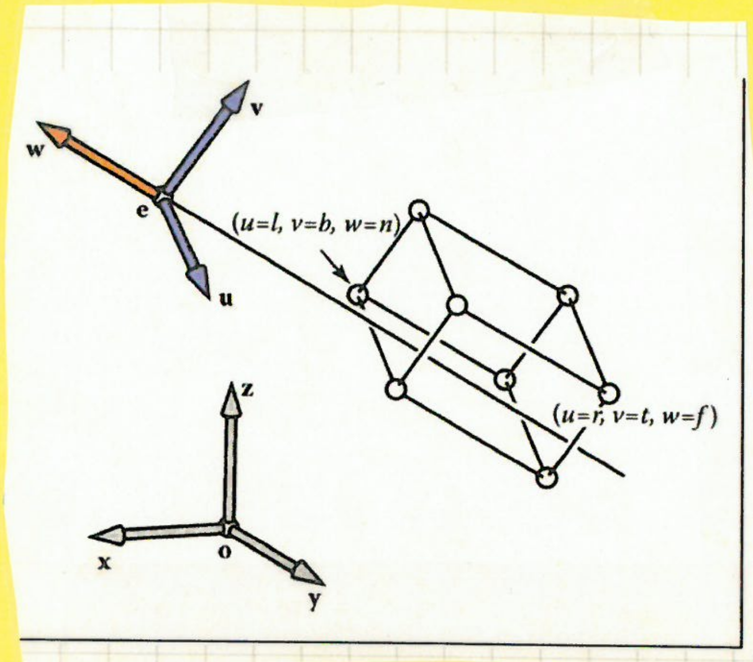
Camera Transform

e = eye position

g = gaze direction

t = up direction



$$w = -\frac{g}{\|g\|} = \begin{bmatrix} w_x \\ w_y \\ w_z \end{bmatrix}$$

$$u = \frac{t \times w}{\|t \times w\|} = \begin{bmatrix} u_x \\ u_y \\ u_z \end{bmatrix}$$

$$v = w \times u = \begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix}$$

"put camera into the world"

$$(TR)^{-1} = R^{-1}T^{T}$$

$$\left(\underbrace{\begin{bmatrix} 1 & 0 & 0 & e_x \\ 0 & 1 & 0 & e_y \\ 0 & 0 & 1 & e_z \\ 0 & 0 & 0 & 1 \end{bmatrix}}_{T} \underbrace{\begin{bmatrix} u_x & v_x & w_x & 0 \\ u_y & v_y & w_y & 0 \\ u_z & v_z & w_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}_{R} \right)^{-1}$$

$$R^{-1}T^{-1} = \overbrace{\begin{bmatrix} u_x & u_y & u_z & 0 \\ v_x & v_y & v_z & 0 \\ w_x & w_y & w_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}^{R^{-1}} \overbrace{\begin{bmatrix} 1 & 0 & 0 & -e_x \\ 0 & 1 & 0 & -e_y \\ 0 & 0 & 1 & -e_z \\ 0 & 0 & 0 & 1 \end{bmatrix}}^{T^{-1}} = M_{camera}$$

Result:

$$\begin{bmatrix} x_{pixel} \\ y_{pixel} \\ z_{canonical} \\ 1 \end{bmatrix} = M_{vp} M_{orth} M_{camera} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

"put camera into the world"

