$$

例 2.1 　 $n = 2$ 　 $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} r\cos\theta \\ r\sin\theta \end{pmatrix}$

$$r = \sqrt{x^2 + y^2} \qquad \theta = \tan^{-1}(y/x).$$

$$\begin{cases} \dfrac{dx}{dt} = V^x(x, y) \\[2mm] \dfrac{dy}{dt} = V^y(x, y) \end{cases}$$

のとき

$$\frac{dr}{dt} = \frac{\partial r}{\partial x} V^x(r\cos\theta, r\sin\theta) + \frac{\partial r}{\partial y} V^y(r\cos\theta, r\sin\theta)$$

$$= \cos\theta\, V^x(r\cos\theta, r\sin\theta) + \sin\theta\, V^y(r\cos\theta, r\sin\theta)$$