Do Non-Compete Clauses Undermine Minimum Wages?*

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Abstract

Many low-wage workers in the United States are subject to non-compete clauses. After the termination of their employments, they are not allowed to work for competitors. Empirical research has found a link between the prevalence of non-compete clauses and minimum wage legislation. To explain this link, we propose a moral hazard model with minimum wages. Non-compete clauses can be used to punish failure. We characterize the optimal contracts with and without the possibility to use a non-compete clause. We find that the principal only uses a non-compete clause if minimum wages are sufficiently high. We show that with non-compete clauses the principal induces an effort level that first decreases in the minimum wage and then increases when the principal starts to use a non-compete clause. If non-compete clauses can be arbitrarily sever, eventually, the principal induces an effort level that exceeds the first-best effort level. If non-compete clauses are bounded, both the principal and the agent might be made better off than without non-compete clauses.

Keywords: non-compete clause, minimum wage, limited liability, moral hazard, rent extraction.

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1 Introduction

49.85 seconds. That's all it takes to make a foot-long turkey breast sandwich.[1] Unimaginable? Imagine you are a sandwich worker and had to sign a non-compete clause that prevents you from working for any fast food franchise for two years after you quit…or are laid off.[2] Does your non-compete clause fire your imagination?

In general, a non-compete clause (*NCC*) is part of an employment contract that prohibits employees from working for a competitor or from starting their own business within specific geographic or temporal boundaries. A vast number of labor force participants in the United States are currently bound by a non-compete clause.[3]

In the literature, there are broadly four rationales for non-compete clauses. Firstly, they mitigate the hold-up problem of investments in human capital. If workers are liquidity constrained and cannot invest in their industry-specific human capital, an NCC allows the employer to recoup her investment. Secondly, NCCs can be used to preserve bad bargaining positions. One third of all workers with NCCs only learn on their first day at work after having rejected all other offers that they have to sign one (Starr et al., 2019a). Thirdly, NCCs lead to more durable employment relations and reduce the turnover. This reduces training and hiring costs. Fourthly, similarly to non-disclosure agreements and non-solicitation agreements, NCCs protect proprietary information and client lists. Some more structured overview over the existing literature is provided by McAdams (2019).

Yet, non-compete clauses are also frequently used in low-skilled jobs to which these rationales do not apply. Colvin and Shierholz (2019) provide descriptive data on the use of NCCs in 634 American businesses in 2017. They find that 29% of the sampled workplaces that pay an average hourly salary of less than 13 dollars, and 20% of the workplaces in which the typical employee has not graduated from high school have each employee sign an NCC. These jobs are neither human capital intensive, nor is there a lot of wage bargaining going on, nor do employees have access to sensitive information.[4] While turnovers are indeed reduced, training and hiring costs are rather low for most low-wage jobs. Why are NCCs frequently found in low-skill jobs? Empirical findings show that an increase in the minimum wage leads to more low-wage workers' being constrained by non-compete clauses (Johnson and Lipsitz, 2020). What is the connection between minimum wages and non-compete clauses?

Empirical evidence (Starr et al., 2019a) suggests several reasons why low-wage workers

[1]That's the time (after penalty seconds) with which Sara Tiegs won the 2019 Subway Global Sub Jammers Championship, for more information see Anthony Kuipers' article `Subway employee from Pullman the fastest in the world...again` or the `Official Rules`.

[2]More accurately "any business which derives more than ten percent (10%) of its revenue from selling submarine, hero-type, deli-style, pita and/or wrapped or rolled sandwiches and which is located with three (3) miles of either [the Jimmy John's location in question] or any such other Jimmy John's Sandwich Shop." Quoted from Dave Jamieson's *Huffington Post* article `Jimmy John's Makes Low-Wage Workers Sign 'Oppressive' Noncompete Agreements`. Jimmy John's settled with the Attorney General in New York State and stopped using non-compete clauses for sandwich workers in 2016. For more details, see Sarah Whitten's article for *CNBC* `Jimmy John's drops noncompete clauses following settlement`.

[3]Starr et al. (2019a) find that 20% of the labor force were restricted by such a clause in 2014 and that 40% had signed one in the past.

[4]One exception are hair dressers who might take their clients with them when leaving the firm.

sign NCCs. Firstly, they might not know that their contract contains a non-compete clause. Secondly, as most employees immediately sign an NCC without consulting legal advice, they might not correctly understand the terms of the non-compete clause. Thirdly, they might not anticipate being asked to sign a non-compete clause on the first day after they have rejected other offers. However, it is an open question why fully rational workers, who understand and anticipate NCCs, would sign such contracts.

In this paper, we show why lower bounds on wages lead to the use of NCCs when workers are rational. In contrast to the existing literature, there is neither human capital, nor inventions, nor bargaining. We argue that NCCs are abused to provide incentives. The implicit threat of having a hard time on the labor market after losing one's job makes workers exert more effort. Minimum wages are a friction that distorts the principal's profit maximization problem by granting the agents a rent. The principal does no longer want to maximize social surplus. Typically, implementing inefficiently little effort becomes profit-maximizing. In this setting, non-compete clauses can be used to both provide incentives and expropriate rents.

Consider efficiency wages. In Shapiro and Stiglitz (1984), employers pay wages above the employees' outside options. The rent of retaining a job is used to provide incentives: The shirker is being dismissed. The fear of losing the rent makes the employees exert effort.

The principal in our model works with sticks rather than carrots (although we also allow for bonus wages). Having an NCC means being unemployed after a dismissal, reducing the agent's payoff. An NCC, hence, increases the agent's incentives to keep his job (*incentive effect*). The additional effort due to an NCC works against the inefficiency from minimum wages. Accordingly, the equilibrium effort first decreases in the minimum wage and then increases again as soon as NCCs are used. For large minimum wages, the equilibrium effort gets inefficiently large. The incentive effect also redistributes from the agent to the principal because the agent is not compensated for his additional effort. As a result, the employer will always make his employee sign the severest NCC that the employee is just willing to sign. Furthermore, activating an NCC makes the agent's labor lie idle which harms him (*idleness effect*). As a result of both effects, the agent never gets a rent as long as NCCs are unbounded. If, however, NCCs are bounded, so is the principal's ability to extract the agent's rent. Thus, for sufficiently large minimum wages, the agent gets a rent.

Our story about NCCs relies on limited liability. Without minimum wages, the principal does not want to use wasteful NCCs as she can sell the firm to the employee and extract the first-best surplus. With limited liability NCCs become attractive as an additional tool. Without an NCC, the agent would get a rent for sufficiently large minimum wages. With an NCC, the principal can seize this rent because the corresponding increase in the equilibrium effort is not compensated. Moreover, seizing the rent makes it smaller due to the principal's not profiting directly from activating the NCCs; the idleness effect is deadweight loss.

To evaluate the welfare effects of NCCs, we consider the utilitarian welfare, the sum of the principal's profit and the agent's rent, that is. The two effects of NCCs also affect the utilitarian welfare. The larger equilibrium effort, due to the incentive effect, has ambiguous effects on the utilitarian welfare: Initially, at the minimum wage, at which the principal starts using NCCs, it is positive since the effort lies between the first-best level and the inefficiently low level in the benchmark without NCCs. As the equilibrium effort increases in the minimum wage above the first-best level, this positive effect decreases until it becomes negative. The idleness effect, merely reduces the agent's rent and, thus, the utilitarian welfare.

Because utilitarian welfare is maximized without minimum wages, we can conclude that a policymaker that has introduced minimum wages uses another welfare criterion that puts a larger weight on the agent's rent.[5] To impose as little structure as possible on the policymaker's true welfare criterion, we additionally look at Pareto dominance of equilibrium outcomes. We find that when NCCs are unbounded, the agent never gets a rent, irrespective of the minimum wage. Moreover, the utilitarian welfare is strictly decreasing in the minimum wage. Both without and with bounded NCCs, minimum wages become redistributive at some level; albeit this level is higher for bounded NCCs. In both cases, as soon as minimum wages become redistributive, the utilitarian welfare is constant in the minimum wage. A one unit increase in the minimum wage transfers one util from the principal to the agent. In other words, the inefficiency from minimum wages is fixed at some level, determined by the effort cost function and the bound on NCCs. We show using an example that the inefficiency with bounded NCCs (possibly non-first-best effort and idleness effect) can be smaller than the inefficiency without NCCs (non-first-best effort). That is, bounded NCCs might lead to a strict Pareto improvement over the benchmark without NCCs. This, however, depends on the effort cost function and a suitably chosen bound.

This paper is organized as follows. Section 2 provides background information on the use of non-compete clauses and their enforcement, and discusses the related literature. Section 3 introduces the model: There is a principal who makes a take-it-or-leave-it offer to an agent. A contract consists of a base wage, a bonus wage conditional on success and an NCC that is activated when the agent is dismissed in case of a failure. There is a minimum wage that affects both the base and the bonus wage. In Section 4, we find the profit maximizing contracts by solving two Kuhn-Tucker problems: One problem, the benchmark, in which we do not allow for NCCs and one problem in which we do. Furthermore, we plot a parameterized example and compare the optimal contracts in both regimes. We also shortly deal with optimal contracts for bounded NCCs. The welfare implications of these contracts are analyzed in Section 5. We find that unbounded NCCs never leave a rent to the agents. If redistribution is the minimum wage's goal, NCCs have to be restricted or abolished. For bounded NCCs, we find that there might be a strict Pareto improvement compared to the case of no NCCs. In Section 6, we discuss the simplifying assumptions of our model, outline a possible generalization of moral hazard with multidimensional payoffs, and summarize empirical predictions of our model. Finally, Section 7 concludes.

[5]With NCCs, it is not sufficient to look at the wages that the agent gets. Although he might get more cash when his contract has an NCC, he is not necessarily better off. The increase in the effort cost, eats up a possible increase in the wages.

2 Background and Related Literature

In this section, we first throw light on where and how NCCs are used in the United States. That is, we describe the world how it is and how it is changing at the moment. Next, we present the academic literature on NCCs and on methodologically similar models, and embed our contribution into it.

2.1 Background

Case law regarding NCCs The legislation on non-compete clauses is very different across the United States. All but three states (California, North Dakota, and Oklahoma) enforce at least some NCCs. Because there is no single law that regulates NCCs, courts' decisions fill the gaps. This introduces some path-dependencies. There are also many differences across industries: For example, lawyers are exempt from NCC in all states, the broadcasting sector is exempted in several states, and Hawaii banned NCCs in the technology sector.

In most states, courts check whether an NCC is "reasonable" before enforcing it. What is reasonable, however, depends on the state. Reasonableness, in general, has something to do with the duration and geographic dimension of the NCC, and with what employments are forbidden. Six states, for example Virginia, follow what is colloquially called the "Janitor rule." Roughly, it says that a salesman is allowed to work for a competitor as a janitor. That is, NCCs are only enforceable if an employee wants to work for a competitor in a position in which he can use whatever the former employer wants to protect with the NCC; for example knowledge about the pricing.

Another difference between the states is what courts do if an NCC contains a clause that a court thinks is too broad to enforce. Imagine a passage that rules out employment for any competitor for 5 years. In general, there are three possibilities, called "red pencil," "blue pencil" or "equitable reform." Under the red pencil doctrine, an invalid passage makes the whole NCC unenforceable. Under the blue pencil doctrine, only the invalid passage is thrown out. If, as in our example, the duration is tossed, this makes the whole NCC unenforceable. If, however, it is a list of competitors, this list might be shortened and the NCC remains intact. Under equitable reform, which most states adopt, the court can change passages and enforce this new NCC. In our example, the court might reduce the duration to two years and enforce this non-compete clause.

One attempt to bring order into this chaos is Malsberger (2019) which has more than 9,000 pages in its twelfth edition including a 2019 supplement. It is a comprehensive survey of how states' courts decide on several dimensions and is intended for legal practitioners. The collection has been used by both Bishara (2011) and Garmaise (2011) to define a one-dimensional measure of enforceability for all states. Both have a list of questions, such as "Question 8: If the employer terminates the employment relationship, is the covenant enforceable?" (Bishara, 2011, p. 777), and award scores if the NCC is enforceable in a state.

Regarding our model, whether a court would enforce an NCC after an employee has been dismissed is a crucial question. Common sense says that threats have to be credible to be effective. Thus, a non-credible threat to remain unemployed after being fired for bad performance does not lead to better performance. On a scale from 0 to 10—where 0 means that a dismissal makes an NCC unenforceable and 10 means that a dismissal makes no difference whatsoever—Bishara (2011) gives only five states a score of less than 6. Moreover, 15 jurisdictions score 10. That is, in most states having an NCC when being dismissed means worse opportunities on the job market. The threats are, in general, credible.

Legislation At the moment, there is a lot of policy making happening regarding NCCs. In the last two years, several states have passed new laws, making NCCs unenforceable for low-wage workers. In Illinois, Maine, Maryland, New Hampshire, and Oregon, there are now income thresholds below which non-compete clauses are invalid. For example, in New Hampshire, NCCs are only enforceable for employees with an hourly wage of at least $14.50. In Washington, the cutoff is an annual income of $100,000 and in Maine, it is four times the federal poverty level.

Furthermore, Massachusetts has passed a law that makes "garden leave" mandatory: The former employer has to continue paying 50% of the annual income for the complete duration of the NCC. Moreover, NCCs have to be announced at least 10 days before the employment begins, and NCCs are not enforced if someone got dismissed without a cause.

In Washington, it is now punishable to include unenforceable NCCs into contracts. If found guilty, the employer has to pay the damage or at least $5,000 and the fees of the employee's attorney. This makes it easier to litigate against obviously unenforceable NCCs.

There are also considerations on the federal level. In 2015, the "Mobility and Opportunity for Vulnerable Employees Act" has been sponsored by Sen. Murphy [D-CT]. The act, which did not make it past the committee, would have banned the use of NCCs with low-wage workers. In 2018, Senators Murphy, Warren [D-MA], and Wyden [D-OR] sponsored the "Workforce Mobility Act" that aimed at banning all NCCs. The bill was also introduced in the House of Representatives by Rep. Crowley [D-NY-14]. In 2019, Senators Murphy and Young [R-IN] sponsored another incarnation of the "Workforce Mobility Act". If passed, the Department of Labor and the FTC were empowered to enforce the ban with fines. Furthermore, Sen. Rubio (R-FL) has introduced the "Freedom to Compete Act" as an amendment to the Fair Labor Standards Act, aiming to ban NCCs for many employees (those who are not exempt by the Fair Labor Standards Act).

In July 2019, 18 state attorney generals have asked the Federal Trade Commission (FTC), one of two U.S. anti-trust agencies, to put a hold on NCCs. Because of this, the FTC has organized a workshop on non-compete clauses in January 2020. In the next months, the FTC will decide whether NCCs violate anti-trust legislation. One possibility would be claiming that NCCs negatively affect third-parties (other firms, other employees, costumers) that have not signed the clause.

2.2 Related Literature

This paper is related to multiple strands of literature. We distinguish between topical relation (non-compete clauses and their effects) and methodological relation (similar models with other applications).

We add to the considerable literature on non-compete clauses. For brevity, we only summarize the most closely related articles. Other theoretical and empirical advances in the area of non-compete clauses are surveyed in McAdams (2019).

NCCs have been argued before to be incentivizing. In research and development, NCCs can have negative effects on the effort level (Kräkel and Sliwka, 2009). The main idea is that without NCCs, there is an additional, potentially more informative signal on an agent's effort: outside job offers. Awarding these with retention payments can provide incentives more cheaply if the other signals are more noisy (in Kräkel and Sliwka, 2009, it is not verifiable who is the inventor). We complement this research by demonstrating that in a different setting the inclusion of a non-compete clause generates incentives. Cici, Hendriock, and Kempf (2019) empirically test the incentive effect of NCCs using legislative changes in Florida, Texas, and Louisiana. The hypotheses are derived without a formal model. They find that mutual fund managers perform better when NCCs get more enforceable.

NCCs have been argued before to redistribute rent from the agent to the principal. Wickelgren (2018) proposes a hold-up model with investments in human capital. Investments increase the agent's outside option which reduces the incentives to invest. Both higher wages and a more severe NCC can be used to prevent the agent from leaving. The agent's participation constraint is always binding in the optimum. Decreasing the wages is (under some technical conditions) more profitable for the principal than making the NCC more severe. Thus, when minimum wages are introduced, the principal might resort to NCCs. Johnson and Lipsitz (2020) provide a model that they test using a survey of owners of hairdresser salons. A minimum wage restricts the transfer of utility from the employee to the employer via money. As a result, it might be an equilibrium outcome to use NCCs to (inefficiently) transfer utility. When signing an NCC, employees incur an exogenous cost while employers receive an exogenous benefit. Employers are heterogeneous in the productivity of the job they offer. By a law of one price, the marginal firm in the market determines whether no contract or all contracts have NCCs. If the marginal firm needs an NCC to satisfy her participation constraint, there will be NCCs. A tighter restriction on utility's transferability, that is a larger minimum wage, makes this more likely. Empirically, they find that higher minimum wages lead to more severe NCCs. Furthermore, minimum wages have no employment effects in states where NCCs are enforceable. This already hints at NCCs' undermining the minimum wages.

The model of Johnson and Lipsitz (2020) is too simplistic to derive policy implications. While the authors mention some aspects that could contribute to the benefits and costs of NCCs, the results rely on ad-hoc assumptions. Wickelgren (2018) provides a microfoundation which, however, is not suitable for low-wage workers because it relies on human capital and innovations. Our contribution to this existing literature is providing a formal model of how

NCCs allow employers to extract rents from low-wage workers. Cici et al. (2019) find evidence that the used channel exists. That is, we have a more suitable microfoundation. Our approach allows us to get clear and plausible welfare implications.

Methodologically, we contribute to the literature of agency models with moral hazard in continuous effort and with limited liability (e.g. Schmitz, 2005, Kräkel and Schöttner, 2010, Ohlendorf and Schmitz, 2012, and Englmaier et al., 2014). Especially, we contribute to the agency literature with multidimensional (monetary and non-monetary) payoffs. In our model, the payoff's dimensions are present and future payoff. Minimum wages affect only present payoffs. NCCs can reduce only future payoffs via unemployment. As in the present paper, Kräkel and Schöttner (2010) show that future rents can be used to incentivize effort in the first period. In their partial market model, minimum wages lead to job rationing, that is having a job means getting a rent. As a result, being dismissed due to bad performance is costly for the agents, providing incentives for good performance.[6] We consider a setting in which the principal also dismisses badly performing agents; in which having a job, however, does not mean that the agent gets a rent. Instead, the NCC worsens the agent's opportunities on the job market such that dismissals are still costly.

There is a multitude of articles in which the second dimension of payoffs is not future payments but pain, unfriendliness, or losing assets. It is pain in the coerced labor settings of Chwe (1990) and Acemoglu and Wolitzky (2011). Chwe (1990) provides a model in which the principal cannot only use monetary but also non-monetary transfers (by inflicting costly pain to the agent). The author finds that a principal might inflict pain on the agent if monetary transfers are limited due to wealth constraints and the reservation utility is relatively low. Acemoglu and Wolitzky (2011) modify the model of Chwe (1990). Besides some simplifications, the principal can reduce the agent's reservation utility by buying guns. Furthermore, the model is later extended from a partial market to a general equilibrium. Dur et al. (2019) use a similar framework to show that under limited liability a leader might, next to monetary incentives, sometimes use an unfriendly leadership style to impose non-monetary costs on the worker. Another approach is collateralized debt (e.g. Stiglitz and Weiss, 1981, Bernanke and Gertler, 1986, Chan and Thakor, 1987, Bester, 1987, and Boot et al., 1991). An agent, who is cash constrained, might pledge an asset in order to improve his access to a credit line. After a signal for low effort (default), the asset is transferred to the bank. This both incentivizes the agent and reduces the loss of the bank. Non-compete clauses in our model are similar to collaterals in lending agreements: The agent pledges his human capital. After a bad performance, the NCC is activated, and the agent is not allowed to sell his human capital to someone else.

One difference in these articles is the efficiency loss from using other payoff dimensions. For example, pledging a perfectly resellable asset is a perfect substitute to monetary payments. Thus, the friction from limited liability vanishes. In the other extreme, the principal inflicts costly pain on the agent (Chwe, 1990), causing inefficiency twice. Still, the existence of the second dimension of the payoff makes the principal better off. Our model is in-between these

[6] The idea is similar to the efficiency wages in Shapiro and Stiglitz (1984), which arise without verifiable signals in the general equilibrium.

extremes: An NCC is costly to the agent but not to the principal.

3 The Model

We use a moral hazard model with continuous effort, binary output, and limited liability. There is a risk-neutral principal P (*she*) who owns a project. The project can be either a success and pay off V or a failure and pay off nothing. P wants to hire a risk-neutral agent A (*he*) to work on the project for one period. The principal offers the agent a contract that consists of three items: a base wage w, that is paid unconditionally, a bonus wage b, that is paid conditionally on a success, and a non-compete clause (NCC). The wages are subject to a minimum wage that introduces limited liability. Our model consists of two parts: the effort provision stage and a continuation in which an NCC might come into play.

We now consider the effort provision stage in more detail. The agent chooses his effort $e \in [0, 1]$ at a strictly convex cost of $c(e)$, where $c(0) = 0$. Additionally to $c''(e) > 0$, we impose the standard Inada conditions that $c'(0) = 0$ and $\lim_{e\to 1} c'(e) = \infty$. Since existence of an additional choice variable, $\bar{v}$, complicates the problem, we need the further assumption that $\frac{c'''(e)}{c''(e)} > \frac{1}{1-e}$ $\forall e \in [0, 1)$ to get a concave objective function (see Lemma 7 in the Appendix)[7]. The effort level that A chooses is private information and, thus, creates a moral hazard problem. The chosen effort is the probability of the project being successful, that is a success payoff V accrues to the principal with probability e, Prob(success$|e$) $= e$. Successes are verifiable and serve as a signal for the agent's effort. In the case of a success, the agent gets the bonus wage b.

We now consider the continuation in more detail. After the project is completed, the agent's continuation payoff is determined. For simplicity, we do not model future periods. Instead, the discounted future life-time payoff depends on the project's outcome, the exogenous payoff of having a job (which we normalize to zero), and on a possible non-compete clause. If the project is a success, the agent is retained. Since there is no dismissal, an NCC does not make any difference. The agent keeps his job and the continuation payoff is zero. If the project ends in a failure, the agent is dismissed. His payoff on the job market depends on his non-compete clause. If the agent's contract does not entail an NCC, his options are not limited, he finds another job, and he gets the same continuation payoff as if he had been retained. If the agent has signed an NCC, he will be unemployed for a while before he is allowed to take a new job. The unemployment reduces his continuation payoff to $\bar{v} \leq 0$. For simplicity, we express the contract's NCC directly as $\bar{v}$. Concerning the principal, we assume that dismissing the agent has no effect on her continuation payoff. That is, hiring a replacement is costless.

Another interpretation is that the agent leaves the firm after any outcome of the project. His NCC, however, is invalidated if he was successful in the effort-provision stage. Thus, he is free to work and receive his outside option. If he was unsuccessful, the NCC is activated and

[7]Chwe (1990) and Acemoglu and Wolitzky (2011) use the same assumption in their models. In the proofs in the Appendix, we will state which assumptions on the cost function we need in the respective steps. The concavity assumption is simpler, intuitive, and implies all of them.

reduces his continuation payoff.

To sum up, a contract between the principal and the agent is defined by the tuple $(w, b, \bar{v})$. The minimum wage law demands that the agent is paid at least the minimum wage $\underline{w}$ for the effort-provision stage; that is, after a success $w + b \geq \underline{w}$, and after a failure $w \geq \underline{w}$. The level of the minimum wage is relative to the agent's outside option that we have normalized to zero.[8] Up to Section 5, we assume that the success V is large enough such that the principal makes a profit, and we ignore the extensive margin. $\bar{v} = 0$ means that the contract does not include a NCC. The lower $\bar{v}$, the longer the agent is unemployed after being dismissed. We refer to a lower $\bar{v}$ as a *more severe* NCC. If the agent rejects an offered contract, the game ends. He, then, finds another job which he keeps forever and gets a utility of zero. The timeline in Figure 1 summarizes the timing of the game.

