The History of the Abacus

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The abacus is a counting tool that has been used for thousands of years. Throughout history, calculating larger numbers has been problematic, especially for the common uneducated merchant. Out of this necessity, the idea of the abacus was born. Solving problems on an abacus is a quick mechanical process rivaling that of modern-day four-function calculators. After first addressing basic counting procedures and memorizing a few simple rules, students can use the abacus to solve a variety of problems. The abacus is a timeless computing tool that is still applicable in today's classrooms.

Introduction

Throughout history, keeping track of numbers has been problematic. How would you keep track of numbers without pen or paper? What if you couldn't read or write? Without a written language, how would you add, subtract, multiply, or divide large numbers accurately and efficiently? All of these problems can be solved with an abacus.

An abacus is a computing tool used for addition, subtraction, multiplication, and division. The abacus does not require pen or paper and works for any base number system. There are two basic forms of the abacus, the first being a counting table, which is a specially marked flat surface that uses small stones or beans as markers, as shown in Figure 1. The second is a bead frame abacus, which is a frame with beads strung on wires or rods, shown in Figure 2.

Counting tables have over 2000 years of documented use, dating back to Greeks and Romans. The earliest document counting boards were simple stone slabs with parallel and horizontal lines serving as place value indicators.

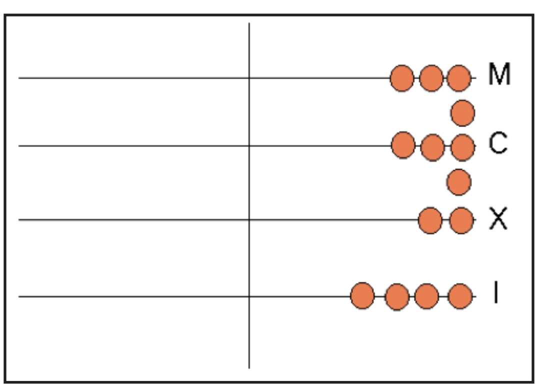

Fig 1 Counting table representing 3874

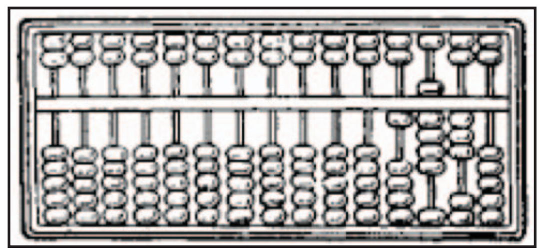

Fig 2 Bead frame abacus

Both computing tools calculate in a similar fashion.

Counting tables have over 2000 years of documented use, dating back to Greeks and Romans. The earliest documented counting boards were simple stone slabs with parallel and horizontal lines serving as place value indicators. The "normal method of calculation in Ancient Greece and Rome was done by moving counters on a smooth board or table suitably marked with lines or symbols to show the 'places'" (Pullan, 1968, p. 18). The counting board's development then stagnated for hundreds of years with the only noticeable change being the move to vertical place-values lines.

The origin of the portable bead frame abacus is not well-known. It was thought to have originated out of necessity for traveling merchants. Some historians credit the Chinese as the inventors of bead frame abacus, while others believe the Romans introduced the abacus to the Chinese through trade (Moon, 1971). From there, Russia and Japan developed their own versions of the abacus.

Today the abacus lives on in rural parts of Asia and Africa and has proven to be a timeless computing tool. Its operation offers everyone the ability to compute large numbers without any electrical devices. As seen in the next section, the simplicity of counting on the abacus provides endless computing possibilities to people of all ages around the world.

History Throughout the Ancient Worlds

It is certain that mechanical aids have been used as calculation devices by ancient civilizations. Merchants may have even used a crude form of the counting table during prehistoric times. Ever since the Phoenicians, Egyptians, and Greeks were involved in Mediterranean trade, the abacus has become a world-wide computing tool (Moon, 1971).

