

## Ray $\Delta$ intersection

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consider parametric rep of a plane  $f(u,v)$

And ray  $\vec{e} + t\vec{d}$

$$e_x + td_x = f_x(u,v)$$

$$e_y + td_y = f_y(u,v)$$

$$e_z + td_z = f_z(u,v)$$

unknowns:  $t, u, v$

3 eqn & 3 unknowns

We have a  $\Delta abc$  we have a plane defined by 3 points  $a, b, c$

$$f(u,v) = \vec{a} + u(\vec{b} - \vec{a}) + v(\vec{c} - \vec{a})$$

On the plane and in  $\Delta$

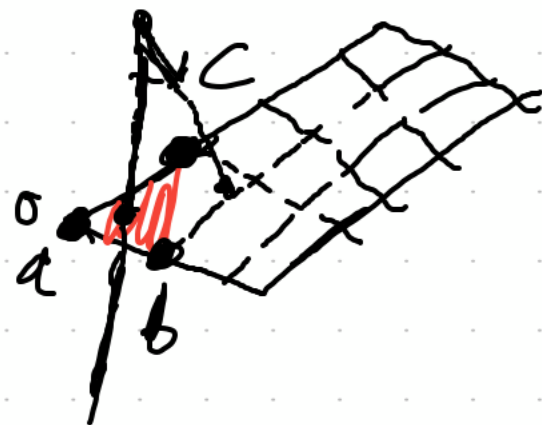
$$u > 0$$

$$v > 0 \text{ and}$$

$$u+v < 1$$

We have an intersection when

$$\vec{e} + t\vec{d} = \vec{a} + u(\vec{b} - \vec{a}) + v(\vec{c} - \vec{a})$$



Solve for  $u, v, t$

Expand into 3 equations

$$\textcircled{1} \quad e_x + t d_x = a_x + u(b_x - a_x) + v(c_x - a_x)$$

$$\textcircled{2} \quad e_y + t d_y = a_y + u(b_y - a_y) + v(c_y - a_y)$$

$$\textcircled{3} \quad e_z + t d_z = a_z + u(b_z - a_z) + v(c_z - a_z)$$

Next rewrite

$$\textcircled{1} \quad e_x - a_x = u(b_x - a_x) + v(c_x - a_x) - t d_x$$

$$\textcircled{2} \quad (\text{sim})$$

$$\textcircled{3} \quad (\text{sim})$$

rewrite as matrix

$$\begin{bmatrix} b_x - a_x & c_x - a_x & -d_x \\ b_y - a_y & c_y - a_y & -d_y \\ b_z - a_z & c_z - a_z & -d_z \end{bmatrix} \begin{bmatrix} u \\ v \\ t \end{bmatrix} = \begin{bmatrix} e_x - a_x \\ e_y - a_y \\ e_z - a_z \end{bmatrix}$$

Solve for  $(u, v \& t)$

e.g using Cramers

ray /  $\Delta$  intersection

RAYINTERSECTRI (ray  $r$ , point  $a$ , point  $b$ , point  $c$ )

compute  $t, u, v$

if ( $u < 0 \parallel 1 < v$ ) NoIntersection

if ( $u < 0 \parallel 1 - v < u$ ) NoIntersection

if ( $t < 0$ ) NoIntersection

intersection at the point  $r \cdot eval(t)$   
 $\hookrightarrow \vec{e} + t \vec{d}$

