

# Tree Isomorphism

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# Isomorphism

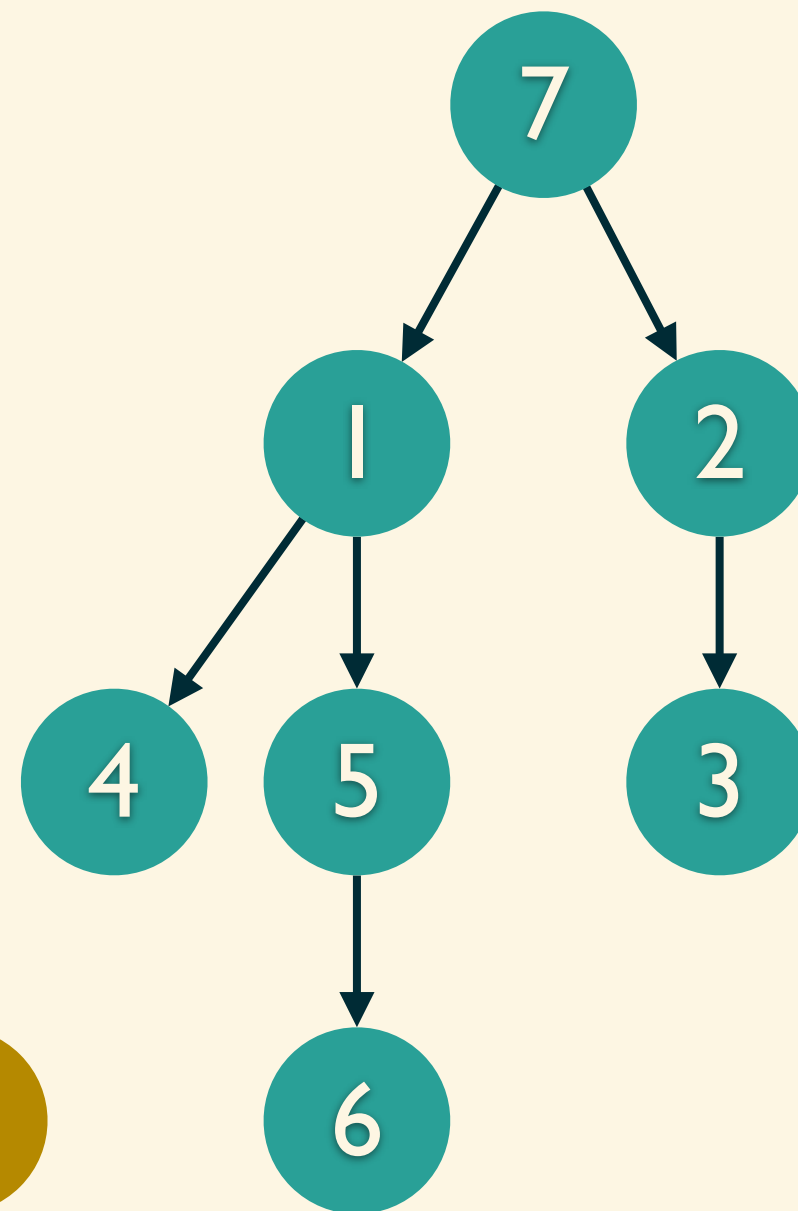
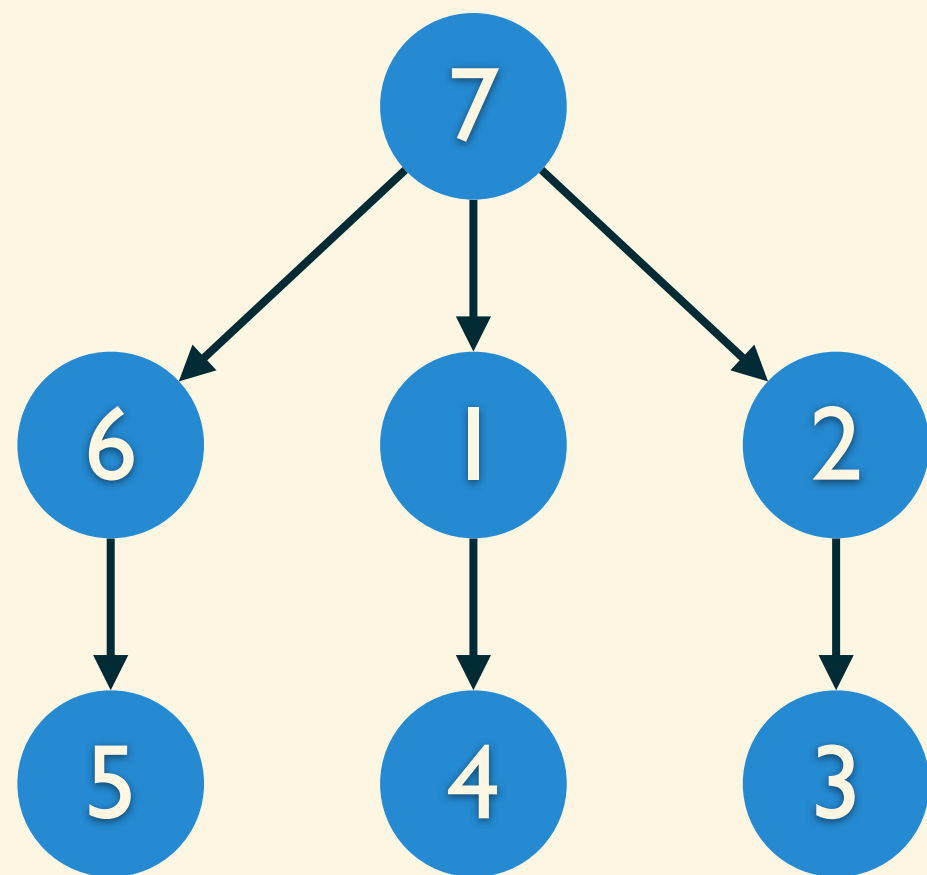
The **structures** of two trees are **equal**.



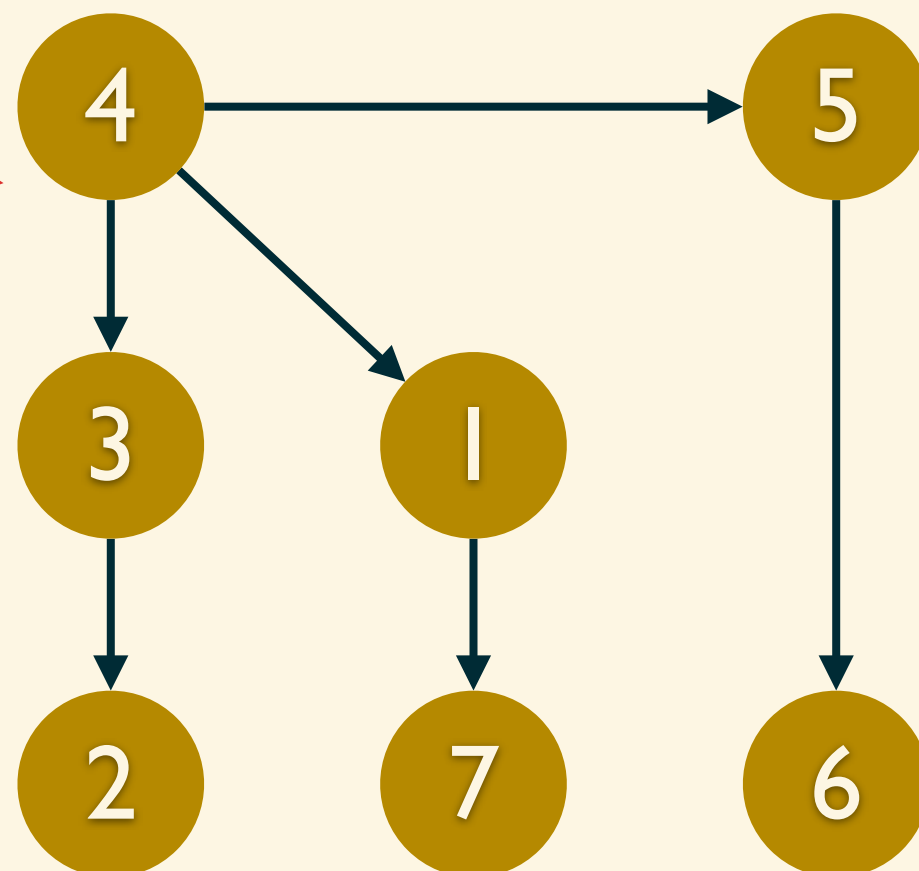
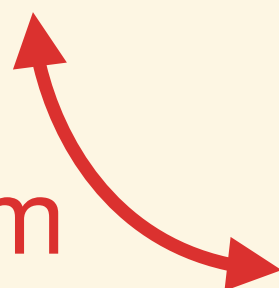
**so abstract...**



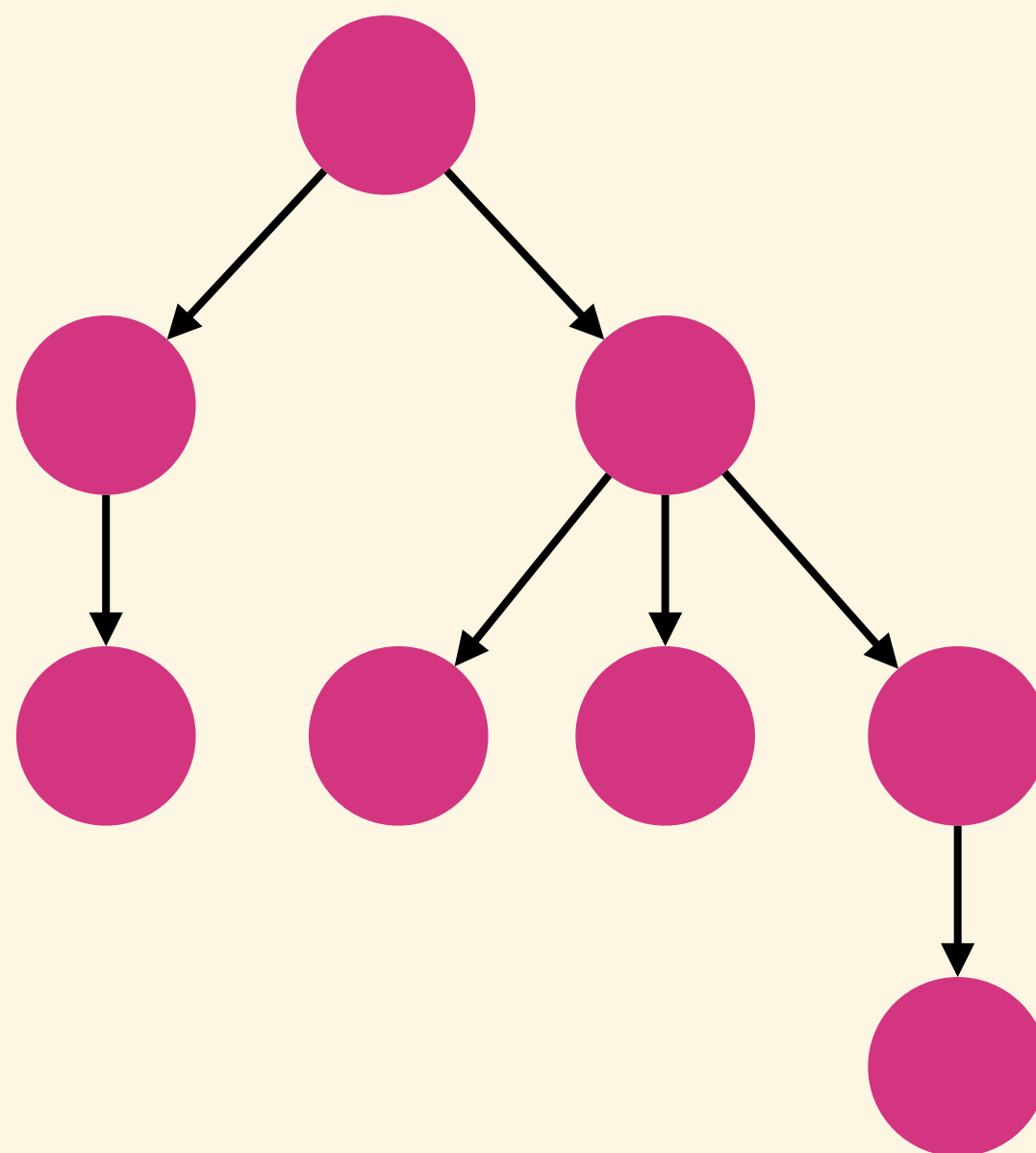
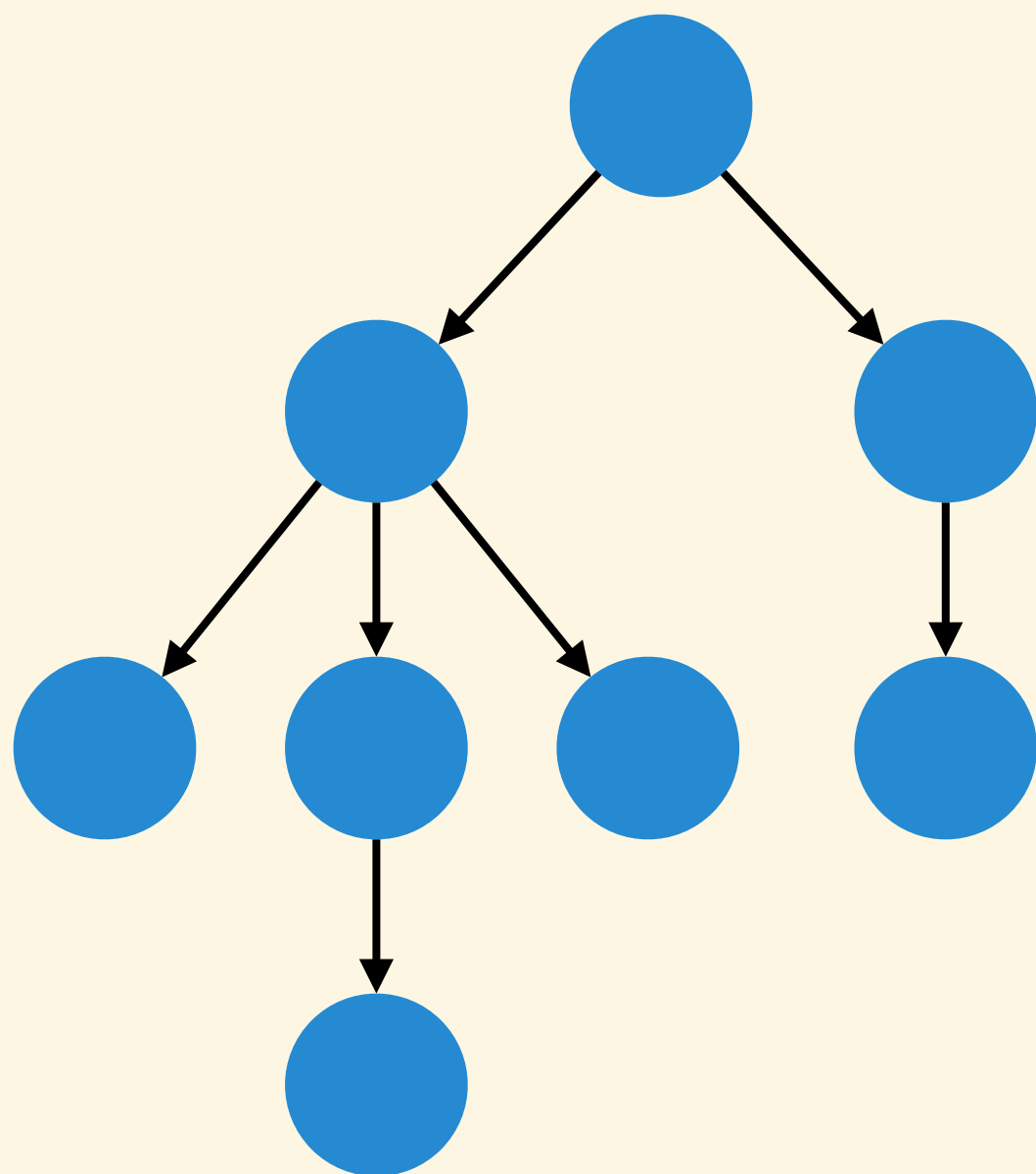