Since the abacus' history is not clear, the origin of the tool can only be speculated. Counting boards are thought to be the predecessor of the beaded abacus. Counting boards were likely crudely constructed wooden tables, lost to decomposition like the wooden huts in which they were used (Pullan, 1968). To unravel the history of the abacus we look next at classical paintings and literature; however, these historical artifacts do not portray the lower class merchants and businessmen who frequently used counting boards. "One could hardly expect the aristocratic Plato, for instance, to write a treatise on a device used by slaves and petty tradesmen" (Moon, 1971, 21). For this reason, the abacus is often left out of Greek and Roman history.

Salamis Counting Board

The oldest existing counting board was discovered on the island of Salamis about a century ago (see Fig 3). It has been estimated that the counting board dates back to the fourth century BC (Pullan, 1968). It's made of white marble and measures 149 × 75 × 4.5 cm. On its top face is a set of eleven parallel lines accompanied with Greek numerals. Metal counters were placed on the board between the lines, and the number of counters on each line represents different values. The vertical spaces were used to indicate base units, tens, hundreds, etc., and intermediate spaces allowed the operator to mark larger values, such as fives, fifties, etc. Figure 3 depicts this idea. Tables could easily be arranged to compute addition and subtraction problems. As time passed and cross-cultural exchange took place, the abacus evolved into a tailored tool that reflected the operator's individual needs.

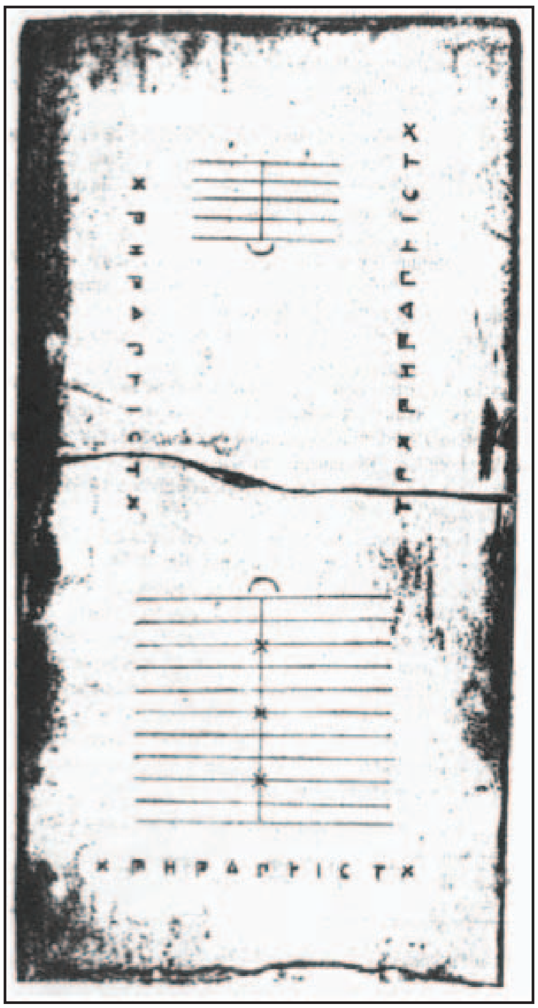

Fig 3 Salamis Counting Board

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Medieval Counting Boards

Counting boards were used throughout Europe during the Middle Ages until the 16th century when they were replaced by pencil-and-paper calculations. Arithmetic books depicting counting boards became the first documentation with the advent of more modern publication methods (Moon, 1971). The illustrations often display two mathematicians solving equations, one using a counting table and the other calculating with pencil and paper. One such painting is shown in Figure 4.

Arithmetic books depicting counting boards . . . often display two mathematicians solving equations, one using a counting table, and the other calculating with pen and paper.

Fig 4 Mathematicians solving equations with a counting table and pen and paper

Medieval counting boards were arranged vertically with the numbers increasing as they moved away from the operator. Counters were placed on lines or grooves signifying a specific place-value while larger values could be marked between the lines.

Other Abaci

The bead frame abacus and a counting board have many features in common but are unique to the cultures that use them. The counting boards are thought to be more flexible than the bead frame abacus because the operator has an unlimited supply of markers, but the bead frame abacus is more compact, portable, and is capable of more rapid computing. In the following sections we will explore the unique features of different forms of beaded abaci as they evolved around the world.