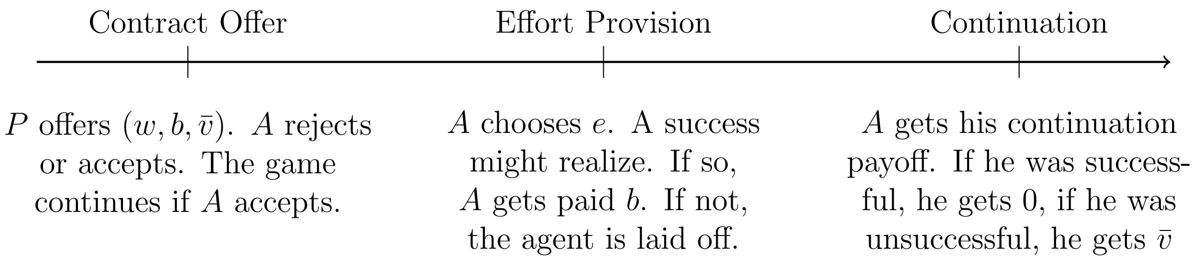

Figure 1: Timing of the Game.

The agent' expected utility is given by

$$\mathbb{E}U = w + e \cdot b + (1 - e) \cdot \bar{v} - c(e). \tag{1}$$

The principal's expected profit is given by

$$\pi = -w + e \cdot (V - b). \tag{2}$$

First-Best Welfare Analysis First, consider the benchmark without any frictions. A social planner maximizes the expected social welfare

$$W^{FB} = \max_{e \in [0,1],\ \bar{v} \leq 0} \ e \cdot V - c(e) + (1 - e) \cdot \bar{v}. \tag{3}$$

The first-order condition shows that in the social optimum there is no NCC because due to the Inada conditions, the effort will be interior. As a result, any NCC comes into action with positive probability and forces the agent's labor to lie idle. This is inefficient. This means that $\bar{v} = 0$ maximizes the social surplus.

[8]In other models with limited liability, it is often assumed that the agents are heterogeneous in their outside options. Instead, the minimum wage (or limited liability) condition is normalized to zero. In our model, on the contrary, we normalize the outside option to zero. This reflects our assumption that ability or human capital plays no role, thus, the agents are homogeneous in their outside options. We are interested in the effects of an increase in the minimum wage. Our modeling choice makes this more easily interpretable. To see the effect of agent's heterogeneity, keep in mind that a better outside option is equivalent to a lower minimum wage.

Given that $\bar{v} = 0$, the first-best effort equates the marginal benefit and the marginal cost, $V = c'(e^{FB})$. This is optimal due to the welfare function's concavity.

4 The Optimal Contract

In this section, we characterize the profit-maximizing contracts for different minimum wages. To bring out the effect of NCCs, firstly, we look at the benchmark in which NCCs are forbidden. We find that, depending on the level of the minimum wage, there are three cases. For low minimum wages, the socially efficient effort is implemented. For intermediate minimum wages, the equilibrium effort is decreasing in the minimum wage. In this range, there is no redistribution. For high minimum wages, the equilibrium effort is inefficiently low and constant. In this range, the minimum wage redistributes from the principal to the agent.

Then, we characterize the profit-maximizing contracts when the principal is allowed to use NCCs. In line with the empirical literature, we find that higher minimum wages lead to more severe NCCs. More specifically, the principal does not want to use NCCs up to some intermediate minimum wage. From thereon, he uses an NCC. The increased incentives from the NCC make the equilibrium effort increase in the minimum wage. Eventually, the principal induces more than first-best effort. For very high minimum wages, if the effort cost function satisfies some technical property, the principal stops paying bonus wages and uses only an NCC to provide incentives. We decompose the NCC's effect into two components. The "idleness effect" always reduces the social welfare. It is the agent's lower continuation payoff, that is the wastefulness of the NCC. The "incentive effect" results from the increasing equilibrium effort. Importantly, the NCC provides incentives while making the participation constraint tight. Intuitively, higher minimum wages grant the agent a higher rent that can be converted into incentives using an NCC. Thus, the incentive effect is also distributional: It redistributes from the agent to the principal.

In the next step, we introduce a bound on NCCs. We find that once the bound is reached, it will also be reached for all larger minimum wages. The bound makes it more difficult for the principal to extract the agent's rent: From some larger minimum wage on, the agent is left a rent. We conclude this section by a comparison of the benchmark and the optimal contracts with unbounded NCCs.

4.1 A World without Non-Compete-Clauses

Outlawing NCCs means that contracts do not affect the continuation payoff. The optimal contracts under limited liability with payoffs in one dimension (money) are well known (see for example Laffont and Martimort, 2002, and Schmitz, 2005). We now derive the optimal contracts in our setting when NCCs are not allowed, that is $\bar{v} \stackrel{!}{=} 0$. Given the compensation scheme (w, b), the agent chooses the effort $e = e^*$ to maximize his expected utility,

$$e^* = \arg\max_{e \in [0,1]} \; w + e \cdot b - c(e). \tag{IC}$$

This is the agent's incentive constraint. Due to the assumption that $c''(e) > 0$, the agent's expected utility is concave, and the equilibrium effort is unique. Due to the Inada conditions, the equilibrium effort is in the interior. For non-negative bonus wages, the effort choice is characterized by the first-order condition

$$b = c'(e^*). \tag{4}$$

The agent participates if his expected utility is weakly larger than the outside option, i.e.

$$w + e^* \cdot b - c(e^*) \geq 0. \tag{5}$$

The principal's problem without access to NCCs is

$$\begin{aligned}
&\max_{w,b} \quad -w + e^* \cdot (V - b) && (6)\\
&\text{subject to} \qquad e^* = \arg\max_{e \in [0,1]} \; w + e \cdot b - c(e) && \text{(IC)}\\
&\qquad\qquad\qquad w + e^* \cdot b - c(e^*) \geq 0 && \text{(PC)}\\
&\qquad\qquad\qquad w \geq \underline{w} \quad \text{and} \quad w + b \geq \underline{w} && \text{(MWC1) and (MWC2)}
\end{aligned}$$

The principal maximizes her expected profit subject to the incentive constraint, the participation constraint and the minimum wage conditions.

Proposition 1. *Let NCCs be outlawed. There exist threshold values in the minimum wage κ_1 and κ_3 such that P offers the following contract to A:*

(i) Let $\underline{w} \leq \kappa_1$. Then P chooses the compensation scheme $(w, b) = (\kappa_1, V)$.

(ii) Let $\kappa_1 < \underline{w} \leq \kappa_3$. Let $e^{**}(\underline{w})$ be implicitly defined by $c(e^{**}) - e^{**} \cdot c'(e^{**}) = \underline{w}$. Then P chooses the compensation scheme $(w, b) = (\underline{w}, c'(e^{**}))$.

(iii) Let $\kappa_3 < \underline{w}$. Let $e^{***}(\underline{w})$ be implicitly defined by $c'(e^{***}) + e^{***} \cdot c''(e^{***}) = V$. Then P chooses the compensation scheme $(w, b) = (\underline{w}, c'(e^{***}))$.

Proof. See Appendix. □ □

Proposition 1 derives the optimal contract that P offers to A if NCCs are prohibited. The three parts of Proposition 1 correspond to the three cases of binding and non-binding constraints; depending on the level of the minimum wage. Figure 2 illustrates which constraints are binding in the optimum depending on the size of the minimum wage.

κ_1 κ_3 $\underline{w}$

Case 1	Case 2	Case 3
PC binds	PC binds	PC is slack
$MWC1$ is slack	$MWC1$ binds	$MWC1$ binds
$MWC2$ is slack	$MWC2$ is slack	$MWC2$ is slack
e^{FB}	$e^{**}(\underline{w})$	e^{***}

Figure 2: The Combinations of Binding and Non-Binding Constraints that Characterize the Optimal Contract when NCCs are outlawed.

Case 1 This case depicts a situation in which the minimum wage is lower than the wages the principal would optimally choose without the minimum wage friction. Therefore, the optimal contract is the same as with unlimited liability. The principal leaves the success payoff to the agent and then adjusts the base wage as to extract the complete surplus from the agent. Therefore, this case is commonly referred to as "selling the firm".

Case 2 Increasing the minimum wage above κ_1 implies that the principal cannot extract all rents anymore if he induces first-best effort. For minimum wages between κ_1 and κ_3, it is optimal to induce less effort, making the social surplus smaller than first-best. More accurately, the principal induces exactly as much surplus as he can extract with the minimum wage by setting the base wage equal to the minimum wage. That is, the agent's participation constraint is kept binding. The minimum wage does not redistribute from the principal to the agent; it solely induces inefficiency. The induced effort $e^{**}(\underline{w})$ is decreasing in the minimum wage as is shown in the proof of Proposition 1 in the Appendix. Since the effort is decreasing in the minimum wage, so is the bonus wage.

Case 3 For minimum wages above κ_3 the principal changes her strategy: The effort is so low that it pays off to leave the agent a rent instead of decreasing incentives further. This implies that the participation constraint is slack. From κ_3 on, the implemented effort does not decrease anymore. The participation constraint is non-binding. The minimum wage, thus, does not enter the problem anymore. As a result, the equilibrium effort is independent of the minimum wage. The social surplus is, thus, constant. A minimum wage now becomes a tool of perfect redistribution: An increase of the minimum wage by one unit translates into an increase of the agent's rent by one unit.

4.2 A World with Non-Compete-Clauses

In this section, we assume that the principal can freely choose the level of the NCC. For simplicity, we assume that there is no bound on the severity of NCCs. This is unrealistic, but as the analysis below will show, this is more tractable and it is straightforward to extend the results to bounded NCCs as is shown below. As above, we solve for the subgame perfect Nash equilibrium by backward induction. To build intuition, we now look at the changes in two conditions due to NCCs: in the incentive compatibility and in the participation constraint.

Given the contract $(w, b, \bar{v})$, the agent chooses the effort level e^* that maximizes his expected utility,

$$e^* = \arg\max_{e \in [0,1]} \; w + e \cdot b + (1-e) \cdot \bar{v} - c(e). \tag{IC}$$

This is the agent's incentive constraint. The NCC becomes active if the agent has not been successful in the effort provision stage. If $b - \bar{v}$ is non-negative, the agent's effort choice is characterized by the first-order condition,

$$b - \bar{v} = c'(e^*). \tag{7}$$

The equilibrium effort is then unique because the marginal cost is strictly increasing. The incentive-compatibility constraint shows that the bonus wage and the NCC are perfect substitutes for giving incentives. Higher bonus wages or more severe non-compete clauses, which means that $\bar{v}$ is lower, provide incentives for the agent. P must now decide to what extent to provide incentives through an NCC and to what extent through a bonus wage. Therefore, NCCs have an *incentive effect* as they generate higher effort incentives.

A only participates if his participation constraint,

$$w + e^* \cdot b + (1-e^*) \cdot \bar{v} - c(e^*) \geq 0, \tag{PC}$$

is satisfied. The bonus wage and the severity of the NCC go into opposite directions in the participation constraint. A higher bonus wage makes the participation constraint slack. A more severe makes the participation constraint tight. This already hints at the use and the distributional effects of NCCs: Whenever the agent would get a rent without an NCC, the principal will add an NCC to the contract and convert the rent into more incentives. The participation constraint will always bind. In the participation constraint, the NCC enters twice. First, it enters indirectly via the equilibrium effort, which is again the incentive effect. Second, the idleness effect can be seen in the participation constraint: The NCC enters directly as $(1-e^*)\bar{v} \leq 0$. That is, the NCC reduces the agent's utility (and, thus, the social surplus) because labor force has to lie idle if there is a failure.

One could also decompose the effect of an NCC in a different way. Rearranging the agent's expected utility yields $(w+\bar{v})+e^*(b-\bar{v})-c(e^*)$. This means that the NCC effectively reduces the base and increases the bonus wage. Thus, the NCC allows the principal to avoid the minimum wage law (that determines the base wage as we show below) to some extent. The reduction in the base wage, however, does not benefit the principal because the NCC does not enter the principal's profit directly. Instead, the principal benefits from the increase in the bonus wage (that is essentially for free, as long as the participation constraint is slack, again because the NCC does not enter the principal's profit directly). In this reformulation, the incentive effect is hidden in the equilibrium effort and the idleness effect is the two $\bar{v}$ in the expected utility.

In Section 5, we will take a closer look at the welfare effects of the incentive and the idleness effect.

With the possibility of imposing an NCC, the principal's problem becomes

$$\max_{w,b,\bar{v}} \quad -w + e^* \cdot (V - b) \tag{8}$$

$$\text{subject to} \qquad e^* = \arg\max_{e\in[0,1]} \; w + e \cdot b + (1-e) \cdot \bar{v} - c(e) \tag{IC}$$

$$w + e^* \cdot b + (1-e^*) \cdot \bar{v} - c(e^*) \geq 0 \tag{PC}$$

$$\bar{v} \leq 0 \tag{NCC}$$

$$w \geq \underline{w} \qquad w + b \geq \underline{w} \qquad \text{(MWC1) and (MWC2)}$$

The principal maximizes her expected profit subject to the incentive-compatibility constraint, the participation constraint, the NCC feasibility constraint and the minimum wage constraints. The differences compared to the benchmark without NCCs are the agent's continuation payoffs in the case of a failure in the incentive and in the participation constraint. The additional inequality constraint (NCC) is a feasibility constraint. An NCC can only decrease an agent's continuation payoff. This constraint binds if the principal does not use an NCC. There is no lower bound on $\bar{v}$, but we consider this case further down.

Proposition 2 summarizes the optimal contracts if the principal can use non-compete clauses.

Proposition 2. *Let NCCs be allowed. There exist threshold values in the minimum wage κ_1 and κ_2. If $\lim_{e\to 1} \frac{c'''(e)}{[c''(e)]^2} \cdot V < 1$, there exist the threshold value in the minimum wage κ_4. P offers the following contract to A:*

(i) Let $\underline{w} < \kappa_1$. Then P offers the contract $(w, b, \bar{v}) = (\kappa_1, V, 0)$.

(ii) Let $\kappa_1 \leq \underline{w} \leq \kappa_2$. Let $e^{**}(\underline{w})$ be implicitly defined by $c(e^{**}) - e^{**} \cdot c'(e^{**}) = \underline{w}$. Then P offers the contract $(w, b, \bar{v}) = (\underline{w}, c'(e^{**}), 0)$.

(iii) Let $\kappa_2 < \underline{w} < \kappa_4$. Let $\hat{e}(\underline{w})$ be implicitly defined by $c(\hat{e})+(1-\hat{e})c'(\hat{e})+\hat{e}(1-\hat{e})c''(\hat{e}) = V+\underline{w}$. Then P offers the contract $(w, b, \bar{v}) = (\underline{w}, (1-\hat{e})c'(\hat{e}) + c(\hat{e}) - \underline{w}, c(\hat{e}) - \underline{w} - \hat{e}c'(\hat{e}))$.

(iv) Let $\kappa_4 \leq \underline{w}$. Let $\tilde{e}(\underline{w})$ be implicitly defined by $(1-\tilde{e}) \cdot c'(\tilde{e}) + c(\tilde{e}) = \underline{w}$. Then P offers the contract $(w, b, \bar{v}) = \left(\underline{w}, 0, -\frac{\underline{w}-c(\tilde{e})}{1-\tilde{e}}\right)$.

Proof. See Appendix. □

The four parts of Proposition 2 correspond the the four combinations of binding and non-binding constraints for different levels of the minimum wage. Figure 3 illustrates which constraints are binding in the optimum depending on the size of the minimum wage.

Another illustration of the optimal contract for a parameterized cost function is given in Figure 4.

We will now consider each combination in more detail.

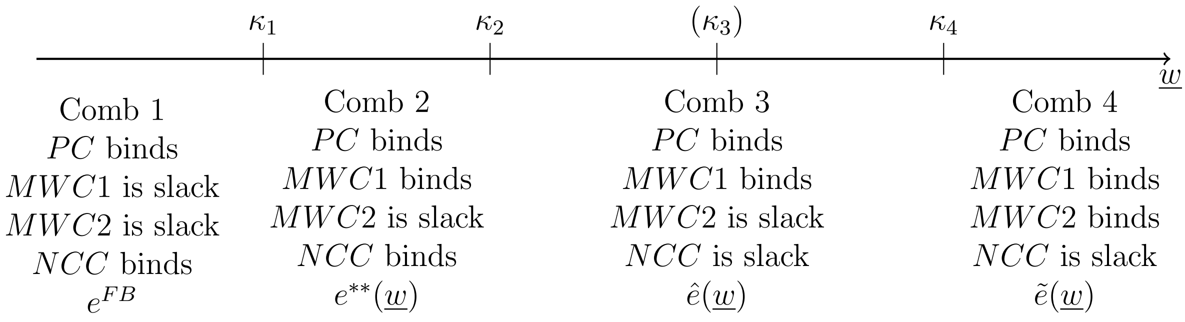

Figure 3: The Combinations of Binding and Non-Binding Constraints that Characterize the Optimal Contract when NCCs are Allowed. The Combinations are from Table 1. κ_3 does not have any meaning in the world with NCCs.

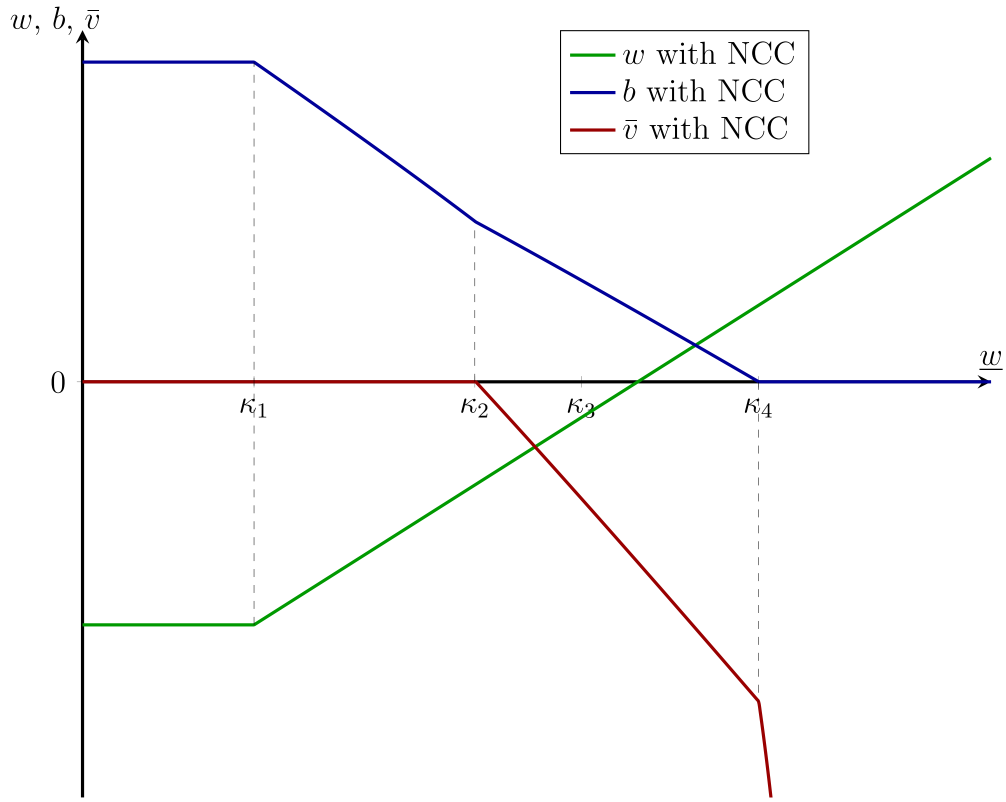

Figure 4: Illustration of the Optimal Contract for Different Minimum Wages for $c(e) = -\ln(1-e) - e$ and $V = 10$.

Combination 1 This Combination corresponds to Case 1 in the previous section because the principal does not use an NCC. Therefore, we get the same results with the additional outcome that the principal prefers not to use an NCC in this case. The reason is that since she can already generate first-best incentives and extract the whole surplus without an NCC, she cannot do any better by introducing an NCC.

Combination 2 In contrast to the benchmark, the principal can now use NCCs. NCCs reduce the agent's continuation payoff after a failure without directly benefiting the principal. We name this burning of social surplus the "idleness effect" because it is due to human capital that cannot be put to use. Instead, the principal benefits from the agent's trying to avoid the punishment by increasing his effort; we name this the "incentive effect" of NCCs. A larger equilibrium effort increases the principal's profit because she keeps some part of the success payoff, $b < V$, if the minimum wage is larger than κ_1. Because the agent's participation constraint always binds, the agent never gets a rent. Thus, the principal's profit is equal to the (utilitarian) social surplus, $(1 - e^*) \cdot \bar{v} + e^* \cdot V - c(e^*)$. For a fixed minimum wage, we measure the idleness and the incentive effect against the benchmark. We discuss these effects in more detail in Section 5. The idleness effect is $(1 - e^{\text{NCC}}) \cdot \bar{v}$, where e^{NCC} denotes the equilibrium effort with an NCC. This is the loss compared to the benchmark. The incentive effect is given by $\int_{e^{\text{No NCC}}}^{e^{NCC}} V - c'(x)\, dx$, where $e^{\text{No NCC}}$ is the equilibrium effort without an NCC. This is the change in the utilitarian efficiency of effort provision. If the effort with an NCC lies between the benchmark and the first-best effort, it is positive. It is maximized if the effort with an NCC is the first-best effort. If the effort with an NCC exceeds the first-best and gets larger, it eventually gets negative. As we will show below, the equilibrium effort will go to one when the minimum wage goes to infinity, which is inefficient. The principal will, nevertheless, do so because the alternative is to make lower profits and to leave the agent a rent. With an NCC, this rent can be used to provide incentives, increase the effort and make larger profits.

For minimum wages between κ_1 and κ_2, it is optimal for the principal not to use NCCs. The optimal contract is the same as in the benchmark in Case 2, although it stops at a lower minimum wage, $\kappa_2 < \kappa_3$. NCCs do not increase the profit because the equilibrium effort is still too large due to large bonus wages. These are needed to satisfy the participation constraint at these low minimum wages. As a result of the convex effort cost, an NCC increases the equilibrium effort little. Furthermore, the social surplus does not react much to an increased equilibrium effort because the marginal cost is close to the success payoff. Thus, the incentive effect is smaller than the idleness effect. Using NCCs would reduce the social surplus (profit, that is).

Combination 3 As the minimum wage increases, the bonus wage and, hence, the equilibrium effort decrease. At a lower equilibrium effort level, an NCC of the same severity increases the equilibrium effort by more. Furthermore, the social surplus reacts stronger to an increased equilibrium effort because the marginal cost is smaller. Thus, the incentive effect gets larger. At the same time, the idleness effect gets more negative as the probability of a failure increases. The change in the incentive effect, however, is stronger due to the convex effort cost. At a minimum wage of κ_2, both effects are equally strong. If the minimum wage is above κ_2, the incentive effect prevails and the principal uses NCCs. As we show in the proof of Proposition 3, the optimal NCC gets monotonically more severe in the minimum wage.

What does the use of an NCC mean for the bonus wage? Knowing that the participation constraint will bind, the principal's profit-maximization problem can be reformulated into one looking for the optimal bonus wage. The NCC follows from the participation constraint. As shown above, a larger bonus wage increases the agent's utility, while a more severe NCC decreases the agent's utility. To make the agent sign an NCC, the bonus wage has to be larger than in Case 2 of the benchmark. By increasing the bonus wage, the principal can provide "double incentives": Higher bonus wages allow for more severe NCCs, both of which provides incentives. In the benchmark, increasing the bonus wage would lead to the agent's getting a rent. With NCCs, the NCC can be made more severe to transfer this rent into incentives which means that the probability of a success increases; which increases the principal's profit. In the proof of Proposition 3, we also show that the optimal bonus wage nevertheless decreases in the minimum wage, although less so than in Case 2 of the benchmark.

Proposition 3 states that the increasing severity of the NCCs dominates the decreasing bonus wages. The total incentives and, hence, the equilibrium effort is increasing in the minimum wage in Combination 3. The non-monotonicity of effort in the minimum wage is a novel result: Standard models in the literature (with only bonus wages as an incentive tool as in the benchmark) find that the equilibrium effort is decreasing and finally stays constant in the minimum wage. The equilibrium effort is decreasing in Combination 2 because of our modelling assumption on how well an NCC transfers utility. Combination 2 exists because the idleness effect is larger than the incentive effect. If the idleness effect were always smaller than the incentive effect, there would be no range of minimum wages, in which the equilibrium effort decreases in the minimum wage. In terms of modelling assumptions, this means that the first unit of an NCC would have to increase the principal's profit to the same degree that it reduces the agent's utility. At the minimum wage κ_2, as soon as the principal uses NCCs, the equilibrium effort starts to increase in the minimum wage. The reason is that the bonus wage decreases slower than the NCC gets more severe in the minimum wage, the sum of incentives $(b - \bar{v})$ is increasing in the minimum wage, as we show in the proof.