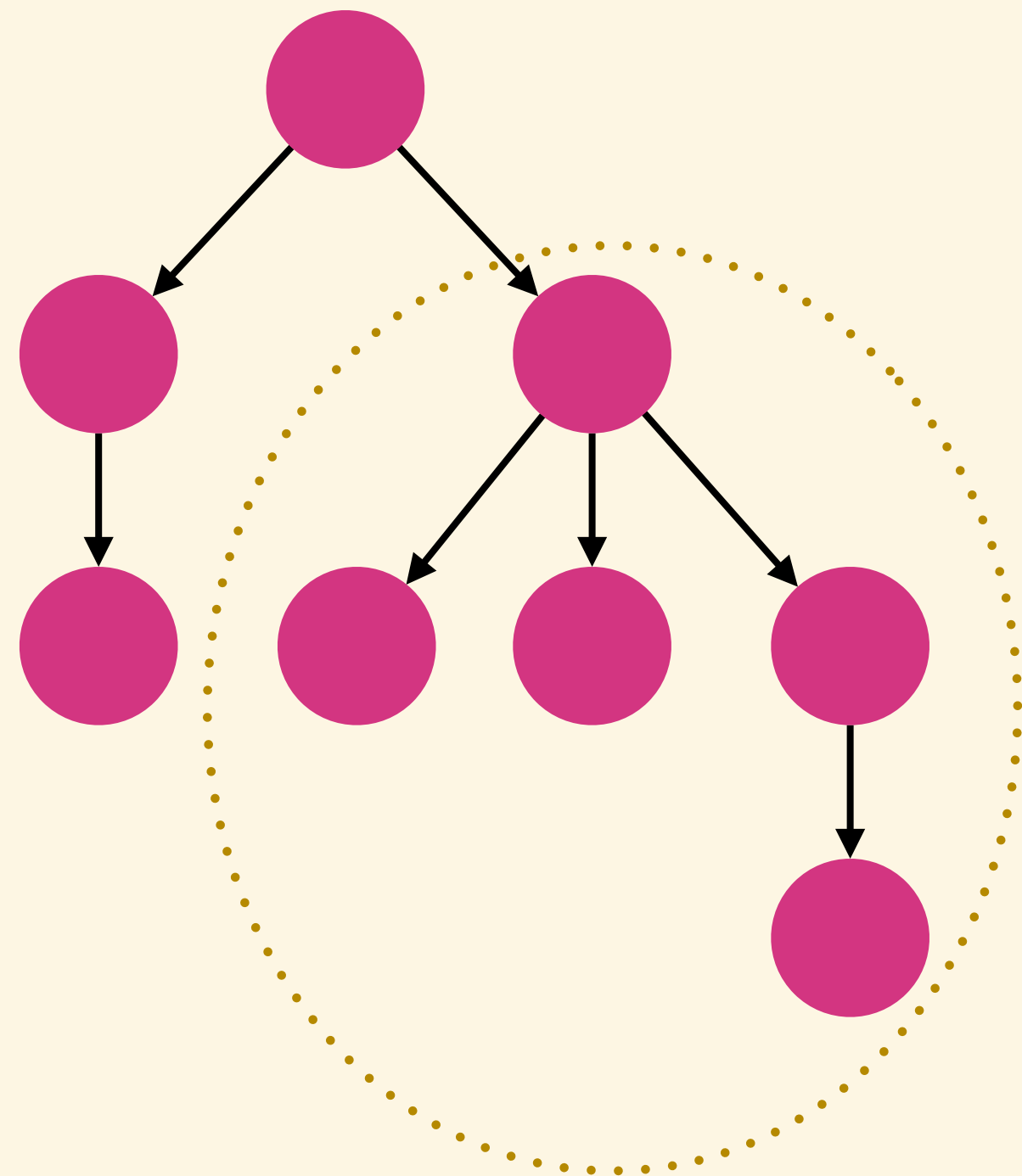
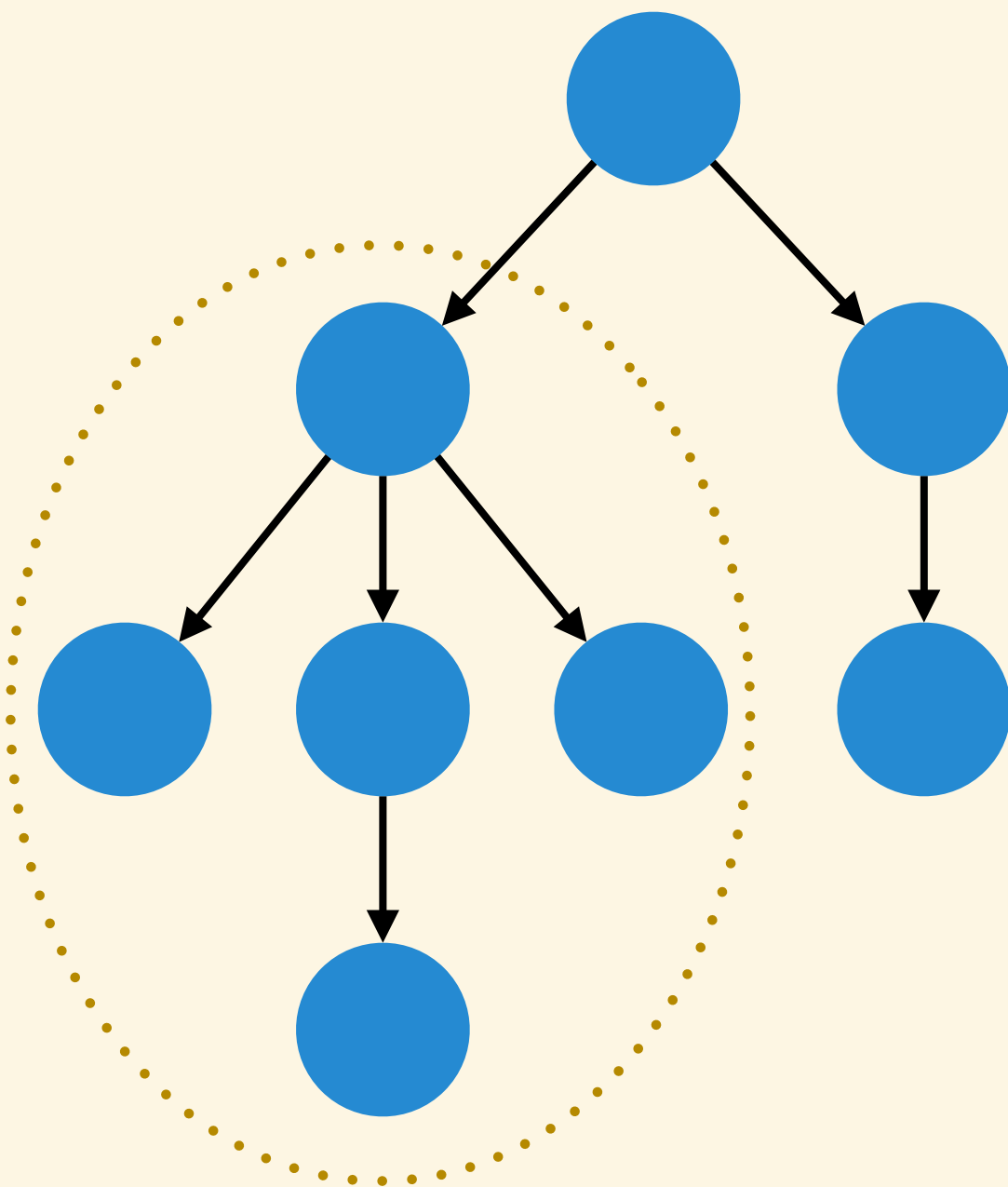


isomorphism

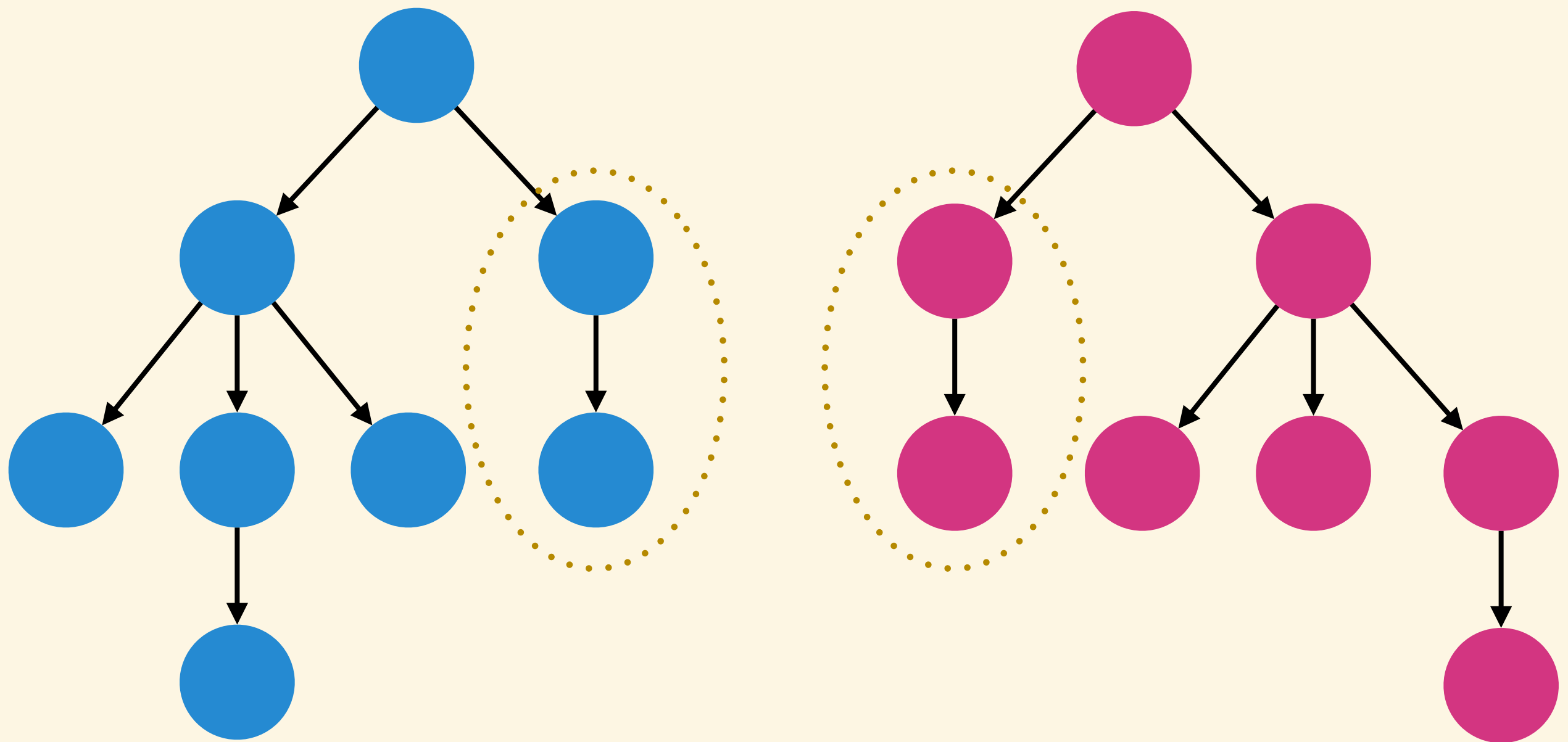


How to judge two rooted  
tree are isomorphic?



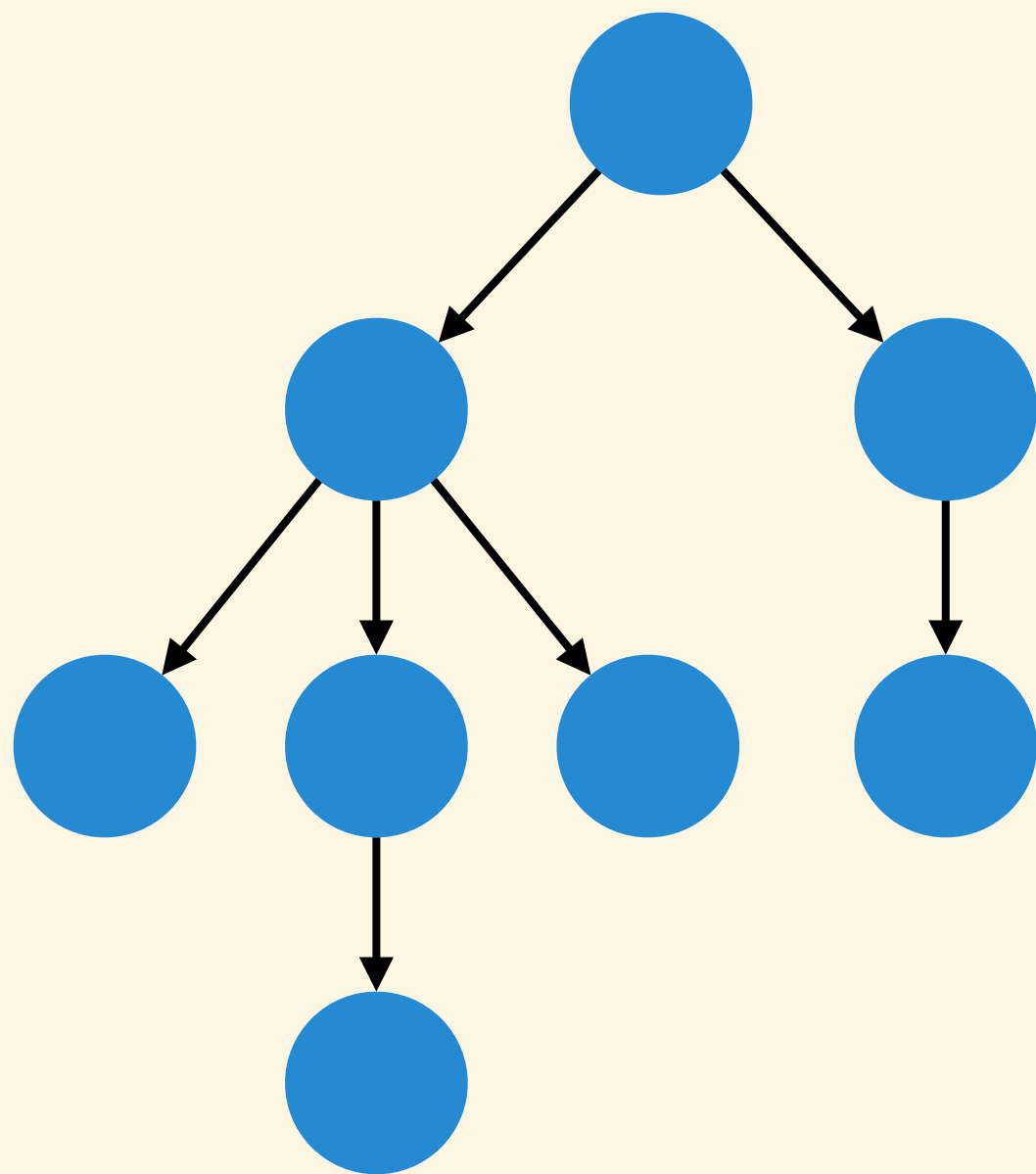


If the trees are isomorphic, all their **sub-trees** are also **isomorphic**.

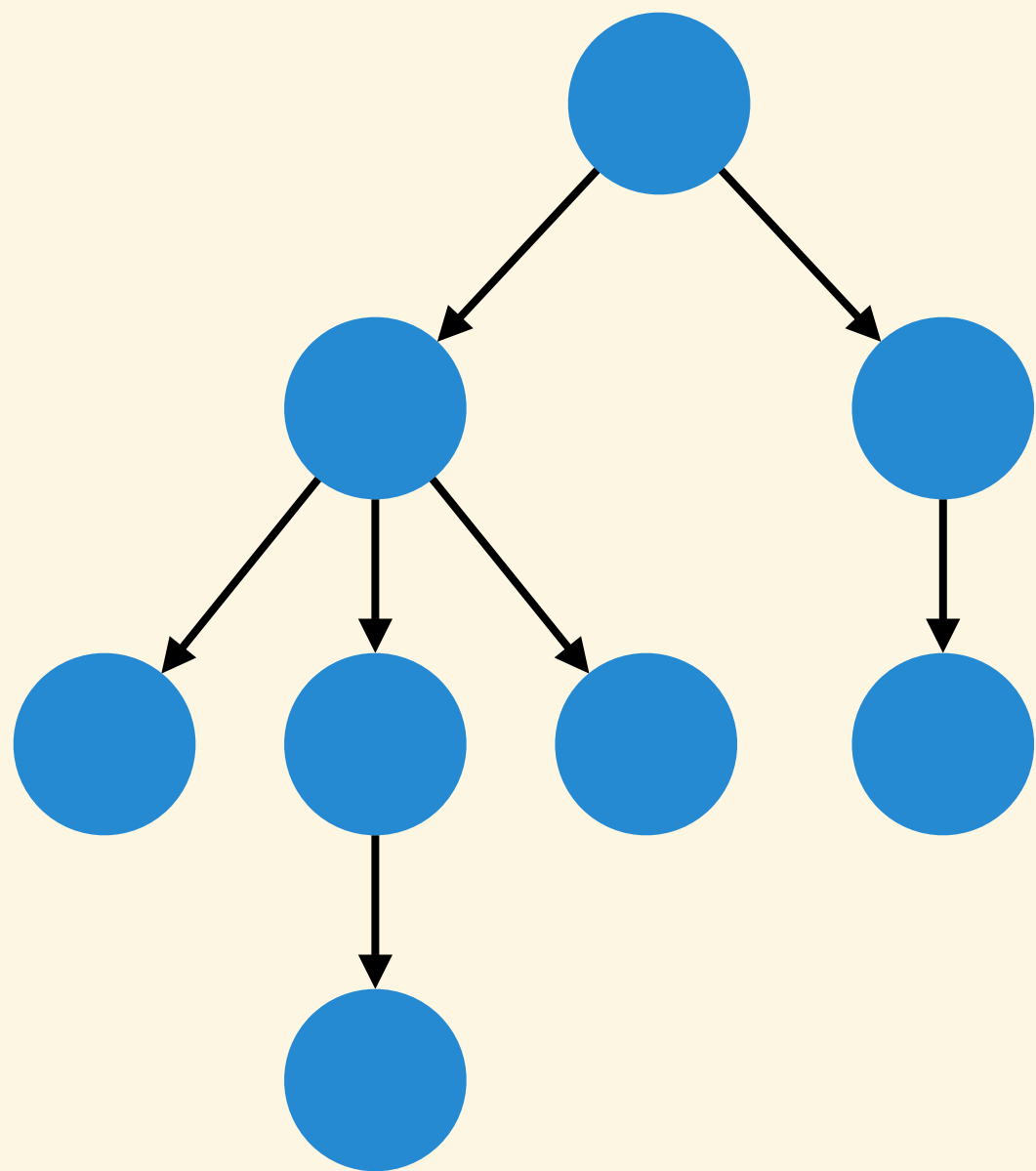


If the trees are isomorphic, all their **sub-trees** are also **isomorphic**.

