

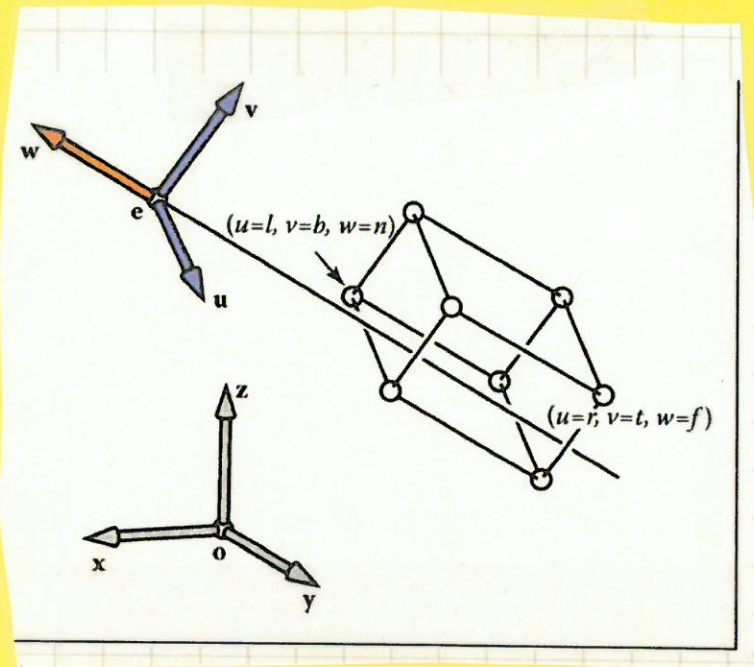
Camera Transform

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e = eye position

g = gaze direction

t = up direction



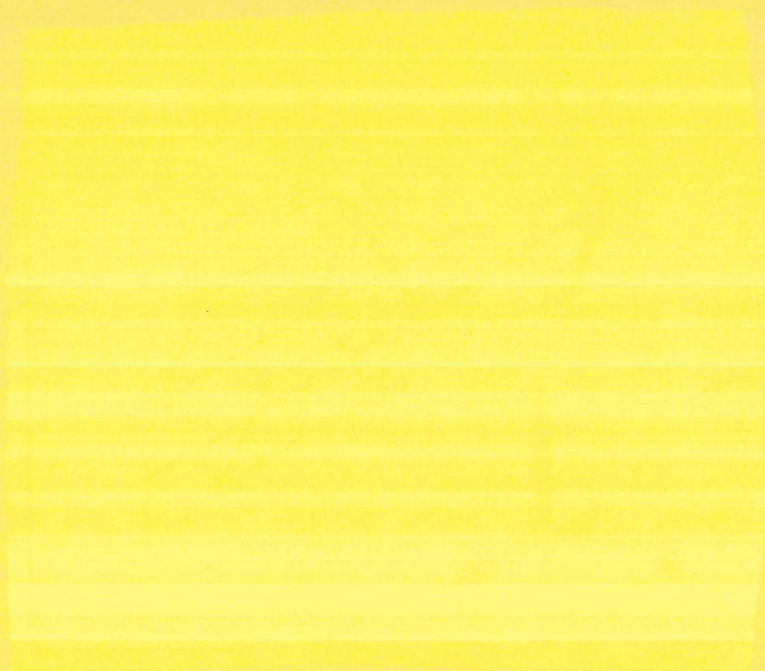
$$w = -\frac{g}{\|g\|} = \begin{bmatrix} w_x \\ w_y \\ w_z \end{bmatrix}$$

$$u = \frac{t \times w}{\|t \times w\|} = \begin{bmatrix} u_x \\ u_y \\ u_z \end{bmatrix}$$

$$v = w \times u = \begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix}$$

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General Transform



normalizing $g \rightarrow \hat{g}$

normalizing $w \rightarrow \hat{w}$

normalizing $x \rightarrow \hat{x}$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \frac{g}{\|g\|} = \hat{g}$$

$$\begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix} = \frac{w \times \hat{g}}{\|w \times \hat{g}\|} = \hat{w}$$

$$\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \hat{w} \times \hat{g} = \hat{v}$$

"Put camera into the world"

$$(TR)^{-1} = R^{-1}T^T$$

$$\left(\begin{array}{c|c} \begin{matrix} \uparrow T \\ \begin{bmatrix} 1 & 0 & 0 & e_x \\ 0 & 1 & 0 & e_y \\ 0 & 0 & 1 & e_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{matrix} & \begin{matrix} \uparrow R \\ \begin{bmatrix} u_x & v_x & w_x & 0 \\ u_y & v_y & w_y & 0 \\ u_z & v_z & w_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{matrix} \end{array} \right)^{-1}$$

$$R^{-1}T^{-1} = \begin{array}{c} \downarrow R^{-1} \qquad \qquad \qquad \downarrow T^{-1} \\ \begin{bmatrix} u_x & u_y & u_z & 0 \\ v_x & v_y & v_z & 0 \\ w_x & w_y & w_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & -e_x \\ 0 & 1 & 0 & -e_y \\ 0 & 0 & 1 & -e_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{array} = M_{\text{Camera}}$$