Roman Abacus

The Roman abacus features sliding counters permanently attached to the device. It consists of a metal plate with fixed counters sliding in grooves. Each lower grove holds four markers, except for the right-most groove which holds five. Similarly, each of the upper grooves holds one marker. Beads in the upper slots have a value five times that of the marker below.

Suan-pan

"Some authorities believe that the Roman abacus was introduced into China early in the Christian era by traveling merchants" (Moon, 1971, p. 30). The earliest mention of the abacus in Chinese literature did not occur until the 12th century. The Suan-pan is widely used throughout China and other parts of Asia. The Suan-pan features five unit beads on each lower rod and two 'five-beads' on each upper rod, as shown in Figure 5.

Soroban

The modern Japanese abacus, known as a Soroban, was developed from the Chinese

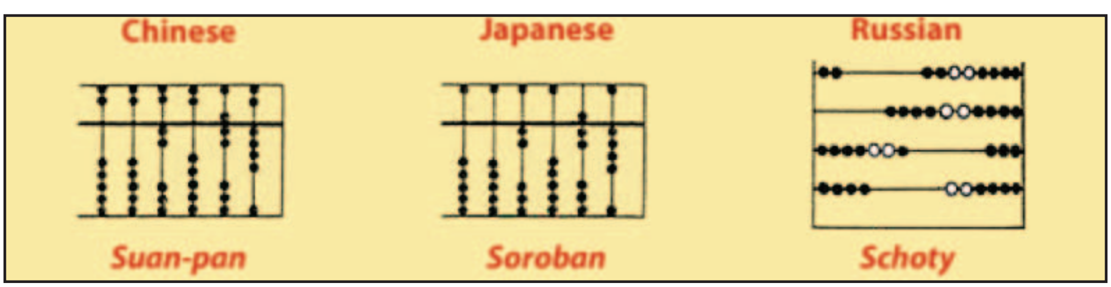

Fig 5 Other abaci

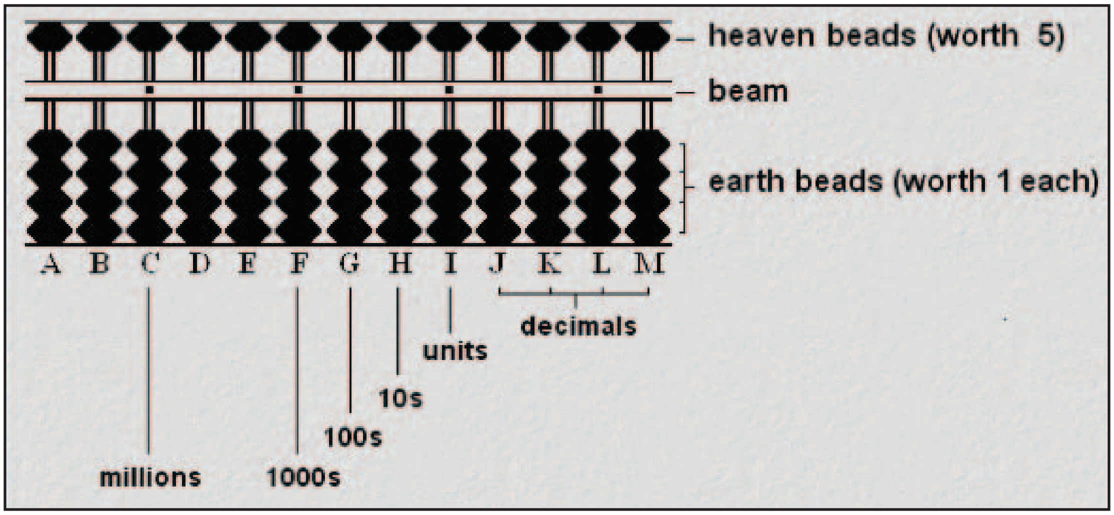

Fig 6 Soroban Abacus

Suan-pan. The Sorban features four unit beads on the lower rods, and one 'five bead' on the upper rods as depicted in Figure 6. The Sorban also features large sharp-edge beads that are easily manipulated, and the distance through which they move is relatively short, allowing for high-speed operation.