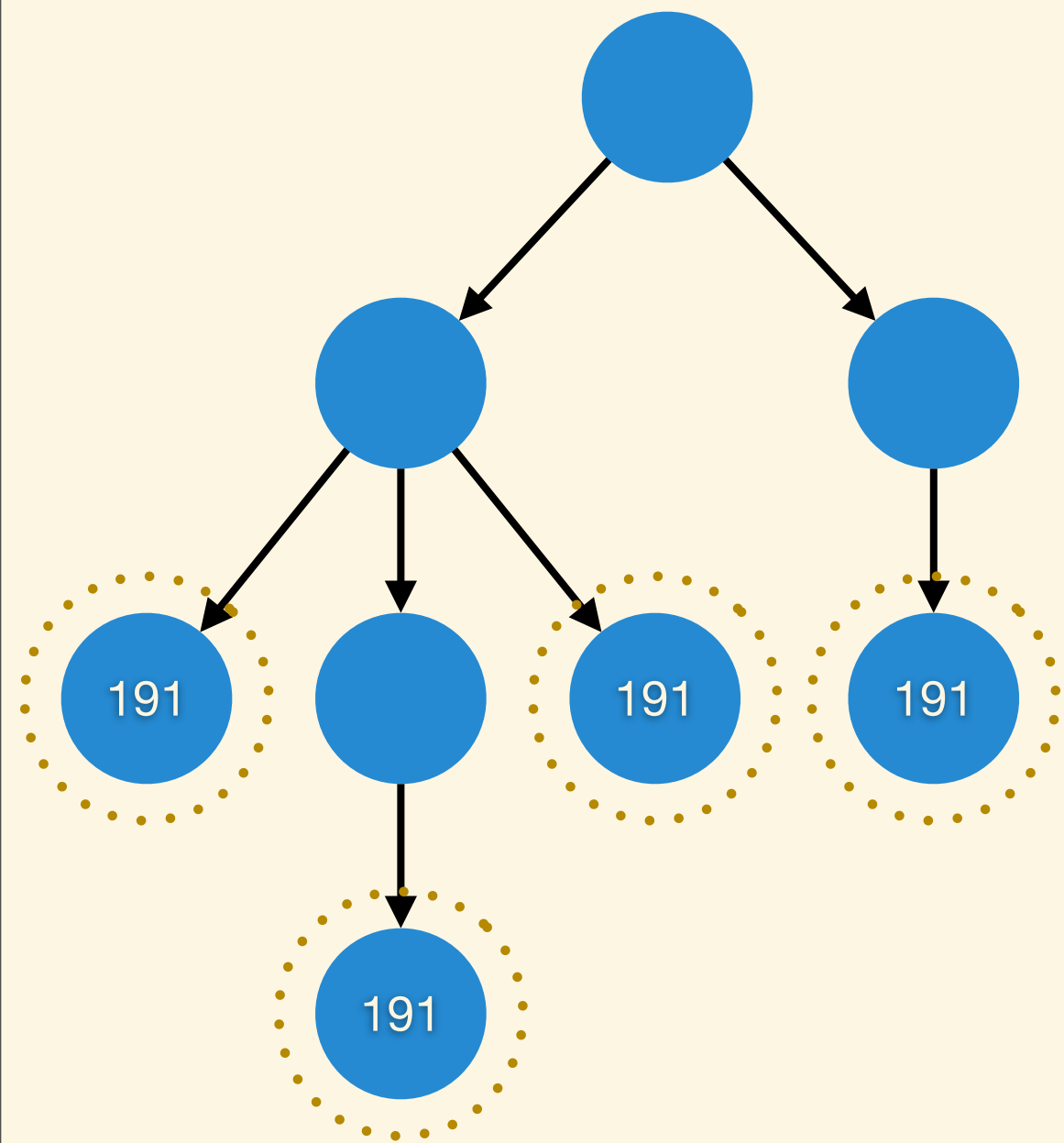




Hash the tree

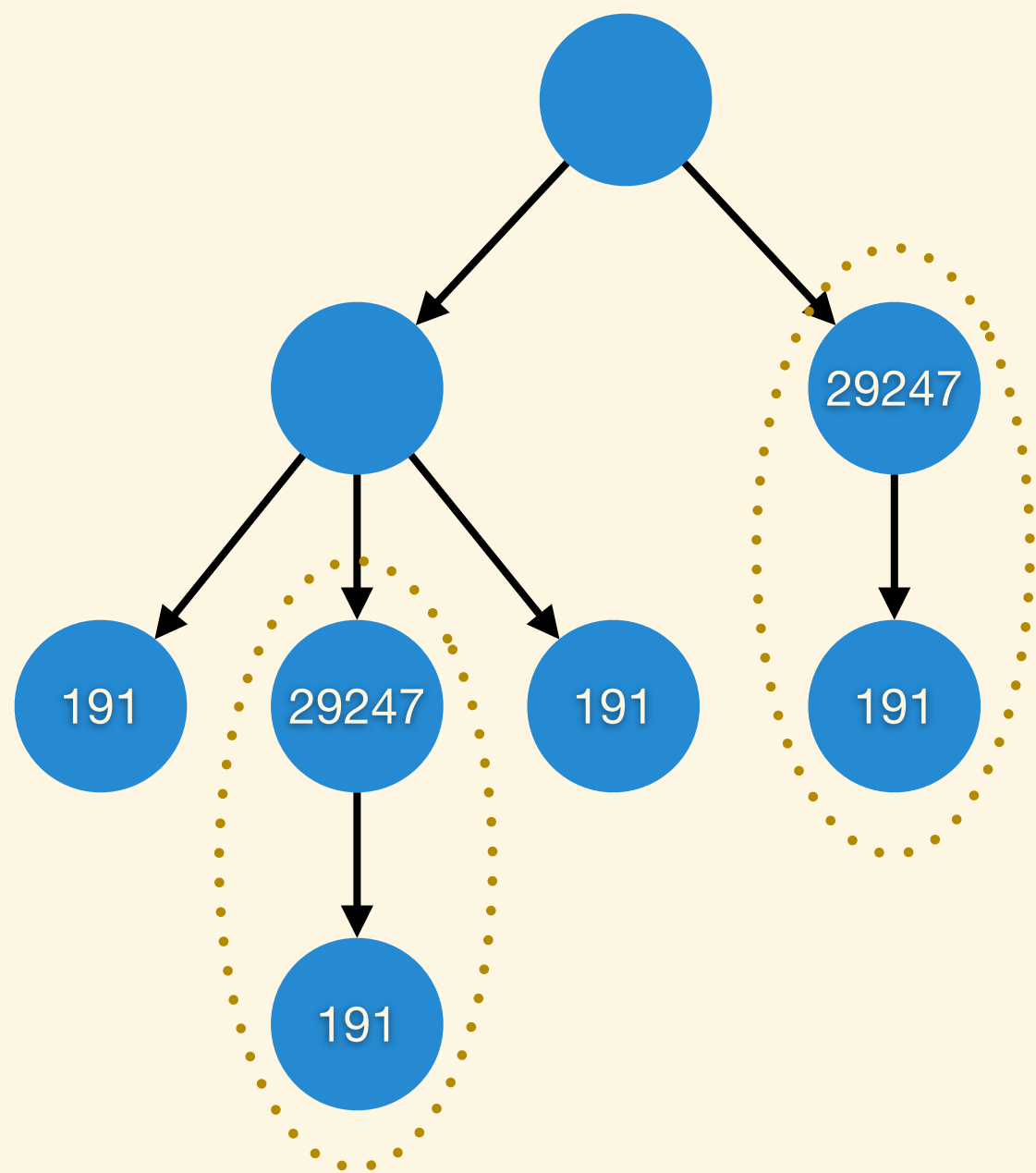


Hash all sub-tree  
recursively



single vertex

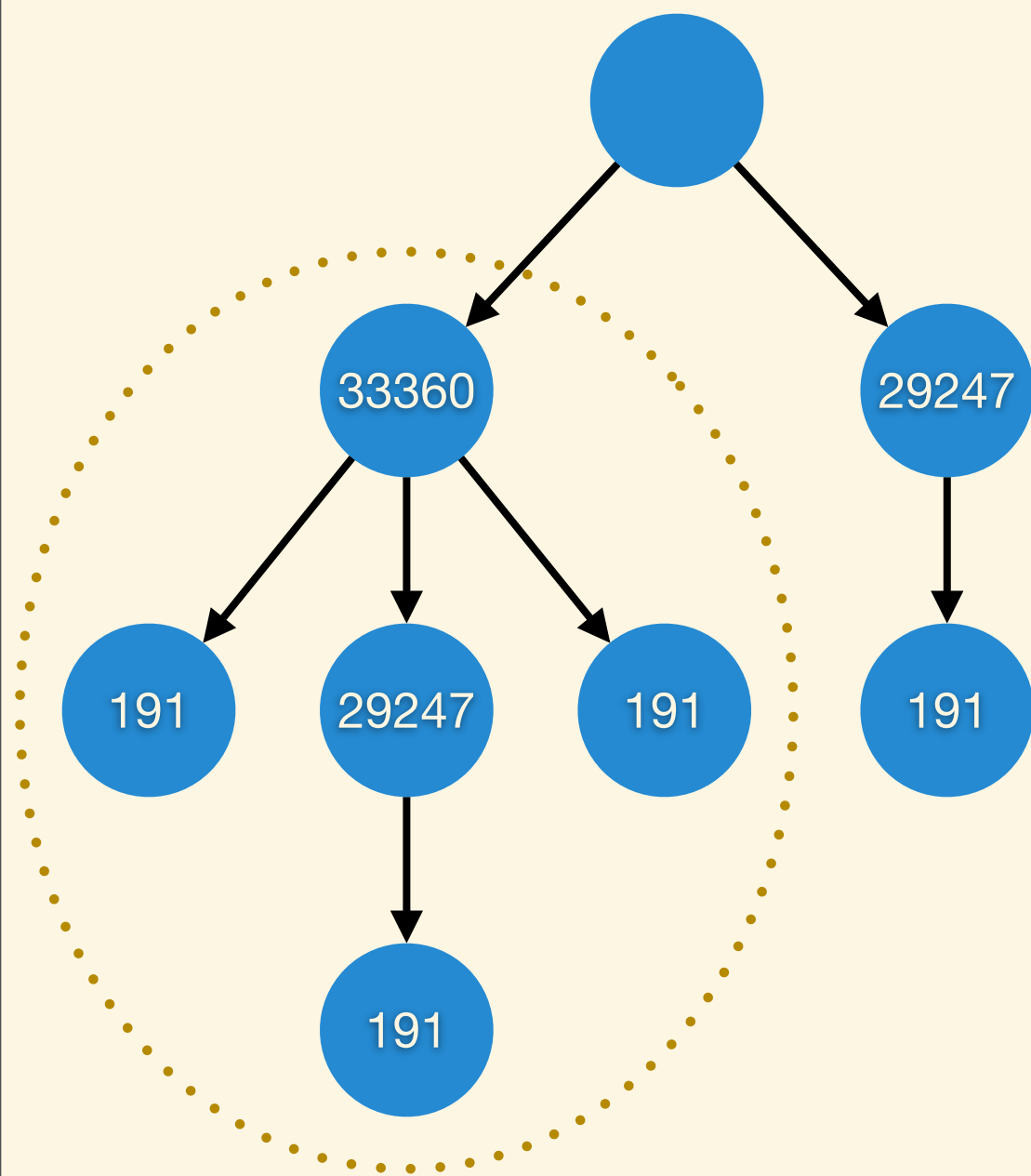
initial value = 191



two-level sub-tree

child = (191)

$$(191 \times 701 \text{ xor } 191) \bmod 34943 = 29247$$



three-level sub-tree

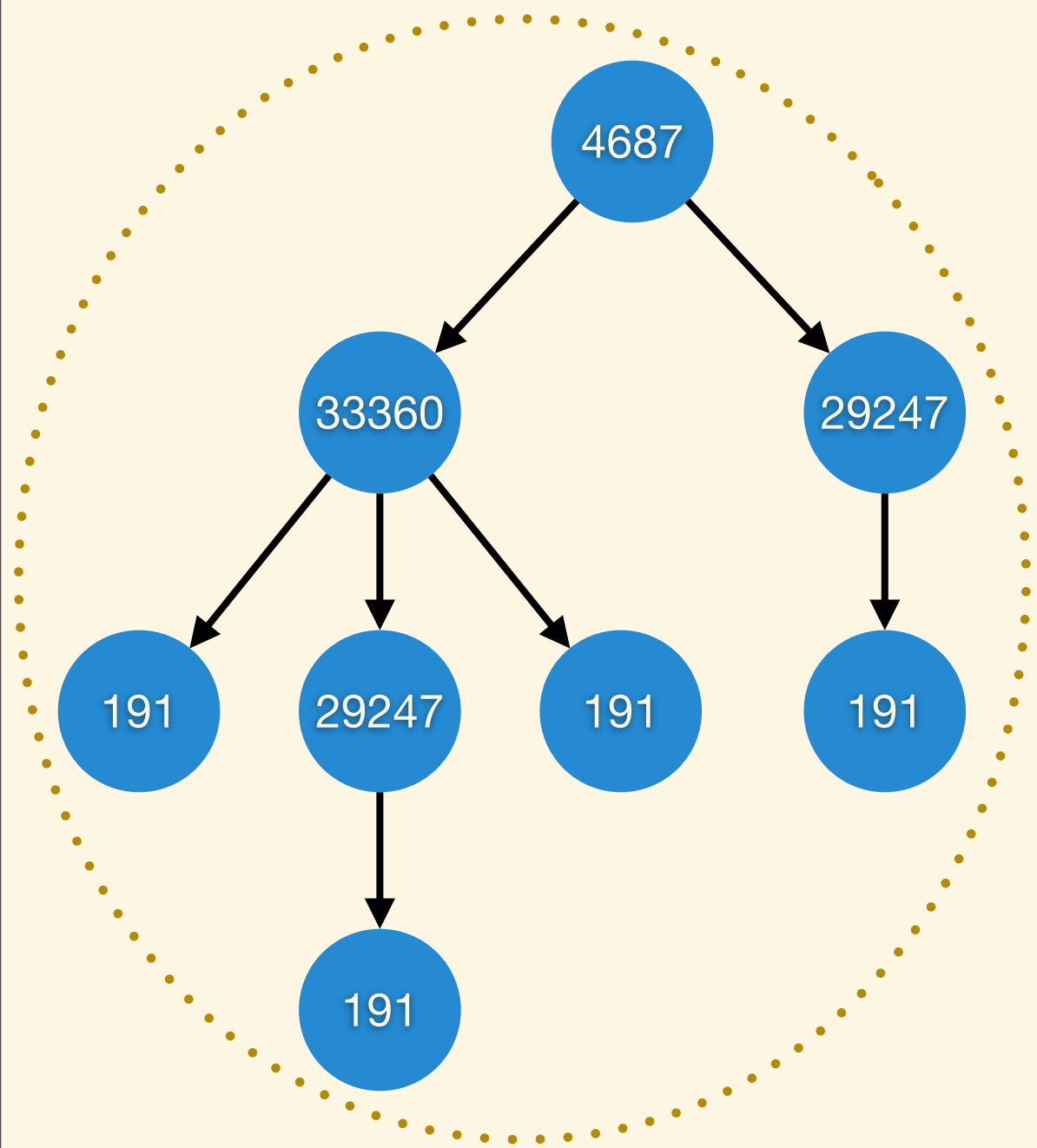
child = (191, 29247, 191)

↓ sort

child = (191, 191, 29247)

$$((((191 \times 701 \text{ xor } 191) \bmod 34943) \times 701 \text{ xor } 191) \bmod 34943) \times 701 \text{ xor } 29247) \bmod 34943 = 33360$$





