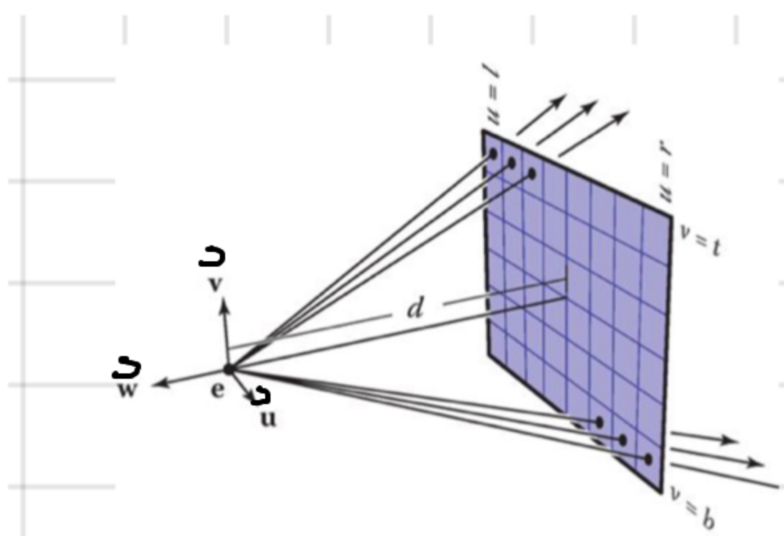
Intro to RayTracing Pt II

Gereate perspective rays

- all rays have the same origin
- directions differ for each pixel

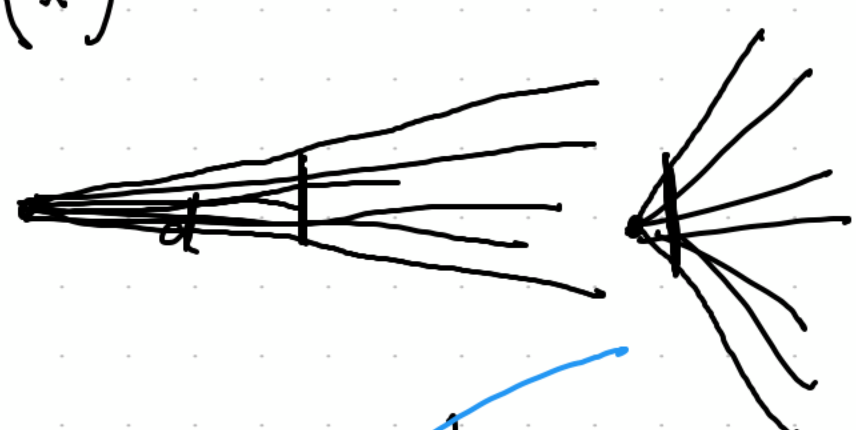


Perspective projection
same origin, different directions

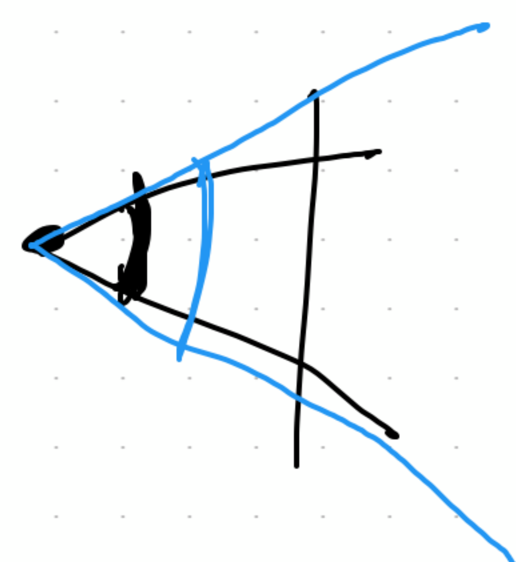
Gereate view rays

$\text{ray.direction} = -d\vec{w} + u\vec{u} + v\vec{v}$ (*)

$\text{ray.origin} = e$



(*) note the u & v are from (eq **) in tue notes



Implicit surface rep of sphere

Implict surfance in 3D ($\mathbb{R}^3$)

$p \in \mathbb{R}^3$ s.t. $f(p_x, p_y, p_z) = 0$

a. line in 2D

$2x + 3y + 4 = 0 \qquad f(x,y) = 2x + 3y + 4$

is $(3,6)$ on the line

$f(3,6) = 2*3 + 3*6 + 4 = 28 \neq 0 \Rightarrow$ not on line

is $(-11,6)$ on the line

$f(-11,6) = 2*-11 + 3*6 + 4 = -22 + 22 = 0$

$\Rightarrow (-11,6)$ is on the line

Shpere as implicit surface

$c = (c_x, c_y, c_z)$ a radius r

$(p_x - c_x)^2 + (p_y - c_y)^2 + (p_z - c_z)^2 - r^2 = 0$

rewrite as

$(p - c) \cdot (p - c) - r^2 = 0$ w/ $p = (x, y, z) = (p_x, p_y, p_z)$

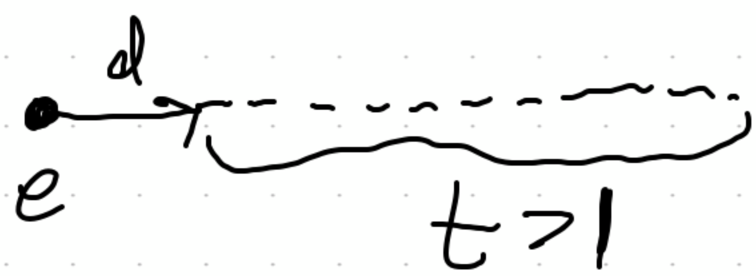
$$\begin{bmatrix} p_x - c_x \\ p_y - c_y \\ p_z - c_z \end{bmatrix} \cdot \begin{bmatrix} p_x - c_x \\ p_y - c_y \\ p_z - c_z \end{bmatrix} - r^2 = 0$$