Schoty

The Russian abacus, the Schoty, features ten beads per rod and no dividing bar, as depicted in Figure 5. This requires the movement of a larger number of beads over a relatively far distance, but is said to be easier to learn than instruments utilizing markers of value five. While all the previous devices were operated horizontally, the Russian instrument can also be operated vertically (Pullan, 1968).

Abacus Operation

In general, all abaci work in a similar fashion. In this section we explore how to operate the popular Soroban abacus. The Soroban abacus is ideal for a base-ten numbering system, in which each rod acts as a placeholder and can represent values 0 through 9. On each rod, the Soroban has one bead in the upper deck, know as a heaven bead, and four beads in the lower deck, know as earth beads (Kojima, 1954). Each heaven bead in the upper deck has a value of 5, each earth bead in the lower deck has a value of 1, as seen in Figure 6. The beads are considered counted when they are moved towards the divider separating the upper and lower decks.

Once it is understood how to count using an abacus, any integer can easily be found. The general rule is that the value of any given bead is ten times that of the bead to its immediate right, or one tenth the value of the bead to its immediate left (Kojima, 1954). Decimals are represented right of the unit rod, also seen in Figure 6.

There are two general rules for quickly and easily solving any addition and subtraction problem with the Soroban abacus. First, the operator should always solve problems from left to right (Kojima, 1954). This may be confusing at first but will drastically increase the speed of operation, and with a little practice can make computing an easy-to-learn mechanical process. Second, the operator must be familiar with how to find complementary numbers, specifically, always with respect to 10 (Kojima, 1954). A number's complement is found through a simple addition problem. The value added to the original number to make 10 is the number's complement. For example, the complement of 7, with respect to 10,

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This lesson built upon the concept of measuring lines using iterated lengths, and successfully built a bridge from this concept to the more abstract ruler.

is 3 and the complement of 6, with respect to 10, is 4.

Addition

When solving addition problems with the Soroban abacus, subtraction is required when the addends sum to a value greater than 9. In such a case, the complement is subtracted and 1 bead is added to the next highest place value (Heffelfinger and Flom, 2011). For example, consider adding 8 and 4. The process begins by registering 4 on the unit rod H, as shown in Figure 7.

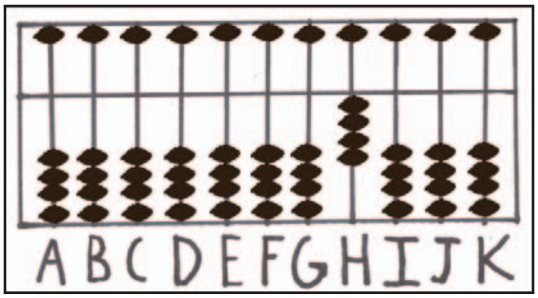


Fig 7 4 on rod H

Because the sum of the two numbers is greater than 9, subtraction must be used. We subtract the complement of 8 - namely 2 - from 4 on rod H and add 1 bead to tens rod G. This is illustrated in Figure 8.

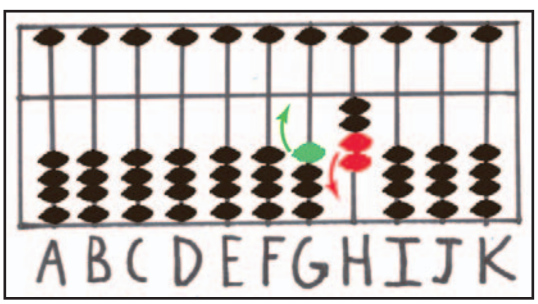


Fig 8 Subtracting 2 and adding 10

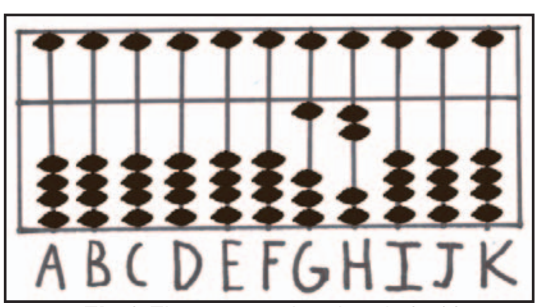


Fig 9 Final result showing 4+8=12

This leaves us with 1 bead registered on rod G (the tens rod) and 2 beads on rod H (the unit rod), as shown in Figure 9. The result shows that 8+4 is 12.