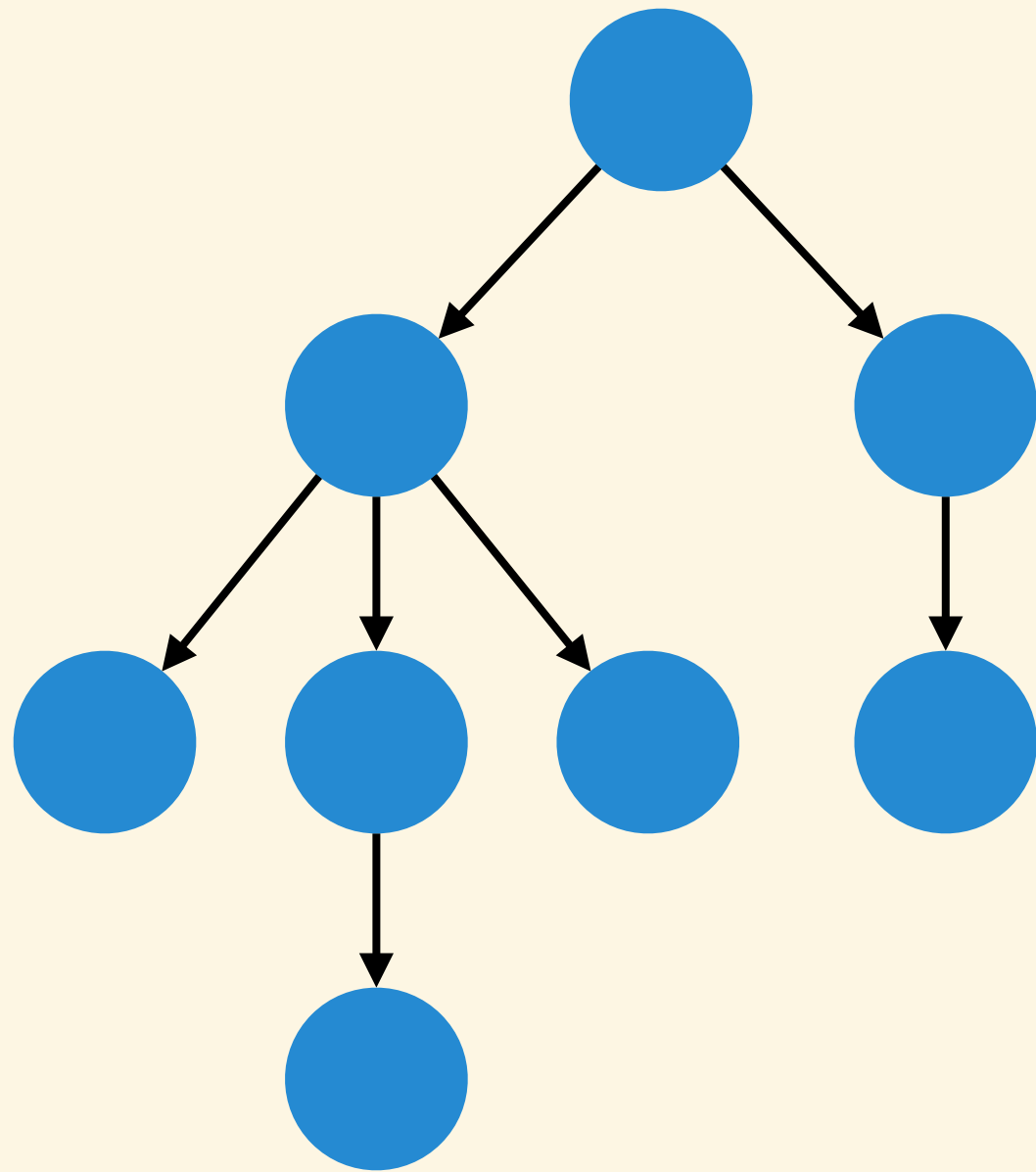
the total tree

child = (33360, 29247)

↓ sort

child = (29247, 33360)

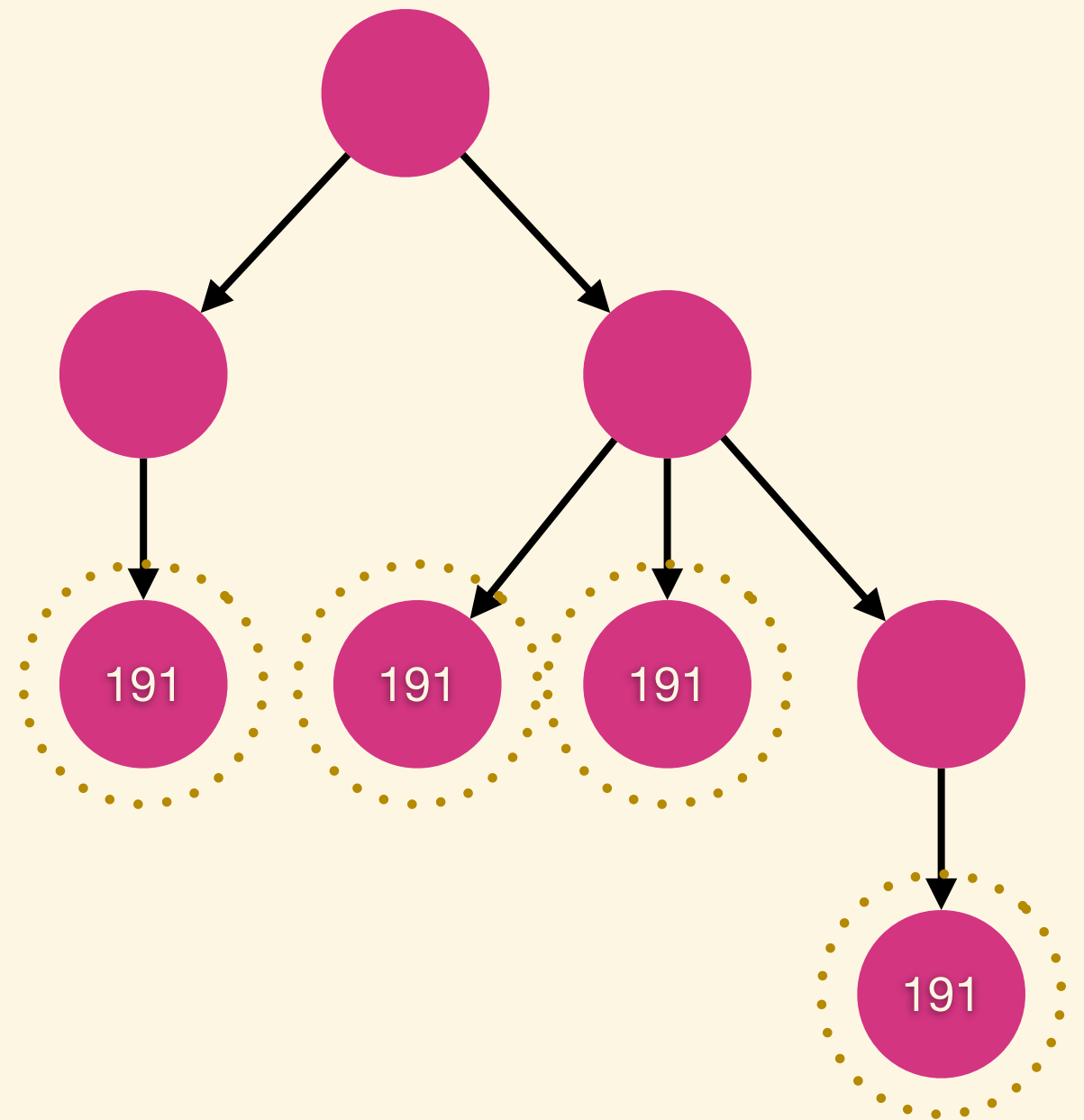
$$\begin{aligned} & (((191 \times 701 \text{ xor } 29247) \bmod 34943) \times 701 \text{ xor } 33360) \\ & \qquad \qquad \qquad \bmod 34943 = 4687 \end{aligned}$$



hash value of  
the tree is **4687**

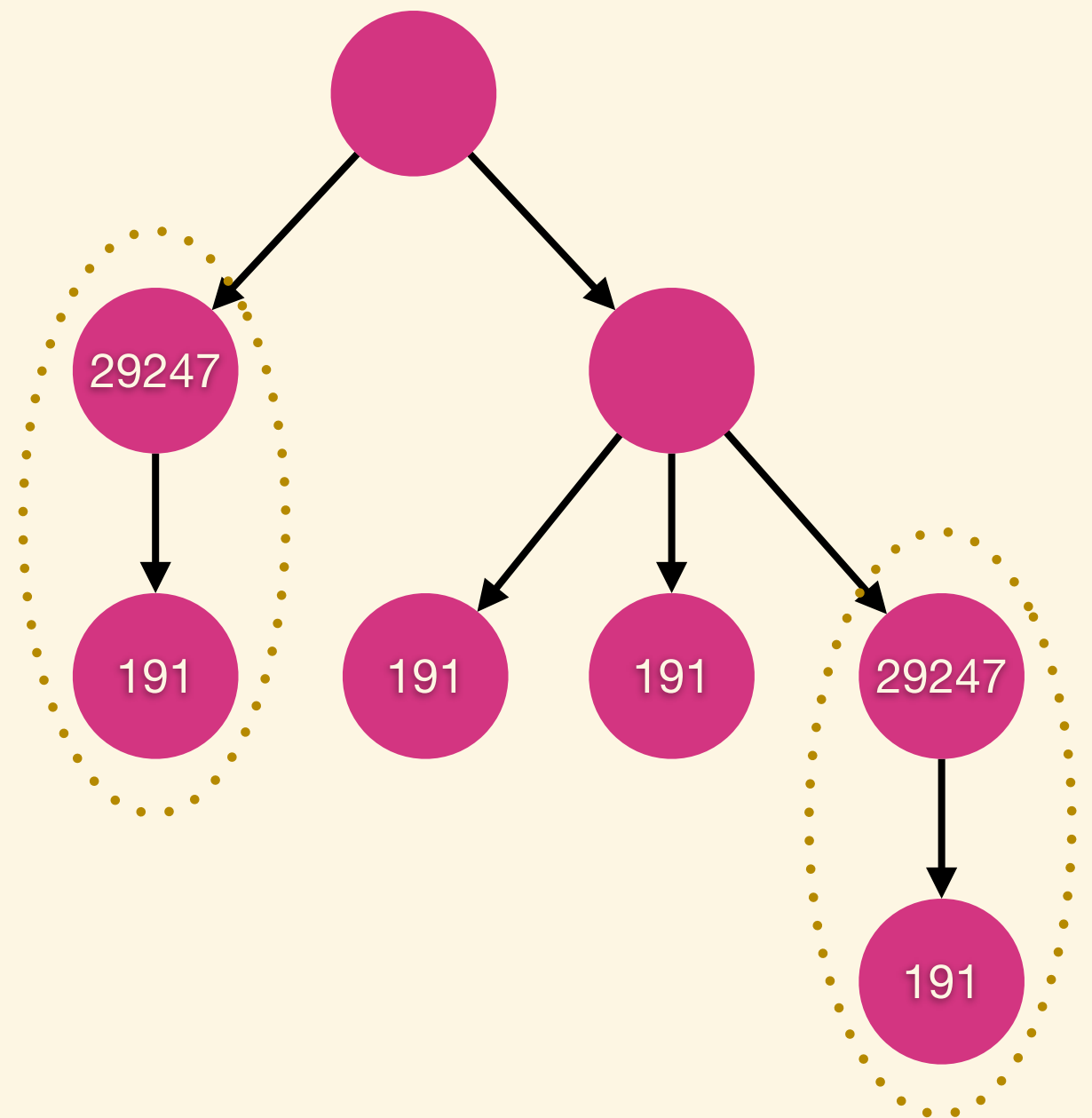
single vertex

initial value = 191



two-level sub-tree

child = (191)



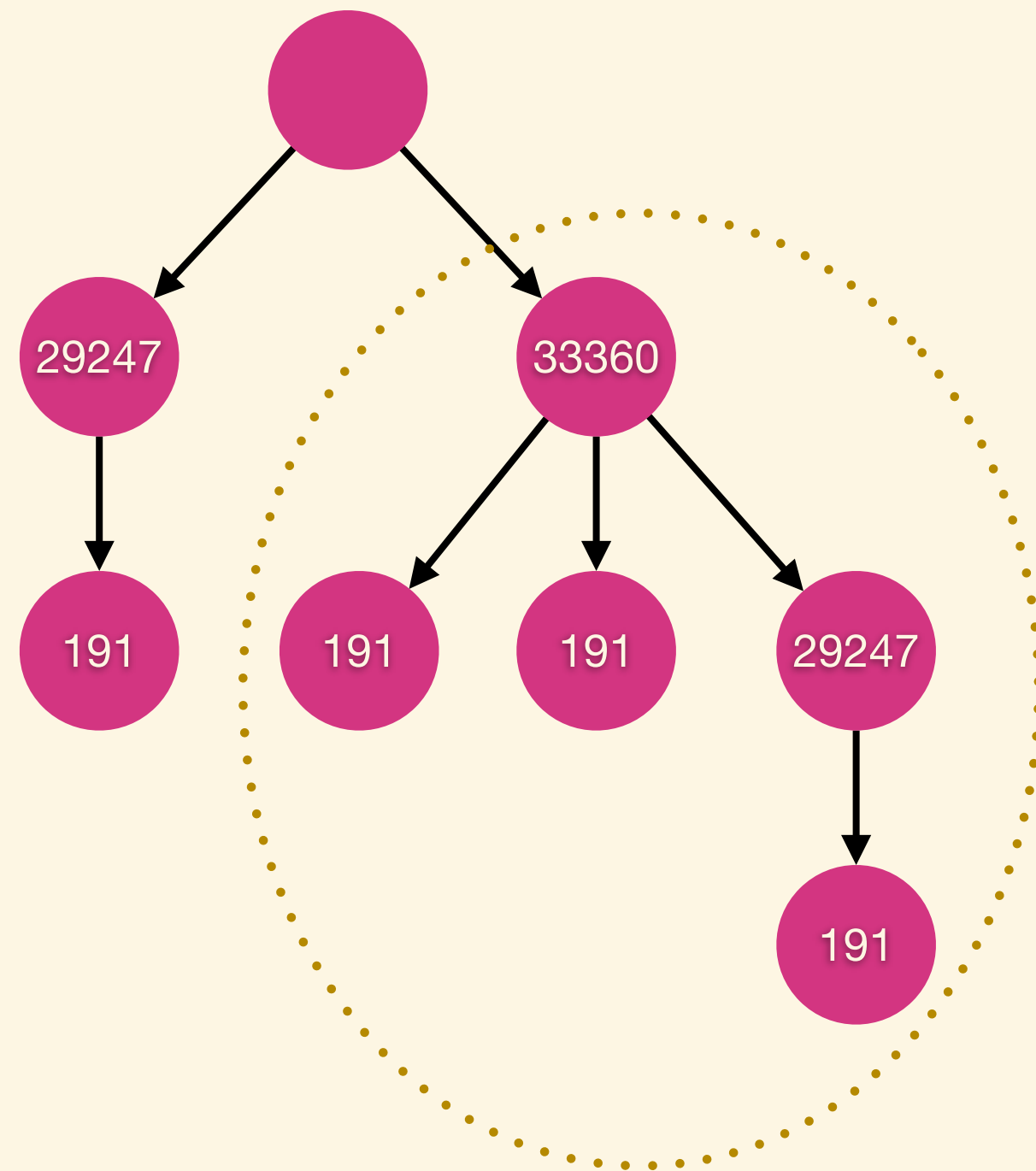
$$(191 \times 701 \text{ xor } 191) \bmod 34943 = 29247$$

three-level sub-tree

child = (191, 191, 29247)

↓ sort

child = (191, 191, 29247)



$$\begin{aligned} &((((191 \times 701 \text{ xor } 191) \bmod 34943) \times 701 \text{ xor } 191) \bmod \\ &34943) \times 701 \text{ xor } 29247) \bmod 34943 = 33360 \end{aligned}$$

