Linear Regression: Comprehensive Explanation and Implementation Guide

1 Introduction

- Linear Regression is a fundamental algorithm in machine learning used for predicting a continuous outcome based on one or more input features.
- The goal is to model the relationship between the input variables (features) and the output variable (target) by fitting a linear equation to observed data.

2 Hypothesis Function

- The hypothesis function represents the linear relationship between input features and the output.
- It is expressed as:

Hypothesis Function

$$\hat{y} = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \cdots + \beta_m X_m$$
$$\hat{y} = X\beta$$

- Here, $\hat{y}$ is the predicted output, X is the feature matrix, and β is the vector of parameters.

3 Cost Function (Mean Squared Error)

- The cost function measures the error between the predicted and actual values.
- The goal is to minimize this error by adjusting the model parameters.
- The cost function is given by:

Cost Function

$$J(\beta) = \frac{1}{2n} \sum_{i=1}^{n} (\hat{y}_i - y_i)^2$$
$$J(\beta) = \frac{1}{2n} (X\beta - y)^T (X\beta - y)$$

4 Gradient Descent Update Rule

- Gradient Descent is an iterative optimization algorithm used to minimize the cost function.
- The parameters are updated in the direction of the negative gradient of the cost function.

- The update rule is:

Gradient Descent Update Rule

$$\beta_j := \beta_j - \alpha \frac{1}{n} \sum_{i=1}^{n} (\hat{y}_i - y_i) X_{ij}$$

$$\nabla J(\beta) = \frac{1}{n} X^T (X\beta - y)$$

$$\beta := \beta - \alpha \nabla J(\beta)$$

5 Normal Equation (Closed-Form Solution)

- The normal equation provides a direct solution for the optimal parameters without the need for iterative optimization.
- It is derived by setting the gradient of the cost function to zero and solving for β.
- The normal equation is:

Normal Equation

$$\beta = (X^T X)^{-1} X^T y$$

6 Conclusion

- Linear Regression is a simple yet powerful method for predictive modeling.
- Understanding the underlying mathematics helps in effectively applying and interpreting the results of Linear Regression.