Result:

$$\begin{bmatrix} x_{\text{pixel}} \\ y_{\text{pixel}} \\ z_{\text{camera}} \\ 1 \end{bmatrix} = M_{\text{up}} M_{\text{orth}} M_{\text{camera}} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

"blow off the name tag"

$$T^{-1} R = (TR)$$

$$\begin{pmatrix} \begin{bmatrix} 0 & sW & sU & sV \\ 0 & sU & sV & sW \\ 0 & sW & sU & sV \\ 1 & 0 & 0 & 0 \end{bmatrix} & \begin{bmatrix} X^S & 0 & 0 & 1 \\ Y^S & 0 & 1 & 0 \\ Z^S & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \end{pmatrix}$$

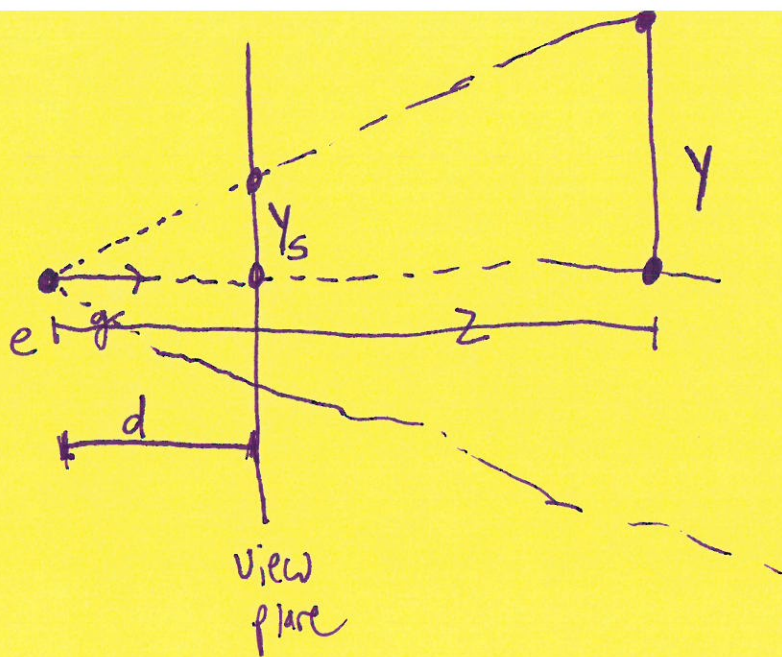
\uparrow $\quad \quad \quad \uparrow$
 q $\quad \quad \quad T$

\swarrow $\quad \quad \quad \swarrow$
 \checkmark $\quad \quad \quad \checkmark$

$$\text{new } M = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & sU & sV & sW \\ 0 & sU & sV & sW \\ 0 & sW & sU & sV \\ 1 & 0 & 0 & 0 \end{bmatrix} = T^{-1} R$$

result

$$\begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix} \text{new } M_{\text{new}} M_{\text{old}} M_{\text{old}} = \begin{bmatrix} 1201X \\ 1201Y \\ 1201Z \\ 1 \end{bmatrix}$$



$$y_s = \frac{d}{z} y$$

Linear trans: $a_1 x + b_1 y + c_1 z$

affine trans: $a_1 x + b_1 y + c_1 z + d_1$

projective trans: $X' = \frac{a_1 x + b_1 y + c_1 z + d_1}{e_1 x + f_1 y + g_1 z + h_1}$

