$= -100(xy+1)\,(dx\wedge dz)\wedge dy$

$= 100(xy+1)\,dx\wedge dy\wedge dz$ □

(b) 3次元空間の中の微分形式の外微分

定義 2.12 f：U 上の関数（微分0形式）の **外微分** とは

$$df = \frac{\partial f}{\partial x^1}dx^1 + \frac{\partial f}{\partial x^2}dx^2 + \frac{\partial f}{\partial x^3}dx^3 \qquad (2.11)$$

微分 1, 2, 3 形式の外微分とは、

$$d(f_1dx^1 + f_2dx^2 + f_3dx^3) = df_1\wedge dx^1 + df_2\wedge dx^2 + df_3\wedge dx^3 \qquad (2.12)$$

$$d(f_{12}dx^1\wedge dx^2 + f_{23}dx^2\wedge dx^3 + f_{13}dx^1\wedge dx^3)$$

$$= df_{12}\wedge dx^1\wedge dx^2 + df_{23}\wedge dx^2\wedge dx^3 + df_{13}\wedge dx^1\wedge dx^3, \qquad (2.13)$$

$$d(f\,dx\wedge dy\wedge dz) = 0 \qquad (2.14)$$

□

（微分 k 形式の外微分は 微分 $k+1$ 形式）

(2.12) より

$$d(f_1dx^1+f_2dx^2+f_3dx^3) = df_1\wedge dx^1 + df_2\wedge dx^2 + df_3\wedge dx^3$$

$$= \left(\frac{\partial f_1}{\partial x^1}dx^1 + \frac{\partial f_1}{\partial x^2}dx^2 + \frac{\partial f_1}{\partial x^3}dx^3\right)\wedge dx^1$$
$$+ \left(\frac{\partial f_2}{\partial x^1}dx^1 + \frac{\partial f_2}{\partial x^2}dx^2 + \frac{\partial f_2}{\partial x^3}dx^3\right)\wedge dx^2$$
$$+ \left(\frac{\partial f_3}{\partial x^1}dx^1 + \frac{\partial f_3}{\partial x^2}dx^2 + \frac{\partial f_3}{\partial x^3}dx^3\right)\wedge dx^3$$

$$= \left(-\frac{\partial f_1}{\partial x^2} + \frac{\partial f_2}{\partial x^1}\right)dx^1\wedge dx^2 + \left(-\frac{\partial f_2}{\partial x^3} + \frac{\partial f_3}{\partial x^2}\right)dx^2\wedge dx^3$$
$$+ \left(-\frac{\partial f_3}{\partial x^1} + \frac{\partial f_1}{\partial x^3}\right)dx^3\wedge dx^1 \qquad (2.15)$$