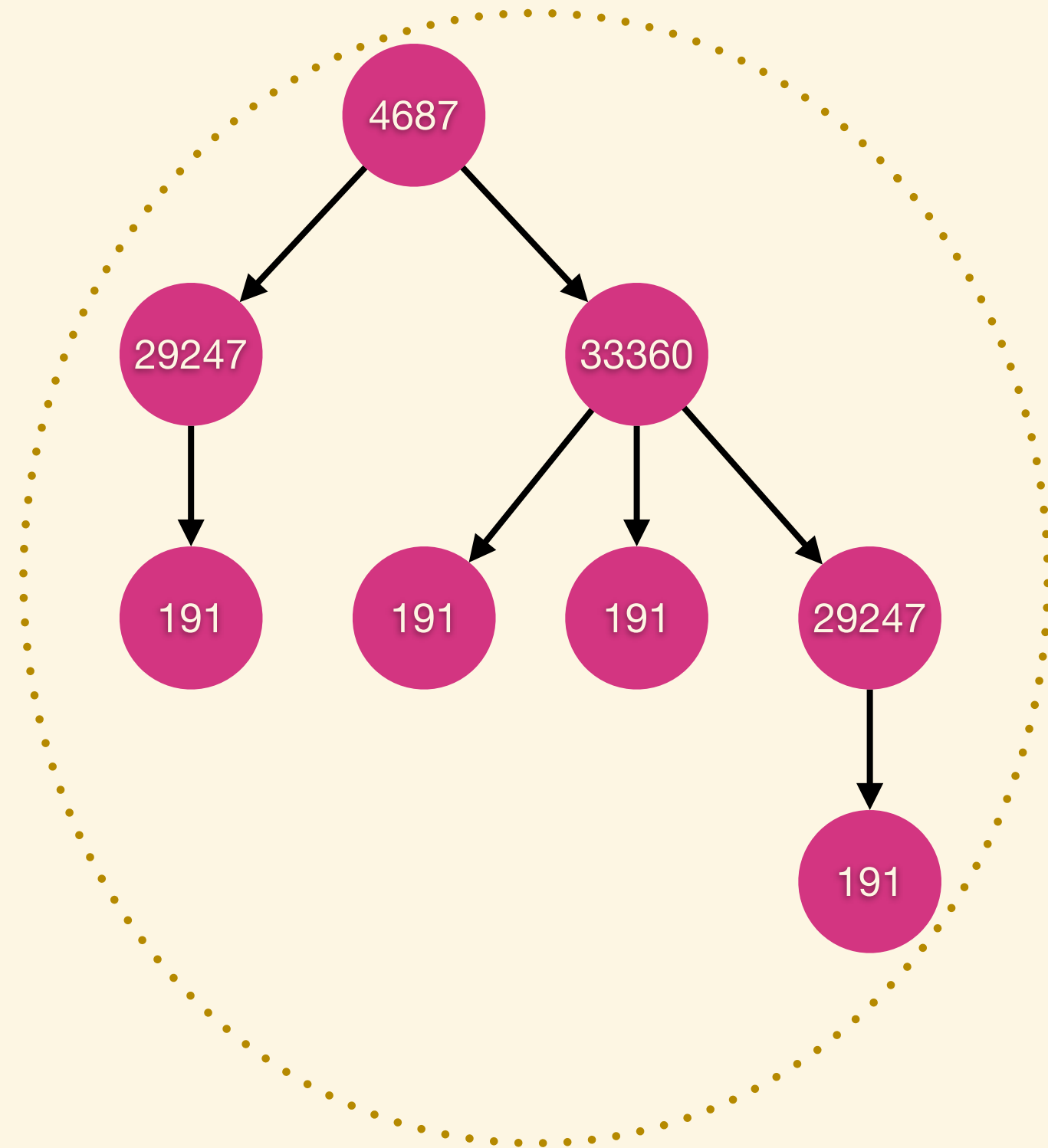


the total tree

child = (33360, 29247)

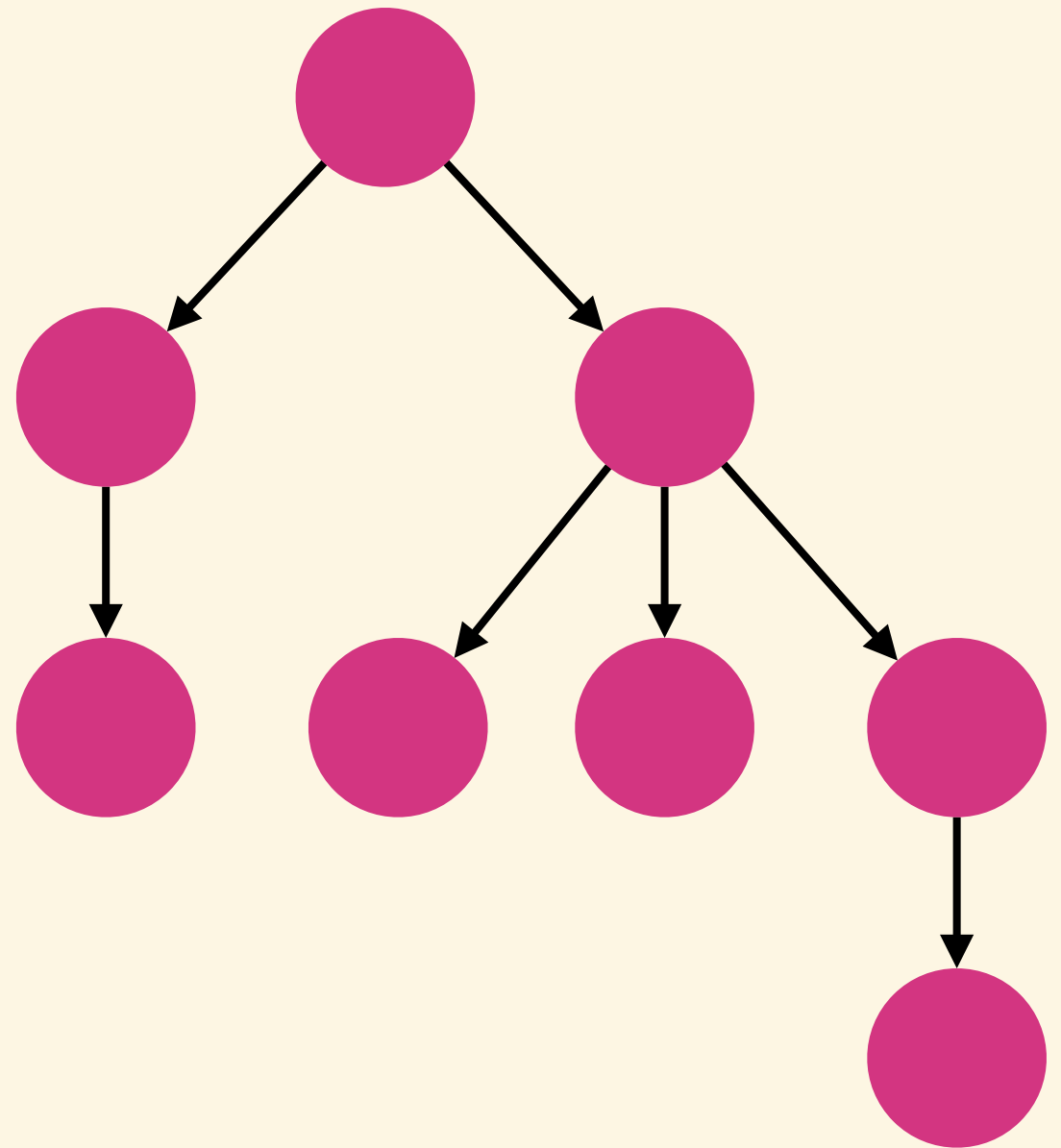
↓ sort

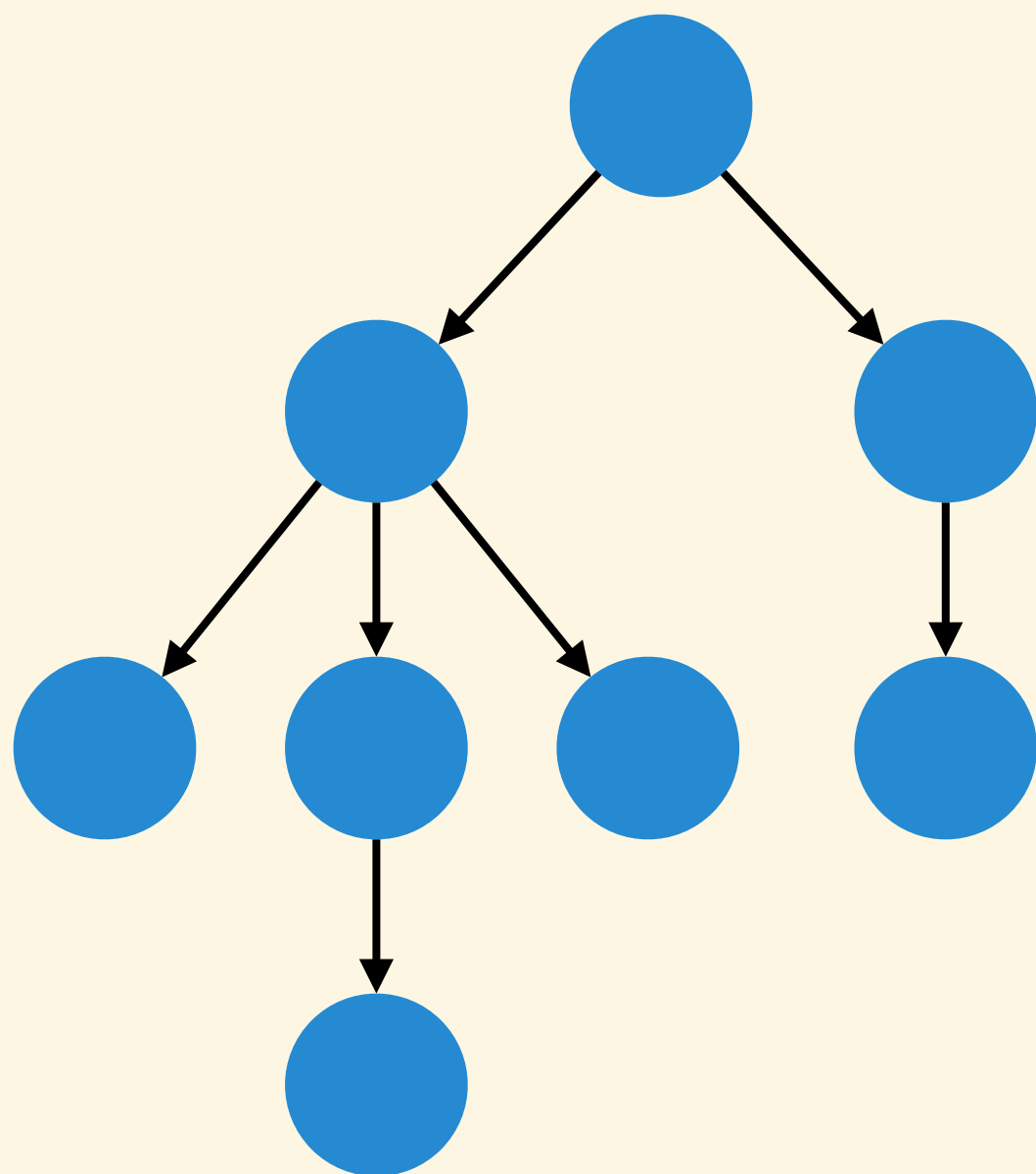
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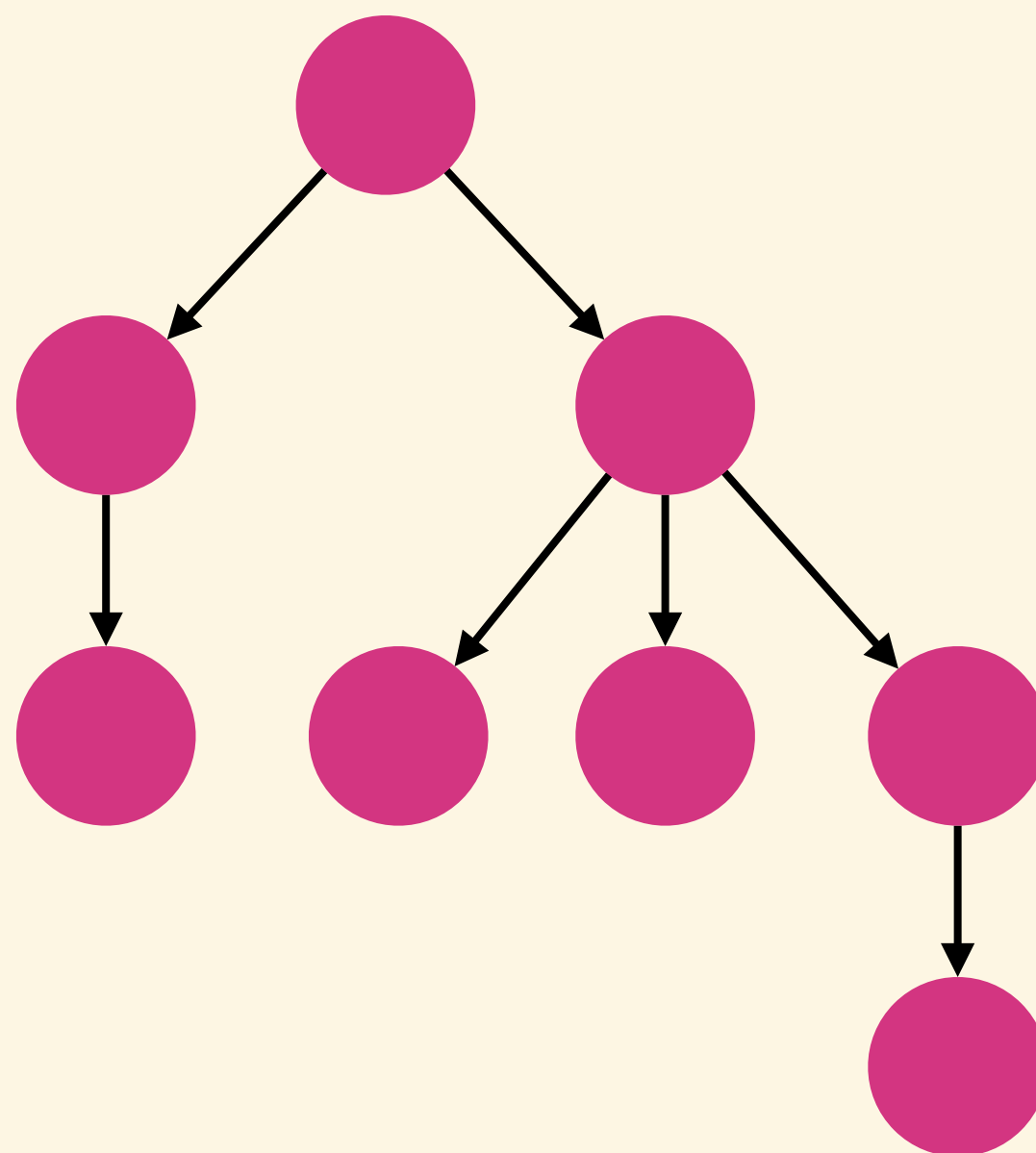
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hash value of  
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$\equiv$

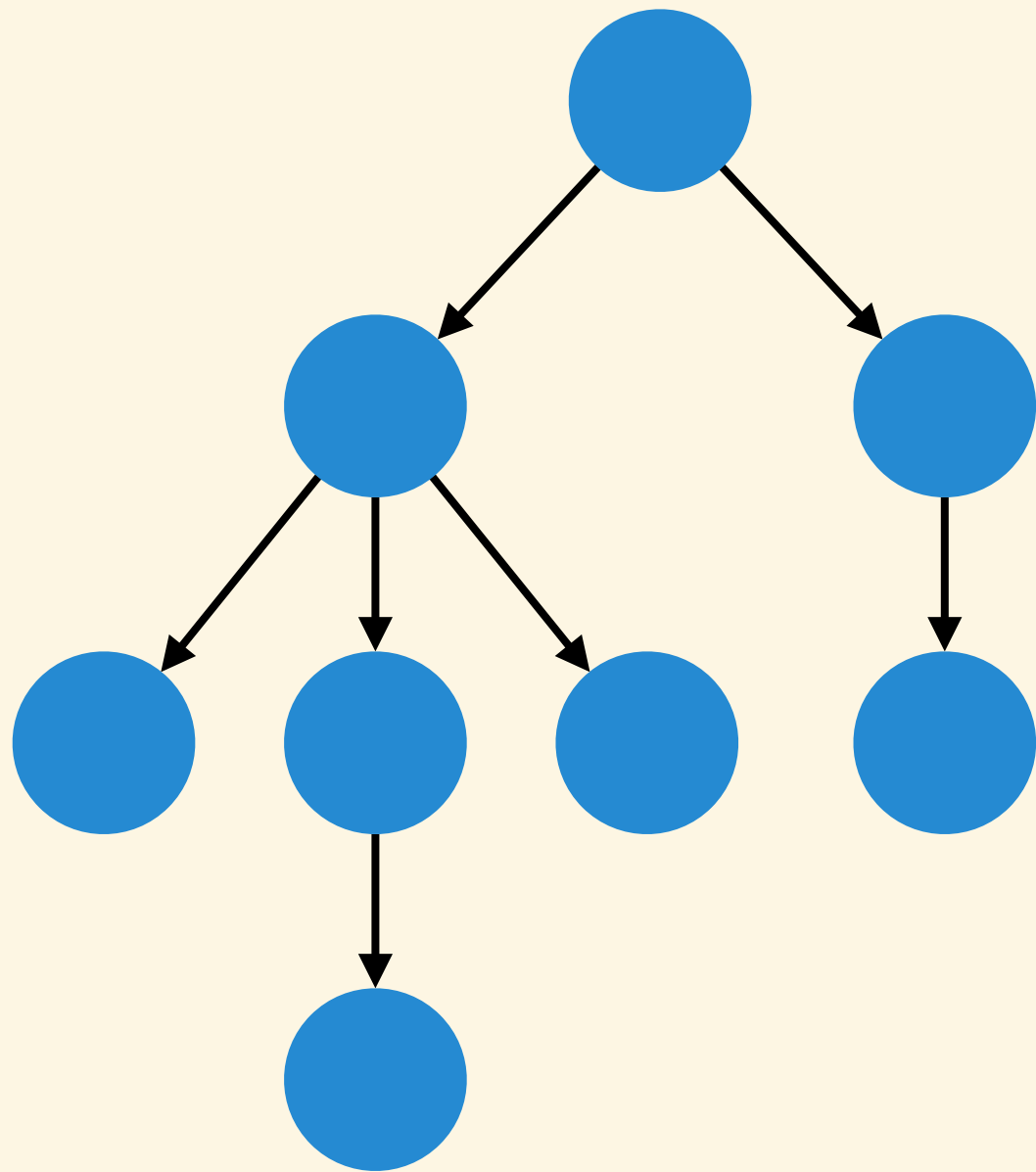


# Algorithm

`HASH_TREE(T):`

1. hash all sub-trees
- 2. sort hash value of sub-trees (unique)**
3. calculate hash value (any hash function)