Proposition 3 (Non-Monotonicity of Optimal Effort)**.** *Let $\underline{w} > \kappa_2$. Then the equilibrium effort level is increasing in the minimum wage.*

Proof. See Appendix. □

The equilibrium effort is not only increasing in the minimum wage, it also gets eventually inefficiently large, as Proposition 4 shows. The intuition behind this is that the principal only gets revenue by achieving successes. Large minimum wages allow her to use very severe NCCs with little bonus payments. The large minimum wages grant the agent a large rent that the principal converts into incentives with severe NCCs. If the minimum wage goes to infinity, the equilibrium effort goes to one. This also means that the incentive effect eventually becomes negative: Due to the Inada condition $\lim_{e \to 1} c'(e) = \infty$, for some minimum wage the inefficiency from too large efforts overcompensates the efficiency gain from a larger effort (remember that we measure the incentive effect starting from the benchmark equilibrium effort, which is inefficiently low). If both idleness and incentive effect are negative, then the social surplus with NCCs is lower than the social surplus in the benchmark.

The optimal bonus wages remain positive as long as the agent's equilibrium effort is not too convex in the incentives. Depending on the effort cost function, this might be for all minimum wages. In this case, the equilibrium effort reacts strongly enough to a bonus wage such that the principal wants to continue using double incentives. Otherwise, the optimal bonus wage becomes zero.

Combination 4 Combination 4 only exists if the agent's equilibrium effort is sufficiently convex in the incentives. If the effort choice is sufficiently convex, the principal stops to use a bonus wage for a high minimum wage as can be seen in Proposition 2. It is then better to provide incentives only through the NCC instead of using double incentives. In other words, for large minimum wages it can be optimal to only use non-monetary incentives. This might be one answer to the empirical question why we do not see that many bonus wages in reality.

If the equilibrium effort does not already get inefficiently large in Combination 3, it does so in Combination 4, as part (ii) of Proposition 4 shows. If Combination 4 exists, the shape of the cost function determines whether the equilibrium effort gets inefficiently large in Combination 3 or 4.[9] The intuition remains the same: Larger minimum wages increase the agent's utility. The principal makes the NCC more severe to extract this rent via an increased equilibrium effort.

As the principal wants to maximize the social surplus given the tools she has, she might prefer a strictly negative bonus wage when the equilibrium effort is inefficiently large. While a negative bonus wage would make the idleness effect larger due to an increased probability of failure, it would also make the incentive effect larger by reducing the inefficiency. If the latter effect prevails, negative bonus wages would be optimal. However, this would violate the minimum wage condition for the case of a success. To use a negative bonus wage, the base wage would have to be above the minimum wage. As we show in the proof of Proposition 2, increasing the base wage is too expensive for the principal. Therefore, the optimal bonus wage can never be negative. Instead, due to our technical assumptions, it remains at zero.

Proposition 4 (Inefficiently Large Optimal Effort)**.** *The equilibrium effort exceeds the first-best effort if the minimum wage is sufficiently large.*

(i) Let $c'(\hat{e}) \geq \hat{e}(1-\hat{e})c''(\hat{e})$. Then, P induces higher than first-best effort already in Combination 3 whenever $\underline{w} \geq c(\hat{e}) - \hat{e}c'(\hat{e}) + \hat{e}(1-\hat{e})c''(\hat{e})$.

(ii) Let $c'(\hat{e}) \leq \hat{e}(1-\hat{e})c''(\hat{e})$. Then, P induces higher than first-best effort in Combination 4 whenever $\underline{w} \geq V - \tilde{e}c'(\tilde{e}) - c(\tilde{e})$.

Proof. See Appendix. □

Figure 5 shows an overview of the optimal contract that P offers to A in the cases in which NCCs are prohibited and allowed. This Figure mainly summarizes the previous results.

[9] Consider the specific cost function that we used to draw the plots: $c(e) = -\ln(1-e) - e$. This cost function has the property that $c'(\hat{e}) = \hat{e}(1-\hat{e})c''(\hat{e})$. That is, part (i) and part (ii) of Proposition 4 coincide. This means that the principal stops to use a bonus wage exactly when she induces first-best effort. We use this property in Section 5.

NO NCC

$\kappa_1 = c(e^{FB}) - e^{FB}c'(e^{FB})$ $\qquad$ $\kappa_3 = c(e^{***}) - e^{***}c'(e^{***})$

Case 1

$$\begin{aligned} w &= \kappa_1 \\ b &= V \\ e &= e^{FB} \end{aligned}$$

Case 2

$$\begin{aligned} w &= \underline{w} \\ b &= c'(e^{**}) \\ e^{**}(\underline{w}) &: \ c(e^{**}) - e^{**} \cdot c'(e^{**}) = \underline{w} \end{aligned}$$

Case 3

$$\begin{aligned} w &= \underline{w} \\ b &= c'(e^{***}) \\ e^{***} &: \ c'(e^{***}) + e^{***} \cdot c''(e^{***}) = V \end{aligned}$$

$\underline{w}$

Combination 1

$$\begin{aligned} w &= \kappa_1 \\ b &= V \\ \bar{v} &= 0 \\ e &= e^{FB} \end{aligned}$$

Combination 2

$$\begin{aligned} w &= \underline{w} \\ b &= c'(e^{**}) \\ \bar{v} &= 0 \\ e^{**}(\underline{w}) &: \\ c(e^{**}) - e^{**} \cdot c'(e^{**}) &= \underline{w} \end{aligned}$$

Combination 3

$$\begin{aligned} w &= \underline{w} \\ b &= (1-\hat{e})c'(\hat{e}) + c(\hat{e}) - \underline{w} \\ \bar{v} &= c(\hat{e}) - \underline{w} - \hat{e}c'(\hat{e}) \\ \hat{e}(\underline{w}) &: \ c(\hat{e}) + (1-\hat{e})c'(\hat{e}) + \\ &\hat{e}(1-\hat{e})c''(\hat{e}) = V + \underline{w} \end{aligned}$$

Combination 4

$$\begin{aligned} w &= \underline{w} \\ b &= 0 \\ \bar{v} &= -\tfrac{\underline{w}-c(\tilde{e})}{1-\tilde{e}} \\ \tilde{e}(\underline{w}) &: \ \underline{w} + \tilde{e}c'(\tilde{e}) - c(\tilde{e}) = c'(\tilde{e}) \end{aligned}$$

$\kappa_1 = c(e^{FB}) - e^{FB}c'(e^{FB})$ $\qquad$ $\kappa_2 : \frac{\partial \pi}{\partial b}\Big|_{b=b_2^{**}} = 0$ $\qquad$ κ_4: $\frac{\partial \pi}{\partial b}\Big|_{b=0} = 0$ (if it exists)

NCC

Figure 5: The Profit-Maximizing Contracts for Different Minimum Wages Without (Above) and With (Below) NCCs.

4.3 Comparison with the Benchmark

In this section, we compare the optimal contracts with and without non-compete clauses. Furthermore, we look at the implemented equilibrium efforts and profits. This will prepare us for the welfare analysis in Section 5.

For illustration, we computed the optimal contracts for a specific cost function. We consider the following functional form of the cost function:

$$c(e) = -\ln(1-e) - e \tag{9}$$

This cost function satisfies all assumptions made above. Moreover, it satisfies the condition under which the bonus wage reaches zero at some minimum wage, and Combination 4 becomes optimal with non-compete clauses. For another specific cost function, Combination 4 and hence κ_4 might not exist. In that case, it does not pay off for the principal to let the bonus wage become 0. Consequently, we would stay in Combination 3 for high minimum wages.

We will now provide a series of plots that show the optimal contracts for different minimum wages. On the x-axis, there is the minimum wage. On the y-axis, there is the corresponding part of the contract, that is base wage, bonus wage, and NCC; respectively the equilibrium effort levels and the principal's profit. The values of the world without NCC are in red, the values of the world with NCC are in blue.

Figure 6 shows the optimal base wages.

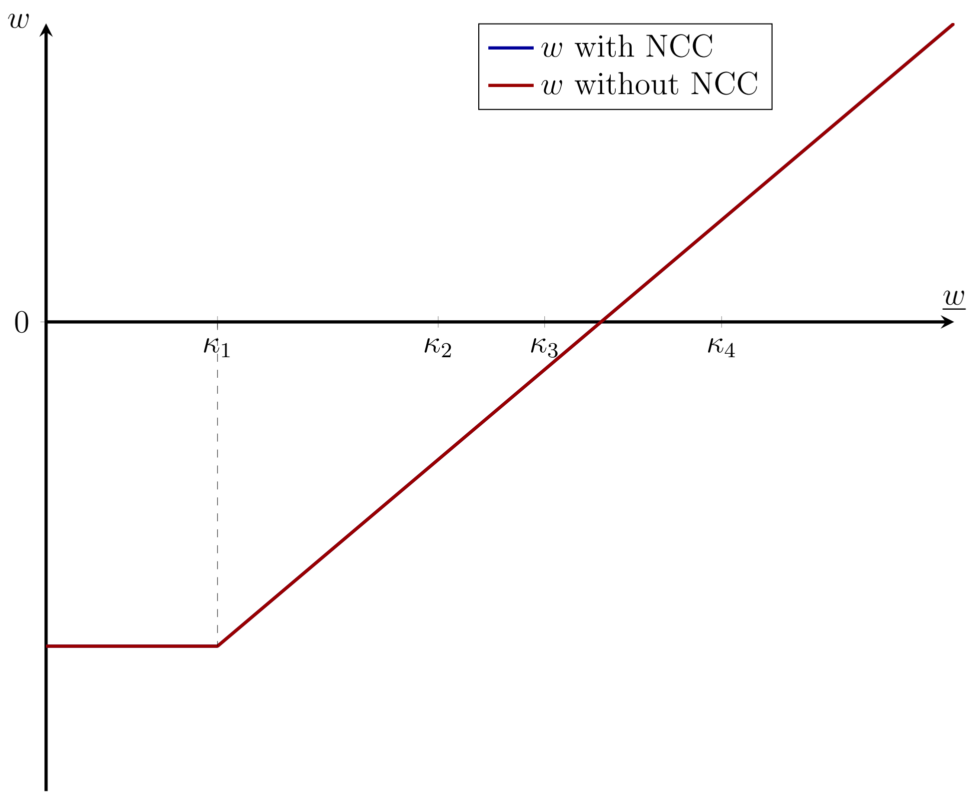

Figure 6: Comparison of the Base Wage.

The base wages are identical with or without an NCC. Hence, introducing an NCC has no effect on the optimal base wage. Left of κ_1, the principal can extract the first-best social surplus. The minimum wage condition for the base wage is slack. Right of κ_1, MWC1 binds.

Figure 7 shows the optimal bonus wages.

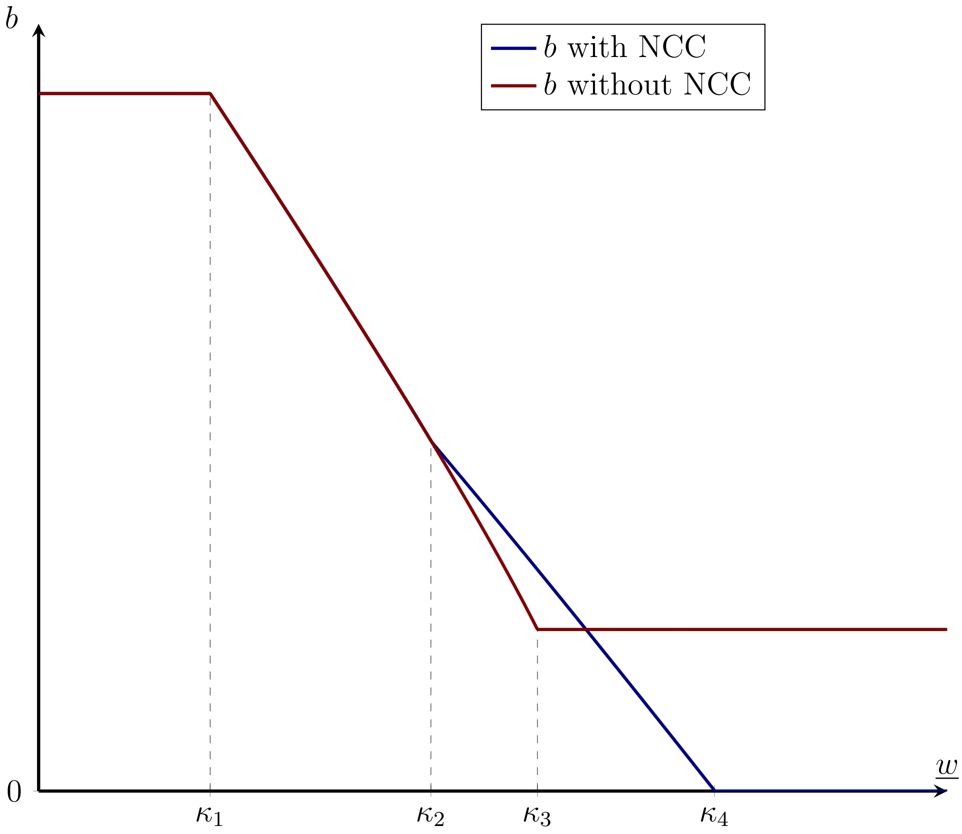

Figure 7: Comparison of the Bonus Wage.

Left of κ_1, P sells the firm to A which implies that the bonus wage is the success payoff, V. To the right of κ_1, the bonus wage gets compressed. The participation constraint is held binding and less than first-best effort is induced.

Between κ_1 and κ_2, the optimal contracts are the same whether the principal may use NCCs or not. The principal does not want to use an NCC. The reason is that the effort level is closer to first-best than at κ_2. Due to the convex cost structure, increasing the equilibrium effort using an NCC is too costly here, and thus the principal refrains from using an NCC.

At κ_2, there is a change: As Figure 8 shows, the principal starts using an NCC. A higher minimum wage leads to the optimal NCC's being more severe.

By setting the bonus wage higher than in the case without an NCC, the agent would get a rent. Instead of granting him the rent, the principal uses a more severe NCC to provide double incentives. This can also be seen in the equilibrium efforts in Figure 9. In other words, the agent pays for the additional effort. This is the redistributive effect of NCCs. For minimum wages between κ_2 and κ_3, allowing NCCs is a Pareto improvement. We will take a closer look on this in the welfare section. The agent gets no rent in either of the worlds. The principal's profit increases when using an NCC. In this sense, the inefficiency from limited liability is counteracted by an NCC.

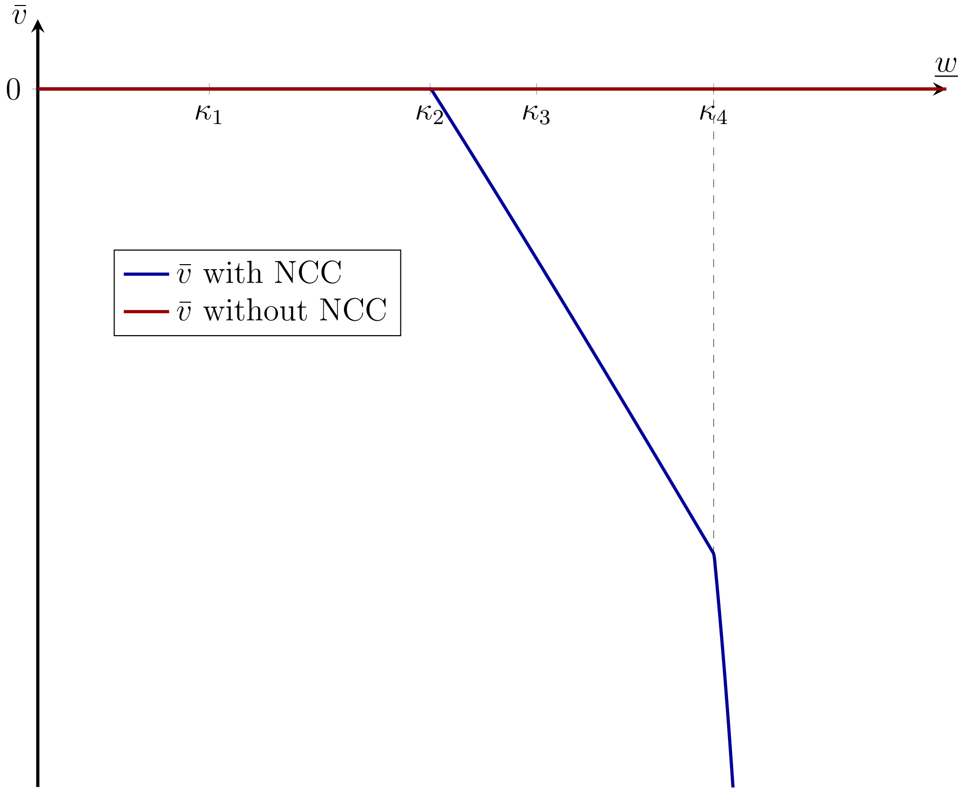

Figure 8: Comparison of the NCC.

At κ_3 in the benchmark, the principal reaches the effort level at which it gets better to leave the agent a rent than to reduce the equilibrium effort further. From then on, the bonus wage without NCCs stays constant. The principal with access to an NCC does not change his behavior at this minimum wage. The reason to use an NCC is more intuitive here. Even without a change in the bonus wages, the agent would get a rent. This rent can be expropriated using NCCs. Thus, the NCC can override the redistributive power of minimum wages.

κ_4 does not always exist, but it does for the chosen cost function. It is characterized by the principal's not wanting to use a bonus wage anymore. This is because the equilibrium effort reacts so little to more incentives that using a bonus wage to implement double incentives decreases the principal's profit. Coincidentally, for this cost function, at κ_4, the first-best effort is implemented. We will use this coincidence as an illustrative example in Section 5. Formally, part (i) and (ii) of Proposition 4 coincide because $c'(\hat{e}) = \hat{e}(1-\hat{e})c''(\hat{e})$ for all minimum wages. In this case, the principal stops using bonus wage when reaching first-best effort.

To the right of κ_4, the rent is so high that the optimal contract implements more than first-best effort. This is an additional social cost of an NCC. If he could, the principal would choose a negative bonus wage. The binding minimum wage condition, however, prevents this.

Figure 9 shows that access to an NCC increases the efforts that the principal optimally induces. At κ_2, using an NCC gets profitable. From then on, the agent is provided with a rent that, however, is converted into incentives. Therefore, the equilibrium effort increases again in the minimum wage if there are NCCs. Without NCCs, the equilibrium effort further decreases. At κ_3, the principal without NCCs reaches the effort at which it gets better to grant the agent a rent. Without NCCs, nothing interesting happens anymore. The optimal contract stays constant and so does the equilibrium effort. With NCCs, at κ_4, the principal stops using a bonus wage. Note that this threshold may not exist for some cost functions.

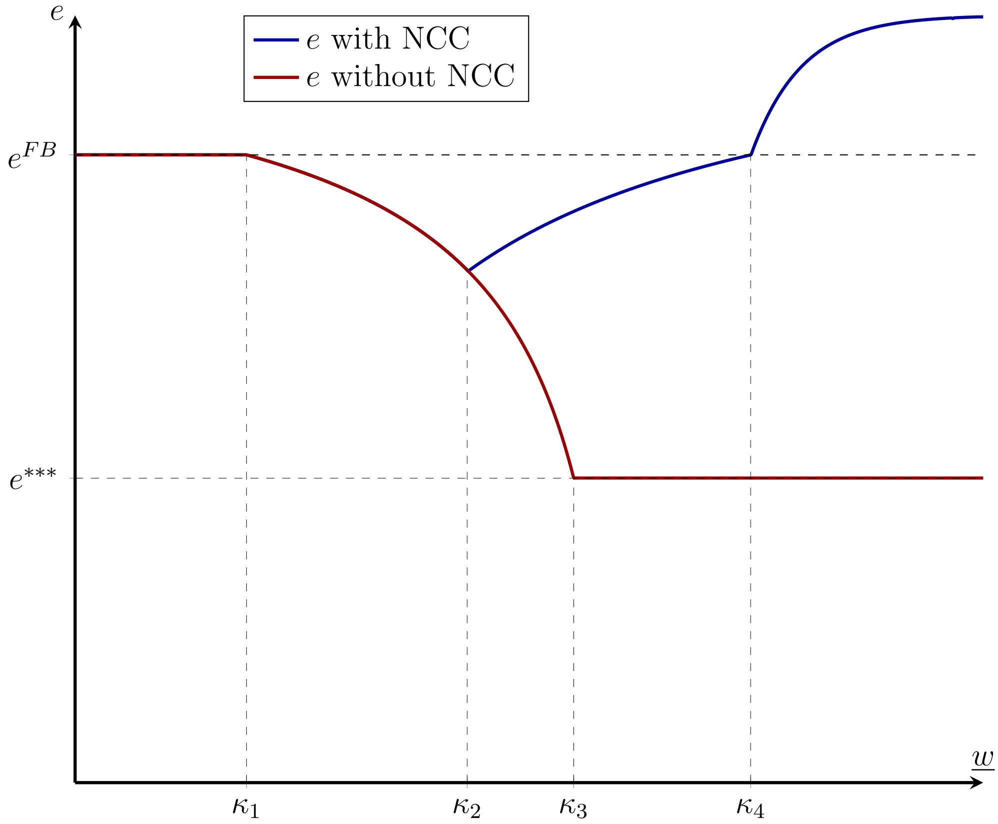

Figure 9: Comparison of the Equilibrium Efforts.

Figure 10 shows the principal's profits. To left of κ_1, the first-best social surplus can be extracted. Up to κ_2, the profits are the same with or without an NCC because the principal does not want to use an NCC. From κ_2 on, the principal with access to an NCC makes higher profits. Note that since the agent never gets a rent in the world with NCCs, the principal's profit is also the social surplus. The social surplus is decreasing between κ_2 and κ_4 although the equilibrium effort is increasing. The negative idleness effect from a more severe NCC outweighs the positive incentive effect from more effort. With an NCC, the slope gets more negative when the equilibrium effort rises above the first-best level. As soon as the effort reaches the first-best level, the positive incentive effect gets smaller again.

Without NCCs, the agent starts getting a rent at κ_3. Until there, the principal's profit is the social surplus. From κ_3 on, the social surplus is constant. Increasing the minimum wage further is frictionless redistribution from the principal to the agent. At some minimum wage, however, the principal does not make profits anymore and would not offer any contract to the agent. This is the extensive margin that we have ignored so far.

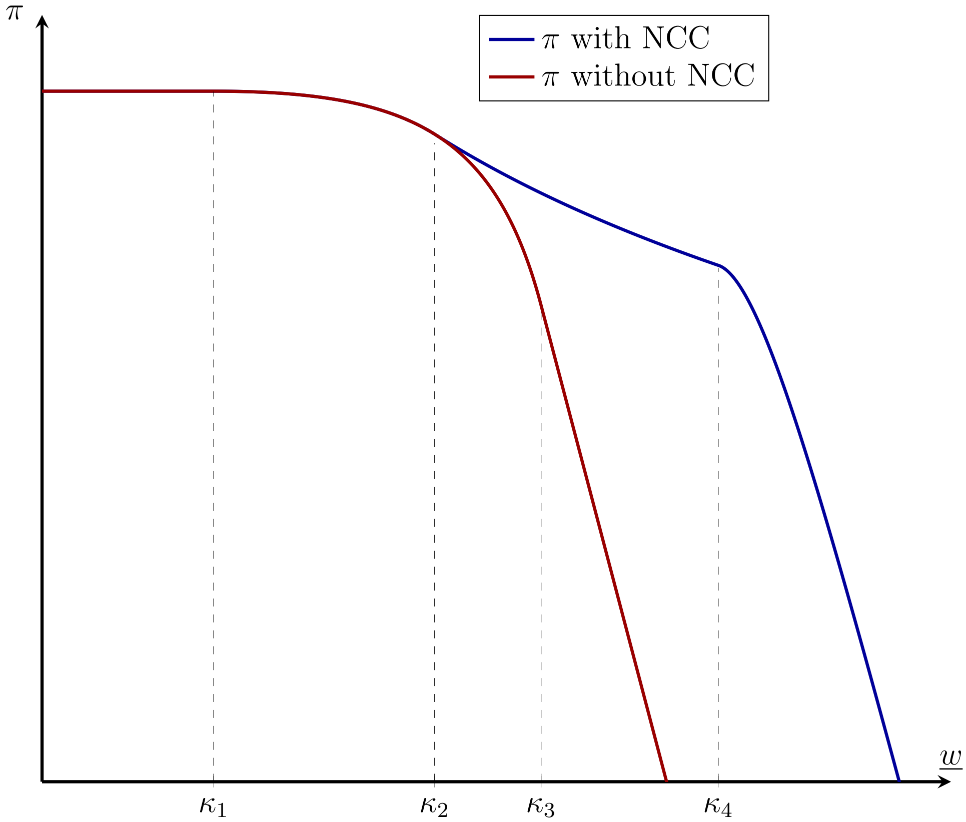

Figure 10: Comparison of Expected Profits.