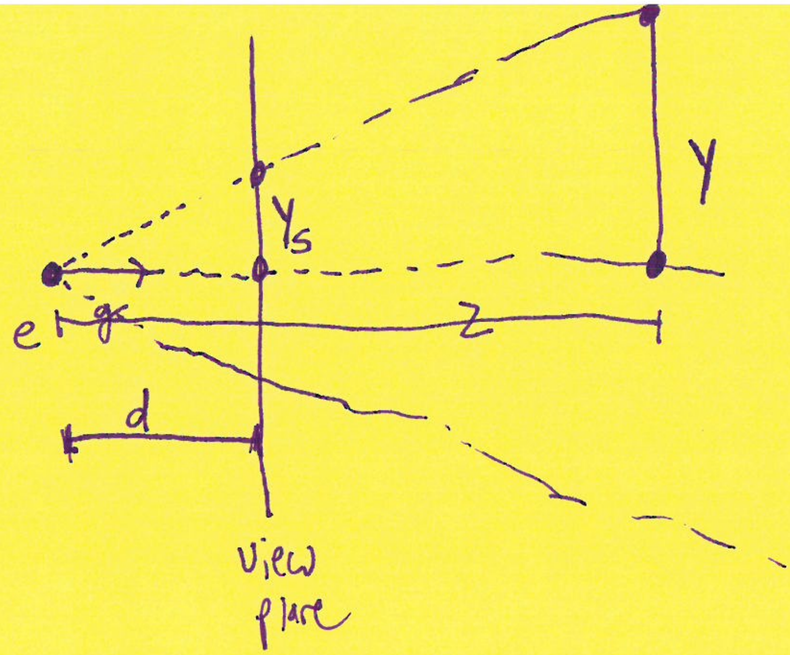
$$(TR)^{-1} = R^{-1}T^{-1}$$

$$\Bigg(\underbrace{\begin{bmatrix}1&0&0&e_x\\0&1&0&e_y\\0&0&1&e_z\\0&0&0&1\end{bmatrix}}_{T} \underbrace{\begin{bmatrix}u_x&v_x&w_x&0\\u_y&v_y&w_y&0\\u_z&v_z&w_z&0\\0&0&0&1\end{bmatrix}}_{R} \Bigg)^{-1}$$

$$M_{camera} = \underbrace{\begin{bmatrix}u_x&u_y&u_z&0\\v_x&v_y&v_z&0\\w_x&w_y&w_z&0\\0&0&0&1\end{bmatrix}}_{R^{-1}} \underbrace{\begin{bmatrix}1&0&0&-e_x\\0&1&0&-e_y\\0&0&1&-e_z\\0&0&0&1\end{bmatrix}}_{T^{-1}} = R^{-1}T^{-1}$$

Result:

$$\begin{bmatrix}x_{pixel}\\y_{pixel}\\z_{canonical}\\1\end{bmatrix} = M_{vp}\, M_{orth}\, M_{camera} \begin{bmatrix}x\\y\\z\\1\end{bmatrix}$$



$$\cancel{\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}}$$

$$\cancel{\begin{bmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{bmatrix}}$$

$$y_s = \frac{d}{z} y$$

Linear trans: $a_1 x + b_1 y + c_1 z$

affine trans: $a_1 x + b_1 y + c_1 z + d_1$

projective trans: $x' = \dfrac{a_1 x + b_1 y + c_1 z + d_1}{ex + fy + gz + h}$ $\quad y' = \dfrac{a_2 x + b_2 y + c_2 z + d_2}{ex + fy + gz + h}$

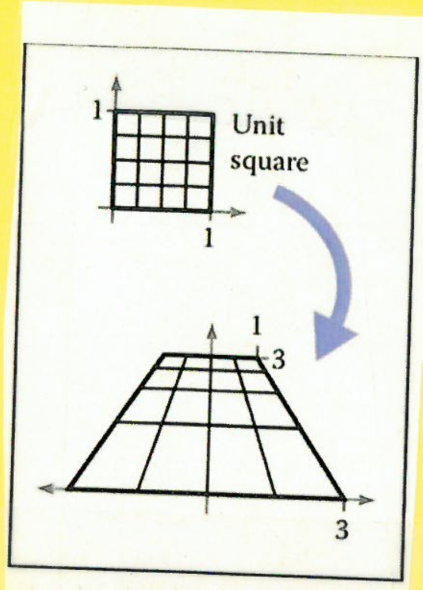
$$z' = \frac{a_3 x + b_3 y + c_3 z + d_3}{ex + fy + gz + h}$$

$$\begin{bmatrix} \tilde{x} \\ \tilde{y} \\ \tilde{z} \\ \tilde{w} \end{bmatrix} = \begin{bmatrix} a_1 & b_1 & c_1 & d_1 \\ a_2 & b_2 & c_2 & d_2 \\ a_3 & b_3 & c_3 & d_3 \\ e & f & g & h \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

w/ $(x', y', z') = \left(\dfrac{\tilde{x}}{\tilde{w}}, \dfrac{\tilde{y}}{\tilde{w}}, \dfrac{\tilde{z}}{\tilde{w}}\right)$

