

HW3

Wanzhu Chen

Problem1:

- (a) $a \rightarrow b$: **No**, since when $a=1$, b could be 2 or 3 or 4.
- (b) $a \rightarrow c$: **Yes**. Since for each a , c only has one value.
- (c) $b \rightarrow a$: **Yes**. Although when $b=2$ or 3 or 4, a has the same value 1, for each b , a only has a specific value, no multiple choices, so it satisfies.
- (d) $b \rightarrow c$: **Yes**. The same reason as (c)
- (e) $c \rightarrow a$: **Yes**. The same reason as (c)
- (f) $c \rightarrow b$: **No**. Since when $c=2$, b could be 2 or 3 or 4 or 5.
- (g) $ab \rightarrow c$: **Yes**. For each combination of ab , there is a specific value of c .
- (h) $ac \rightarrow b$: **No**. Since when $a=1$, $c=2$, b could be 2 or 3 or 4. It is not a single result for b .
- (i) $bc \rightarrow a$: **Yes**. Since a only has one value for each combination of bc .

Problem2:

- ca , cb , cd and ce are the candidate keys.
- (1) ca has a closure including $\{abcde\}$, ca is candidate key.
 - (2) cb has a closure including $\{cbdea\}$, cb is candidate key.
 - (3) Since $cd \rightarrow e$, $e \rightarrow a$, $a \rightarrow b$, $b \rightarrow d$, with Transitivity rule from Armstrong's axioms, $cd \rightarrow e, a, b, d$. Thus cd is the candidate key.
 - (4) Since $e \rightarrow a$, $a \rightarrow b$, $b \rightarrow d$, with Transitivity rule, $e \rightarrow abd$.
Then apply Augmentation rule by adding c to both sides at the same time, so I have $ce \rightarrow cabd$. Thus ce is one candidate key.

Problem3:

- (a) **af is the candidate key.**

Reason:

- $a \rightarrow bcd$ ----- (FD1)
 - $ad \rightarrow e$ ----- (FD2)
 - $efg \rightarrow h$ ----- (FD3)
 - $f \rightarrow gh$ ----- (FD4)
- Start with af , put af into its closure: $\{af\}$.
Firstly, considering $a \rightarrow bcd$, and $f \rightarrow gh$, I can safely add bcd and gh into the closure: $\{abcd fgh\}$
Next consider $ad \rightarrow e$. Since both ad is in the closure of af , add e into the closure: $\{abcde fgh\}$, which is R .
Thus, af is the candidate key.

- (b) **$efg \rightarrow h$ can be removed.**

Reason:

- $f \rightarrow gh$ could separate as $f \rightarrow g$ and $f \rightarrow h$ using Decomposition Rule.

Also, when compare $f \rightarrow g$, $f \rightarrow h$ to $efg \rightarrow h$, I could find e and g are extraneous attributes. If $efg \rightarrow h$ is deleted, it will make no differences for F^+ .
So, $efg \rightarrow h$ can be removed.

Problem4:

(a) **The only possible candidate key is abe.**

Start with {abe} as the closure of abe itself.

Firstly adding c and d into the closure after considering $a \rightarrow c$ and $b \rightarrow d$:

{abecd};

Then since I have $de \rightarrow f$ and both d and e are in the closure, I could safely add f into the closure: {abecdf}, which is R.

Thus, abe is candidate key.

(b) **Add $a \rightarrow e$ as a new FD.**

Since $a \rightarrow b$, $b \rightarrow d$, with Transitivity rule, $a \rightarrow d$.

Since $a \rightarrow d$ and $a \rightarrow e$, $a \rightarrow de$.

Sine $a \rightarrow de$, $de \rightarrow f$, so $a \rightarrow f$.

Since $a \rightarrow c$, $a \rightarrow b$ (additional), $a \rightarrow d$, $a \rightarrow f$, $a \rightarrow e$ (added), with Union rule $a \rightarrow bcdef$, which means a is the candidate key.