4.4 Bounded Non-Compete Clauses

Having characterized the optimal contracts with unbounded non-compete clauses, it is straightforward to extend to bounded non-compete clauses. This additional constraint takes the form $\bar{v} \geq \underline{\bar{v}}$, where $\underline{\bar{v}}$ is the most severe enforceable NCC. Since the optimal NCC is getting more severe in the minimum wage, we can express the results in terms of minimum wages. If the minimum wage is small enough such that the optimal NCC is less severe than the bound, this NCC remains optimal. The bound introduces one more constraint to the maximization problem, however, it is non-binding. If the minimum wage is so large that the optimal NCC would be more severe than the bound, then the optimal NCC is the bound. The optimal bonus wage with a bound is initially lower than the optimal bonus wage without a bound, and decreases faster. Eventually it reaches a constant level that might be positive and, thus, larger than the optimal bonus wage without a bound.

We formalize these findings as Proposition 5.

Proposition 5 (Bounded Non-Compete Clauses)**.** *Let $\underline{\bar{v}} < 0$ be a lower bound on the NCC. If, without a bound on NCCs, the optimal NCC is $\bar{v} \geq \underline{\bar{v}}$, then the optimal contract remains the same with a bound on NCCs. If, without a bound on NCCs, the optimal NCC is $\bar{v} < \underline{\bar{v}}$, then the optimal contract with a bound on NCCs has $\bar{v} = \underline{\bar{v}}$. If the optimal bonus wage is positive, when the bound on the NCC starts binding, the bonus wage decreases more steeply than without a bound. At some larger minimum wage, the optimal bonus wage becomes constant at a non-negative level. If the optimal bonus wage is zero, when the bound on the NCC starts binding,* *the bonus wage remains at zero for all larger minimum wages.*

Proof. See Appendix. □

The intuition behind the proof is about double incentives. Remember that the principal always profits from making the NCC more severe as long as this does not violate either the participation constraint or the bound on NCCs.

Double incentives or the lack of it play a central role in the determination of the optimal bonus wage if NCCs are bounded. Consider an increase in the bonus wage. As long as the NCC has not yet reached the bound, a higher bonus wage allows a more severe NCC while keeping the participation constraint satisfied. These are the double incentives. At the moment, in which the NCC hits the bound, a higher bonus wage does not allow for a more severe NCC anymore as this would violate the bound. This also means that there are no double incentives anymore.

As has been shown in the proof of Proposition 2, the NCC is getting more severe in the minimum wage. When the NCC hits the bound, it stays equal to the bound for larger minimum wages. Assume that the NCC hits the bound in Combination 3. When the NCC is equal to the bound, a change in the bonus wage due to higher minimum wages does not trigger a change in the NCC anymore. Hence, there are no double incentives. This means that the bonus wage will decrease more steeply in the minimum wage. The participation constraint will then be kept binding. In fact, this range corresponds to Case 2 of the benchmark because the bonus wage is the only instrument to induce incentives. Consequently, effort will also decrease for higher minimum wages. At some point, however, the bonus wage will become constant. Either the bonus wage becomes constant because it reaches 0 and $MWC2$ becomes binding or P does not want to decrease the effort level further and keeps the bonus wage constant at a positive level. In both cases, the agent starts to receive a rent from this minimum wage onward. This corresponds to Case 3 of the benchmark.

Assume that the NCC hits the bound in Combination 4. The principal keeps the bonus wage at 0 for higher minimum wages and only provides incentives through the bounded NCC. The agent also starts to receive a rent.

For illustration, Figure 11 shows the optimal contract with bounded NCCs for a specific effort cost function and a specific bound. In the depicted case, the optimal constant bonus wage is positive. The optimal contract is the same as without a bound up to a minimum wage slightly above κ_2. Then, the bound on the NCC starts to bind and the optimal bonus wage has a kink. Somewhere to the right of κ_3, the participation constraint gets slack and the optimal bonus wage gets constant. If the bound on the NCC were further away from zero, it might also be that the optimal constant bonus wage is zero.

5 Welfare Analysis

Having characterized the profit-maximizing contracts, we can now look at the welfare effects of NCCs. To do so, we use the notions of (utilitarian) efficiency and of Pareto efficiency. The

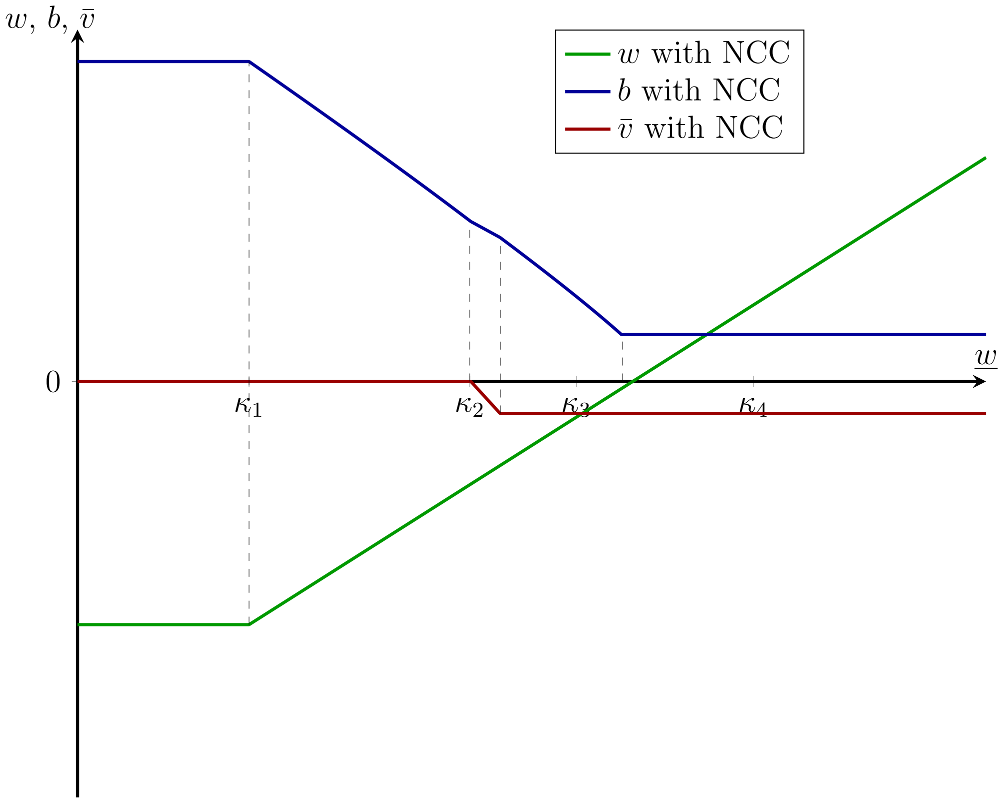

Figure 11: Illustration of the Optimal Contract for Different Minimum Wages for $c(e) = -\ln(1-e) - e$, $V = 10$ and a Bound on the NCC of $\underline{v} = -1$.

former is the sum of the agent's rent and the principal's profit; the social surplus. We also use Pareto efficiency because it puts slightly more weight on the distribution of the social surplus. It makes sense to use Pareto efficiency: If the policymaker were only interested in utilitarian efficiency, she would not introduce a minimum wage. Still, Pareto efficiency is relatively uncontroversial as it remains agnostic about how the policymaker aggregates profits and rents in her welfare measure. An equilibrium outcome strictly Pareto dominates another if both rent and profit are strictly larger; it weakly Pareto dominates another if either rent or profit are strictly larger and the other one is equal. An equilibrium outcome that (weakly) Pareto dominates another is also (strictly) more efficient. We do not use the agent's expected remuneration because it ignores both the effort level and the NCC.

We start by recapping the welfare effects of non-compete clauses: the incentive and the idleness effect. Next, we compare the outcomes of the benchmark with the outcomes of the setting with unbounded NCCs for fixed minimum wages. As we already know, the agent's participation constraint is always binding with unbounded NCCs. The agent, thus, never gets a rent; with unbounded NCCs, minimum wages do not redistribute. We also find that NCCs mitigate the inefficiency from minimum wages. Then, we will consider bounded NCCs. In this case, the welfare implications are not as clear as without a bound. Particularly, the results depend on the effort cost function. Nevertheless, we can demonstrate that a correctly chosen combination of bounds on NCCs and minimum wages might Pareto dominate minimum wages without NCCs.

NCCs affect the utilitarian efficiency through two channels. The incentive effect works indirectly through an NCC's increasing the equilibrium effort. Its effect on the utilitarian efficiency is positive for minimum wages, slightly above κ_2, and, thus, small NCCs. Without an NCC, the equilibrium effort is inefficiently low. With an NCC, the equilibrium effort is closer to the first-best. As long as the equilibrium effort with an NCC lies below the first-best, the incentive effect is certainly positive. For large minimum wages, however, the equilibrium effort increases above the first-best. This leads to an inefficiency that eats up the gained efficiency. From some minimum wage on, the equilibrium effort is so large such that the incentive effect gets negative. The incentive effect can be expressed as

$$\int_{e^{\text{No NCC}}}^{e^{NCC}} V - c'(x)\ dx, \tag{10}$$

where $e^{\text{No NCC}}$ denotes the equilibrium effort without an NCC and e^{NCC} denotes the equilibrium effort with an NCC.

The idleness effect directly affects the utilitarian inefficiency by reducing the agent's payoff. In the case of a failure, the NCC gets activated and burns $\bar{v}$ of the agent's rent. Thus, this effect is unambiguously negative. The effect can be expressed as

$$(1 - e^{\text{NCC}}) \cdot \bar{v}, \tag{11}$$

where e^{NCC} again denotes the equilibrium effort with an NCC. The total effect on the utilitarian efficiency is the sum of the two effects.

Non-compete clauses redistribute utility from the agent to the principal via the incentive effect: The agent bears the increased effort cost. At the same time, the incentive effect changes the social surplus, as explained above. Furthermore, the agent has to bear the idleness effect, which does not redistribute but only wastes social surplus.

For minimum wages $\underline{w} \leq \kappa_2$, the optimal contract does not have an NCC. If the minimum wage is between between κ_2 and κ_3, NCCs leads to a weak Pareto improvement: With NCCs, the principal is better off, while the agent is no worse off. The principal induces more effort but keeps less of a success payoff. Nevertheless, her profit is strictly larger than without an NCC.[10] The agent is not worse off because even without NCCs, minimum wages between κ_2 and κ_3 do not redistribute; in both settings, he does not get a rent. With an NCC, the agent gets paid higher wages, however, this is exactly eaten up by the increased effort cost and the worse outcome after a failure. The benchmark outcome without NCCs is weakly Pareto dominated by the outcome with unbounded NCCs, which is in turn weakly Pareto dominated by the outcome without a minimum wage. NCCs mitigate the inefficiency that accompanies minimum wages.

Above κ_3, the situations with and without NCCs cannot be compared by Pareto efficiency anymore. On the one hand, with NCCs, the principal is still better off. On the other hand, the agent is now strictly worse off: Without an NCC, he gets a rent. With an NCC, he does not.

[10] In the Appendix, this is shown more formally. It is due to the profit's concavity in the bonus wage after pinning down the optimal non-compete clause.

The effect of the NCC on the utilitarian efficiency is ambiguous.

Whether the incentive effect is positive and outweighs the idleness effect depends on the functional form of the effort cost. For the functional form that we have used above to draw the plots, NCCs increase the utilitarian efficiency in the whole range from κ_3 to κ_4 and up to some minimum wage beyond κ_4. The fact that the utilitarian efficiency decreases due to NCCs for some high minimum wages is independent of the functional form because it follows from the Inada conditions and the fact that the equilibrium effort goes to 1 if the minimum wage goes to infinity. The principal provides inefficiently much incentives because this is the only way to extract the agent's rent.

Extensive margin There might, however, be a weak Pareto improvement and, thus, efficiency gain on the extensive margin. Above, we have only considered the intensive margin, assuming that the principal always wants to offer a contract to the agent. That is, the success is large enough such that the profits are larger than the principal's outside option with or without NCCs for all minimum wages. For this paragraph, we drop this simplifying assumption. As a result, there are minimum wages for the benchmark and for the case with NCCs at which the principal stops offering a contract to the agent. With NCCs, the principal's profit is strictly larger and so is the minimum wage at which the principal stops offering a contract to the agent. If the principal does not offer a contract, the agent gets his outside utility, 0. For all minimum wages for which the principal does not participate in the benchmark but does participate with NCCs, there is a weak Pareto improvement. The agent is no worse off, whereas the principal makes profits exceeding her outside option.

Bounded non-compete clauses With bounded NCCs, the welfare effects get more nuanced. The bound on NCCs brings back redistribution from the principal to the agent: The agent gets a rent as soon as the optimal bonus wage gets constant in the minimum wage. Intuitively, this bonus wage allows to use the most severe NCC. Increasing the minimum wage further increases the agent's utility, allowing for a more severe NCC without violating the participation constraint. The bound on the NCC, however, forbids the principal to do this. Thus, the participation constraint gets slack.

In general, it is not possible to analytically examine welfare effects of bounds on NCCs. Therefore, we reconsider the functional form of the cost function that we have plotted above. We now provide an example for the bound on NCCs that makes the welfare analysis particularly simple. This relies on the peculiar fact that the principal coincidentally induces first-best effort at κ_4; that is without a bonus wage, using only an NCC. This is due to the NCC's being $-V$ at κ_4. We choose this NCC as the bound, $\bar{\underline{v}} = -V$.

Consider first the utilitarian efficiency for $\underline{w} \geq \kappa_4$. One aspect makes this example particularly simple (while still illustrative): Because the optimal bonus wage is already at 0, the bound on NCCs will not change the optimal bonus wage as has been shown in Proposition 5.[11]

[11]This also implies that the agent starts getting a rent at a minimum wage of κ_4. That's why we look at these minimum wages and this example.

This implies that the equilibrium effort remains constant at the first-best level for all larger minimum wages. As the effort is at the first-best level, the incentive effect is maximized and constant in the minimum wage. The inefficiency of the minimum wage without NCCs due to reduced effort is canceled out. This leaves only the inefficiency from the idleness effect, which is also constant in the minimum wage because neither the equilibrium effort nor the NCC change in the minimum wage. To sum up, with the logarithmic cost function and the bound $\underline{\bar{v}} = -V$ for $\underline{w} \geq \kappa_4$, the utilitarian efficiency is $(1 - e^{FB}) \cdot V$ below the first-best utilitarian efficiency (that is achieved with $\underline{w} \leq \kappa_1$).

Consider now the utilitarian efficiency in the benchmark for minimum wages above κ_3; that is minimum wages for which there is redistribution from the principal to the agent. There is one source of inefficiency: too low effort. The equilibrium effort is the same for all minimum wages above κ_3, thus, the utilitarian efficiency is also the same. The utilitarian efficiency is $\int_{e^{***}}^{e^{FB}} V - c'(x)\, dx$ compared to the first-best.

We can now compare the utilitarian efficiency with bounded NCCs for $\underline{w} \geq \kappa_4$ with the utilitarian efficiency in the benchmark for $\underline{w} \geq \kappa_3$. In both cases, the utilitarian efficiency is constant. With a bounded NCC, the equilibrium effort is efficient. The idleness effect reduces the utilitarian efficiency by $(1 - e^{FB}) \cdot V$ compared to the first-best. In the benchmark, the equilibrium effort is inefficiently little. The utilitarian efficiency is reduced by $\int_{e^{***}}^{e^{FB}} V - c'(x)\, dx$ compared to the first-best. When V is large enough,[12] the outcome with a bounded NCC is more efficient! This is illustrated by Figure 12. On the x-axis, the minimum wage is plotted. On the y-axis, the principal's expected profit is plotted. For the benchmark up to κ_3 and for the case of bounded NCCs up to κ_4, the expected profit is the same as the utilitarian efficiency. Beyond the respective thresholds, the utilitarian efficiency is constant; these levels are marked by dotted horizontal lines. The principal's profit gets a slope of -1, while the agent's rent is the remainder between the utilitarian efficiency and the principal's profit. For comparison, we added the case of unbounded NCCs. In this case, the agent never gets a rent and the principal's profit always is the utilitarian efficiency. Due to the increasing marginal effort cost, utilitarian efficiency decreases fast when the equilibrium effort gets close to one. This is driven by the incentive effect.

Consider now the Pareto efficiency. Remember that in the benchmark, minimum wages above κ_3 redistribute perfectly. The agent's rent is $\underline{w} - \kappa_3$. With bounded NCCs, the above-mentioned aspect makes it particularly simple to determine the agent's rent in this example. Because the bonus wage stays at zero above κ_4, the minimum wage redistributes perfectly. An increase in the minimum wage by one unit increases the agent's rent by one unit; starting with a rent of 0 at κ_4. The agent's rent is $\underline{w} - \kappa_4$.

Therefore, whenever the outcome with a bounded NCC at κ_4 is more utilitarian efficient than the outcome without NCCs at κ_3, NCCs allow strict Pareto improvements. Intuitively,

[12] When $V \to 0$, both social losses go to zero. The loss with a bounded NCC is coincidentally equal to e^{FB}; it is concave in V. The loss in the benchmark is a more complicated expression, $\sqrt{1+V} - 1 - \frac{1}{2} \cdot \ln(1+V)$. It is the area between V and the marginal cost in the range from e^{***} to e^{FB}; it is convex in V. When increasing V, the loss with a bounded NCC increases initially faster than the loss in the benchmark. For larger V, the loss in the benchmark increases faster. Numerically, they intersect at $V \approx 7.873$.

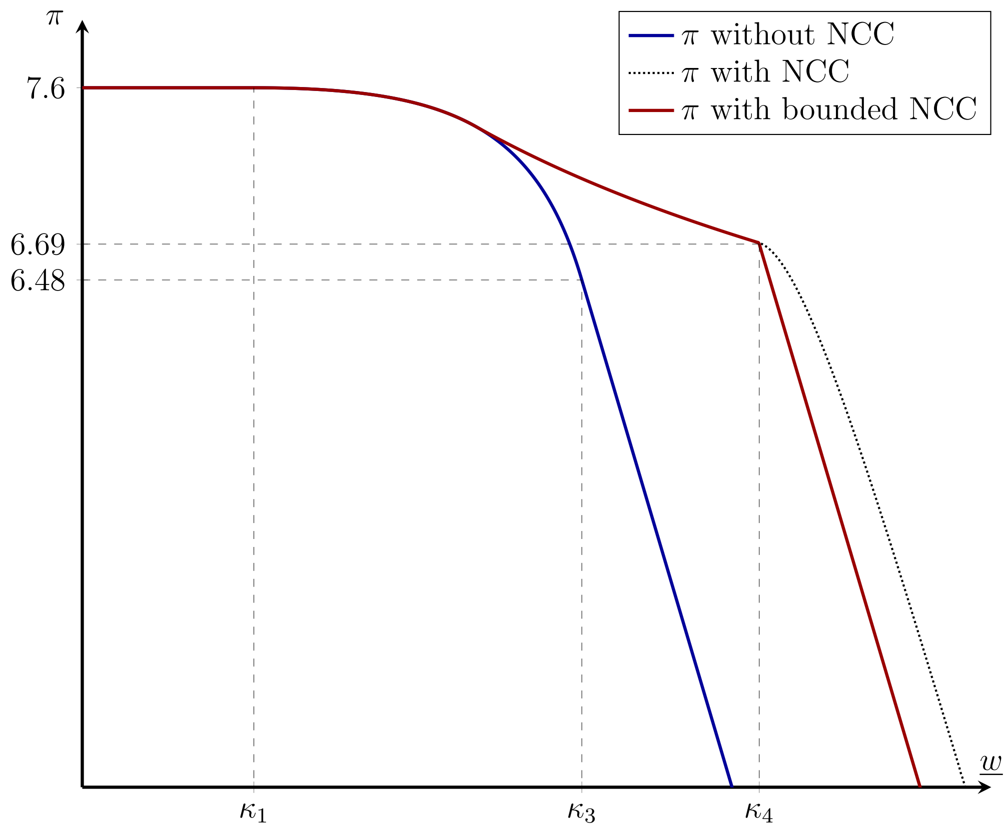

Figure 12: Bounded Non-Compete Clauses Potentially Allow for Strict Pareto Improvements. We choose $c(e) = -\ln(1-e) - e$, $V = 10$ and $\underline{v} = -10$.

the pie that can be redistributed is constant and the minimum wage determines the distribution. Whenever the pie is larger, both principal and agent can be made strictly better off. Importantly, to give the agent the same rent as in the benchmark, minimum wages have to be increased in this example by $\kappa_4 - \kappa_3$.

It is difficult to make general statements on the welfare effects of bounded NCCs. There is an interplay between the effort cost function and the bound. Furthermore, equilibrium effort levels that change in the minimum wage further complicate everything. Our example, however, shows that NCCs might be a tool to mitigate the inefficiency that is associated with redistribution in agency problems. The inefficiently low effort is being replaced by barred labor force and another effort level that might be closer to first-best. The level of the minimum wage has to be chosen taking into account the interactions of non-compete clauses with the minimum wage. The example has shown, that larger minimum wages are needed for redistribution with bounded non-compete clauses than without non-compete clauses. There remains the caveat that the minimum wage's redistributory effect strongly depends on the effort cost function that is unknown in the real world. Heterogeneity in agents could make it impossible to find a minimum wage that suits all. Due to all these uncertainties, an ambiguity averse (in a maxmin preferences sense) policymaker might prefer to ban NCCs.

6 Discussion

In this section, we argue that some key aspects of our model are not as restrictive as they seem at first sight. These entail our partial market setting, the continuation payoff and the enforcability of NCCs in the real world. Furthermore, we stress that a minimum wage might not be the only friction that prompts employers to use NCCs. In general, employers want to use NCCs as soon as employee's receive some kind of rent that can be expropriated via NCCs. Another generalization of our work lies in the agent's different kinds of payoffs: In our model, there is a failure-contingent punishment. Minimum wages might also affect, for example, non-monetary unconditional payments. Finally, we derive empirical predictions from our model that researchers could take to the data.

Partial Market Setting Our principal agent model is a partial market model as is standard in the literature on moral hazard problems. There are only one principal and one agent, and the principal owns all the bargaining power. We assume that the agent has an exogenous outside option, $\bar{u}$, which we normalize to 0 before signing any contract.

A crucial aspect of our model is that an increase in the minimum wage does not change the outside option. However, economic theory asserts that, in a general equilibrium framework, there are at least two effects of higher minimum wages. First, a higher minimum wage means that the firms that pay minimum wages increase them. Therefore, a worker who is employed at a certain firm has a higher outside option at other firms. This implies that the outside option should increase as the minimum wage increases. Second, a higher minimum wage leads to an oversupply of workers. This implies that the outside option should decrease as the minimum wage increases. However, our results continue to hold as long as an increase in the minimum wage does not increase the outside option by too much, by which we mean that the outside option does not increase by the same amount. This is no unrealistic assumption.

One starting point to extend our results to a general equilibrium setting is Acemoglu and Wolitzky (2011) who use a similar principal-agent framework. They use a reduced form matching model to extend their partial market to a general equilibrium and to corroborate their findings from the partial market model.

Continuation Payoff We simplify our analysis by considering a one-period setting with an additional continuation. The advantage is that we can exclusively focus on the effects of NCCs. Hence, we can identify the incentive and idleness effects.

One line of critique involves having a more dynamic model by incorporating a second period with an explicit incentive problem to see how the NCCs interact with future incentives. We argue that it is not crucial not to model future periods with additional incentive problems. If there was a second period with an explicit effort provision problem without NCCs (because the world ends after the second period), our results would not substantially change. The reason is as follows. The agent would get a rent in the second period if the minimum wage is high enough ($\underline{w} > \kappa_3$). With NCCs, the agent only gets access to this rent after a first-period success and hence after not being laid off after the first period. The principal would then choose a more severe NCC to extract the future rent. The results, however, change in the benchmark in which the principal cannot use an NCC. In the case of an incentivized second period, the principal can make use of the second period rents to induce incentives in the first period as has been shown by Kräkel and Schöttner (2010) and Schmitz (2005). Hence, the principal would already be able to induce higher incentives due to future rents without an NCC. This implies that effort is already closer to the first-best level and hence, an NCC would relatively fast induce an inefficiently large effort level. The qualitative results, however, stay the same which is why our setting is not too restrictive. To make the exposition easier, we choose not to model an explicit second period but rather a continuation in which the agent gets his outside option.