# Time Complexity



$$O(\mathbf{N} \log_2 \mathbf{N})$$

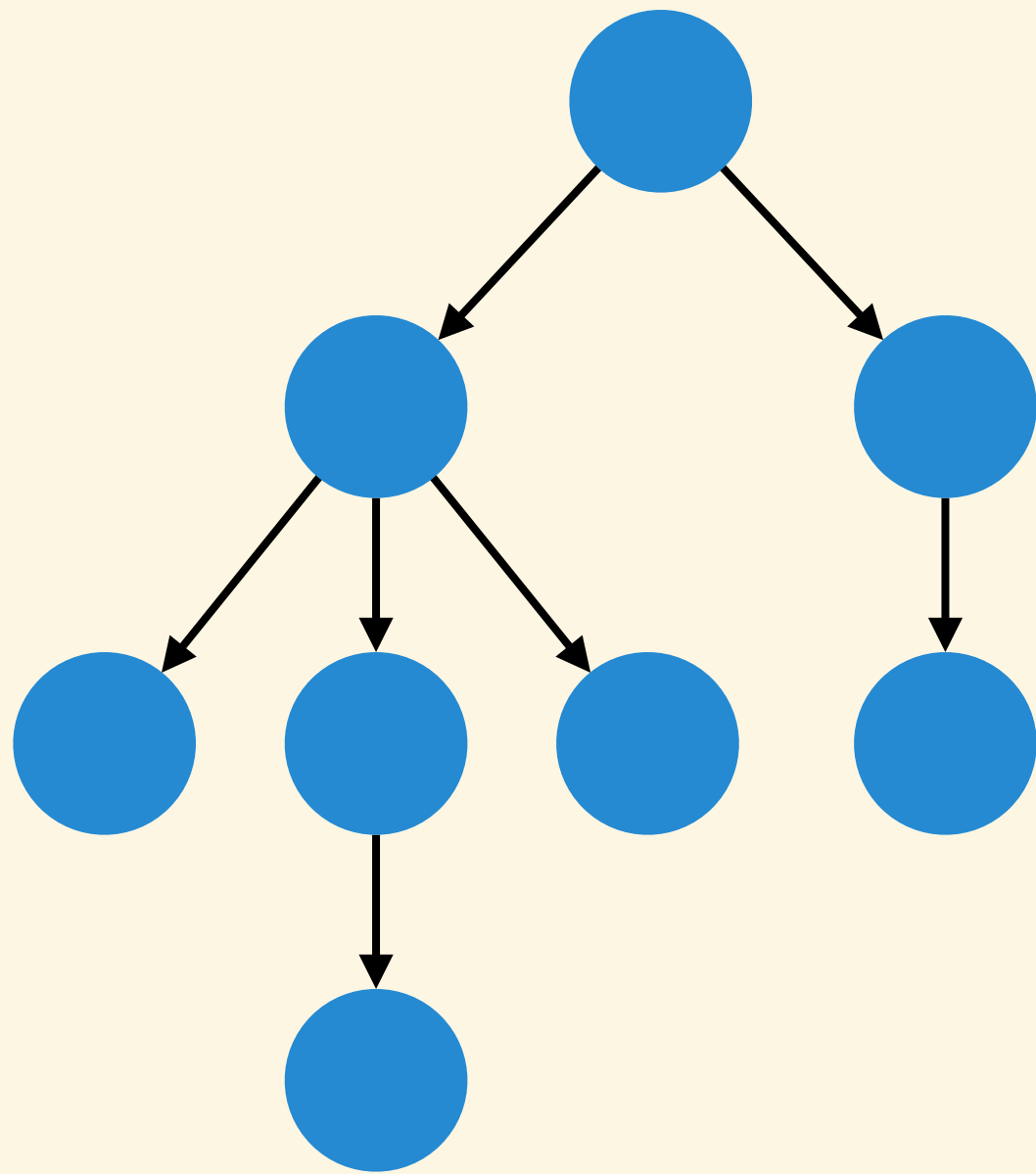
number of vertices  
height of tree



# Source Code

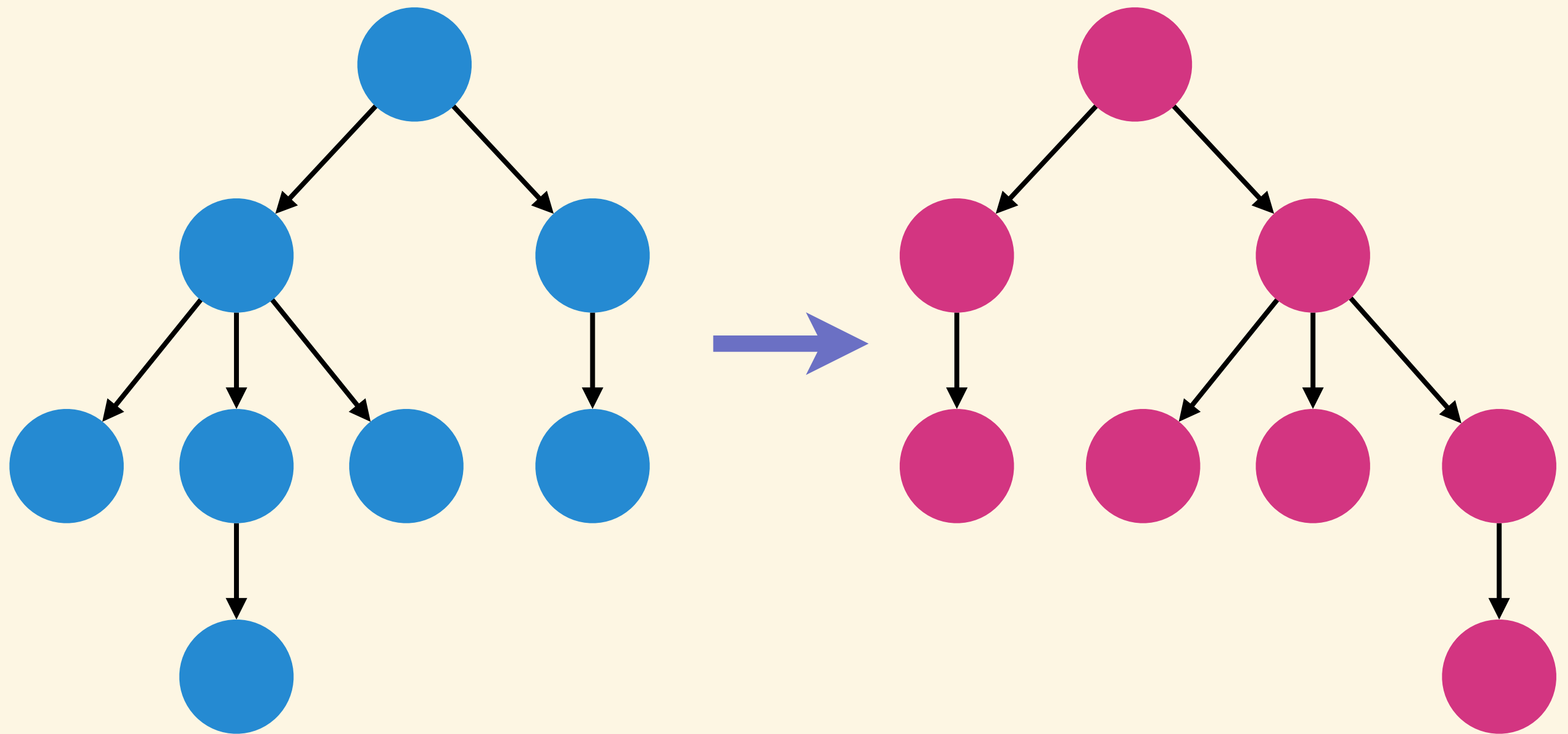
```
int hash( TREE &now, int root ) {  
    int value = INIT;  
    vector< int > sub;  
    //get all hash value of subtree  
    for ( int i = 0; i < now[ root ].size(); ++i )  
        sub.push_back( hash( now, now[ root ]  
[ i ] ) );  
    //sort them to keep unique order  
    sort( sub.begin(), sub.end() );  
    //hash this tree  
    for ( int i = 0; i < sub.size(); ++i )  
        value = ( ( value * P1 ) ^ sub[ i ] ) % P2;  
    return value % P2;  
}
```

# Representation of Tree

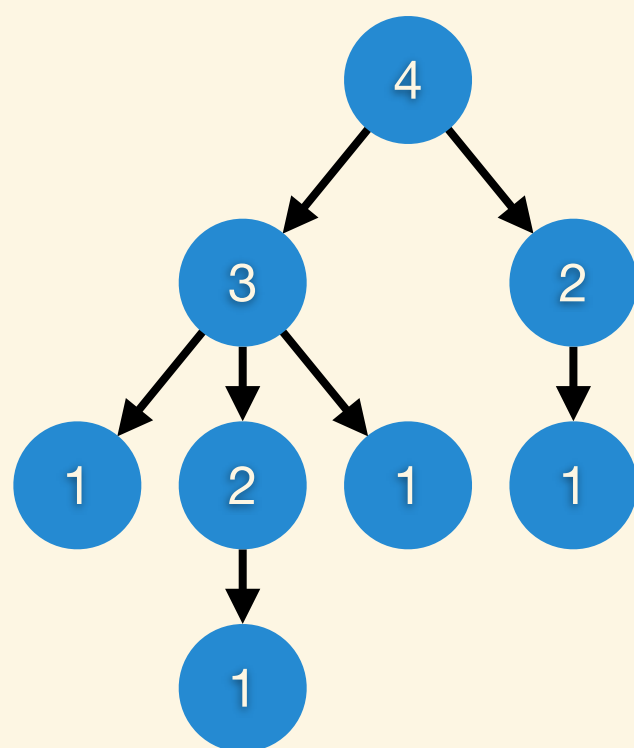


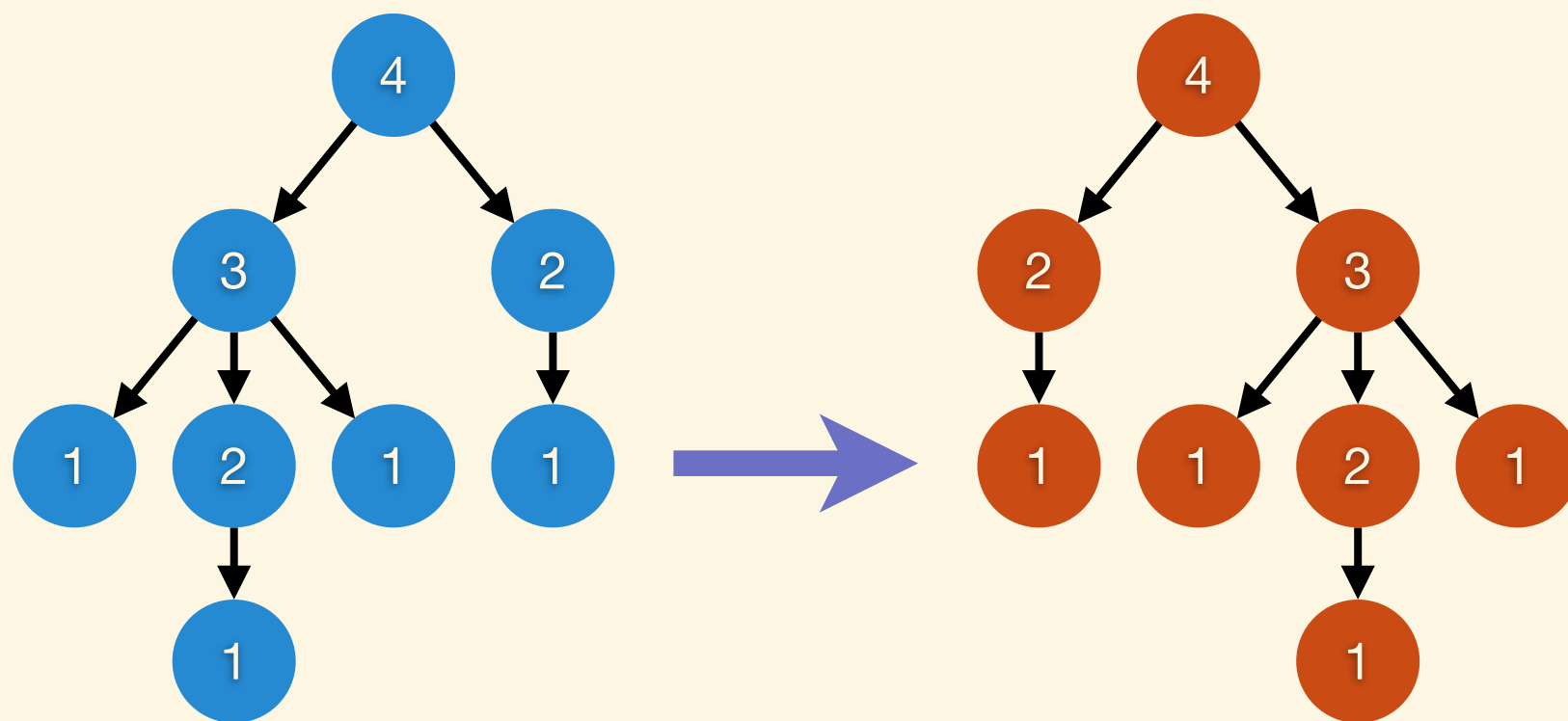
Let the **height** of left child **less than** the right one.

# Representation of Tree

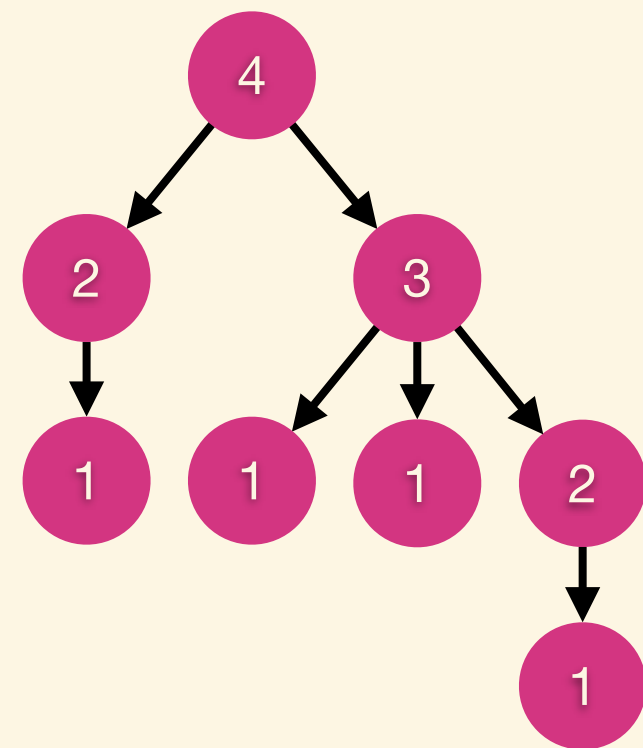
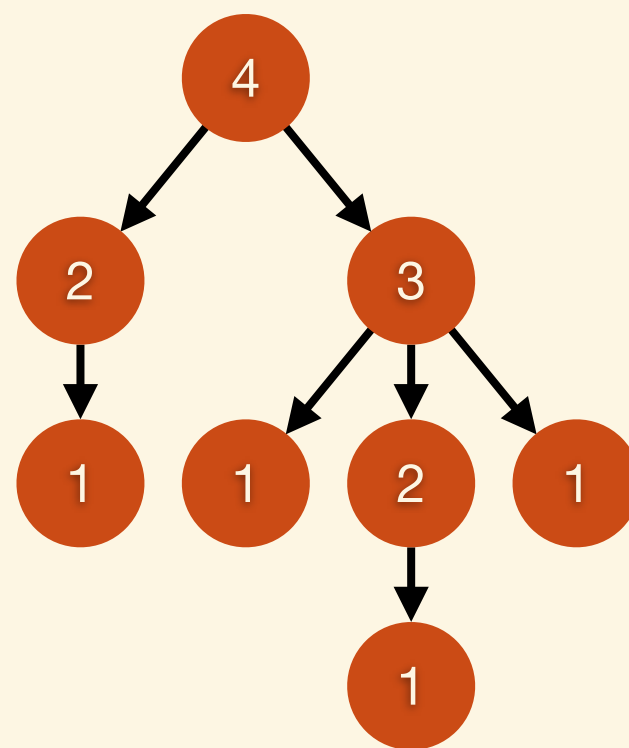
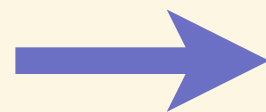
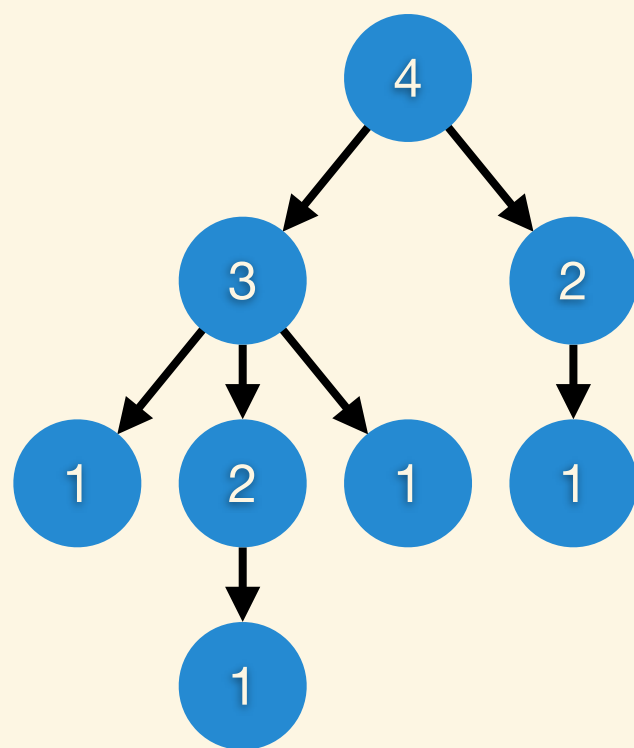


Let the **height** of left child **less than** the right one.