This rule remains the same regardless of the numbers used. Consider, for instance the addition problem 356+472. First register 356 on rods F, G, and H, with H as the unit rod, as shown in Figure 10.

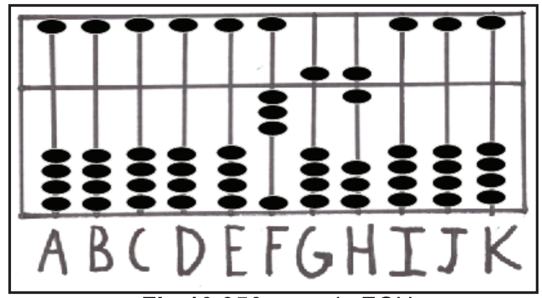


Fig 10 356 on rods FGH

Recall that when performing operations, we work from left to right. Next, add 4 hundreds to rod F (adding 5 and subtracting 1). The abacus now reads 756, as shown in Figure 11.

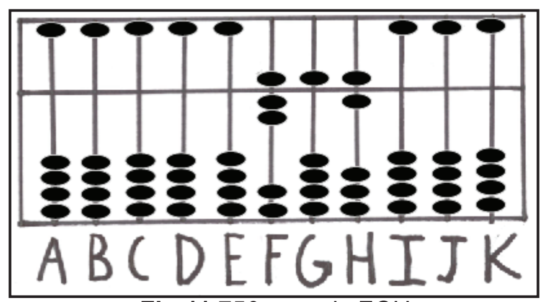


Fig 11 756 on rods FGH

Now add 7 tens to rod G. Because 5+7 > 9, the complement of 7 - namely 3 - is subtracted from rod G, and 1 bead added to rod F. The resulting value, 826, is illustrated in Figure 12.

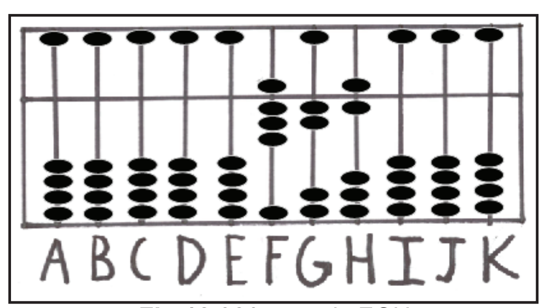


Fig 12 826 on rods FGH

The final step is to add 2 to the unit rod H. This yields the final solution 828 illustrated in Figure 13.

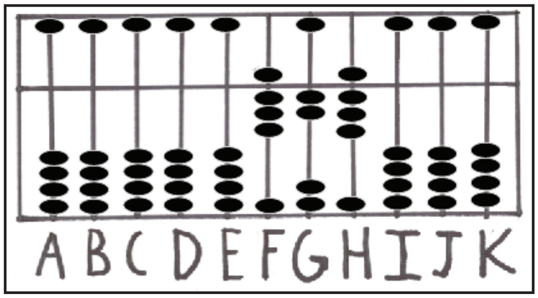


Fig 13 828 on rods FGH

Subtraction

As we all know, subtraction is the opposite operation of addition. Thus, when subtracting with the Soroban abacus, we add the complement and subtract 1 bead from the next highest place value. For example, when subtracting 8 from 12, start by registering 1 on the tens rod G and 2 on the units rods H. No tens are subtracted, so we begin by subtracting 7 from rod H. Because $2 - 8 < 0$, the complement of 8 - namely 2 - is added to the units rod H, and 1 bead is subtracted from the tens rod G. This is shown in Figure 14. This leaves the answer 4.