Enforcability of NCCs Another line of critique is the enforceability of an NCC that is used in this way. Implicitly threatening the agent to work more hardly qualifies as a "legitimate business interest." Would a court take someone's means of making a living to honor a NCC? This critique can be defended in three ways.

Firstly, as discussed in Section 2, the states differ widely in their enforcing of NCCs. Most states enforce NCCs after employees were dismissed for whatever or even without any reasons. Some states, above all Florida, instruct courts to disregard harm done to former employees by enforcing NCCs. As a result, there almost always is some probability that a court rules in favor of the employer. If NCCs have terrible consequences, they do not have to be enforced after each dismissal; $\bar{v}$ is the expected loss due to NCCs.

Secondly, often, courts do not even come into play. Dismissed employees might receive a threatening letter from their past employer's attorney, reminding them of their NCC. If this is not enough, past employers get courts to issue preliminary injunctions or inform new employers about the NCC which is often sufficient to break up the employee's new employment. Many employees shy away from costly, lengthy, and uncertain litigation and just sit their NCC out. Furthermore, the litigation we see might not be all there is. Some NCCs specify that they will not be trialed by courts, but by mandatory arbitration. This is legal and intended to reduce the burden on the courts. As a downside, the outcomes of arbitration are usually confidential. It is questionable whether arbitrators are as lenient towards employees as some courts are.

Thirdly, NCCs are equally prevalent in California and North Dakota, two states that do not enforce NCCs at all (Starr et al., 2019a). Moreover, these unenforceable NCCs have been shown to still affect the employees' behavior (Starr et al., 2019b). In fact, Californian NCCs are still effective because many employees shy away from litigation (Colvin and Shierholz, 2019, p. 5-6).

Employees might have wrong beliefs about the enforceability of their NCCs for several reasons. If the equilibrium effort is close to one, dismissals might be rare, limiting the learning of the enforceability. The availability bias might make employees overestimate the probability of enforcement due to the media reports on (few) dramatic cases. Employees might be uninformed about the laws in their state and, hence, fall for the bluff of their employer's attorney. The beliefs of the new employer are also important: Former employers might get a judge to enter a preliminary injunction against the new employment. Preliminary injunctions try to freeze the status quo before a case has been decided. This delay and the risk of litigation might be enough to break the new employment.

Other Sources of Rent We have shown that the principal can expropriate an agent's rent using NCCs. In our model, the agent gets a rent because of minimum wages. There are other reasons for which agents get a rent that can lead to the use of NCCs. For example, the rigidity of nominal wages might prevent the principal from reducing the wages but not from adding an NCC. Cici et al. (2019) have shown that NCCs have incentive effects on funds managers, which suggests that funds managers receive some rent.

Note that there are also rents that cannot be taken away. One example are information rents. When the principal tries to take away an information rent with an NCC, he distorts the screening and loses the information.

Multidimensional payoffs Without minimum wages, assuming risk neutrality, money is sufficient to implement the first-best outcome and to extract all rents. Minimum wages restrict the set of wages that the principal can offer to the agent. This has two effects: On the one hand, it protects the agent from complete rent extraction. On the other hand, this introduces inefficiencies because it means that the principal no longer wants to maximize social surplus.

In reality, payoffs are multidimensional. Each of these dimensions, one of which is money, is subject to constraints such as limited liability. Some models that use this observation have been mentioned in the literature review in Section 2. They all make the same finding, as do we: Constraints lead to spillovers into other dimensions of the payoff. In our model, restricting the monetary dimensions leads to the use of NCCs to reduce the agent's payoff in the dimension "continuation payoff". One can embed our model into a more general framework.

Unconditionally, agents receive some vector $\vec{w}$. In our model, this is the one-dimensional base wage. This dimension is constrained by the minimum wage. Other examples of unconditional rewards are health benefits, working equipment, office space, casual Fridays, or flexible working hours. Conditional on success, agents receive some vector $\vec{b}$. In our model, these are both the bonus wage and the non-compete clause. The bonus wage is constrained by the minimum wage and the NCC is constrained by the NCC feasibility constraint (NCCs may only harm). Other examples of contingent rewards or punishments are the working climate (Dur et al. (2019)), perks (Marino and Zábojník (2008)), reputation, or career prospects.

Besides the constraints on the dimensions, it is particularly important how well they transfer utility. With risk neutrality, money perfectly transfers utility. There are other dimensions that might be quasi-cash: Passing on the office's rent to the agent is perfectly transferable. For each unit that the agent pays, the principal saves one unit.[13]

[13]If there is a quasi-cash, minimum wages are immaterial: Increasing the minimum wage by one unit just lets the principal pass one unit more of the office's rent. There is neither inefficiency nor redistribution. Additional laws are needed that introduce constraints in the dimension of office space like forbidding employers to charge the employees for office space.

Broadly, there are two kinds of inefficiencies in this generalized framework: loss from utility transfer and suboptimal effort. In our model, NCCs do not transfer utility at all. While an NCC harms the agent, the principal gets nothing (directly) out of the reducing the agent's utility. This is the idleness effect. For each util that is "transferred" from the agent to the principal, there is a loss of one unit. If NCCs could be used only unconditionally (that is in $\vec{w}$ instead of in $\vec{b}$), they would be useless. Because NCCs can be used conditionally, they have an incentive effect. NCCs affect the second type of inefficiency by increasing effort. This is used by the principal because he gains from successes and does not have to pay for more effort as long as the agent gets a rent; which redistributes to the principal. The flip side is that the loss in this kind of indirect redistribution grows very large when transferring a large rent due to large minimum wages.

Our modelling assumption is losing one unit when transferring one unit.[14] Let's take a look at two different modelling assumptions: One with less and one with more loss than unity.

Imagine there are base and bonus wages, and another unconditional dimension, say health benefits, which transfers utility imperfectly. The agent has a concave utility in his health benefits, whereas the principal has a linear cost. Assume that health benefits can only be provided by the principal, that is the agent cannot spend his own money on health benefits. Without a minimum wage, the provision of health benefits is efficient; the agent's utility is equal to the principal's cost. Now, a minimum wage is introduced and it is large enough to constrain the monetary base wage. To extract more rent, the principal reduces the health benefits. Because of the agent's concave utility, in the new optimum, for each util the agent loses, the principal gains less than one util.

The principal might have to pay to reduce the agent's utility: Beating the agent after a failure reduces (probably) both the agent's and the principal's utility (Chwe, 1990). As NCCs, this dimension is only used if it can be made conditional on success.

Of what use is a generalized framework? Theory can improve our understanding of interactions between the principal's two motives, utility transfer and incentives. Does the principal first reduce health benefits and then use NCCs when minimum wages are increased? Does this depend on how well they transfer utility? Is it the other way around when the conditional payoff perfectly transfers utility while the unconditional payoff has a large loss? Which effects do different constraints have? With empirical findings, a generalized model can be calibrated: Which dimensions exist? Are they conditional or unconditional? How well do they transfer utility? With such a model, policies can be evaluated. Does the increase in the minimum wage really benefit the minimum wage workers or does it just increase the inefficiencies? Do constraints on non-monetary dimensions help?

[14] We could also have allowed for renegotiation between the agent and the principal after a failure. Ripping apart the non-compete clause generates a social surplus of $-\bar{v}$, which the principal and the agent can split. Assuming that the principal has the full bargaining power in the renegotiation is equivalent to assuming that NCCs transfer utility perfectly. Assuming that the agent has full bargaining power in the renegotiation would make NCCs useless as they do not reduce the agent's continuation payoff. Thus: no incentives.

Empirical predictions Our model yields some (potentially) testable empirical predictions that researchers could bring to the data.

On the macro level, our model predicts that the effect of minimum wages on the employment is lower when NCCs can be used. Both with bounded or unbounded NCCs, the principal makes weakly larger profits. Importantly, the profits also decrease slower in the minimum wage (at least before the bound is hit). Therefore, when NCCs can be used, there should be fewer market exits in general.

Wages are affected by NCCs in different ways. On the one hand, because they mitigate the hold-up problem, employers can invest more in their employees' human capital, increasing the marginal productivity. On the other hand, the employees' bargaining position deteriorates. There might also be effects on the macro level such as reduced match quality due to reduced labor mobility or a generally different wage level. Moreover, employees that do not anticipate being asked to sign an NCC seem to receive lower wages (Starr et al., 2019a), suggesting that irrational expectations play a role. Unsurprisingly, the empirical findings on the wage effects of NCCs are mixed (see McAdams, 2019, pp. 17-19, for more information).

Our partial market model predicts another ambiguous effect on top of the known discordant effects. It might be helpful to take another look at Figure 7. Consider the minimum wage, κ_2, at which the principal starts using an NCC and slightly above. For this minimum wage, the principal has to increase the bonus wage to keep the participation constraint satisfied. Because the NCC also increases the equilibrium effort, this larger bonus is being paid more often, implying larger (total) wages than without an NCC. For larger minimum wages, if there is no tight bound on NCCs, the prediction reverses. Without NCCs, the bonus wage remains constant for minimum wages larger than κ_3. With NCCs, the bonus wage falls below this constant level for some minimum wage. For even larger minimum wages, the bonus wage either falls to zero or, if the bound on the NCC binds earlier, it remains constant at some lower level. In the first case, the agent surely gets a lower total wage than he would get without NCCs. In the second case, this also happens, if the bound is not too tight (remember that the success probability with an NCC is larger such that the agent receives the bonus more often).

These predictions might be tested if the data are detailed enough. Especially, information on the composition of wages is needed. The base wage is always the minimum wage. Our predictions concern only bonus wages, incentive pay that is. This might also help to separate out the other, aforementioned effects. Empirical research could test whether with NCCs there is more incentive pay for low minimum wages and less incentive pay for high minimum wages.

Furthermore, our model predicts that the wages are actually not that informative for the well-being of employees. Although they might receive higher wages, the agent loses his rent due to an NCC because he has to exert more effort. Thus, another prediction is that workers with a binding minimum wage and an NCC should be less happy with their job than workers with a binding minimum wage without an NCC, if the minimum wages are sufficiently large $\underline{w} > \kappa_2$. The reason is that without an NCC, workers earn a rent, whereas with an NCC the worker's rent gets eaten up by an increased effort.[15]

One last prediction is on who signs NCCs. Not all minimum wage workers are subject to NCCs. One explanation might be timing: Some workers have signed their contracts before NCCs have come into vogue. Because of replacement costs, they might not have to give up their rents later by signing an NCC.[16] Our model suggests heterogeneity as an additional reason. Consider a single principal with several agents who work on independent projects. Assume that for various reasons (for example education, age, mobility, or health), these agents have different outside options. Remember that in our model, the minimum wage is defined as "minimum wage minus the outside option", because we normalized the outside option. With heterogeneous agents, thus, the same minimum wage is "low" for those with good outside options, and "high" for those with bad outside options. Therefore, we expect to see more and more severe NCCs in the contracts of agents with bad outside options. Surprisingly, those who would have trouble finding a new job anyway are predicted to be bound by NCCs.

7 Conclusion

Non-compete clauses not only secure returns to investments, conserve bad bargaining positions, or protect proprietary information. They can also be used for incentive provision by (implicit) threats of being dismissed. In this paper, we consider a moral hazard model with continuous effort and binary output to examine the use of non-compete clauses for minimum-wage workers. Non-compete clauses have two economic effects, the incentive effect and the idleness effect. The threat of reduced options after a dismissal induces the agent to exert a higher effort level (incentive effect). Without NCCs, the agent exerts too little effort as compared to first-best. Through the incentive effect, NCCs make the equilibrium effort non-monotonic in the minimum wage. For small minimum wages the incentive effect makes effort come closer to the first-best effort level. This increases social surplus. For large minimum wages, optimal effort will become inefficiently large. This decreases social surplus. The agent has to pay for the increased incentives with his rent. In this sense, NCCs can be (ab-)used to redistribute from the agent to the principal via increased incentives. Therefore, the incentive effect states that NCCs induce a higher effort level. The idleness effect is that human capital has to be idle after a lay-off. This reduces social surplus.

The incentive effect explains why NCCs get more pervasive when minimum wages are increased. Without minimum wages, base and bonus wages are enough to shift the entire rent to the principal and social surplus can be maximized. With minimum wages, however, the principal's rent extraction is limited—which is the goal of minimum wages. Now, NCCs enter the

[15] Another measure than happiness might be (self-reported) effort at work or stress-related health issues. In the fast food industry, work effort of minimum-wage workers could be measured as cleanliness, customer satisfaction with the service (or amount of complaints), customer waiting time (or number of sales during peak hours). These measures are imperfect but might be used to draw some conclusions about the effort level.

[16] The cost of using NCCs has drastically decreased since there are ready-to-go forms on the internet. Until 2020, there was no punishment in any state for having employees sign unenforceable NCCs. They just would have been not or only partially enforced if it ever came to litigation. Since 2020, there is a punishment in the state of Washington.

stage. The principal uses them as an redistribution device. Including NCCs into the contract, makes the agent work harder. The principal profits from the increased effort as she does not have to compensate the agent for it. Together, incentive and idleness effect eat up all of the agent's rent.

We have shown that NCCs are used from a minimum wage on that is below the minimum wage from which on the agent receives a rent. That is, if the minimum wage intends to redistribute from the principal to the agent, there will also appear NCCs that counteract. With unbounded NCCs, the agent might receive a higher monetary payoff but also exerts more effort. Therefore, with unbounded NCCs, minimum wages are both inefficient and ineffective: the agent never gets a rent.

In the real world, NCCs are bounded. Courts refuse to enforce NCCs that run for an unreasonably long time span; typically more than two years. Furthermore, NCCs only prevent employment in the same industry. That is, they can only take the industry-specific human capital hostage.

The welfare analysis shows that bounded NCCs might lead to a (strict) Pareto improvement compared to the world without NCCs under some technical conditions. The bound on NCCs has to be chosen optimally such that the smaller inefficiency is achieved. To get a Pareto improvement, the minimum wage has to be increased compared to a world without NCCs. The redistributional effect of the minimum wage only arises after the NCC has hit the upper bound, which is for higher minimum wages than without NCCs.

In general, policymakers do not have all of this information. Furthermore, the shape of the effort cost function might not allow Pareto improvements. Heterogeneity of the agents complicates things further. As a result, banning NCCs for low-wage workers might be the best option.

8 Appendix

8.1 Proof of Proposition 1

Proof of Proposition 1. First, we show that the objective function is strictly concave in the bonus wage. Let $E(b)$ be the maximizer of the agent's utility, that is, the equilibrium effort.

$$E(b) = \begin{cases} (c')^{-1}(b) & \text{if } b \geq 0 \\ 0 & \text{if } b < 0 \end{cases} \tag{12}$$

If the bonus wage is non-negative, the equilibrium effort is determined by the solution of the agent's first-order condition. Furthermore, $E(b)$ is strictly increasing in this range. If the bonus wage is negative, a corner solution, $E(b) = 0$, is optimal. We will use this function with a different argument again, when NCCs are allowed. The first and second derivative of $E(b)$ with respect to its positive argument are $E'(b) = \frac{1}{c''(E(b))}$ and $E''(b) = -\frac{c'''(E(b))}{(c''(E(b)))^3}$.

Remember that the expected profit is $\pi = -w + E(b) \cdot (V - b)$. The first and second derivatives with respect to the bonus wage are then given by:

$$\frac{\partial \pi}{\partial b} = E'(b) \cdot (V - b) - E(b) \tag{13}$$

$$\frac{\partial^2 \pi}{\partial b^2} = E''(b) \cdot (V - b) - 2E'(b) \tag{14}$$

Since $E''(\cdot) < 0$ and $E'(\cdot) > 0$, the second derivative is negative. This implies that P's objective function is strictly concave in the bonus wage.

Next, we look at the constraints of P's problem. We now show that $MWC2$ is always slack. Assume to the contrary that $MWC2$ binds. Rearranging $MWC2$ yields $b = \underline{w} - w$. By $MWC1$ we know that $w \geq \underline{w}$, which then implies that $b \leq 0$. A non-positive bonus wage, however, implies that the equilibrium effort is zero, which cannot be optimal.[17] Hence, $MWC2$ is always slack.

This leaves two constraints that can either bind or be slack, the PC and $MWC1$. We now show that it cannot be the case that both PC and $MWC1$ are slack. Assume to the contrary that both PC and $MWC1$ are slack. This means that there is a profitable deviation: Decreasing w by ϵ still leaves PC and $MWC1$ slack but increases P's expected profit. Therefore, we can decrease w until either PC or $MWC1$ binds.

This leaves us with the following three possible cases:
Case 1: PC binds and $MWC1$ is slack.
Case 2: PC binds and $MWC1$ binds.
Case 3: PC is slack and $MWC1$ binds.

[17] The maximum profit is zero for negative minimum wages and $-\underline{w}$ for positive minimum wages. As we assume that the success payoff is sufficiently large for the principal to be able to achieve a positive profit, a non-positive bonus wage cannot be optimal.

Next, we focus on each case in more detail.

Case 1 P's problem is given by:

$$\max_{w,b} \quad -w + E(b) \cdot (V - b) \tag{15}$$

$$\text{subject to} \qquad w + E(b) \cdot b - c(E(b)) = 0 \tag{PC}$$

$$w > \underline{w} \quad \text{and} \quad w + b > \underline{w} \qquad \text{(MWC1) and (MWC2)}$$

We will ignore the slack constraints for the moment and later check for which minimum wages they are not violated. The PC can be rewritten such that $E(b) \cdot b = c(E(b)) - w$. We plug this into P's objective function and maximize over the equilibrium effort instead of the bonus wage. The first-order condition is $V = c'(E(b)) = b$. Since the objective function is concave, we know that the first-order condition yields the global maximum. Therefore, $b = V$, $E(V) = e^{FB}$, and $w = c(e^{FB}) - e^{FB} \cdot c(e^{FB})$. Now, we check the constraints. Because $V > 0$, $MWC2$ is slack. $MWC1$ is slack if $\underline{w} < c(e^{FB}) - e^{FB}c'(e^{FB}) \equiv \kappa_1$.

Case 2 P's problem is given by:

$$\max_{w,b} \quad -w + E(b) \cdot (V - b) \tag{16}$$

$$\text{subject to} \qquad w + E(b) \cdot b - c(E(b)) = 0 \tag{PC}$$

$$w = \underline{w} \quad \text{and} \quad w + b > \underline{w} \qquad \text{(MWC1) and (MWC2)}$$

There are two unknowns and two binding constraints. Plugging $MWC1$ into PC implicitly characterizes $E(b)$ and the bonus wage. There are three subcases: negative minimum wages, $\underline{w} = 0$, and positive minimum wages.

For each negative $\underline{w}$, there are exactly one b and one $E(b)$ such that the participation constraint binds. The reason is the following: Rearrange the binding participation constraint to get

$$E(b) \cdot b - c(E(b)) = -\underline{w} \tag{17}$$

The left-hand side is the part of the agent's utility that is generated by exerting effort. Graphically, it is the area above an increasing function ($c'(e)$), between 0 and $E(b)$, another increasing function. It is zero for a bonus wage of zero, and is strictly increasing in the bonus wage because $c''(e) > 0$. Therefore, there can be at most one bonus wage for each negative minimum wage such that this holds. Furthermore, for negative minimum wages, there is a bijection between b and $E(b)$. Since the right-hand side is strictly positive, so is the bonus wage, which implies $MWC2$.

Consider the minimum wage $\underline{w} = 0$. Since the right-hand side of equation (17) is zero, so is the equilibrium effort, which means that the bonus wage has to be non-positive. $MWC2$ is only slack if the bonus wage is positive. Thus, there is no bonus wage such that PC binds and $MWC2$ is slack.

Consider positive minimum wages. The participation constraint is always slack. That is, there are no bonus wage and no equilibrium effort that satisfy equation (17).

Summing up the optimal contract in Case 2: For negative minimum wages, let $e^{**}(\underline{w})$ denote the effort that makes the participation constraint (17) binding. Then, $e^{**}(\underline{w})$ is implicitly defined by $e^{**}(\underline{w}) \cdot c'(e^{**}(\underline{w})) - c(e^{**}(\underline{w})) = -\underline{w}$. We also get that $b = c'(e^{**}(\underline{w}))$ and from $MWC1$ we get $w = \underline{w}$.

Case 3 P's problem is given by:

$$\max_{w,b} \quad -w + E(b) \cdot (V - b) \tag{18}$$

$$\text{subject to} \qquad w + E(b) \cdot b - c(E(b)) > 0 \tag{PC}$$

$$w = \underline{w} \quad \text{and} \quad w + b > \underline{w} \qquad \text{(MWC1) and (MWC2)}$$

We will ignore the slack constraints for the moment and later check for which minimum wages they are not violated. We plug $MWC1$ into the objective function and take the derivative. The optimal bonus wage is characterized by the marginal profit's being 0. The solution to the first-order condition implicitly defines the optimal effort in Case iii, e^{***}: $c'(e^{***}) + e^{***} \cdot c''(e^{***}) = V$. Hence, $e^{***} < e^{FB}$. We also get that $w = \underline{w}$ and $b = c'(e^{***})$. Next, we check the constraints. As $e^{***} > 0$, $MWC2$ is slack. PC is slack if $\underline{w} > c(e^{***}) - e^{***}c'(e^{***}) \equiv \kappa_3$.

The Optimal Contract We have verified that the optimal contract from Case 1 is feasible if $\underline{w} < \kappa_1$, the optimal contract from Case 2 is feasible if $\underline{w} < 0$, and the optimal contract from Case 3 is feasible if $\underline{w} > \kappa_3$. These thresholds are $\kappa_1 = c(e^{FB}) - e^{FB}c'(e^{FB}) < 0$ and $\kappa_3 = c(e^{***}) - e^{***}c'(e^{***}) < 0$. Because $e^{***} < e^{FB}$, it follows that $\kappa_1 < \kappa_3$.

Thus, for $\underline{w} < \kappa_1$, we have two candidates: Case 1 and Case 2. The maximization problem in Case 2 has two binding constraints, while the maximization problem in Case 1 has none. As a result, the profit from the optimal contract in Case 1 is weakly larger. The concavity of the objective function and the fact that the bonus wages from Case 1 and Case 2 are different for all $\underline{w} < \kappa_1$ imply that the profit is strictly larger. For $\kappa_1 \leq \underline{w} \leq \kappa_3$, the only candidate is Case 2; thus, this contract is optimal. For $\kappa_3 < \underline{w}$, we have again two candidates: Case 2 and Case 3. Since the maximization problem in Case 3 has only one binding constraint, the profit from the optimal contract in Case 3 is weakly larger. Again, concavity and different solutions imply strictly larger profits.

□

8.2 Proof of Proposition 2

Proof of Proposition 2. The proof proceeds in two main parts. The first part is about simplifying the problem. Since there are four inequality constraints, there are 16 possible combinations.

First, we identify those four combinations that can be optimal. In all of those combinations, the participation constraint is binding; the agent does not get a rent. We use this fact to reduce the problem's dimensionality by using the participation constraint to express the optimal NCC in terms of the minimum wage and the bonus wage. The first combination is the same as Case 1 in the benchmark, which means that this contract is profit maximizing for $\underline{w} < \kappa_1$. For all $\underline{w} \geq \kappa_1$, the base wage has to be the minimum wage. This fact and an additional piece of notation simplify the problem further. This yields a strictly quasi-concave objective function in only the bonus wage with one inequality constraint on the bonus wage. The optimal bonus wage and whether the inequality constraint binds shows into which combination the contract falls. In the second part, we solve the simplified problem for negative minimum wages. We find that there is a unique minimum wage $\kappa_2 < 0$ such that the principal uses no NCCs for all lower minimum wages and she uses NCCs (and bonus wages) for all larger minimum wages. For a minimum wage of 0, it is also optimal to use NCCs and bonus wages. We then solve the simplified problem for positive minimum wages. Under some conditions on the effort cost function, there is a unique minimum wage κ_4 such that the principal uses both NCCs and bonus wages for all lower minimum wages and only an NCC without bonus wages for all larger minimum wages.

