(2.21) $$di_1(V) = d(V^x dx + V^y dy + V^z dz)$$

$$= dV^x dx + dV^y dy + dV^z dz$$

$$= \left(\frac{\partial V^x}{\partial x}dx + \frac{\partial V^x}{\partial y}dy + \frac{\partial V^x}{\partial z}dz\right)\wedge dx$$

$$+ \left(\frac{\partial V^y}{\partial x}dx + \frac{\partial V^y}{\partial y}dy + \frac{\partial V^y}{\partial z}\right)\wedge dy$$

$$+ \left(\frac{\partial V^z}{\partial x}dx + \frac{\partial V^z}{\partial y}dy + \frac{\partial V^z}{\partial z}dz\right)\wedge dz$$

$$= \frac{\partial V^x}{\partial y}dy\wedge dx + \frac{\partial V^x}{\partial z}dz\wedge dx + \frac{\partial V^y}{\partial x}dx\wedge dy + \frac{\partial V^y}{\partial z}dz\wedge dy$$

$$+ \frac{\partial V^z}{\partial x}dx\wedge dz + \frac{\partial V^z}{\partial y}dy\wedge dz$$

$$= \left(\frac{\partial V^y}{\partial x} - \frac{\partial V^x}{\partial y}\right)dx\wedge dy + \left(\frac{\partial V^z}{\partial y} - \frac{\partial V^y}{\partial z}\right)dy\wedge dz$$

$$+ \left(\frac{\partial V^x}{\partial z} - \frac{\partial V^z}{\partial x}\right)dz\wedge dx$$

$$= *\left(\left(\frac{\partial V^y}{\partial x} - \frac{\partial V^x}{\partial y}\right)dz + \left(\frac{\partial V^z}{\partial y} - \frac{\partial V^y}{\partial z}\right)dx\right.$$

$$\left. + \left(\frac{\partial V^x}{\partial z} - \frac{\partial V^z}{\partial x}\right)dy\right)$$

$$= *\left(\left(\left(\frac{\partial V^z}{\partial y} - \frac{\partial V^y}{\partial z}\right)\frac{\partial}{\partial x} + \left(\frac{\partial V^x}{\partial z} - \frac{\partial V^z}{\partial x}\right)\frac{\partial}{\partial y} + \left(\frac{\partial V^y}{\partial x} - \frac{\partial V^x}{\partial y}\right)\frac{\partial}{\partial z}\right)^*\right)$$

$$= *\left((\mathrm{rot}\, V)^*\right)$$

$$= i_2(\mathrm{rot}\, V) \qquad \circ$$

(2.22) $$di_2(V) = d*(V^*)$$

$$= d*(V^x dx + V^y dy + V^z dz)$$

$$= d(V^x dy\wedge dz + V^y dz\wedge dx + V^z dx\wedge dy)$$