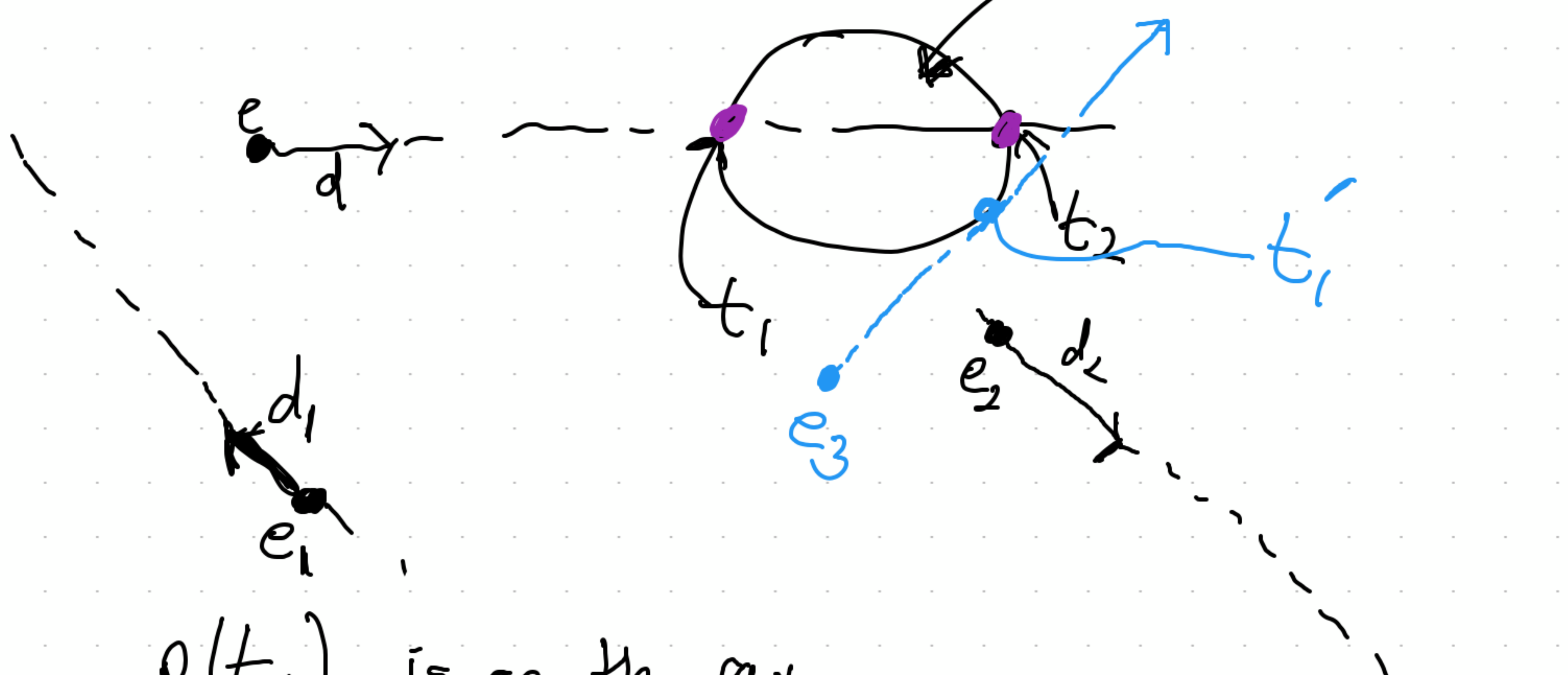
Eq $(**)$

$$f(p) = (p_x - c_x)^2 + (p_y - c_y)^2 + (p_z - c_z)^2 - r^2$$

any point $p \in \mathbb{R}^3$ s.t $f(p) = 0$
is on the sphere

ray $p(t) = e + td$ w/ e is "eye"
d is "direction"





$p(t_1)$ is on the ray
$p(t_2)$ is on the ray
$f(p(t_1))$ ray and sphere
$f(p(t_2))$ ray and sphere

$\Rightarrow$ $f(p(t)) \stackrel{?}{=} 0$
is on sphere
goal: find
t_i's

$f(p(t)) = f(\vec{e}+t\vec{d})$ def of our ray

$e, d \in \mathbb{R}^3$

$((e+td)-c)\cdot((e+td)-c)-r^2 \quad t \in \mathbb{R}$

some alg

$(p-c)\cdot(p-c) = r^2$

$$= \underbrace{(d\cdot d)}_{\substack{a \\ \Uparrow \\ \mathbb{R}}} t^2 + \underbrace{2d\cdot(e-c)}_{\substack{b \\ \Uparrow \\ \mathbb{R}}} t + \underbrace{(e-c)\cdot(e-c)-r^2}_{\substack{c \\ \Uparrow \\ \mathbb{R}}}$$

$$= at^2 + bt + c$$

solve for t (one not numerically good way)

$$t = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

Recall: discriminant : $b^2-4ac = D$

$D<0 \Rightarrow$ no real solutions (ray does not intersect the sphere)

$D=0 \Rightarrow$ 1 solution (ray intersects tangentially)

$D>0 \Rightarrow$ 2 solutions (ray intersects the sphere 2x)

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⇒ .~~12345678910~~000000000000.

"Numerical Receipes" good explanation of how to correctly solve quadratics by a computer