The Possibly Optimal Combinations The agent's first-order condition for the optimal effort with NCCs is

$$b - \bar{v} = c'(e). \tag{19}$$

Whenever the left-hand side is non-negative, the first-order condition yields the optimal equilibrium effort, which we, express as $E(b - \bar{v}) \equiv (c')^{-1}(b - \bar{v})$. As above, a negative left-hand side implies that the corner solution $E(b - \bar{v}) = 0$ is optimal.

The principal's problem is

$$\max_{w,b,\bar{v}} \quad -w + \quad E(b - \bar{v}) \cdot (V - b) \tag{20}$$

$$\text{subject to} \qquad w + E(b - \bar{v}) \cdot b + (1 - E(b - \bar{v})) \cdot \bar{v} - c(E(b - \bar{v})) \geq 0 \tag{PC}$$

$$\bar{v} \leq 0 \tag{NCC}$$

$$w \geq \underline{w} \qquad w + b \geq \underline{w} \qquad \text{(MWC1) and (MWC2)}$$

To solve the principal's problem, one has to know which constraints bind and which are slack for different minimum wages. In total, there are 16 combinations. They are summarized in Table 1. The combinations' order in Table 3 reflects their occurrence when the minimum wage increases. We will now prove that the optimal contract can never fall into the Combinations 5 to 16 for three distinct reasons.

Firstly, the participation constraint has to bind. Otherwise, there is a profitable deviation: Make the NCC more severe, keeping everything else fixed. Note that the bonus wage is optimally

No.	PC	NCC	MWC1	MWC2	Relevant?
1	binds	binds	slack	slack	$\underline{w} \leq \kappa_1$
2	binds	binds	binds	slack	$\kappa_1 < \underline{w} \leq \kappa_2$
3	binds	slack	binds	slack	$\kappa_2 < \underline{w} \leq \kappa_4$
4	binds	slack	binds	binds	$\kappa_4 < \underline{w}$
5	slack	binds	binds	binds	no, PC
6	slack	binds	binds	slack	no, PC
7	slack	binds	slack	binds	no, PC
8	slack	binds	slack	slack	no, PC
9	slack	slack	binds	binds	no, PC
10	slack	slack	binds	slack	no, PC
11	slack	slack	slack	binds	no, PC
12	slack	slack	slack	slack	no, PC
13	binds	binds	binds	binds	no, no effort
14	binds	binds	slack	binds	no, no effort
15	binds	slack	slack	binds	no, deviation
16	binds	slack	slack	slack	no, deviation

Table 1: The 16 Combinations of Binding Constraints.

never larger than the success. Then, the agent exerts more effort which leads to more successes and more profit.

Secondly, it cannot be that $MWC2$ and NCC bind simultaneously. If they did, the agent would exert no effort. Then, the principal has no revenue. This cannot be optimal by our assumption that the success payoff is sufficiently large to allow for positive profits.

Thirdly, $MWC1$ can only be slack when the NCC feasibility constraint binds. Otherwise, there is a profitable deviation. In these combinations, the principal uses an NCC and pays a larger than necessary base wage. This cannot be optimal because there is a profitable deviation: Decrease the base wage by one unit and increase the bonus wage and make the NCC less severe by one unit. Because bonus wage and the NCC's severity are perfect substitutes, the equilibrium effort stays the same. Furthermore, the participation constraint remains satisfied: The agent loses one unit on the base wage but gains one unit both if there is a success and if there is a failure. The principal's profit increases because he saves on the base wage one unit with certainty and loses on the bonus wage one unit with the success probability (less than one by the Inada conditions). The principal can repeat this deviation until either MWC1 or NCC binds.

When is the first combination optimal? In the benchmark, we have seen that in the first combination the optimal contract implements the first-best effort. Additionally, the principal extracts the whole surplus. Therefore, this contract is profit-maximizing whenever it is feasible.

As we have seen in the benchmark, the contract in the first combination is only feasible if

$\underline{w} < \kappa_1 = c(e^{FB}) - e^{FB}c'(e^{FB}) < 0$. This implies that for all $\underline{w} \geq \kappa_1$, the optimal contract is from either the second, the third, or the fourth combination. In all of these combinations, the base wage optimally is the minimum wage; $MWC1$ binds. Using that both $MWC1$ and PC bind, allows us to simplify the maximization problem to one in only the bonus wage with two inequality constraints, $MWC2$ and NCC. Simplifying the problem helps us to characterize the optimal contracts in the remaining three combinations.

The fact that $MWC1$ binds, eliminates w from the problem. Thus, b and $\bar{v}$ remain. Furthermore, the participation constraint PC has to bind. This lets us express $\bar{v}$ as an implicit function of $\underline{w}$ and b:

$$\bar{v}(\underline{w}, b) = -\frac{\underline{w} + E(b - \bar{v}(\underline{w}, b)) \cdot b - c(E(b - \bar{v}(\underline{w}, b)))}{1 - E(b - \bar{v}(\underline{w}, b))}. \tag{21}$$

Note that $(b - \bar{v})$ is non-negative because of $MWC1$ binds, which simplifies $MWC2$ to $b \geq 0$ and NCC, $\bar{v} \leq 0$. Thus, the agent's first-order condition yields the equilibrium effort.

$\bar{v}(\underline{w}, b)$ is the most severe NCC that the agent is willing to accept given a base wage of $\underline{w}$ and a bonus wage b. Lemma 6 shows that the higher the minimum wage is, the more severe is this NCC for a given bonus wage. The higher the bonus wage is, the more severe is this NCC for a given minimum wage. Furthermore, due to monotonicity, the values of $\bar{v}(\underline{w}, b)$ are unique in b for a fixed $\underline{w}$ and the other way around.

Therefore, the principal's problem can also be expressed as

$$\max_{b} \quad -\underline{w} + E(b - \bar{v}(\underline{w}, b))\,(V - b) \tag{22}$$

$$\text{subject to} \quad \bar{v}(\underline{w}, b) = -\frac{\underline{w} + E(b - \bar{v}(\underline{w}, b)) \cdot b - c(E(b - \bar{v}(\underline{w}, b)))}{1 - E(b - \bar{v}(\underline{w}, b))} \tag{PC'}$$

$$\bar{v} \leq 0 \tag{NCC}$$

$$b \geq 0 \tag{MWC2}$$

In the second combination, $MWC2$ is slack and NCC binds. In the third combination, $MWC2$ and NCC are both slack. In the fourth combination, $MWC2$ binds and NCC is slack.

Lemma 6. *i) Fix a minimum wage. The NCC that makes the participation constraint bind $\bar{v}(\underline{w}, b)$ is strictly decreasing in the bonus wage: $\frac{\partial \bar{v}(\underline{w}, b)}{\partial b} < 0$.*
ii) Fix a bonus wage. The NCC that makes the participation constraint bind $\bar{v}(\underline{w}, b)$ is strictly decreasing in the minimum wage: $\frac{\partial \bar{v}(\underline{w}, b)}{\partial \underline{w}} < 0$.

Proof. Rearrange the binding participation constraint to

$$Z \equiv \underline{w} + E(b - \bar{v}) \cdot (b - \bar{v}) + \bar{v} - c(E(b - \bar{v})) = 0, \tag{23}$$

Because this is continuously differentiable, the implicit function theorem can be used to get the derivatives of $\bar{v}$ with respect to $\underline{w}$ and b.

$$\frac{\partial \bar{v}(\underline{w}, b)}{\partial \underline{w}} = -\frac{\frac{\partial Z}{\partial \underline{w}}}{\frac{\partial Z}{\partial \bar{v}}} = -\frac{1}{-E'(b-\bar{v}) \cdot (b-\bar{v}) - E(b-\bar{v}) + 1 + c'(E(b-\bar{v})) \cdot E'(b-\bar{v})} \tag{24}$$

$$= -\frac{1}{1 - E(b-\bar{v})} \tag{25}$$

The simplification is due to the agent's first-order constraint, $(b - \bar{v} - c'(E)) = 0$.

$$\frac{\partial \bar{v}(\underline{w}, b)}{\partial b} = -\frac{\frac{\partial Z}{\partial b}}{\frac{\partial Z}{\partial \bar{v}}} = -\frac{E'(b-\bar{v}) \cdot (b-\bar{v}) + E(b-\bar{v}) - c'(E(b-\bar{v})) \cdot E'(b-\bar{v})}{-E'(b-\bar{v}) \cdot (b-\bar{v}) + 1 - E(b-\bar{v}) + c'(E(b-\bar{v})) \cdot E'(b-\bar{v})} \tag{26}$$

$$= -\frac{E(b-\bar{v})}{1 - E(b-\bar{v})} \tag{27}$$

Again, the agent's first-order constraint simplifies the expression. □

We will now define a useful term to simplify the maximization problem further. Let $b_2^{**}(\underline{w})$ denote the optimal bonus wage in Case 2 of the benchmark (binding PC, binding $MWC1$, slack $MWC2$). The case conditions imply a property of $b_2^{**}(\underline{w})$: It makes the participation constraint binding in the absence of an NCC.

To use this particular bonus wage to simplify the problem, we have to extend the definition of $b_2^{**}(\underline{w})$ to minimum wages above κ_3 for which it is not the optimal bonus wage. Let $b_2^{**}(\underline{w})$ denote the *minimum non-negative* bonus wage that keeps the participation constraint *satisfied* in the absence of an NCC.

$$\forall \underline{w} \geq \kappa_1 \qquad b_2^{**}(\underline{w}) \equiv \min \left\{ b \in \mathbb{R}_0^+ \;\middle|\; \underline{w} + E(b) \cdot b - c(E(b)) \geq 0 \right\} \tag{28}$$

For non-positive minimum wages, $b_2^{**}(\underline{w})$ is determined by the minimum wage that makes the participation constraint binding. For positive minimum wages the participation constraint is always slack without an NCC; there is no bonus wage that makes the participation constraint binding. Thus, if $\underline{w} \geq 0$, then $b_2^{**}(\underline{w}) = 0$. Furthermore, $b_2^{**}(\underline{w})$ has the nice property that it exists and it is strictly decreasing in the minimum wage between κ_1 and 0.

To simplify the problem, we now replace the inequality constraints using $b_2^{**}(\underline{w})$: As long as PC' holds, the conditions NCC and $MWC2$ are equivalent to another condition, $b \geq b_2^{**}(\underline{w})$.

Consider $\underline{w} < 0$. In this case, PC' and NCC imply $MWC2$. The bonus wage has to be at least $b_2^{**}(\underline{w})$, even without an NCC, to satisfy the participation constraint. If $\underline{w} < 0$, then $b_2^{**}(\underline{w}) > 0$, implying $MWC2$. In this case, the new constraint $b \geq b_2^{**}(\underline{w})$ is binding if and only if NCC is binding.

Consider $\underline{w} \geq 0$. In this case, PC' and $MWC2$ imply NCC. If $\underline{w} \geq 0$, then $b_2^{**}(\underline{w}) = 0$; for $\underline{w} = 0$ the participation constraint is binding without an NCC, for $\underline{w} > 0$, the participation constraint is slack without an NCC. In both cases, the binding PC means that $\bar{v} \leq 0$, implying NCC. In this case, the new constraint is binding if and only if $MWC2$ is binding.

The problem is, thus, equivalent to

$$\max_b \quad -\underline{w}+E(b-\bar{v}(\underline{w},b))\cdot(V-b) \tag{29}$$

$$\text{subject to} \qquad \bar{v}(\underline{w},b) = -\frac{\underline{w}+E(b-\bar{v}(\underline{w},b))\cdot b - c(E(b-\bar{v}(\underline{w},b)))}{1-E(b-\bar{v}(\underline{w},b))} \tag{PC'}$$

$$b \geq b_2^{**}(\underline{w}). \tag{30}$$

The problem (29) is simpler because it has only one inequality constraint which is on the only argument of the objective function. Under the assumptions made in Section 3, moreover, the objective function is strictly concave, as Lemma 7 shows. We introduced this assumption because it implies all assumptions that we need in this proof. To make the proof tighter, however, we make weaker assumptions wherever possible. Thus, for determining whether the second or the third combination is optimal, we will use a weaker assumption and the notion of strict quasi-concavity that is sufficient to derive the results. In Lemma 8, we determine the necessary and sufficient condition that makes the objective function strictly quasi-concave in the bonus wage.

Lemma 7. *(29) is strictly concave in b if for all bonus wages*

$$\frac{c'''(E(b,\bar{v}(\underline{w},b)))}{c''(E(b,\bar{v}(\underline{w},b)))} > \frac{1}{1-E(b,\bar{v}(\underline{w},b))}. \tag{31}$$

Proof. The objective function's first and second derivatives with respect to the bonus wage are

$$\frac{\partial\pi}{\partial b} = \frac{E'(b,\bar{v}(\underline{w},b))}{1-E(b,\bar{v}(\underline{w},b))}\cdot(V-b) - E(b,\bar{v}(\underline{w},b)) \tag{32}$$

and (omitting the argument of $E(b,\bar{v}(\underline{w},b))$ for readability)

$$\frac{\partial^2\pi}{\partial b^2} = \left[\frac{E''}{(1-E)^2} + \frac{(E')^2}{(1-E)^3}\right]\cdot(V-b) - \frac{2E'}{1-E}. \tag{33}$$

Because $E'(b,\bar{v}(\underline{w},b)) > 0$, a sufficient condition for the concavity of the objective function is that $\frac{E''}{(1-E)^2} + \frac{E'E'}{(1-E)^3} < 0$. Rearranging and simplifying shows that this is true under our assumption on the cost function.

$$E''(b,\bar{v}(\underline{w},b)) + \frac{(E'(b,\bar{v}(\underline{w},b)))^2}{1-E(b,\bar{v}(\underline{w},b))} < 0 \quad \Longrightarrow \quad \frac{\partial^2\pi}{\partial b^2} < 0 \tag{34}$$

Plugging in for $E'(\cdot) \equiv \frac{1}{c''(E(\cdot))}$ and $E''(\cdot) \equiv -\frac{c'''(E(\cdot))}{(c''(E(\cdot)))^3}$ yields

$$\frac{c'''(E(b,\bar{v}(\underline{w},b)))}{c''(E(b,\bar{v}(\underline{w},b)))} > \frac{1}{1-E(b,\bar{v}(\underline{w},b))} \tag{35}$$

□

Lemma 8. *(29) is strictly quasi-concave in b if for all bonus wages*

$$\frac{c'''(E(b-\bar{v}(\underline{w},b)))}{c''(E(b-\bar{v}(\underline{w},b)))} > \frac{1}{1-E(b-\bar{v}(\underline{w},b))} - \frac{2}{E(b-\bar{v}(\underline{w},b))}. \tag{36}$$

Proof. The objective function, $\pi(\underline{w},b)$, is twice continuously differentiable. It is strictly quasi-concave in b if the second derivative is negative at each critical point.

For readability, we will omit the argument of $E(b-\bar{v}(\underline{w},b))$ and its derivatives, and instead write $E(\cdot)$. The objective function's first derivative with respect to b is

$$\begin{aligned}\frac{\partial\pi(\underline{w},b)}{\partial b} &= E'(\cdot)\cdot\left(1-\frac{\partial\bar{v}(\underline{w},b)}{\partial b}\right)\cdot(V-b)-E(\cdot)\\ &= \frac{E'(\cdot)}{1-E(\cdot)}\cdot(V-b)-E(\cdot).\end{aligned} \tag{37}$$

Since $1-E(\cdot)$ is the equilibrium probability of a failure, it is positive due to the Inada conditions. Critical points are characterized by

$$V-b=\frac{E(\cdot)\cdot(1-E(\cdot))}{E'(\cdot)}. \tag{38}$$

It is easier to work with an expression with the same sign. We call it "expression 1":

$$E'(\cdot)\cdot(V-b)-E(\cdot)\cdot(1-E(\cdot)). \tag{Expression 1}$$

Expression 1 is zero at critical points of the bonus wage. The objective function is strictly quasi-concave in the bonus wage if the derivative of expression 1 with respect to the bonus wage is negative at all critical points.[18] Expression 1's derivative is

$$\begin{aligned}&E''(\cdot)\cdot\left(1-\frac{\partial\bar{v}(\underline{w},b)}{\partial b}\right)\cdot(V-b)-E'(\cdot)-E'(\cdot)\cdot(1-2E(\cdot))\cdot\left(1-\frac{\partial\bar{v}(\underline{w},b)}{\partial b}\right)\\ &=\frac{E''(\cdot)}{1-E(\cdot)}\cdot(V-b)-E'(\cdot)-\frac{E'(\cdot)\cdot(1-2E(\cdot))}{1-E(\cdot)}\\ &=\frac{E''(\cdot)}{1-E(\cdot)}\cdot(V-b)-\frac{E'(\cdot)\cdot(2-3E(\cdot))}{1-E(\cdot)}\end{aligned} \tag{39}$$

Since we only care about the sign at the critical points, we can now plug in the solution of the first-order condition (38) for $(V-b)$. This yields an expression that we would like to be negative.

$$\frac{E''(\cdot)}{1-E(\cdot)}\cdot\frac{E(\cdot)\cdot(1-E(\cdot))}{E'(\cdot)}-\frac{E'(\cdot)\cdot(2-3E(\cdot))}{1-E(\cdot)}<0 \tag{40}$$

[18] Let's work on the intuition of this step: We want to show that $f'(x)$ crosses zero from above in all critical points x_0. It is easier to work with the expression $f'(x)\cdot g(x)$, where $g(x)$ is positive for all x (it is $(1-E(\cdot))$). $f'(x)$ crosses zero from above if $f''(x_0)<0$. The chain rule yields that the sign of $f''(x_0)$ is equal to the sign of $f''(x_0)g(x_0)+f'(x_0)g'(x_0)$, as $f'(x_0)=0$.

Rearranging yields

$$E''(\cdot) < \frac{(E'(\cdot))^2 \cdot (2 - 3E(\cdot))}{E(\cdot) \cdot (1 - E(\cdot))} \tag{41}$$

Using the definition of $E(\cdot)$, this can be simplified.

$$E(\cdot) = (c')^{-1}(\cdot) \quad \Longrightarrow \quad E'(\cdot) = \frac{1}{c''(E(\cdot))} \quad \Longrightarrow \quad E''(\cdot) = -\frac{c'''(E(\cdot))}{(c''(E(\cdot)))^3} \tag{42}$$

Therefore, (41) is equivalent to our assumption

$$\frac{c'''(E(\cdot))}{c''(E(\cdot))} > \frac{1}{1 - E(\cdot)} - \frac{2}{E(\cdot)} \tag{43}$$

□

For equilibrium efforts below $\frac{2}{3}$, the assumption is always satisfied. For equilibrium efforts above $\frac{2}{3}$, the assumption says that the marginal cost has to be convex enough. As a result, the equilibrium effort reacts not too strongly to increased incentives and the strict quasi-concavity is preserved when introducing NCCs.

Strict quasi-concavity in the bonus wage implies that the maximum is unique if it exists. To see that the maximum exists, note that the maximum is equivalent to the maximum of the problem constraining $b_2^{**}(\underline{w}) \leq b \leq V$, since the optimal bonus wage cannot be above V. Because of the extreme value theorem we know that the latter problem has a solution ($b_2^{**}(\underline{w}) \leq b \leq V$ is a compact set and the objective function is continuous).

This last simplification concludes the first part of the proof. In the second part of the proof, we look at the three remaining combinations and determine for which minimum wages they are optimal. We first characterize the different combinations in the simplified problem. Then, we use the monotonicity of the marginal profit in the bonus wage evaluated at the bonus wage $b_2^{**}(\underline{w})$ to find the minimum wages for which the second combination is optimal. Lastly, we derive a condition under which the fourth combination is optimal for some minimum wages.

In the second combination, the principal uses a bonus wage but no NCC. In the simplified problem, this means that $b = b_2^{**}(\underline{w}) > 0$. This is only possible for negative bonus wages; otherwise $b_2^{**}(\underline{w}) = 0$. Furthermore, this means that the global optimum of the strictly quasi-concave objective function is at a bonus wage $b \leq b_2^{**}(\underline{w})$.

In the third combination, the principal uses both a bonus wage and an NCC. In the simplified problem, this means that $b > b_2^{**}(\underline{w})$. The bonus wage is positive and the NCC that makes the participation constraint binding has a negative $\bar{v}(\underline{w}, b)$. Furthermore, the global optimum of the strictly quasi-concave objective function is at a bonus wage $b > b_2^{**}(\underline{w})$.

In the fourth combination, the principal uses no bonus wage but an NCC. In the simplified problem, this means that $b = 0$ and that $\underline{w} > 0$. If $\underline{w} < 0$, then $b_2^{**}(\underline{w}) > 0$, such that $b = 0$ is not feasible. If $\underline{w} = 0$ and $b = 0$, then the only NCC that does not violate the participation constraint or the NCC feasibility constraint is $\bar{v} = 0$, which means using no NCC. Furthermore, this means that the global optimum of the strictly quasi-concave objective function is at a bonus wage $b \leq b_2^{**}(\underline{w}) = 0$.

Negative minimum wages Consider negative minimum wages first. For $\kappa_1 \leq \underline{w} < 0$, only the second or the third combination can be optimal. The sign of the derivative of the objective function with respect to the bonus wage at the lower bound $b_2^{**}(\underline{w})$ shows whether there is an inner solution or not. If the derivative is non-positive, there is a corner solution and, thus, no NCC. The second combination is optimal. If the derivative is positive, there is an inner solution and, thus, an NCC. The third combination is optimal. The monotonicity of the derivative evaluated at $b_2^{**}(\underline{w})$ in the minimum wage yields uniqueness of minimum wage at which a switch happens.

Lemma 9. *Assume that* $\frac{c'''(E(b-\bar{v}(\underline{w},b)))}{c''(E(b-\bar{v}(\underline{w},b)))} > \frac{1}{1-E(b-\bar{v}(\underline{w},b))} - \frac{2}{E(b-\bar{v}(\underline{w},b))}$. *There is a unique cutoff* $\kappa_2 < 0$ *in the minimum wage such that for all* $\kappa_1 \leq \underline{w} \leq \kappa_2$, *the optimal contract has* $b = b_2^{**}(\underline{w})$, *and for all* $\kappa_2 < \underline{w} < 0$, *the optimal contract has* $b > b_2^{**}(\underline{w})$.

Proof. The derivative of the profit with respect to the bonus wage evaluated at the lower bound is

$$\begin{aligned}\frac{\partial \pi(\underline{w}, b)}{\partial b}\Big|_{b=b_2^{**}(\underline{w})} &= \frac{\partial E(b - \bar{v}(\underline{w}, b))}{\partial b}\Big|_{b=b_2^{**}(\underline{w})} \cdot (V - b_2^{**}(\underline{w})) - E(b_2^{**}(\underline{w})) \\ &= \frac{E'(b_2^{**}(\underline{w}))}{1 - E(b_2^{**}(\underline{w}))} \cdot (V - b_2^{**}(\underline{w})) - E(b_2^{**}). \end{aligned} \tag{44}$$

The term is the change in the profit from increasing the bonus wage at the lowest minimum wage for which the agent participates. The product is the marginal benefit. The first factor is the increase in equilibrium effort due to a marginal increase of the bonus wage. The denominator follows from the existence of NCC. An increase in b increases the incentives $(b - \bar{v}(\underline{w}, b))$ by $(1 - \frac{\partial \bar{v}(\underline{w},b)}{\partial b}) = (1 + \frac{E}{1-E})$. We call this “double incentives” because a larger bonus wage allows to use a more severe NCC, both of which provides incentives. Thus, without access to NCCs, the denominator is 1. The next factor is the principal’s share of a success. The second term is the marginal cost of increasing the bonus wage. In the case of a success, the payment to the agent gets one unit larger.

We will now look at different minimum wages and show that there is exactly one minimum wage at which the optimum switches from a corner to an inner solution. The corresponding minimum wage is the minimum wage from which on NCCs are used, κ_2. Technically, at κ_2, the objective function’s first derivative evaluated at the lowest possible bonus wage $b_2^{**}(\underline{w})$ switches the sign from negative (corner solution) to positive (inner solution).

We use the same strategy as when proving quasi-concavity: We show that in all candidates for κ_2, the derivative goes from negative to positive. By continuity, there can be only one candidate.