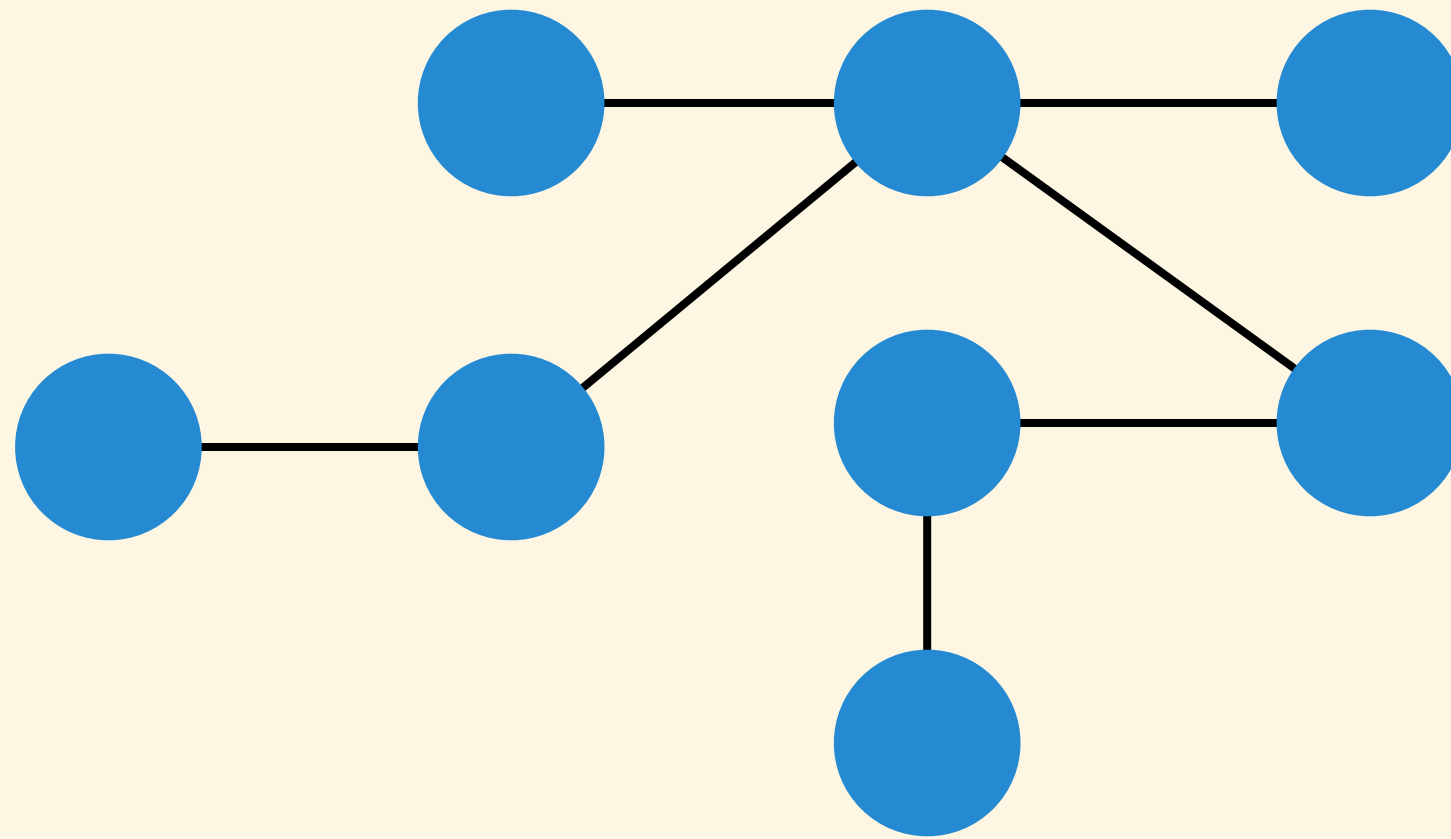




# Algorithm

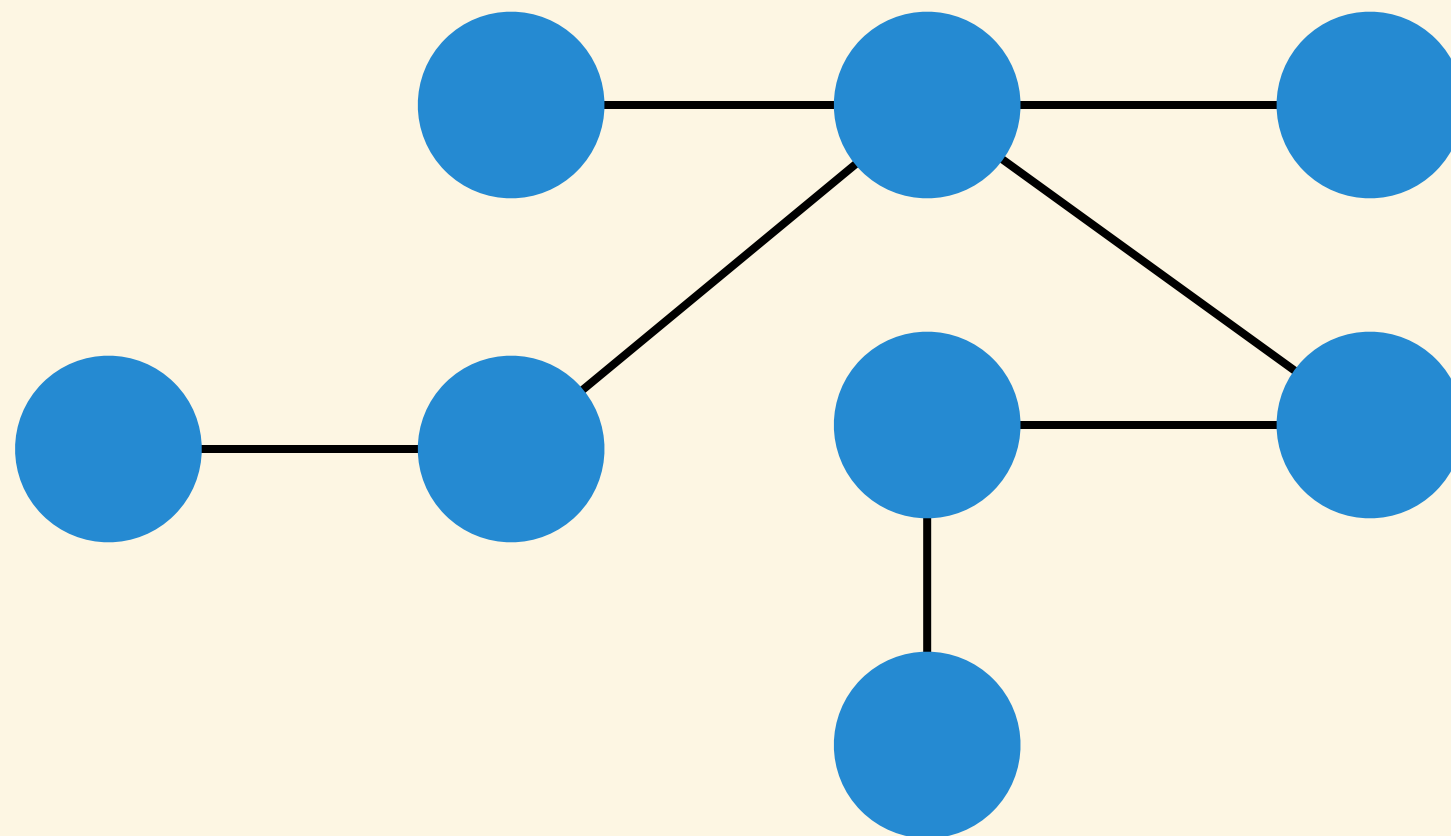
`SORT_CHILD(T):`

1. sort all sub-trees
2. compare the height
3. if height is equal, compare child recursively
4. put the lower at left and the higher at right

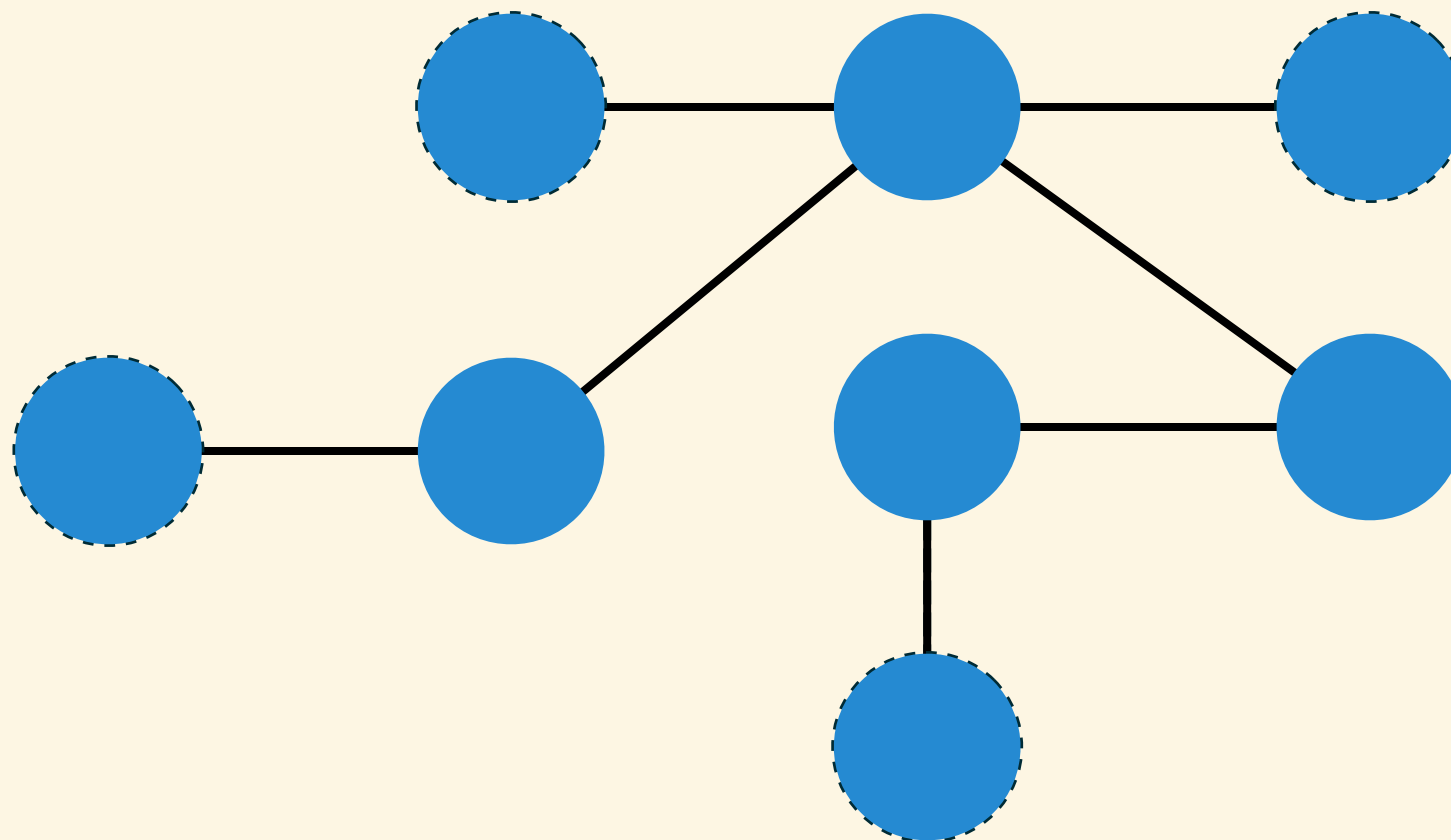


How about unrooted tree?