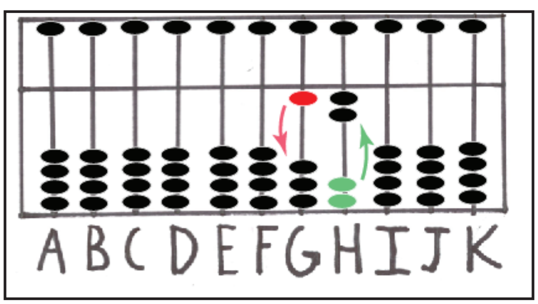


Fig 14 Adding 2 and subtracting 10

Next, we examine a more challenging problem, 6432-5361. First register 6432 on rods E, F, G, and H, with H as the units rod (Fig 15). Subtract 5 thousands from rod E, leaving 1432 (Fig 16). Next, subtract 3 hundreds from rod F. As Figure 17 illustrates, the abacus now reads 1132.

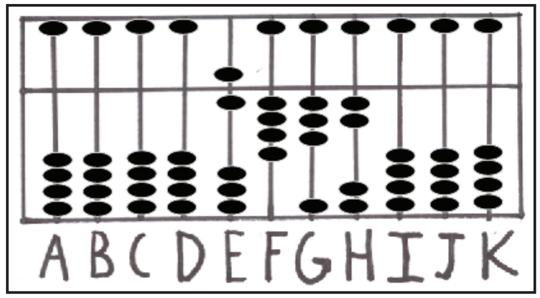


Fig 15 6432 on rods EFGH

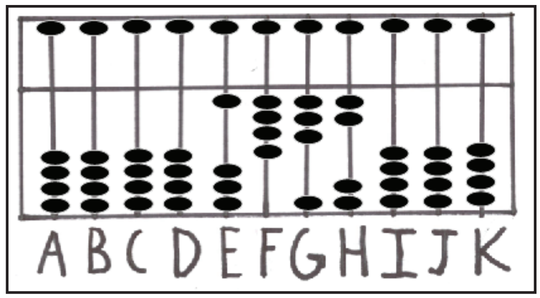


Fig 16 1432 on rods EFGH

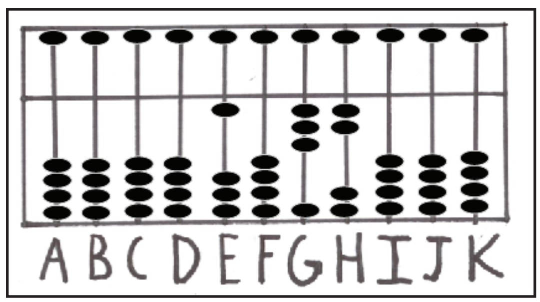


Fig 17 1132 on rods FGH

When subtracting 6 tens from rod G, the complement of 6 - namely 4 - is added to rod G, and 1 bead is subtracted from rod F. The abacus now reads 1072 as shown in Figure 18. The final step is to subtract 1 from the unit rod H. The abacus depicts the final solution, 1071 (left to the reader).

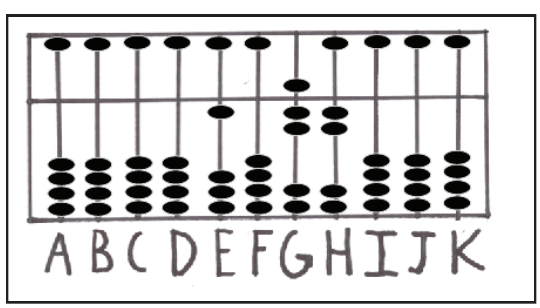


Fig 18 1072 on rods FGH

Multiplication

Multiplication problems, although potentially more difficult than addition and subtraction, can be easily computed with the Soroban abacus. Before students can successfully complete multiplication problems, they must first be familiar with multiplication tables through 9×9. Registering the multiplicand and the multiplier is the most critical step in the process. This ensures the ones value of the product falls neatly on the unit rod.

As an example, let's consider the multiplication problem 36×4, with multiplicand 36 and multiplier 4. We begin by placing our finger on unit rod H and count left one rod for every digit in the multiplier (1 position to rod G) and one rod for each digit in the multiplicand (2 positions to rod E) (Heffelfinger and Flom, 2011). Next, register 36 on rods E and F. Then place 4 on rod B. This leaves enough space to help students distinguish the multiplicand from the multiplier, as suggested in Figure 19.

