Face Recognition Using PCA / Eigenface Approach

1 Overview

The Eigenface method is a face recognition technique that represents faces as a linear combination of a set of basis faces (eigenfaces). The algorithm reduces the dimensionality of the face space using Principal Component Analysis (PCA) and then compares the projection of the input face onto this reduced space.

2 Algorithm

2.1 1. Prepare the Dataset

Collect a set of face images, each of which is represented as a vector. Suppose we have m face images, each with dimensions $h \times w$.

$$\text{Image vector} \quad \mathbf{x}_i \in \mathbb{R}^{h \times w}, \quad i = 1, 2, \ldots, m$$

Flatten each face image into a 1D vector.

$$\mathbf{x}_i \in \mathbb{R}^n, \quad \text{where} \quad n = h \times w$$

2.2 2. Compute the Mean Face

Calculate the mean face vector by averaging all face vectors.

Formula 2.1: Mean Face

$$\bar{\mathbf{x}} = \frac{1}{m} \sum_{i=1}^{m} \mathbf{x}_i$$

- $\bar{\mathbf{x}}$ is the mean face, computed as the average of all face images.

2.3 3. Center the Data

Subtract the mean face from each face vector to center the data.

Formula 2.2: Center the Data

$$\mathbf{x}_i^{\text{centered}} = \mathbf{x}_i - \bar{\mathbf{x}}, \quad i = 1, 2, \ldots, m$$

- $\mathbf{x}_i^{\text{centered}}$ is the centered face vector obtained by subtracting the mean face from each face vector.

2.4 4. Compute the Covariance Matrix

Calculate the covariance matrix of the centered data. Since the data is high-dimensional, we use a trick to compute it efficiently.

Formula 2.3: Covariance Matrix

$$\mathbf{C} = \frac{1}{m}\mathbf{A}\mathbf{A}^T$$

- $\mathbf{A} = [\mathbf{x}_1^{\text{centered}}, \mathbf{x}_2^{\text{centered}}, \dots, \mathbf{x}_m^{\text{centered}}] \in \mathbb{R}^{n \times m}$

2.5 5. Compute Eigenvalues and Eigenvectors

Compute the eigenvalues and eigenvectors of the covariance matrix $\mathbf{C}$. Instead of computing the eigenvectors of $\mathbf{C}$ directly, solve for the eigenvectors of the smaller matrix $\mathbf{L}$:

Formula 2.4: Eigenvalues and Eigenvectors

$$\mathbf{L} = \frac{1}{m}\mathbf{A}^T\mathbf{A} \in \mathbb{R}^{m \times m}$$

$$\mathbf{L}\mathbf{v}_i = \lambda_i \mathbf{v}_i$$

The eigenvectors of $\mathbf{C}$ are obtained by:

$$\mathbf{u}_i = \mathbf{A}\mathbf{v}_i$$

$$\mathbf{u}_i = \frac{\mathbf{u}_i}{\|\mathbf{u}_i\|}$$

- $\mathbf{u}_i$ are the eigenvectors of the covariance matrix $\mathbf{C}$.
- $\mathbf{v}_i$ are the eigenvectors of the matrix $\mathbf{L}$.
- λ_i are the eigenvalues.

2.6 6. Sort Eigenvectors and Select Top k Components

Sort the eigenvalues in descending order and select the top k eigenvectors corresponding to the top k eigenvalues.

Formula 2.5: Sorting and Selecting Top k Eigenvectors

$$\text{Sort } \lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_m$$

$$\mathbf{W}_k = [\mathbf{u}_1, \mathbf{u}_2, \dots, \mathbf{u}_k]$$

- $\mathbf{W}_k$ is the matrix of top k eigenvectors (eigenfaces).

2.7 7. Project Faces into the Eigenface Space

Project the centered face vectors onto the eigenface space to obtain the PCA coefficients (weights).

Formula 2.6: Project Faces

$$\mathbf{y}_i = \mathbf{W}_k^T \mathbf{x}_i^{\text{centered}}, \quad i = 1, 2, \dots, m$$

- $\mathbf{y}_i$ are the projections of the face vectors onto the eigenface space.

2.8 8. Recognition (Classification)

For a new test image $\mathbf{x}_{\text{test}}$:
1. **Center the Test Image:**

Formula 2.7: Center the Test Image

$$\mathbf{x}_{\text{test}}^{\text{centered}} = \mathbf{x}_{\text{test}} - \bar{\mathbf{x}}$$

2. **Project the Test Image:**

Formula 2.8: Project the Test Image

$$\mathbf{y}_{\text{test}} = \mathbf{W}_k^T \mathbf{x}_{\text{test}}^{\text{centered}}$$

3. **Compute the Distance:**

Formula 2.9: Compute Distance

$$d(\mathbf{y}_{\text{test}}, \mathbf{y}_i) = \|\mathbf{y}_{\text{test}} - \mathbf{y}_i\|, \quad i = 1, 2, \ldots, m$$

4. **Identify the Closest Match:**

Formula 2.10: Identify Closest Match

$$\text{Label}(\mathbf{x}_{\text{test}}) = \text{Label}(\mathbf{x}_j), \quad \text{where} \quad j = \arg\min_i d(\mathbf{y}_{\text{test}}, \mathbf{y}_i)$$

2.9 9. Performance Evaluation

Evaluate the model using metrics like accuracy, precision, recall, F1-score, and confusion matrix.