Problem 5:

(a) Nontrivial functional dependencies:

$a \rightarrow b$

$b \rightarrow a$

$c \rightarrow b$

$ac \rightarrow b$

(b) **It is not BCNF. it is $a \rightarrow b$, $b \rightarrow a$ that violate BCNF.**

Since the candidate key is c, and a or b are not superkeys while they are determinants in some of the FDs($a \rightarrow b$, $b \rightarrow a$). Thus, it is $a \rightarrow b$, $b \rightarrow a$ that violate BCNF.

(c) **Valid decomposition:**

R1(a b), R2(a b c), and a, b in R2 are foreign keys reference to a, b in R1 respectively.

Problem 6:

(a) **ab, bc, bde are candidate keys.**

$ab \rightarrow c$, so given ab, I could have abc to closure of ab.

$bc \rightarrow d$, so, add d to closure of ab.

$cd \rightarrow e$, so add e to closure of ab.

Thus, ab is a candidate key, since its closure is {abcde} that covers all the attributes of R.

$bc \rightarrow d$, so add d to closure of bc. Now the closure of bc is {bcd}

$cd \rightarrow e$, so add e to closure of bc. Now the closure of bc is {bcde}

$de \rightarrow a$, so add a to closure of bc. Now the closure of bc is {abcde}, which is R.

Thus, bc is a candidate key.

$de \rightarrow a$, so add a to closure of bde. Now the closure of de is {abde}

$ab \rightarrow c$, so add c to closure of bde. Now the closure of de is {abcde}, which is R.

Since neither bd or be could has a closure containing all attributes of R, bde is a candidate key.

(b) **$cd \rightarrow e$ and $de \rightarrow a$** violates BCNF, since neither cd nor de is a superkey,

(c) **Decomposition1: $R_1(\underline{c} \ \underline{d} \ e)$, $R_2(\underline{b} \ \underline{c} \ d)$, $R_3(\underline{a} \ \underline{b} \ c)$**

Decomposition2: $R_1(\underline{c} \ \underline{d} \ e)$, $R_2(\underline{b} \ \underline{c} \ d)$, $R_3(\underline{d} \ \underline{e} \ a)$

Problem7

(a)

FD1: course_id, sec_id, semester, year → building, room_no

FD2: course_id, sec_id, semester, year → time_slot_id

FD3: course_id, sec_id, semester, year, s_ID → grade

FD4: course_id, sec_id, semester, year → i_ID

(b) **Candidate key:** {course_id, sec_id, semester, year, s_ID}

(c) **No, it is not BCNF.**

The reason is that on the left hand side of FD1, FD2, FD4, they only have {course_id, sec_id, semester, year} and it is not a superkey. Also, FD1, FD2 and FD4 all three are not trivial.

Decomposition:

R1(course_id, sec_id, semester, year, building, room_no)

R2(course_id, sec_id, semester, year, time_slot_id)

R3(course_id, sec_id, semester, year, i_ID)

R4(course_id, sec_id, semester, year, s_ID, grade)

(d)

update anomalies:

It may contain records in takesSecTaughtBy table that have the same course_id, sec_id, semester, year, building, room_no while have different s_IDs, which reflects one section is scheduled in exactly one building and room and can have many students enrolled. Then if I want to change the place of this section, I have to update the building and room_no values in all tuples that are related to this section.

delete anomalies:

If I have to delete one section, which has many tuples in takesSecTaughtBy table, since there may be many students register it, then I have to delete all the records. It is really not efficient to do such redundant work like this. Also, when I delete this section, I have to delete other information like who have registered this section at the same time.

insert anomalies:

If I want to insert a new section with only course_id, sec_id, semester and year value while who registered for this section is not set, then it may be no way to insert a new record like this, since the candidate key is {course_id, sec_id, semester, year, s_ID}, and the candidate key can't have null value.