A candidate for κ_2 is a minimum wage such that the derivative is zero:

$$\frac{\partial \pi(\underline{w}, b)}{\partial b}\bigg|_{b=b_2^{**}(\underline{w})} = \frac{E'(b_2^{**}(\underline{w})}{1 - E(b_2^{**}(\underline{w}))} \cdot (V - b_2^{**}(\underline{w})) - E(b_2^{**}(\underline{w})) \stackrel{!}{=} 0 \tag{45}$$

$$\iff \quad (V - b_2^{**}(\underline{w})) = \frac{E(b_2^{**}(\underline{w})) \cdot (1 - E(b_2^{**}(\underline{w})))}{E'(b_2^{**}(\underline{w}))} \tag{46}$$

To see how the derivative of the profit with respect to the bonus wage at the lower bound changes, take the derivative with respect to the minimum wage. Note that although $\bar{v}(b, \underline{w})$ is a function of both the bonus and the minimum wage, it will not change: At $b_2^{**}(\underline{w})$, the participation constraint binds without an NCC. Thus, $\bar{v}(b_2^{**}(\underline{w}), \underline{w}) = 0$ for all negative minimum wages.

Again, we work with another expression that has the same sign as the first derivative but which is easier to work with. "Expression 2" is

$$E'(b_2^{**}(\underline{w})) \cdot (V - b_2^{**}(\underline{w})) - E(b_2^{**}(\underline{w})) \cdot (1 - E(b_2^{**}(\underline{w}))) \tag{Expression 2}$$

The derivative of expression 2 with respect to the minimum wage (where we express $E(b_2^{**}(\underline{w}))$ and its derivatives as E to improve readability) is

$$\begin{aligned}
\frac{\partial\left(\frac{\partial \pi}{\partial b}\Big|_{b=b_2^{**}(\underline{w})}\right)}{\partial \underline{w}} &= E'' \cdot (V - b_2^{**}(\underline{w})) \cdot \frac{\partial b_2^{**}(\underline{w})}{\partial \underline{w}} - E' \cdot \frac{\partial b_2^{**}(\underline{w})}{\partial \underline{w}} \\
&\quad - (1 - E) \cdot E' \cdot \frac{\partial b_2^{**}(\underline{w})}{\partial \underline{w}} + E' \cdot E \cdot \frac{\partial b_2^{**}(\underline{w})}{\partial \underline{w}} \\
&= \frac{\partial b_2^{**}(\underline{w})}{\partial \underline{w}} \left(E'' \cdot \frac{E(1-E)}{E'} - 2E' \cdot (1 - E)\right) > 0
\end{aligned} \tag{47}$$

The second line follows from plugging (46) in. At the critical point, the derivative of the profit with respect to the bonus wage evaluated at the lower bound is increasing because $\frac{\partial b_2^{**}}{\partial \underline{w}} < 0$; the lowest bonus wage to satisfy the participation constraint is decreasing in the minimum wage because a higher minimum wage makes the participation constraint already slack. Moreover, it is globally true that $E' > 0$, and $E'' < 0$.

We have shown that any switches between corner and inner solutions have to be from corner to inner solutions. Moreover, there can be at most one switching point. That is, conditional on existence, κ_2 is unique.

To show that there is at least one critical point, we use that the derivative of the profit with respect to the bonus wage is continuous in the minimum wage. There is a minimum wage for which the derivative is negative and there is a minimum wage for which the derivative is positive. Thus, there also is a minimum wage for which the derivative is zero.

The derivative is negative for the minimum wage κ_1. The principal implements first-best effort and extracts all surplus by selling the firm. Because all of the success payoff goes to the agent, increasing the bonus wage further reduces the profit. Plugging κ_1 in, yields $b_2^{**}(\kappa_1) = V$.

The derivative is

$$\left.\frac{\partial \pi}{\partial b}\right|_{b=b_2^{**}(\kappa_1)} = -E(V) < 0. \tag{48}$$

The derivative is positive for the minimum wage κ_3. Following a similar argument as above, we know from the benchmark that the derivative of the profit with respect to the bonus wage without access to NCC at the minimum wage κ_3 is zero: Left of κ_3, the optimal bonus wage just satisfies the participation constraint, right of κ_3, the optimal bonus wage makes the participation constraint slack. The derivative of the profit with respect to the bonus wage without NCC is

$$\left.\frac{\partial \pi^{\text{No NCC}}}{\partial b}\right|_{b=b_2^{**}(\kappa_3),\bar{v}=0} = E'(b_2^{**}(\kappa_3)) \cdot (V - b_2^{**}(\kappa_3)) - E(b_2^{**}(\kappa_3)) = 0 \tag{49}$$

With NCCs, there are double incentives. Thus, the derivative with NCCs is strictly larger: The marginal benefit gets multiplied with $\frac{1}{1-E} > 1$. Therefore, the positive term is larger. The negative term is the same. Since at κ_3 the derivative without NCC is zero, the derivative with NCC is positive.

$$\left.\frac{\partial \pi(\underline{w}, b)}{\partial b}\right|_{b=b_2^{**}(\kappa_3),\bar{v}=0} = \frac{E'(b_2^{**}(\kappa_3))}{1 - E(b_2^{**}(\kappa_3))} \cdot (V - b_2^{**}(\kappa_3)) - E(b_2^{**}(\kappa_3)) > 0 \tag{50}$$

To sum up: The profit's first derivative evaluated at the bonus wage $b_2^{**}(\underline{w})$ is continuous and monotonically increasing. It is strictly negative at κ_1 and strictly positive at κ_3. Thus, its root, κ_2, exists and lies strictly in-between, $\kappa_1 < \kappa_2 < \kappa_3 < 0$.

□

For all minimum wages below κ_2, the optimal contract and, thus, the profit is the same as in the benchmark. For minimum wages above κ_2, an NCC is used and the principal's profits are strictly larger than in the benchmark: Strict quasi-concavity of the profit in the bonus wage means that the maximum is unique. The principal could mimic the world without NCC. He does, however, not want to. Uniqueness of the maximimum means that the optimal contract with NCC is strictly better than the optimal contract without NCC.

A minimum wage of zero For $\underline{w} = 0$, the second combination is not feasible. The binding participation constraint with no NCC implies that the bonus wage has to be zero. In the second combination, the bonus wage has to be strictly positive. Furthermore, the fourth combination is not feasible. The binding participation constraint with no bonus wage implies that the most severe NCC is no NCC. In the fourth combination, the NCC has to be strictly negative. Thus, the optimal contract has to have both a bonus wage and an NCC.

Having established that the first, the second, and then the third combination are optimal in an increasing minimum wage, we now turn to positive minimum wages.

Positive minimum wages For positive minimum wages, contracts from the second combination are not feasible: It is not possible to make the participation constraint binding without an NCC. In this range, only the third or the fourth combination can be optimal. We show that starting at a minimum wage of 0, the third combination is optimal. We derive one condition on the effort cost function for the existence and one condition for the uniqueness of there being a minimum wage $\kappa_4 > 0$ such that for all $\underline{w} < \kappa_4$, the third combination is optimal and for all $\underline{w} \geq \kappa_4$, the fourth combination is optimal. At κ_4, the principal stops using a bonus wage. Instead, all incentives follow from an NCC. If the condition is not met, the third combination is optimal for all positive minimum wages.

To get uniqueness of κ_4, we need an assumption on the cost function. For all bonus wages, it has to hold that $\frac{c'''(E(b-\bar{v}(\underline{w},b)))}{c''(E(b-\bar{v}(\underline{w},b)))} > \frac{1}{1-E(b-\bar{v}(\underline{w},b))} - \frac{1}{E(b-\bar{v}(\underline{w},b))}$. While this assumption is stronger than the assumption to get strict quasi-concavity, it is also implied by our assumptions in Section 3 that imply strict concavity of the objective function. With this assumption, we can show that there is at most one minimum wage at which the principal switches between the third and the fourth combination. Furthermore, this assumption implies that the switch is such that for lower minimum wages there is a positive bonus wage, while for higher minimum wages, the optimal bonus wage is zero.

The strategy of the proof is to determine the sign of the marginal profit of the bonus wage, evaluated at a bonus wage of 0. If it is positive, there is an inner solution and the optimal bonus wage is positive. To make the participation constraint binding, an NCC is needed. The optimal contract is, thus, from the third combination. Using no bonus wage is optimal if it is negative. Then, the first unit of the bonus wage is not worth the marginal cost. The optimal contract is, thus, from the fourth combination. The assumption on the uniqueness implies that the every switch of the sign goes from the positive to the negative.

To prove existence, we show that the sign of the marginal profit of the bonus wage, evaluated at a bonus wage of 0, is initially positive. We assume that the condition for uniqueness is met. The marginal profit of the first unit of bonus wage is continuous in the minimum wage. Because its sign is initially positive, can switch its sign at most once, and the marginal profit's continuity, the sign in the limit is negative if and only if the switch happened for a finite minimum wage. We then derive the (necessary and sufficient) condition under which the sign is negative in the limit. This is the conition for the existence of κ_4. To determine the sign in the limit, we use L'Hôpital's rule.

Lemma 10. *If for all bonus wages* $\frac{c'''(E(b-\bar{v}(\underline{w},b)))}{c''(E(b-\bar{v}(\underline{w},b)))} > \frac{1}{1-E(b-\bar{v}(\underline{w},b))} - \frac{1}{E(b-\bar{v}(\underline{w},b))}$, *then there is at most one minimum wage for which* $\frac{\partial \pi}{\partial b}\Big|_{b=0} = 0$

Proof. Again, we will employ the same strategy of proof as above to show the uniqueness of a critical point. The critical point in the minimum wage is characterized by

$$\frac{\partial \pi}{\partial b}\Big|_{b=0} = \frac{E'(-\bar{v}(\underline{w},0))}{1 - E(-\bar{v}(\underline{w},0))} \cdot V - E(-\bar{v}(\underline{w},0)) \stackrel{!}{=} 0. \tag{51}$$

This is the marginal profit by increasing the bonus wage starting at a bonus wage of zero. That is, for which minimum wage it is optimal not to use the bonus wage $b = 0$. Since $\underline{w} > 0$, the principal will use an NCC to provide incentives. The optimal contract falls into the fourth combination.

Thus, a critical point is defined by

$$V = \frac{E(-\bar{v}(\underline{w}, 0)) \cdot (1 - E(-\bar{v}(\underline{w}, 0)))}{E'(-\bar{v}(\underline{w}, 0))} \tag{52}$$

As above, we show that this critical point is unique if it implies that the marginal profit from the first unit of bonus wage hits zero from above. Then, to the left of the critical point, it is optimal to use positive bonus wages; to the right of the critical point, it is optimal to use no bonus wages. We want to show that

$$\left.\frac{\partial \pi}{\partial b}\right|_{b=0} \overset{!}{=} 0 \quad \Longrightarrow \quad \frac{\partial \left(\left.\frac{\partial \pi}{\partial b}\right|_{b=0} \right)}{\partial \underline{w}} < 0 \tag{53}$$

To do so, we compute this derivative (we again omit the arguments and express $E(-\bar{v}(\underline{w}, 0))$ as E to improve readability)

$$\frac{\partial \left(\left.\frac{\partial \pi}{\partial b}\right|_{b=0} \right)}{\partial \underline{w}} = \frac{(1 - E)E'' + E' \cdot E'}{(1 - E)^3} \cdot V - \frac{E'}{1 - E} \tag{54}$$

Plugging in the characterization of a critical point ($V = \frac{E \cdot (1-E)}{E'}$) and simplifying yields

$$\frac{c'''(E)}{c''(E)} > \frac{1}{1 - E} - \frac{1}{E}, \tag{55}$$

which holds by assumption. $\square$

Lemma 11. *Assume that for all bonus wages* $\frac{c'''(E(b-\bar{v}(\underline{w},b)))}{c''(E(b-\bar{v}(\underline{w},b)))} > \frac{1}{1-E(b-\bar{v}(\underline{w},b))} - \frac{1}{E(b-\bar{v}(\underline{w},b))}$. *If*

$$\lim_{\underline{w} \to \infty} \frac{c'''(E(-\bar{v}(\underline{w}, 0)))}{[c''(E(-\bar{v}(\underline{w}, 0)))]^2} \cdot V < 1, \tag{56}$$

then there is a minimum wage $\kappa_4 > 0$ such that the optimal contract uses a bonus wage for all lower minimum wages and the optimal contract uses no bonus wage for all larger minimum wages.

Proof. κ_4 exists if there is a positive minimum wage that equates the marginal benefit and the marginal cost of the first unit of bonus wage.

$$\left.\frac{\partial \pi}{\partial b}\right|_{b=0} = \frac{E'(-\bar{v}(\underline{w}, 0))}{1 - E(-\bar{v}(\underline{w}, 0))} \cdot V - E(-\bar{v}(\underline{w}, 0)) = 0. \tag{57}$$

We have shown above that there is at most one such minimum wage. Furthermore, we have shown that the intersection has to be such that the marginal benefit intersects the marginal cost from above. Now we show under which conditions there is at least one such intersection.

Initially, the marginal benefit is larger than the marginal cost. Consider the minimum wage $\underline{w} = 0$. Together with $b = 0$, this implies that $\bar{v} = 0$ to make the PC binding and that the equilibrium effort is 0. The marginal benefit is $\frac{E'(0)}{1} \cdot V$. Since $E'(\cdot) \equiv \frac{1}{c'(E(\cdot))}$, this is strictly positive for a minimum wage of 0. The marginal cost is $E(0) = 0$ at a minimum wage of 0. Hence, we showed that for $\underline{w} = 0$, the bonus wage's marginal benefit is higher than the marginal cost. By continuity, this also holds for some positive minimum wages.

Since the marginal benefit is initially larger, can intersect the marginal cost only from above, and both are continuous, it is sufficient to look at the limits of the minimum wage's going to infinity. Without a bonus wage, the non-compete clause will then become ever stronger which implies that the equilibrium effort will go to 1.

First, consider the marginal cost of increasing the bonus wage starting at $b = 0$. When the minimum wage goes to infinity, the equilibrium effort goes to 1 and the marginal cost goes to 1. Second, consider the marginal benefit of increasing the bonus wage starting at $b = 0$.. When the minimum wage goes to infinity, the equilibrium effort goes to 1 and the marginal benefit goes to $\lim_{\underline{w}\to\infty} \frac{E'(-\bar{v}(\underline{w},0))}{1-E(-\bar{v}(\underline{w},0))} \cdot V$. Let us consider numerator and denominator separately. The numerator goes to zero because $\lim_{\underline{w}\to\infty} E'(-\bar{v}(\underline{w}, 0)) = \lim_{\underline{w}\to\infty} \frac{1}{c''(E(-\bar{v}(\underline{w},0)))}$ and $\lim_{\underline{w}\to\infty} c''(E(-\bar{v}(\underline{w}, 0))) = \infty$. This follows because $\underline{w} \to \infty$ implies that $E(-\bar{v}(\underline{w}, 0)) \to 1$ which implies that $c'(e) \to \infty$. For the same reason, the denominator also goes to zero.

Thus, we use L'Hôpital's rule to evaluate $\lim_{\underline{w}\to\infty} \frac{E'(-\bar{v}(\underline{w},0))}{1-E(-\bar{v}(\underline{w},0))} \cdot V$. In order to use L'Hôpital's rule we need to check two conditions:

First, we must check that for all (positive) finite minimum wages $\frac{\partial(1-E(-\bar{v}(\underline{w},0)))}{\partial \underline{w}} \neq 0$. This condition is fulfilled because $\frac{\partial(1-E(-\bar{v}(\underline{w},0)))}{\partial \underline{w}} = -\frac{E'(-\bar{v}(\underline{w},0))}{1-E(-\bar{v}(\underline{w},0))}$. By assumption, the numerator is positive.

Second, we must check that limit of the ratio of the derivatives exists. This condition is fulfilled because

$$\lim_{\underline{w}\to\infty} \frac{\frac{\partial E'(-\bar{v}(\underline{w},0))}{\partial \underline{w}}}{\frac{\partial(1-E(-\bar{v}(\underline{w},0)))}{\partial \underline{w}}} \cdot V = \lim_{\underline{w}\to\infty} \frac{c'''(E(-\bar{v}(\underline{w}, 0)))}{[c''(E(-\bar{v}(\underline{w}, 0)))]^2} \cdot V < 1 \tag{58}$$

by assumption.

All in all, L'Hôpital's rule yields

$$\lim_{\underline{w}\to\infty} \frac{E'(-\bar{v}(\underline{w}, 0))}{1 - E(-\bar{v}(\underline{w}, 0))} \cdot V = \lim_{\underline{w}\to\infty} \frac{\frac{\partial E'(-\bar{v}(\underline{w},0))}{\partial \underline{w}}}{\frac{\partial(1-E(-\bar{v}(\underline{w},0)))}{\partial \underline{w}}} \cdot V = \lim_{\underline{w}\to\infty} \frac{c'''(E(-\bar{v}(\underline{w}, 0)))}{[c''(E(-\bar{v}(\underline{w}, 0)))]^2} \cdot V \tag{59}$$

Therefore, there is a critical minimum wage κ_4 if and only if

$$\lim_{\underline{w}\to\infty} \frac{c'''(E(-\bar{v}(\underline{w}, 0)))}{[c''(E(-\bar{v}(\underline{w}, 0)))]^2} V < 1. \tag{60}$$

The assumption can also be expressed in properties of the effort cost function. It is an assumption on the convergence speeds of the second and the third derivative. Note that both

$c''(\cdot)$ and $c'''(\cdot)$ go to infinity when the minimum wage goes to infinity because the equilibrium effort goes to 1 and then $c'(\cdot)$ goes to infinity. Therefore, if $(c''(\cdot))^2$ goes to infinity strictly faster than $c'''(\cdot)$, the marginal benefit converges to zero. If the convergence of $(c''(\cdot))^2$ and $c'''(\cdot)$ has the same speed, the limit is some number. If this number times V is less than 1, the assumption is also satisfied. Whenever the convergence of $c'''(\cdot)$ is faster than that of $(c''(\cdot))^2$, the assumption does not hold.

$\square$

Having characterized which constraints bind in which combination, we can now characterize the optimal contract in each combination. Note that the contract in the first (second) combination mirrors the one in Case 1 (2). The base and bonus wages are equal and the principal does not want to use a NCC. The computations of base and bonus wage is therefore identical to the computations in Case 1 and 2 in Proposition 1 and therefore are skipped here for clarity. We now characterize the optimal bonus wage and the optimal non-compete clause depending on the the effort level that will be chosen in each combination.

Next, we consider the third combination.

Third Combination Let E be the effort level that the agent chooses given the contract. $MWC1$ binds which implies that $w = \underline{w}$. PC binds as well. We substitute IC and $MWC1$ into PC and rewrite to get

$$\bar{v} = c(E) - E \cdot c'(E) - \underline{w} \tag{61}$$

where we suppress the arguments of E and $\bar{v}$ for simplicity.

Combining $MWC1$, PC and IC by substituting for $\bar{v}$ gives

$$b = (1 - E) \cdot c'(E) + c(E) - \underline{w} \tag{62}$$

Now, we substitute for w and b in P's objective function to get.

$$\pi = E \cdot V - (1 - E) \cdot \underline{w} - E \cdot (1 - E) \cdot c'(E) - E \cdot c(E) \tag{63}$$

P maximizes over the incentive-compatible effort level and hence $E = \hat{e}$ is chosen such that

$$c(\hat{e}) + (1 - \hat{e}) \cdot c'(\hat{e}) + \hat{e} \cdot (1 - \hat{e}) \cdot c''(\hat{e}) = V + \underline{w}. \tag{64}$$

Next, we consider the fourth combination.

Fourth Combination Let E be the effort level that the agent chooses given the contract. $MWC1$ binds which implies that $w = \underline{w}$. $MWC2$ binds which together with the binding

$MWC1$ implies that $b = 0$. $\bar{v}$ is then determined by the binding participation constraint

$$\bar{v} = -\frac{\underline{w} - c(E)}{1 - E} \tag{65}$$

The optimal effort choice is then determined by the IC and hence $E = \tilde{e}$ is characterized by

$$\underline{w} + \tilde{e} \cdot c'(\tilde{e}) - c(\tilde{e}) = c'(\tilde{e}) \tag{66}$$

□

8.3 Proof of Proposition 3

Proof of Proposition 3. We show that the equilibrium effort is increasing in the minimum wage if P uses a NCC, that is, in the third and fourth combination.

We start with the third combination, in which the optimal contract has both a bonus wage and an NCC. We, therefore, need to evaluate their combined effect on the effort. The equilibrium effort is defined by $c'(E) = b(\underline{w}) - \bar{v}(\underline{w}, b(\underline{w}))$. Since the marginal cost is increasing, the equilibrium effort gets larger if the right hand side gets larger. Thus, we need to show that the right hand side is increasing in the minimum wage. Taking the derivative with respect to the minimum wage of the right hand side yields

$$\frac{\partial b(\underline{w})}{\partial \underline{w}} - \left(\frac{\partial \bar{v}(\underline{w}, b(\underline{w}))}{\partial \underline{w}} + \frac{\partial \bar{v}(\underline{w}, b(\underline{w}))}{\partial b(\underline{w})} \cdot \frac{\partial b(\underline{w})}{\partial \underline{w}} \right). \tag{67}$$

To show that this expression is positive, we look at its parts in turn. We already calculated the effect of a change in the minimum wage and in the bonus wage on the NCC that makes the participation constraint bind in Lemma 6. For convenience, we reproduce the result here:

$$\frac{\partial \bar{v}(\underline{w}, b)}{\partial \underline{w}} = -\frac{1}{1 - E(b - \bar{v})} \qquad \frac{\partial \bar{v}(\underline{w}, b)}{\partial b} = -\frac{E(b - \bar{v})}{1 - E(b - \bar{v})} \tag{68}$$

It remains to characterize how the optimal bonus wage changes in the minimum wage. Again, we use the implicit function theorem on the first-order condition of the expected profit maximization problem. The FOC of P's expected profit with respect to the bonus wage is

$$Z \equiv E'(b - \bar{v}) \cdot \left(1 - \frac{\partial \bar{v}}{\partial b}\right) \cdot (V - b) - E(b - \bar{v}) = 0 \tag{69}$$

We will from now on skip the argument of E for clarity. Before we apply the implicit function theorem to this equation to see how b changes in $\underline{w}$, we need two intermediary derivatives: $\frac{\partial^2 \bar{v}}{\partial b \partial \underline{w}}$ and $\frac{\partial^2 \bar{v}}{\partial b^2}$. And again, we can use Lemma 6, which shows that $\frac{\partial \bar{v}}{\partial b} = -\frac{E}{1-E}$.

Thus,

$$\frac{\partial^2 \bar{v}}{\partial b \partial \underline{w}} = -\frac{E' \cdot (1 - E) \cdot \frac{\partial \bar{v}}{\partial \underline{w}} + E' \cdot E \cdot \frac{\partial \bar{v}}{\partial \underline{w}}}{(1 - E)^2} = -\frac{E'}{(1 - E)^3}, \tag{70}$$

and

$$\frac{\partial^2 \bar{v}}{\partial b^2} = \frac{-E' \cdot (1-E) \cdot \left(1 - \frac{\partial \bar{v}}{\partial b}\right) - E' \cdot E \cdot \left(1 - \frac{\partial \bar{v}}{\partial b}\right)}{(1-E)^2} = -\frac{E'}{(1-E)^3}. \tag{71}$$

Since Z is continuously differentiable, the implicit function theorem can be used to get the derivative of b with respect to $\underline{w}$.

$$\frac{\partial b}{\partial \underline{w}} = -\frac{\frac{\partial Z}{\partial \underline{w}}}{\frac{\partial Z}{\partial b}} = -\frac{-E'' \cdot \frac{\partial \bar{v}}{\partial \underline{w}} \cdot \left(1 - \frac{\partial \bar{v}}{\partial b}\right) \cdot (V-b) - E' \cdot \frac{\partial^2 \bar{v}}{\partial b \partial \underline{w}} \cdot (V-b) + E' \cdot \frac{\partial \bar{v}}{\partial \underline{w}}}{E'' \cdot \left(1 - \frac{\partial \bar{v}}{\partial b}\right)^2 \cdot (V-b) - E' \cdot \frac{\partial^2 \bar{v}}{\partial b^2} \cdot (V-b) - 2E' \cdot \left(1 - \frac{\partial \bar{v}}{\partial b}\right)} \tag{72}$$

$$= -\frac{\left(\frac{1}{1-E} - \frac{c'''(E)}{c''(E)}\right) \cdot \frac{V-b}{(1-E)^2 \cdot (c''(E))^2} - \frac{1}{(1-E) \cdot c''(E)}}{\left(\frac{1}{1-E} - \frac{c'''(E)}{c''(E)}\right) \cdot \frac{V-b}{(1-E)^2 \cdot (c''(E))^2} - \frac{2}{(1-E) \cdot c''(E)}} \tag{73}$$

Since $E(\cdot) < 1$, $c''(\cdot) > 0$, $c'''(\cdot) > 0$, $b \leq V$ and concavity $\left(\frac{c'''(E)}{c''(E)} > \frac{1}{1-E}\right)$, we get that $\frac{\partial b}{\partial \underline{w}} < 0$. Hence, a higher minimum wage implies a lower bonus wage.