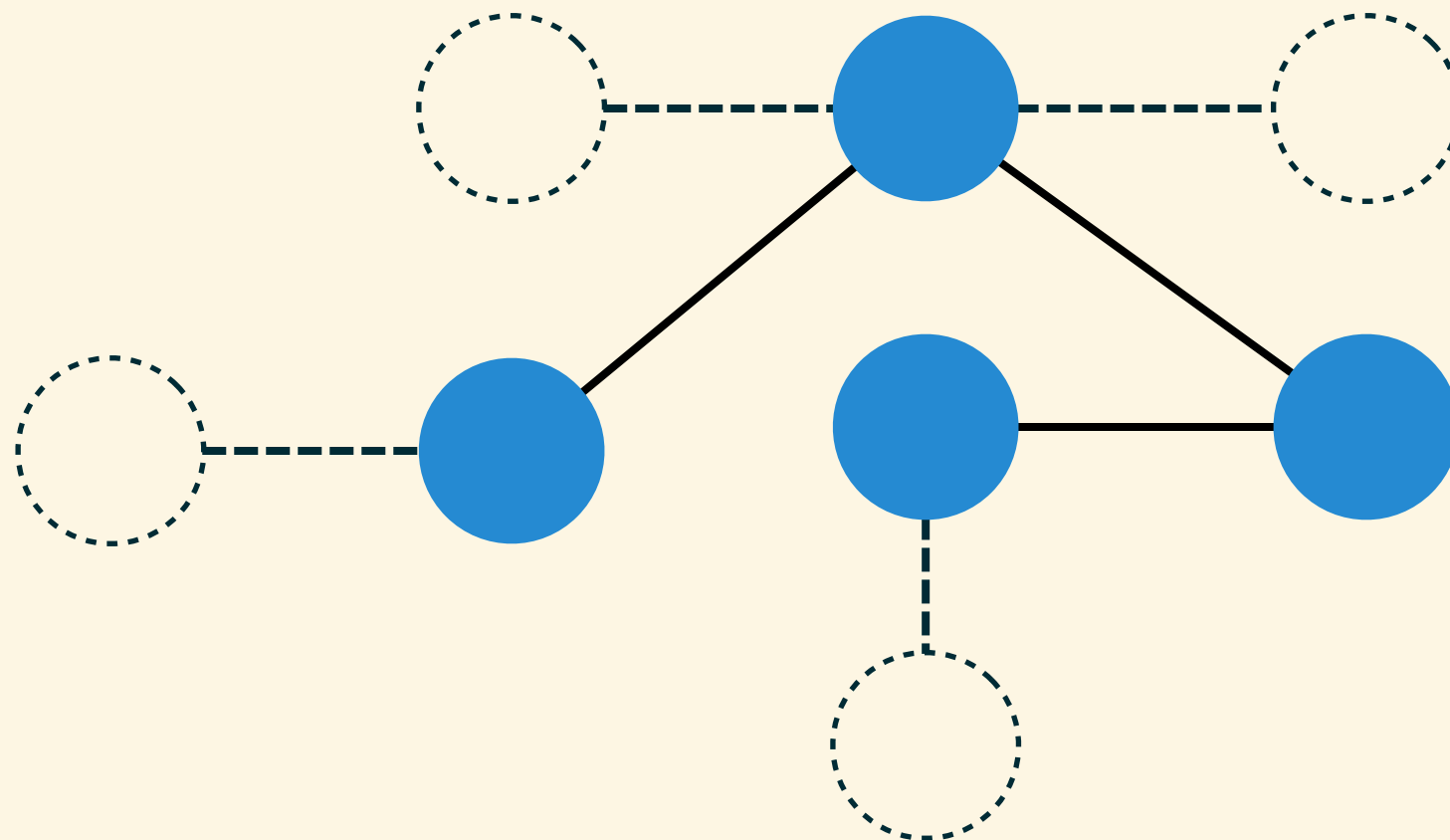
# find a root



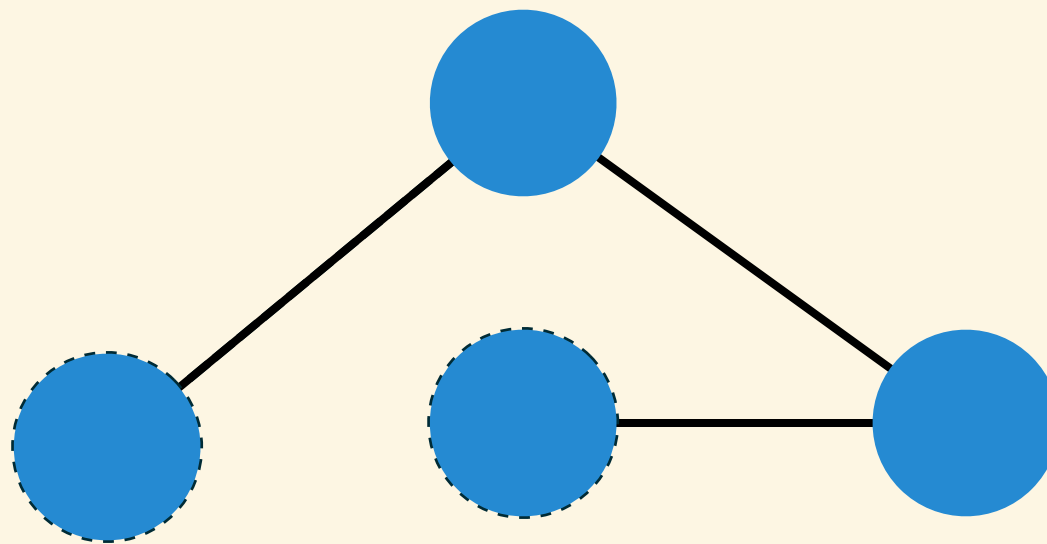
# eliminate leaves



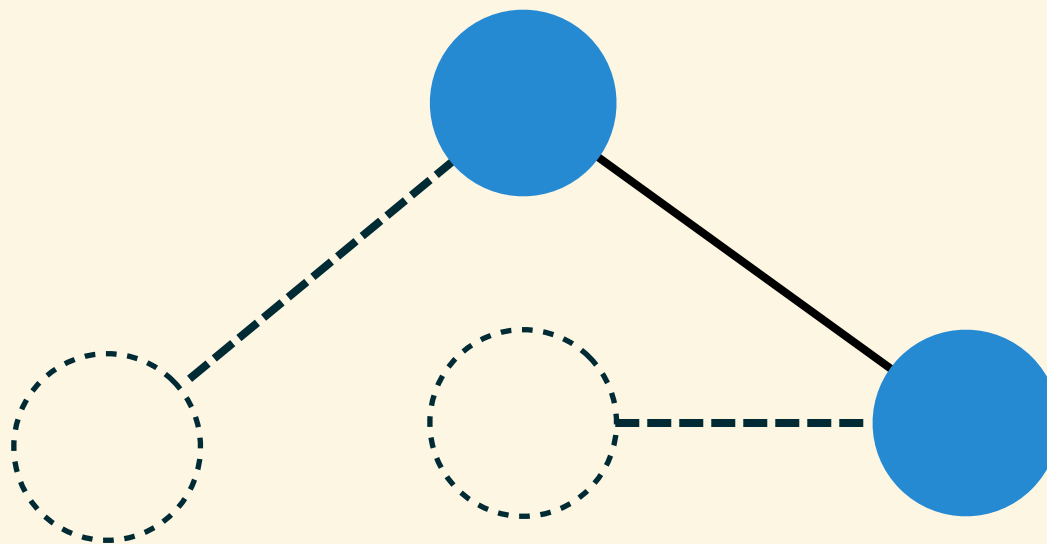
# eliminate leaves



# eliminate leaves

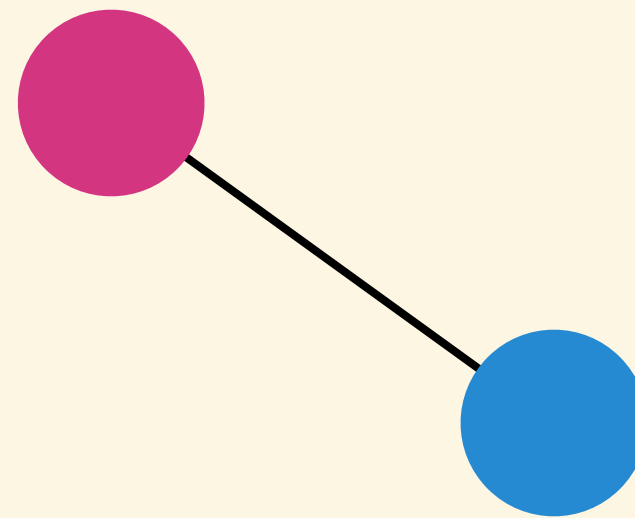


# eliminate leaves

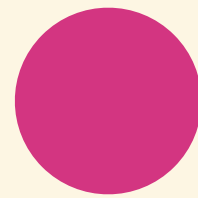




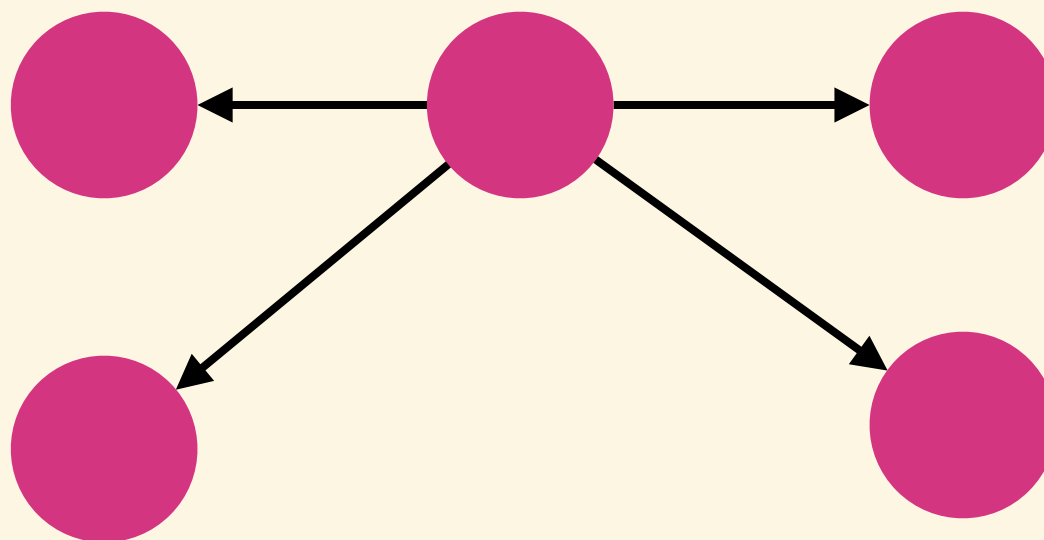
try each root



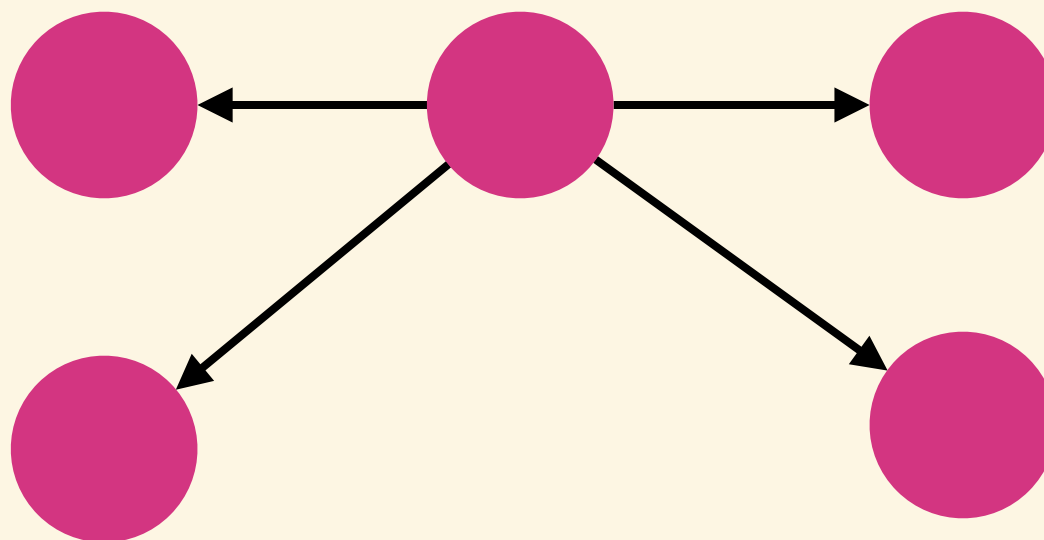
# rebuild the tree



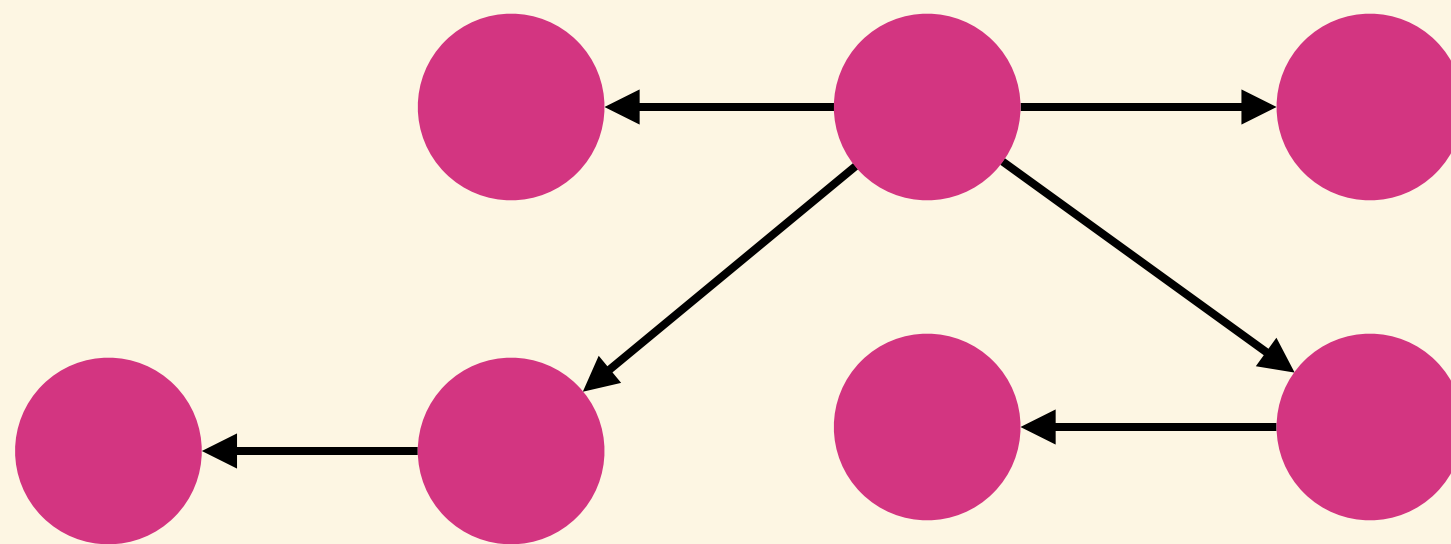
# rebuild the tree



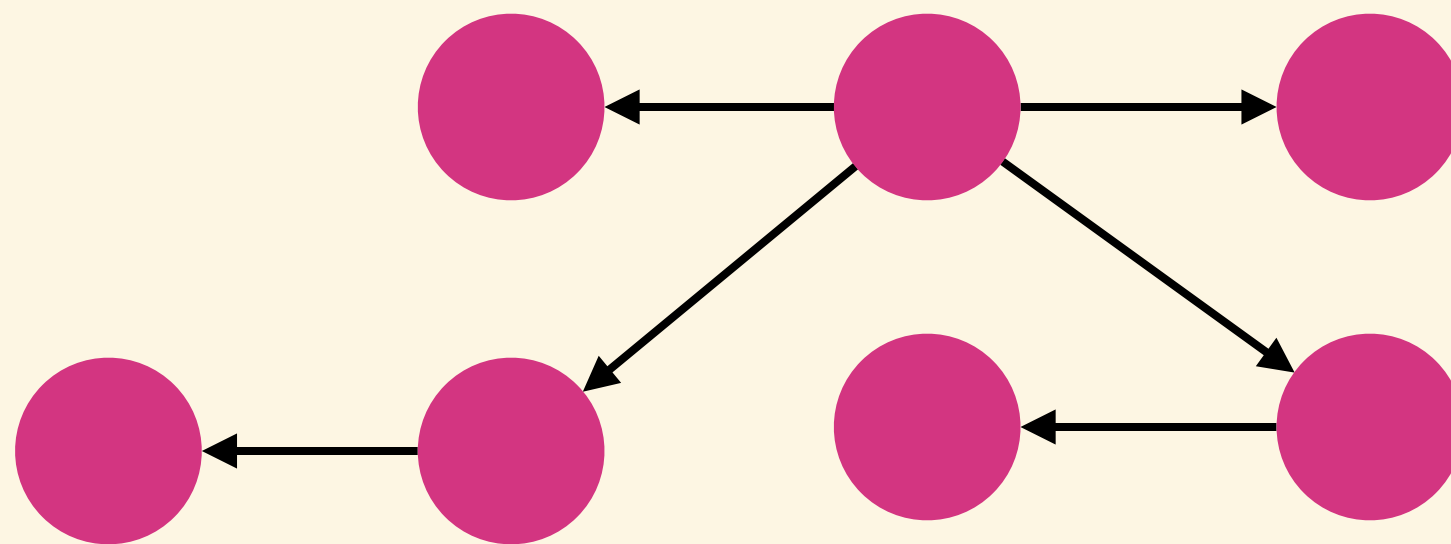
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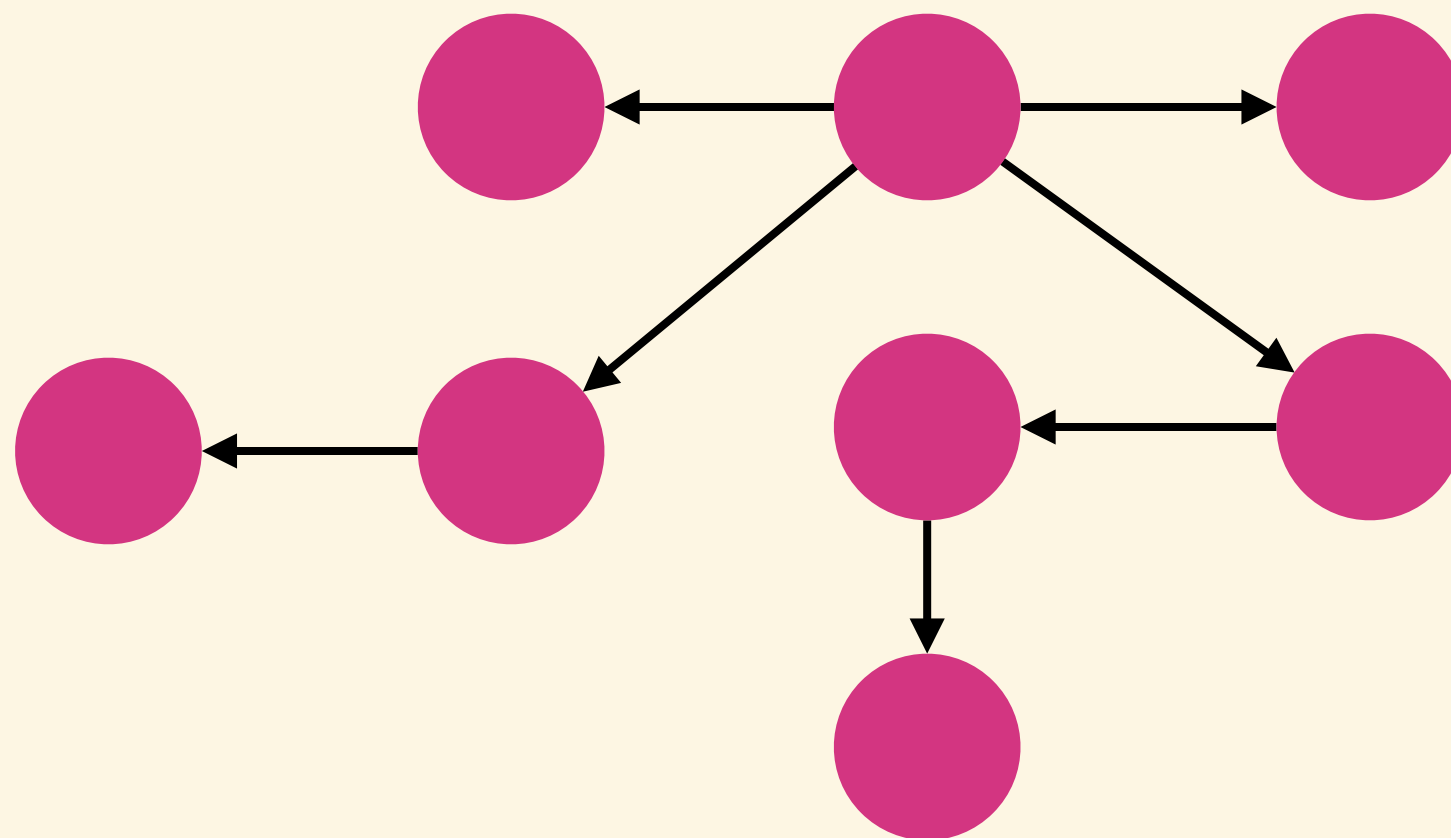
# rebuild the tree



# rebuild the tree



# rebuild the tree



apply the isomorphism detection



# Practice Now

POJ 1635 - Subway tree systems

Thank You for Your  
Listening.