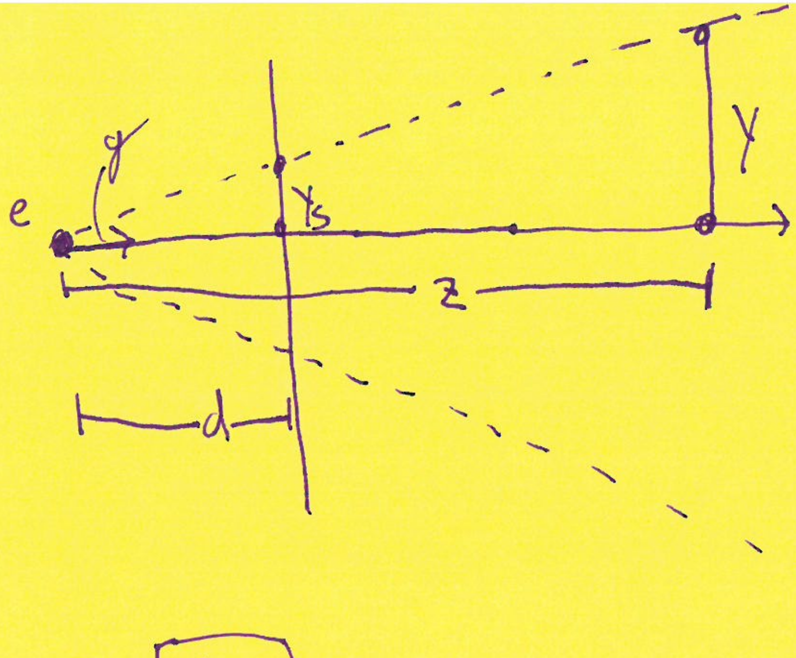
proj trans
w/ unit square

- scale x by 2
- scale y by 3
- translate x by -1
- (add stuff on bottom)



$$\begin{bmatrix} 2 & 0 & -1 \\ 0 & 3 & 0 \\ 0 & \frac{2}{3} & \frac{1}{3} \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ \frac{1}{3} \end{bmatrix}$$

$$\left(\frac{1}{(\frac{1}{3})}, \frac{0}{(\frac{1}{3})} \right) = (3, 0)$$



$$y_s = \boxed{\frac{d}{z}\,y}$$

what matrix gives w/ proj trans & homogenous coords

$$\begin{bmatrix} \tilde{y} \\ \tilde{w} \end{bmatrix} = \begin{bmatrix} \quad & \quad & \quad \end{bmatrix} \begin{bmatrix} y \\ z \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} y_s \\ 1 \end{bmatrix} = \underset{\uparrow\; z}{\begin{bmatrix} \tilde{y} \\ \tilde{w} \end{bmatrix}} = \begin{bmatrix} d & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} dy \\ z \end{bmatrix}$$

$$\Downarrow$$

$$\frac{dy}{z} = y_s$$

make "offical" projection matrix

$z = n$: near plane (view plane)

$z = f$: far plane

$$M_{pers} = \begin{bmatrix} n & 0 & 0 & 0 \\ 0 & n & 0 & 0 \\ 0 & 0 & n+f & -nf \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

perspective

brings z coords allong for ride

$$M^{GL}_{perspective} = M_{orth} M_{pers} = \begin{bmatrix} \frac{2|n|}{r-l} & 0 & \frac{r+l}{r-l} & 0 \\ & \frac{2|n|}{t-b} & \frac{t+b}{t-b} & 0 \\ & & \frac{|n|+|f|}{|n|-|f|} & \frac{2|f||n|}{|n|-|f|} \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

w/o perspective $M_1 = M_{vp} M_{orth} M_{cam}$

w/ perspective $M_2 = M_{vp} \underbrace{M_{orth} M_{pers}}_{M_{perspective}} M_{cam}$