$$\Phi_*(fW)(y) = D\Phi_x(fW)(y)$$

$$= \left(\sum_j \frac{\partial y^i}{\partial x^j} (f(x) W^j) \right)(y)$$

$$= f(\Phi^{-1}(y)) \left(\sum_j \frac{\partial y^i}{\partial x^j} W^j \right)(y)$$

$$= f(\Phi^{-1}(y))\, \Phi_*(W)(y) \quad .$$

$$(\Psi\Phi)_*(W) = D(\Psi\Phi)_x(W)$$

$$= \begin{pmatrix} \cdots \\ \sum_{j=1}^n \frac{\partial z^k}{\partial x^j} W^j \\ \cdots \end{pmatrix}$$

$$= \begin{pmatrix} \cdots \\ \frac{\partial z^k}{\partial x^1} W^1 + \cdots + \frac{\partial z^k}{\partial x^n} W^n \\ \cdots \end{pmatrix}$$

$$= \begin{pmatrix} \cdots \\ \frac{\partial z^k}{\partial y^1} \frac{\partial y^1}{\partial x^1} W^1 + \cdots + \frac{\partial z^k}{\partial y^n} \frac{\partial y^n}{\partial x^1} W^1 + \cdots \\ + \frac{\partial z^k}{\partial y^1} \frac{\partial y^1}{\partial x^n} W^n + \cdots + \frac{\partial z^k}{\partial y^n} \frac{\partial y^n}{\partial x^n} W^n \\ \cdots \end{pmatrix}$$

$$= \begin{pmatrix} \cdots \\ \frac{\partial z^k}{\partial y^1} \sum_{i=1}^n \frac{\partial y^1}{\partial x^i} W^i + \cdots + \frac{\partial z^k}{\partial y^n} \sum_{i=1}^n \frac{\partial y^n}{\partial x^i} W^i \\ \cdots \end{pmatrix}$$

$$= \begin{pmatrix} \cdots \\ \sum_{\ell=1}^n \frac{\partial z^k}{\partial y^\ell} \sum_{i=1}^n \frac{\partial y^\ell}{\partial x^i} W^i \\ \cdots \end{pmatrix}$$

$$= D\Psi_{y} \begin{pmatrix} \vdots \\ \sum_{i=1}^n \frac{\partial y^\ell}{\partial x^i} W^i \\ \vdots \end{pmatrix} = \Psi_* \left(D\Phi_x(W) \right)$$

$$= \Psi_*(\Phi_*(W))$$