$$
\begin{tikzcd}
(s,t) \arrow[rr, "\varphi"] \arrow[dr, "\Phi"'] & & p \\
& (u,v) \arrow[ur, "\psi"'] &
\end{tikzcd}
$$

$\because$ $\varphi : (s,t) \mapsto \begin{pmatrix} \varphi^1(s,t) \\ \varphi^2(\ ,\ ) \\ \varphi^3(\ ,\ ) \end{pmatrix}$ $\quad \psi : (u,v) \mapsto \begin{pmatrix} \psi^1(u,v) \\ \psi^2(\ ,\ ) \\ \psi^3(\ ,\ ) \end{pmatrix}$

$$\Phi : (s,t) \mapsto \begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} \Phi^1(s,t) \\ \Phi^2(s,t) \end{pmatrix}, \quad \psi\Phi = \varphi$$

$\forall p \in S$ について　法ベクトル

$$n(p) = n(\psi(u,v)) = +\frac{\partial \psi}{\partial u} \times \frac{\partial \psi}{\partial v} \Big/ \left\| \frac{\partial \psi}{\partial u} \times \frac{\partial \psi}{\partial v} \right\|$$

$$= \begin{pmatrix} \frac{\partial \psi^1}{\partial u} \\ \frac{\partial \psi^2}{\partial u} \\ \frac{\partial \psi^3}{\partial u} \end{pmatrix} \times \begin{pmatrix} \frac{\partial \psi^1}{\partial v} \\ \frac{\partial \psi^2}{\partial v} \\ \frac{\partial \psi^3}{\partial v} \end{pmatrix} \Big/ \| \qquad \|$$

$$= \begin{pmatrix} \frac{\partial \varphi^1}{\partial s} \\ \frac{\partial \varphi^2}{\partial s} \\ \frac{\partial \varphi^3}{\partial s} \end{pmatrix} \times \begin{pmatrix} \frac{\partial \varphi^1}{\partial t} \\ \frac{\partial \varphi^2}{\partial t} \\ \frac{\partial \varphi^3}{\partial t} \end{pmatrix} \det D\Phi \Big/ \| \qquad \|$$

$$\because \begin{pmatrix} \frac{\partial \psi}{\partial u} & \frac{\partial \psi}{\partial v} \end{pmatrix} = \begin{pmatrix} \frac{\partial \varphi}{\partial s} & \frac{\partial \varphi}{\partial t} \end{pmatrix} D\Phi$$

$$(\mathbb{a}\ \mathbb{b}) = (\mathbb{c}\ \mathbb{d}) A$$

$$\Rightarrow \mathbb{a} \times \mathbb{b} = (\mathbb{c} \times \mathbb{d}) \det A$$

$\square$

定理 2.28 より

$$\int_U \varphi^* u = \int_U \Phi^* \psi^* u = \int_V \psi^* u$$

$\therefore$ $\int_U \varphi^* u$ が座標 φ によらない。