(e)

Based on the requirements, the FDs are:

FD1: course_id, sec_id, semester, year → building, room_no

FD2: $\text{course_id, sec_id, semester, year} \rightarrow \text{time_slot_id}$
FD3: $\text{course_id, sec_id, semester, year, s_ID} \rightarrow \text{grade}$
FD4: $\text{course_id, sec_id, semester, year} \rightarrow \text{i_ID}$

FD5: $\text{s_ID, i_ID} \rightarrow \text{course_id}$
FD6: $\text{s_ID, course_id} \rightarrow \text{i_ID}$
FD7: $\text{course_id} \rightarrow \text{i_ID}$

Simplify these FDs by deleting FD4 and FD6. So I have:

FD1: $\text{course_id, sec_id, semester, year} \rightarrow \text{building, room_no}$
FD2: $\text{course_id, sec_id, semester, year} \rightarrow \text{time_slot_id}$
FD3: $\text{course_id, sec_id, semester, year, s_ID} \rightarrow \text{grade}$
FD5: $\text{s_ID, i_ID} \rightarrow \text{course_id}$
FD7: $\text{course_id} \rightarrow \text{i_ID}$

(f) **Candidate keys are:**

$\{\text{course_id, sec_id, semester, year, s_ID}\}$ and
 $\{\text{s_ID, i_ID, sec_id, semester, year}\}$

(g) **No, it is not BCNF**, since FD1, FD2, FD5, FD7 are not trivial and don't have superkeys on the left hand side.

Decomposition to BCNF:

R1(course_id, sec_id, semester, year, building, room_no)
R2(course_id, sec_id, semester, year, time_slot_id)
R3(course_id, sec_id, semester, year, s_ID, grade)
R4(course_id, i_ID)
R5(course_id, sec_id, semester, year, s_ID)

Problem 8

(a)

Answer:

It is neither BCNF nor 3NF.

Decomposition: $R1(\underline{b} \ c)$, $R2(\underline{a} \ b)$. It is lossless join and dependency preserved.

Analysis:

Since $a \rightarrow b$, $b \rightarrow c$, so a is the only candidate key. Thus, it is not BCNF since $b \rightarrow c$ violates BCNF rule that b is not a superkey.

Since $b \rightarrow c$ violates 3NF that b is not a superkey while c on the right hand side is not part of candidate key, it is not 3NF.

Decomposition: $R1(\underline{b} \ c)$, $R2(\underline{a} \ b)$

$R1 \cap R2 = b$, which is the primary key of $R1$, so it is lossless join decomposition.

Checking each FD finds that every FD holds on either $R1$ or $R2$, thus it is dependency-preserved.

(b)

Answer:

It is not BCNF but it is 3NF.

Analysis:

Candidate keys are ab , cb , db .

Since either c or d is a superkey while both of them are on the left hand side of $c \rightarrow d$ and $d \rightarrow a$, so $c \rightarrow d$, $d \rightarrow a$ violates BCNF. It is not BCNF.

However, since the right hand side of $c \rightarrow d$, $d \rightarrow a$ are d and a , and both of them are part of candidate keys, specifically part of db or ab , thus it is a 3NF.

(c)

Answer:

It is neither BCNF nor 3NF.

Decomposition: $R1(\underline{b} \ d)$, $R2(\underline{b} \ c)$, $R3(\underline{a} \ b)$, and it is a dependency-preserved and lossless join decomposition.

Analysis:

Candidate key is ab .

Both FDs violates BCNF, since b itself is not a superkey.

Also, the right hand side of these two FDs are c and d , but neither of them are part of candidate key, so it is not 3NF.

Decomposition: $R1(\underline{b} \ d)$, $R2(\underline{b} \ c)$, $R3(\underline{a} \ b)$

It is a lossless join decomposition, since

$R1 \cap R2 = b$ and b is the candidate key of $R1$;

$R2 \cap R3 = b$ and b is the candidate key of $R2$;

$R1 \cap R3 = b$ and b is the candidate key of $R1$.

It is a dependency-preserved decomposition since each FD holds on one of the relationships.

(d)

Answer:

It is BCNF. It is 3NF.

Analysis:

Candidate keys are ab , bc , ad and cd .

For those four FDs, they all have superkeys on the left hand side, thus it is BCNF. There is no doubt that if it is BCNF, it is 3NF.