$$y' = \frac{a_2 x + b_2 y + c_2 z + d_2}{e_1 x + f_1 y + g_1 z + h_1}$$

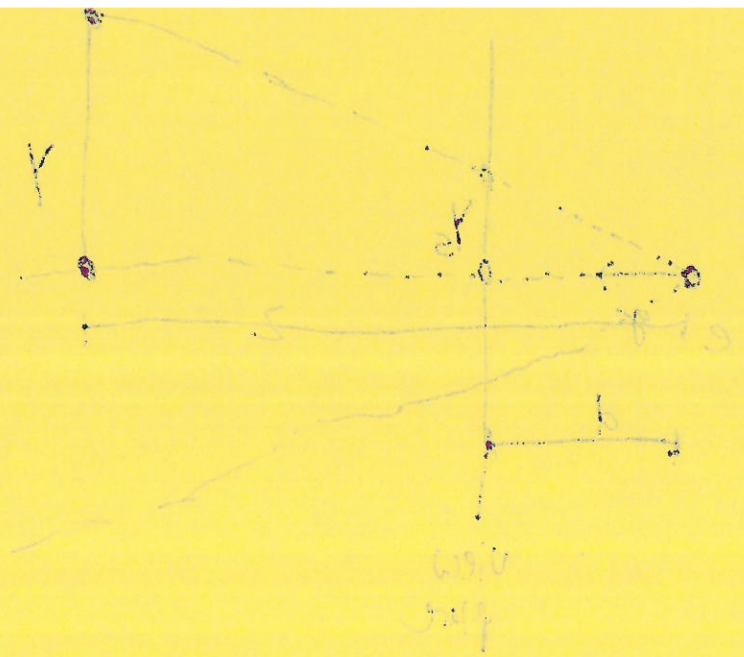
$$z' = \frac{a_3 x + b_3 y + c_3 z + d_3}{e_1 x + f_1 y + g_1 z + h_1}$$

~~$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{bmatrix}$$~~

$$\begin{bmatrix} \tilde{x} \\ \tilde{y} \\ \tilde{z} \\ \tilde{w} \end{bmatrix} = \begin{bmatrix} a_1 & b_1 & c_1 & d_1 \\ a_2 & b_2 & c_2 & d_2 \\ a_3 & b_3 & c_3 & d_3 \\ e & f & g & h \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$$\begin{aligned} &w/ (x', y', z') \\ &= \left(\frac{\tilde{x}}{\tilde{w}}, \frac{\tilde{y}}{\tilde{w}}, \frac{\tilde{z}}{\tilde{w}} \right) \end{aligned}$$



~~$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 \\ 1 & 4 & 9 & 16 \\ 1 & 9 & 16 & 25 \end{bmatrix}$$~~

$$Y = \frac{a}{b} = X$$

$$Y = a + bX + cX^2 + dX^3$$

$$Y = a + bX + cX^2 + dX^3$$

$$Y = \frac{a + bX + cX^2 + dX^3}{1 + X + X^2 + X^3}$$

$$Y = \frac{a + bX + cX^2 + dX^3}{1 + X + X^2 + X^3}$$

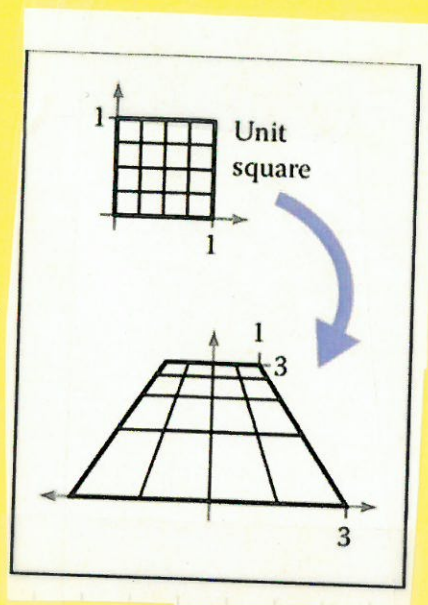
$$Y = \frac{a + bX + cX^2 + dX^3}{1 + X + X^2 + X^3}$$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 \\ 1 & 4 & 9 & 16 \\ 1 & 9 & 16 & 25 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} Y_1 \\ Y_2 \\ Y_3 \\ Y_4 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 \\ 1 & 4 & 9 & 16 \\ 1 & 9 & 16 & 25 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} Y_1 \\ Y_2 \\ Y_3 \\ Y_4 \end{bmatrix}$$

proj trans
w/ unit square

- Scale x by 2
- Scale y by 3
- translate x by -1
- (add stuff on bottom)



$$\begin{bmatrix} 2 & 0 & -1 \\ 0 & 3 & 0 \\ 0 & \frac{2}{3} & \frac{1}{3} \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ \frac{1}{3} \end{bmatrix}$$

$$\left(\frac{1}{\left(\frac{1}{3}\right)}, \frac{0}{\left(\frac{1}{3}\right)} \right) = (3, 0)$$

