定理3.25は，（read the rest aloud）～ 例をやってみよう。

例3.26 $H = \dfrac{p_1^2}{2} + (p_1^2 + q_1^2)p_2^2 + \dfrac{q_1^2}{2} - q_2$.（tに依存しない場合）

↑

巡回座標を求める例

このとき H・J方程式は，（(3.24)を参考にして，Sは未知）

$$K(Q_1, Q_2) = \frac{1}{2}\left(\frac{\partial S}{\partial q_1}\right)^2 + \left(\left(\frac{\partial S}{\partial q_1}\right)^2 + q_1^2\right)\left(\frac{\partial S}{\partial q_2}\right)^2 + \frac{q_1^2}{2} - q_2 \qquad (3.25)$$

$$= \frac{1}{2}\left\{\left(\frac{\partial S}{\partial q_1}\right)^2 + q_1^2\right\} + \left(\left(\frac{\partial S}{\partial q_1}\right)^2 + q_1^2\right)\left(\frac{\partial S}{\partial q_2}\right)^2 - q_2$$

$$\begin{cases} Q_1^2 = \left(\dfrac{\partial S_1(q_1; Q_1)}{\partial q_1}\right)^2 + q_1^2 \\ Q_2^2 = Q_1^2\left(\dfrac{\partial S_2(q_2; Q_1, Q_2)}{\partial q_2}\right)^2 - q_2 \end{cases} \qquad (3.26)$$

$\boxed{\dfrac{\partial S}{\partial q_1} = \dfrac{\partial S_1}{\partial q_1},\ \dfrac{\partial S}{\partial q_2} = \dfrac{\partial S_2}{\partial q_2}}$ ， $S(q_1, q_2; Q_1, Q_2) = S_1(q_1; Q_1) + S_2(q_2; Q_1, Q_2)$ とおくと

$$K = Q_2^2 + \frac{Q_1^2}{2}$$ と書ける．

(3.26)を解くと，

$$\begin{cases} S_1(q_1; Q_1) = \displaystyle\int_0^{q_1} \sqrt{Q_1^2 - x^2}\, dx \\ S_2(q_2; Q_1, Q_2) = \dfrac{1}{Q_1}\displaystyle\int_0^{q_2} \sqrt{Q_2^2 + x}\, dx \end{cases}$$

よって
$$\begin{cases} p_1 = \dfrac{\partial S}{\partial q_1} = \sqrt{Q_1^2 - q_1^2} \\ p_2 = \dfrac{\partial S}{\partial q_2} = \dfrac{\sqrt{Q_2^2 + q_2}}{Q_1} \end{cases}$$

$$\Rightarrow \begin{cases} Q_1 = \sqrt{p_1^2 + q_1^2} \\ Q_2 = \sqrt{Q_1^2 p_2^2 - q_2} = \sqrt{(p_1^2 + q_1^2)p_2^2 - q_2} \end{cases}$$

が 巡回座標（第1積分）（定理3.25） □