Let us recap what we have shown so far. On the one hand, we found that a higher minimum wage leads to a lower bonus wage which provides less incentives. On the other hand, we found that a higher minimum wage implies a more severe NCC which provides more incentives. It remains to show that the effect on the NCC is stronger than on the bonus wage. Rearranging the marginal change of the incentives in the minimum wage (67) and plugging in yields

$$-\frac{\partial \bar{v}(\underline{w}, b(\underline{w}))}{\partial \underline{w}} + \frac{\partial b(\underline{w})}{\partial \underline{w}} \cdot \left(1 - \frac{\partial \bar{v}(\underline{w}, b(\underline{w}))}{\partial b}\right) \tag{74}$$

$$= \frac{1}{1-E} + \frac{\partial b(\underline{w})}{\partial \underline{w}} \cdot \left(1 + \frac{E}{1-E}\right) \tag{75}$$

$$= \frac{1}{1-E} \cdot \left(1 + \frac{\partial b(\underline{w})}{\partial \underline{w}}\right). \tag{76}$$

To show that this is positive, it now suffices to show that the bracket is positive. That is, $\frac{\partial b(\underline{w})}{\partial \underline{w}} > -1$.

Consider $-\frac{\partial b}{\partial \underline{w}}$ as it is characterized in equation (73). For simplicity, let

$$x \equiv \left(\frac{1}{1-E} - \frac{c'''(E)}{c''(E)}\right) \frac{V-b}{(1-E)^2 (c''(E))^2} \quad \text{and} \quad y \equiv \frac{1}{(1-E)c''(E)}. \tag{77}$$

We have that $x < 0$ and $y > 0$. It is then easy to check that $-\frac{\partial b}{\partial \underline{w}} = \frac{x-y}{x-2y} < 1$. Which was to be shown. Therefore, the equilibrium effort is increasing in the minimum wage in the third combination.

We now show that in the fourth combination the equilibrium effort is also increasing in the minimum wage. The principal does not use a bonus wage anymore. Lemma 6 shows that $\frac{\partial \bar{v}}{\partial \underline{w}} = -\frac{1}{1-E}$ where $E(-\bar{v}(\underline{w}))$ is the solution to the agent's incentive problem. This shows that higher minimum wages lead to more severe NCCs which then leads to higher effort through the incentive constraint.

To sum up, if $\underline{w} > \kappa_2$, then higher minimum wages lead to more effort incentives, and, thus a non-monotonicity of the equilibrium effort.

$\square$

8.4 Proof of Proposition 4

Proof of Proposition 4. We show that the principal induces a higher effort level than first-best effort if the minimum wage is sufficiently large. Intuitively, it is clear that as the minimum wage becomes higher, the NCC becomes stricter which then leads to higher effort. Therefore, eventually, the agent exerts more than first-best effort. We distinguish two different cases. First, more than first-best effort might be achieved in the third combination. Second, more than first-best effort might be achieved in the fourth combination.

First, we start with the third combination. Note that Proposition 3 states that the equilibrium effort level increases in the minimum wage. Using the implicit characterization of $\hat{e}$ in Proposition 2 gives us that $\hat{e} \geq e^{FB}$ whenever $\underline{w} \geq c(\hat{e}) - \hat{e} \cdot c'(\hat{e}) + \hat{e} \cdot (1-\hat{e}) \cdot c''(\hat{e})$. Furthermore, for a given minimum wage, the third combination only applies if the bonus wage is strictly positive. From Proposition 2 we get that $b > 0$ whenever $\underline{w} < (1-\hat{e}) \cdot c'(\hat{e}) + c(\hat{e})$. Taken these two together, we get that P induces higher effort than first-best in the third combination, if there exist minimum wages for which the following condition is satisfied:

$$c(\hat{e}) - \hat{e} \cdot c'(\hat{e}) + \hat{e} \cdot (1-\hat{e}) \cdot c''(\hat{e}) \leq \underline{w} < (1-\hat{e}) \cdot c'(\hat{e}) + c(\hat{e}) \tag{78}$$

To satisfy this condition, we need that the right bound must be strictly higher than the left bound which then boils down to

$$c(\hat{e}) - \hat{e} \cdot c'(\hat{e}) + \hat{e} \cdot (1-\hat{e}) \cdot c''(\hat{e}) < (1-\hat{e}) \cdot c'(\hat{e}) + c(\hat{e}) \tag{79}$$

$$\hat{e} \cdot (1-\hat{e}) \cdot c''(\hat{e}) < c'(\hat{e}) \tag{80}$$

To sum up, under this condition the principal induces higher than first-best effort level (in the third combination) whenever $\underline{w} \geq c(\hat{e}) - \hat{e} \cdot c'(\hat{e}) + \hat{e} \cdot (1-\hat{e}) \cdot c''(\hat{e})$. Note that it does not matter if the fourth combination exists or not if this condition is fulfilled.

Now, consider the case that this condition does not hold. We have that $\hat{e} \cdot (1-\hat{e}) \cdot c''(\hat{e}) \geq c'(\hat{e})$. We now show that the fourth combination must exist and that first-best effort will eventually be reached in the fourth combination. First, assume (by contradiction) that the fourth combination does not exist, i.e. the bonus wage is always strictly positive. As discussed above this means that $\underline{w} < (1-\hat{e}) \cdot c'(\hat{e}) + c(\hat{e})\ \forall \hat{e}$. But this is a contradiction because the minimum wage goes to infinity. If the minimum wage is large enough the inequality is not satisfied anymore. Once it is not satisfied anymore, it will continue to not be satisfied for even higher minimum wages. Note that the LHS ($\underline{w}$) goes to infinity when the minimum wage goes to infinity whereas the RHS ($(1-\hat{e}) \cdot c'(\hat{e}) + c(\hat{e})$) does not go to infinity when the minimum wage goes to infinity because $\hat{e} < e^{FB} < 1$.

We showed that if $\hat{e} \cdot (1-\hat{e}) \cdot c''(\hat{e}) \geq c'(\hat{e})$, we enter the fourth combination for a high enough minimum wage and will then stay in the fourth combination for higher minimum wages. Using the implicit characterization of $\tilde{e}$ in Proposition 2 gives us that $\tilde{e} \geq e^{FB}$ whenever $\underline{w} \geq V - \tilde{e} \cdot c'(\tilde{e}) + c(\tilde{e})$. Assume (by contradiction) that effort in the fourth combination is always strictly lower than first-best effort, $\underline{w} < V - \tilde{e} \cdot c'(\tilde{e}) + c(\tilde{e})\ \forall \tilde{e}$. But this is again a contradiction when the minimum wage goes to infinity. If the minimum wage is large enough, the inequality is not satisfied anymore. Once it is not satisfied anymore, it will continue to not be satisfied for even higher minimum wages. Note that the LHS ($\underline{w}$) goes to infinity when the minimum wage goes to infinity whereas the RHS ($V - \tilde{e} \cdot c'(\tilde{e}) + c(\tilde{e})$) does not go to infinity when the minimum wage goes to infinity because $\tilde{e} < e^{FB} < 1$.

To sum up, if $\hat{e} \cdot (1-\hat{e}) \cdot c''(\hat{e}) \geq c'(\hat{e})$, P induces higher effort than first-best (in the fourth combination) whenever $\underline{w} \geq V - \tilde{e} \cdot c'(\tilde{e}) - c(\tilde{e})$. □

8.5 Proof of Proposition 5

Proof of Proposition 5. Let the principal's expected profit be strictly quasi-concave in the bonus wage, that is,

$$\frac{c'''(E(b-\bar{v}(\underline{w},b)))}{c''(E(b-\bar{v}(\underline{w},b)))} > \frac{1}{1-E(b-\bar{v}(\underline{w},b))} - \frac{2}{E(b-\bar{v}(\underline{w},b))} \tag{81}$$

holds for all minimum wages.

With a bounded NCC we have the additional constraint that $\bar{v} \geq \underline{\bar{v}}$. This changes P's maximization problem to

$$\max_{b} \quad -\underline{w} + E(b-\bar{v}) \cdot (V-b) \tag{82}$$

$$\text{subject to} \quad \bar{v} = \max\{\bar{v}(\underline{w},b), \underline{\bar{v}}\} \tag{NCC}$$

$$b \geq b_2^{**}(\underline{w}), \tag{83}$$

where again $\bar{v}(\underline{w},b) = -\frac{\underline{w}+E(b-\bar{v}(\underline{w},b))\cdot b - c(E(b-\bar{v}(\underline{w},b)))}{1-E(b-\bar{v}(\underline{w},b))}$. The NCC condition already uses that the profit is increasing in more severe non-compete clauses (because the optimal bonus wage is smaller than the success payoff). Thus, the principal would never use a NCC that is less severe than the one NCC that makes the PC binding ($\bar{v}(\underline{w},b)$), except this would violate the bound on NCCs ($\underline{\bar{v}}$). As a result, the optimal NCC is determined by which constraint binds first: the participation constraint ($\bar{v}(\underline{w},b)$) or the bound on NCCs ($\underline{\bar{v}}$).

We now split the minimum wages into two ranges. One for which the bound on NCCs is insubstantial and one for which the bound on NCCs makes the formerly optimal contracts infeasible. This is possible because the optimal $\bar{v}$ without a bound decreases continuously and strictly monotonically in the minimum wage above κ_2. Moreover, $\bar{v}$ lies between zero and minus infinity such that any bound binds for some minimum wages. We define $\underline{w}_{bound}$ as the minimum wage for which the optimal contract without a bound on NCCs uses an NCC that is exactly the bound. That is, the optimal contract is $(\underline{w}_{bound}, b(\underline{w}_{bound}), \underline{\bar{v}})$. As argued above, $\underline{w}_{bound}$ exists and is unique for each bound $\underline{\bar{v}}$.

Case i) $\underline{w} < \underline{w}_{bound}$. For these minimum wages, the optimal contract without a bound on the NCCs does not violate the bound on the NCCs. Since the bound only introduces another constraint, these contracts remain optimal. The bound on NCCs can be ignored.

Case ii) $\underline{w} \geq \underline{w}_{bound}$. For all minimum wages above $\underline{w}_{bound}$, the optimal contracts without a bound on the NCCs are not feasible anymore: they violate the bound on NCCs. In the simplified problem, the only choice variable of the principal is the bonus wage. Thus, the optimal NCC is implicitly defined by the optimal bonus wage.

For $\underline{w} \geq \underline{w}_{bound}$, the constraint $b \geq b_2^{**}(\underline{w})$ can be ignored. The constraint said that, firstly, the participation constraint must not be violated if $\bar{v} = 0$, and, secondly, that the bonus wage must be non-negative. Since the optimal NCC at $\underline{w}_{bound}$ is strictly negative (and because of its comparative statics), we know that the participation constraint without an NCC would be satisfied. Furthermore, the optimal bonus wage can never be negative because there is a profitable deviation, as argued in the proof of Proposition 2; this deviation exists independently of a bound on the NCC.

For minimum wages $\underline{w} \geq \underline{w}_{bound}$, the optimal contract without a bound is either the one from the third combination or the one from the fourth combination. We can distinguish these as different cases. For each case, we show that once the binding NCC is optimal, it will remain optimal for all larger minimum wages, and we characterize the optimal bonus wage.

a) The optimal contract for the minimum wage $\underline{w}_{bound}$ is from the third combination. That is, the optimal bonus wage without a bound is strictly positive. Thus, the optimal bonus wage is determined by the first-order condition; the bonus wage for which the marginal profit gets zero. It is unique because the objective function is quasi-concave by assumption. For $\underline{w} = \underline{w}_{bound}$, the optimal contract remains optimal and just makes the bound on the NCC binding. Thus, the marginal profit at the bonus wage $b(\underline{w}_{bound})$ is 0. We will reconsider this particular minimum wage after describing the marginal profit in the bonus wage in general.

How does the marginal profit with respect to the bonus wage behave for a fixed minimum wage $\underline{w} > \underline{w}_{bound}$? For a sketch of the marginal profit, see Figure 13.

As mentioned above, starting at $b_2^{**}(\underline{w})$, the marginal profit is positive. When increasing the bonus wage, it keeps being positive. Moreover, it has the same values as in the problem without a bound. Then, the bonus wage, $b_{bound}(\underline{w})$, is reached that allows the principal to reach the bound $\bar{v}(\underline{w}, b_{bound}(\underline{w})) \stackrel{!}{=} \underline{\bar{v}}$. Importantly, at this minimum wage, the derivative is still positive: The optimal bonus wage is $b(\underline{w})$ and by the case assumption it is true that $\bar{v}(\underline{w}, b(\underline{w})) < \underline{\bar{v}}$. Because $\bar{v}(\underline{w}, b)$ is decreasing in the bonus wage, and because the root of the first-order condition is unique, we know that $b_{bound}(\underline{w}) < b(\underline{w})$. From $b_{bound}(\underline{w})$ on, the principal cannot make the NCC more severe when increasing the bonus wage. Therefore, there are no double incentives

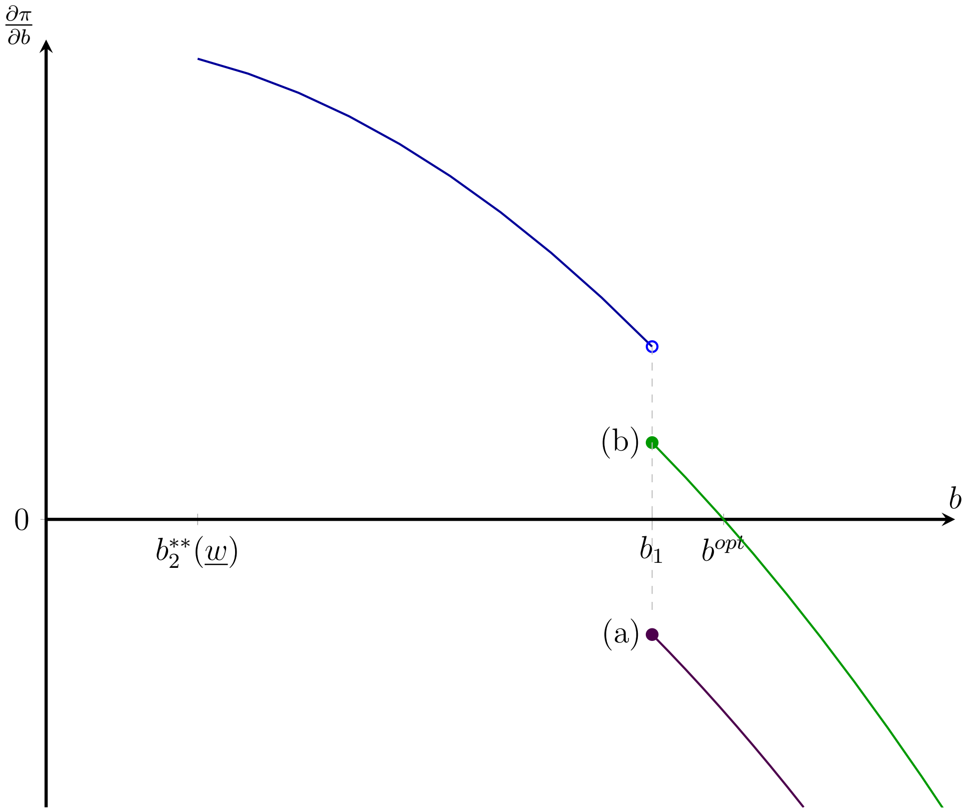

Figure 13: The Derivative of the Profit with Respect to the Bonus Wage Jumps Down as Soon as the Bound on the NCC Binds. If (a) the Jump Ends below Zero, the Agent Gets no Rent. If (b) the Jump Ends above Zero, the Agent Gets a Rent and the Optimal Bonus Wage is the Same for Higher Minimum Wages. Drawn for a Concave Objective Function.

anymore. The marginal profit, thus, jumps downwards to its level in the benchmark; formally

$$\lim_{b' \to b_{bound}(\underline{w})^-} \frac{\partial \pi}{\partial b}(\underline{w}, b') = \frac{E'(b_{bound}(\underline{w}) - \underline{v})}{1 - E(b_{bound}(\underline{w}) - \underline{v})} \cdot (V - b_{bound}(\underline{w})) - E(b_{bound}(\underline{w}) - \underline{v}) > 0 \quad (84)$$

$$\lim_{b' \to b_{bound}(\underline{w})^+} \frac{\partial \pi}{\partial b}(\underline{w}, b') = E'(b_{bound}(\underline{w}) - \underline{v}) \cdot (V - b_{bound}(\underline{w})) - E(b_{bound}(\underline{w}) - \underline{v}). \quad (85)$$

For bonus wages after the jump, the profit function is strictly concave in the bonus wage, as in the benchmark.[19] The marginal profit is, thus, strictly decreasing.

The optimal bonus wage is now either $b_{bound}(\underline{w})$, if the marginal profit jumps (weakly) below zero, or a higher bonus wage if the marginal profit remains positive after the jump. In any case, this implies that $\bar{v}(\underline{w}, b) \leq \underline{v}$ in the optimum. Therefore, the bound on the NCC is the binding constraint; thus $\underline{v}$ is the optimal NCC.

To find the optimal bonus wage, we have to find out which constraints will bind. This depends on whether the optimal bonus wage is at the jump point or not. If it is at the jump point, the participation constraint binds ($\bar{v}(\underline{w}, b_{bound}(\underline{w})) = \underline{v}$); which implies that the agent gets no rent. If it is to the right of the jump point, the participation constraint is slack because the NCC that would make the participation constraint binding lies outside the bound. Therefore, it is slack; which implies that the agent gets a rent.

[19]The second derivative is $E''(\cdot) \cdot (V - b) - 2E'(\cdot) < 0$. $E''(\cdot)$ is globally negative and $E'(\cdot)$ is globally positive.

For the other constraints (MWC_1, MWC_2, NCC), the same reasoning as above, in the proof of Proposition 2, applies. The minimum wage condition on the base wage binds. Otherwise, there is a profitable deviation. Due to the case assumption, the optimal bonus wage without a bound is positive, thus MWC_2 is slack. With a bound, it might also be that MWC_2 binds if ignoring the constraint leads to a violation. Due to the case assumption, an NCC is used, which means that the NCC feasibility constraint is slack. As a result, the optimal base wage always is the minimum wage and, as shown above, the optimal NCC is the binding NCC.

Firstly, we now determine the optimal bonus wage depending on where the jump ends and then, secondly, we show that there always is a range of minimum wages for which the jump ends in the negative.

We start with the case in which the marginal profit's jump ends in the non-positive. In this case, the optimal bonus wage is at the jump point and makes the participation constraint binding. Thus, the participation constraint pins down the optimal bonus wage. How does the optimal bonus wage change in the minimum wage? We use the implicit function theorem to show that the bonus wage that makes the participation constraint binding is strictly decreasing in the minimum wage. Rearrange the binding participation constraint to

$$Z \equiv \underline{w} + E(b_{bound} - \underline{v}) \cdot (b_{bound} - \underline{v}) + \bar{v} - c(E(b_{bound} - \underline{v})) = 0. \tag{86}$$

Because this is continuously differentiable, the implicit function theorem can be used to get the derivatives of b_{bound} with respect to $\underline{w}$.

$$\frac{\partial b_{bound}(\underline{w})}{\partial \underline{w}} = -\frac{\frac{\partial Z}{\partial \underline{w}}}{\frac{\partial Z}{\partial b_{bound}}} = -\frac{1}{E'(\cdot) \cdot (b_{bound} - \underline{v}) + E(\cdot) - c'(E(\cdot)) \cdot E'(\cdot)} \tag{87}$$

$$= -\frac{1}{E(\cdot)} \tag{88}$$

where we suppress the argument of E for clarity. The simplification is due to the agent's first-order constraint, $b_{bound} - \underline{v} - c'(E) = 0$. Since $E(b_{bound} - \underline{v}) > 0$, the bonus wage that makes the participation constraint binding is strictly decreasing in the minimum wage.

Further, we can say that the optimal bonus wage with a bound lies below the optimal bonus wage without a bound on the NCC. In both cases, the participation constraint is binding and the bonus wage is positive (due to the case assumption). Without a bound on the NCC, the optimal NCC is weakly more severe than the bound because $\underline{w} \geq \underline{w}_{bound}$; strictly more severe if $\underline{w} > \underline{w}_{bound}$. For a fixed minimum wage, a strictly more severe NCC needs a strictly larger bonus wage to keep the participation constraint satisfied. Thus, with a bound on the NCC, the optimal bonus wage is smaller.

When the optimal bonus wage hits zero, it stays at zero for all larger minimum wages. It can never become negative because of the profitable deviation. Note that when the bonus wage hits zero, for all larger minimum wages the participation constraint is slack and the agent gets a rent.

We now look at the optimal bonus wage if the jump in the marginal benefit ends in the positive and the participation constraint can be ignored. The optimal bonus wage is constant because the minimum wage does not enter the problem anymore. The optimal bonus wage is determined by the marginal profit's being zero or the minimum wage condition on the bonus wage. We define b_3 as the root.

$$\frac{\partial \pi}{\partial b} \overset{!}{=} 0 \quad \iff \quad b_3 : \quad E'(b_3 - \underline{v}) \cdot (V - b_3) - E(b_3 - \underline{v}) = 0 \tag{89}$$

Note that $E'(\cdot)$ is decreasing in its arguments because $E''(\cdot) < 0$. Furthermore, $E(\cdot)$ is increasing in its arguments. Therefore, compared to the third case in the benchmark, the marginal benefit of the bonus wage is smaller and the marginal cost is larger for all bonus wages. We shift $E'(\cdot)$ to the left and $E(\cdot)$ to the right. Thus, $b_3 < b^{***}$. If the marginal profit is zero for a negative bonus wage, the optimal bonus wage is zero because of the minimum wage condition. Thus, the optimal bonus wage is $b_3^+ \equiv \max\{0, b_3\}$.

What is the relation between the solution when the jump ends in the negative and when it ends in the positive? The maximization problem when ignoring the participation constraint yields a weakly larger maximum than taking into account the participation constraint. Therefore, the profit with b_3^+ is weakly larger than the profit with $b_{bound}(\underline{w})$. b_3^+ is optimal whenever it does not violate the participation constraint.

We now show that there are some minimum wages for which b_3^+ does violate the participation constraint, such that $b_{bound}(\underline{w})$ is the optimal solution. Reconsider the minimum wage $\underline{w}_{bound}$. The optimal contract is $(\underline{w}_{bound}, b(\underline{w}_{bound}), \underline{v})$. By the case assumption, $b(\underline{w}_{bound}) > 0$. Thus, without a bound on NCCs, the marginal profit of an additional unit of bonus wage is 0 at $b(\underline{w}_{bound})$. With a bound on NCCs, this is the bonus wage at which the jump from double incentives to incentives (only through bonus wage) happens. The jump, thus, has to end in the negative. Thus, this is one minimum wage for which the participation constraint would be violated for b_3^+. Furthermore, the point at which the jump ends, moves continuously in the minimum wage: The marginal profit is a continuous function in the bonus wage and the bonus wage at which the jump happens is a continuous function of the minimum wage. Thus, the jump also ends in the negative for some larger minimum wages.

b) The optimal contract for the minimum wage $\underline{w}_{bound}$ is from the fourth combination, that is, the optimal bonus wage is 0. With a bound on the NCC, the optimal contract now is $(\underline{w}, 0, \underline{v})$. A positive bonus wage cannot increase the profits. The optimal contract only falls into the fourth combination if the marginal profit from the first unit of bonus wage is negative. Because the binding NCC does not violate the participation constraint even without a bonus wage, there never are double incentives. Thus, the marginal profit is smaller than without a bound on the NCC (intuitively, the jump happened for a negative bonus wage). Since the marginal profit was negative with double incentives, the marginal profit is still negative. It is optimal not to use a positive bonus wage. A negative bonus wage cannot increase the profits because this means increasing the base wage above the minimum wage (otherwise the minimum wage constraint on the bonus would be violated). Then, there is a profitable deviation (making the NCC less severe, the bonus wage larger and the base wage lower by one marginal unit).

□

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