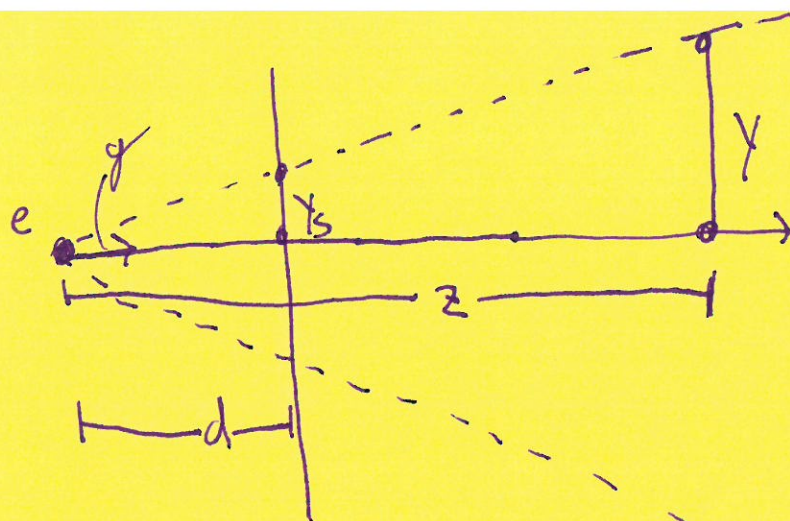
for time
w/ out space



1 - pd x slope -
2 - pd x slope -
1 - pd x slope -
(add shift or bottom) -

$$\begin{bmatrix} 1 \\ 0 \\ 1/2 \\ 1/2 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \\ 0 & 0 & 0 \end{bmatrix}$$

$$(0, 1, 2) = \left(\frac{1}{2}, \frac{1}{2}, 0 \right)$$



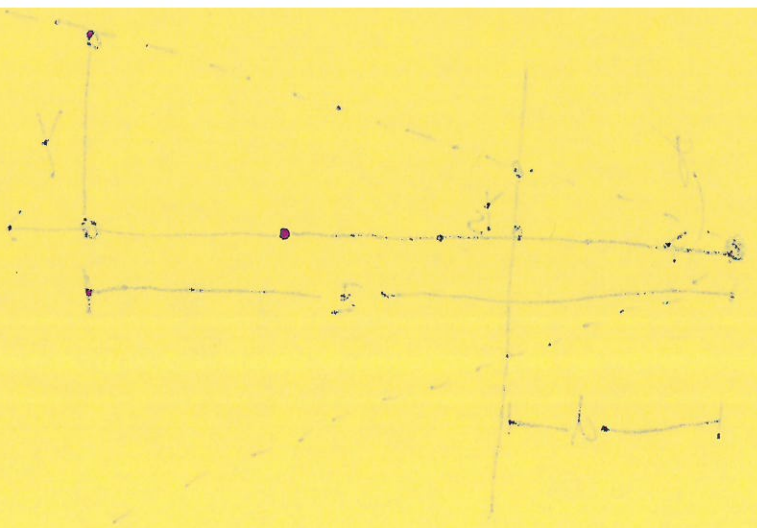
$$y_s = \frac{d}{z} y$$

what matrix gives w/ proj trans & homogeneous coords

$$\begin{bmatrix} \tilde{y} \\ \tilde{w} \end{bmatrix} = \begin{bmatrix} \\ \end{bmatrix} \begin{bmatrix} y \\ z \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} y_s \\ 1 \end{bmatrix} = \begin{bmatrix} \tilde{y} \\ \tilde{w} \end{bmatrix} \Rightarrow \begin{bmatrix} d & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} dy \\ z \end{bmatrix}$$

$$\frac{dy}{z} = y_s$$



$$P = \begin{bmatrix} 1 & 1 \\ 4 & 5 \\ 1 & 5 \end{bmatrix} = X$$
 copy matrix into the table

$$\begin{bmatrix} 1 \\ 4 \\ 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 4 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 \\ 4 & 5 \\ 1 & 5 \end{bmatrix} = \begin{bmatrix} 1 \\ 4 \\ 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 4 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 4 \\ 1 \end{bmatrix}$$

$$P = \begin{bmatrix} 1 & 1 \\ 4 & 5 \\ 1 & 5 \end{bmatrix}$$

make "official" projection matrix

$z = n$: near plane (view plane)

$z = f$: far plane

~~proj~~ $M_{pers} = \begin{bmatrix} n & 0 & 0 & 0 \\ 0 & n & 0 & 0 \\ 0 & 0 & n+f & -nf \\ 0 & 0 & 1 & 0 \end{bmatrix}$

perspective

brings z coords along for ride

$$M_{perspective}^{GL} = M_{orth} M_{pers} = \begin{bmatrix} \frac{z|n|}{r-l} & 0 & \frac{r+l}{r-l} & 0 \\ 0 & \frac{z|n|}{t-b} & \frac{t+b}{t-b} & 0 \\ 0 & 0 & \frac{|n|+f}{|n|-f} & \frac{2f|n|}{|n|-f} \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

w/o perspetia

$$M_1 = M_{up} M_{orth} M_{cam}$$

w/ perspective

$$M_2 = M_{up} \underbrace{M_{orth} M_{pers}}_{M_{perspective}} M_{cam}$$

unit "offset" projection matrix
 $S = U$: look plane
 $S = T$: fr plane

projection
 fr plane
 look plane
 fr plane

$$M_{proj} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$M_{proj} = M_{look} M_{fr} = M_{proj}$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

no fr plane
 no fr plane

$$M_{proj} M_{look} M_{fr} = M_{proj}$$