# Intersecting a group of objects

FINDINTERSECTION

$hitObj = \emptyset$

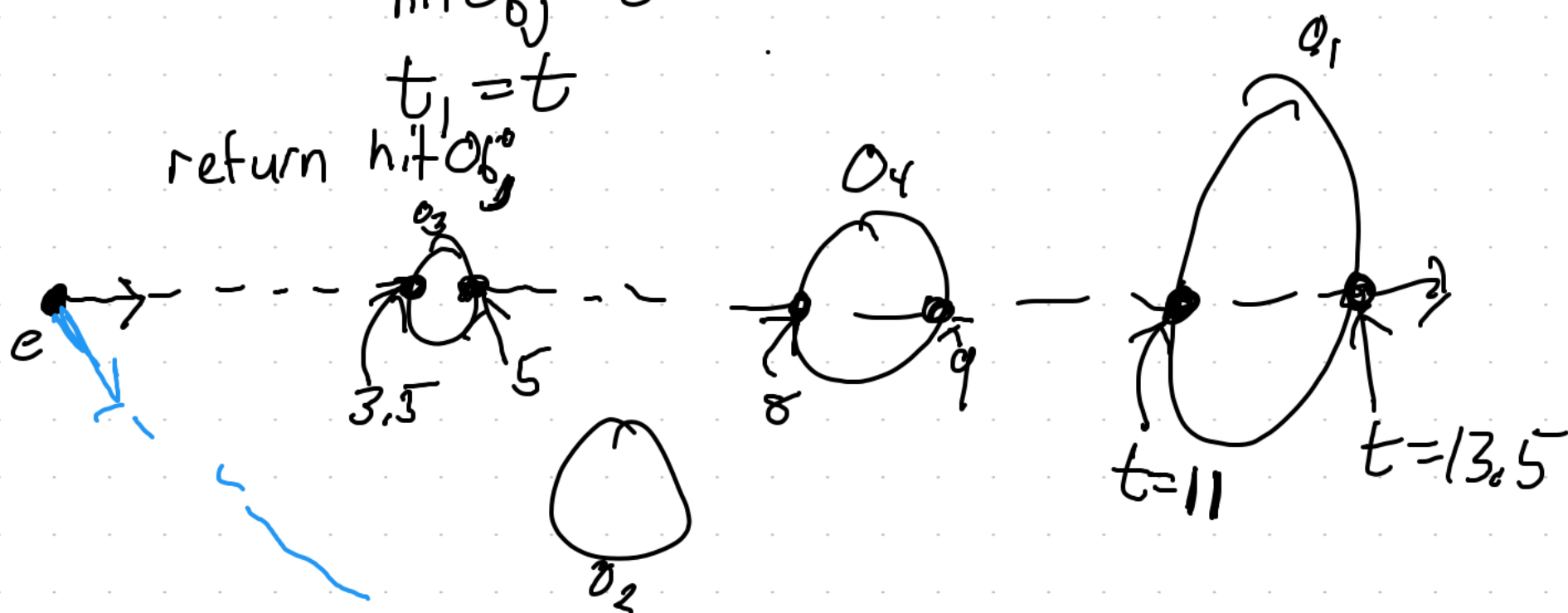
for obj  $o$  in Object Group

if ( $o$  is hit by a ray param  $t$   
w/  $t \in [t_0, t_1]$ )

$hitObj = o$

$t_1 = t$

return  $hitObj$



$t_0 = 0$   
 $t_1 = \infty$   
 $hitObj = \emptyset$

$O_1$   
 $\rightarrow$

$t_0 = 0$   
 $t_1 = 11$   
 $hitObj = O_1$

$O_2$   
 $\rightarrow$

$t_0 = 0$   
 $t_1 = 11$   
 $hitObj = O_1$

$O_3$   
 $\rightarrow$

$t_0 = 0$   
 $t_1 = 3.5$   
 $hitObj = O_3$

$\rightarrow$

$t_0 = 0$   
 $t_1 = 3.5$   
 $hitObj = O_3$

$t_0 = 0$   
 $t_1 = \infty$   
 $hitObj = \emptyset$

$O_1$   
 $\rightarrow$

$t_0 = 0$   
 $t_1 = \infty$   
 $hitObj = \emptyset$

$O_2$   
 $\rightarrow$

$O_3$   
 $\rightarrow$

$O_4$   
 $\rightarrow$

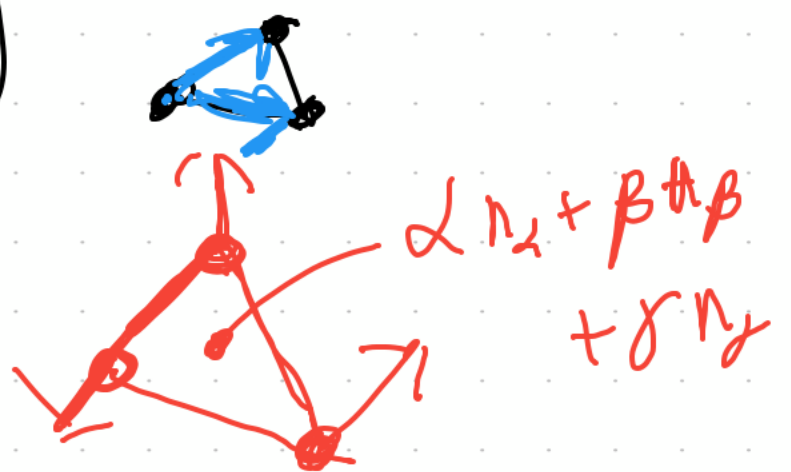
$t_0 = 0$   
 $t_1 = \infty$   
 $hitObj = \emptyset$

# Shading

• Use Phong Shading (lab 5 (ish))

What did we need

- intersection point  $p$
- surface normal at  $p$ 
  - Sphere (implemented in lab 5 (ish))
  - $\Delta$  (lab 6 & exam)



## Simple Raytracer

```
for each pixel do
  compute viewing ray
  if (ray hits an object with  $t \in [0, \infty)$ ) then
    Compute  $n$ 
    Evaluate shading model and set pixel to that color
  else
    set pixel color to background color
```