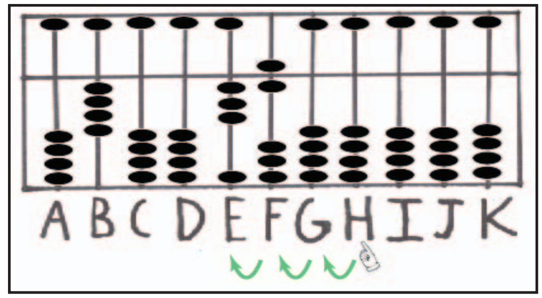


Fig 19 Registering 36 x 4

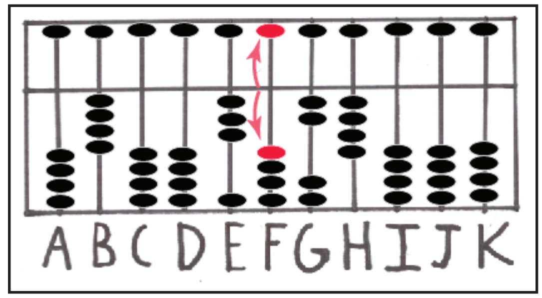


Fig 20 Partial product 24 on GH with F reset to 0

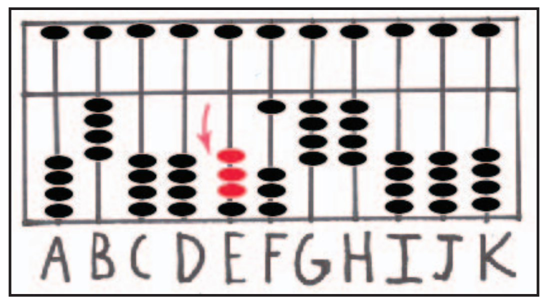


Fig 21 Final product 144 on FGH with D reset to 0

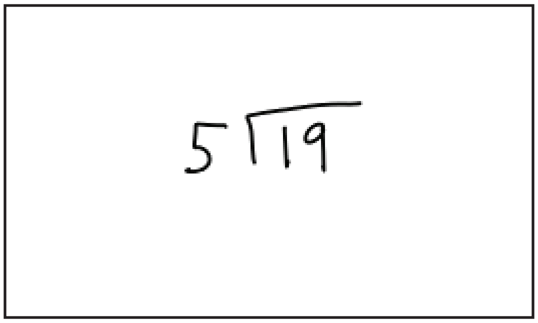
Performing multiplication on the abacus involves nothing more than the addition of partial products. Our first step is multiplying 6 by 4, and adding the partial product on the two rods, GH, to the right of the multiplicand. Since we've accounted for the 6, we reset rod F to zero. These steps are illustrated in Figure 20.

A similar process is followed to multiply 30 by 4. Its product, 120, is added to rods EFG. Since we've accounted for the 30 in our calculation, we reset rod D to zero. This leaves the final product, 144, on rods FGH as shown in Figure 21. Once addition is mastered, the reader is encouraged to try multiplication problems that involve carrying, such as 36×9.

Division

Solving division problems on the Soroban abacus mirrors familiar paper-and-pencil calculations. Furthermore, the process utilizes addition, subtraction, and multiplication we've discussed in previous sections of this paper. As such, we explore the following example, $19 \div 5$, by means of a series of visual steps that connect our abacus work to the traditional paper-and-pencil procedure familiar to many teachers.

As Figures 22-25 suggest, division on the abacus provides students with an alternative to paper-and-pencil calculation that provides a more hands-on experience for young learners. Our example is rudimentary. For a more in-depth treatment of the topic, readers are encouraged to explore resources provided in the reference list of this paper.



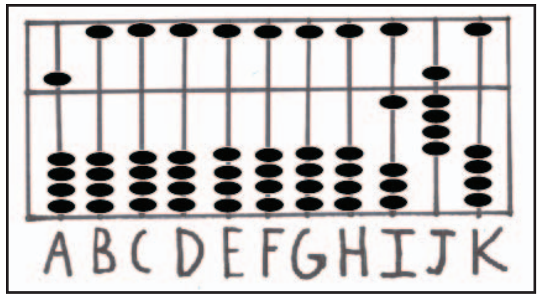
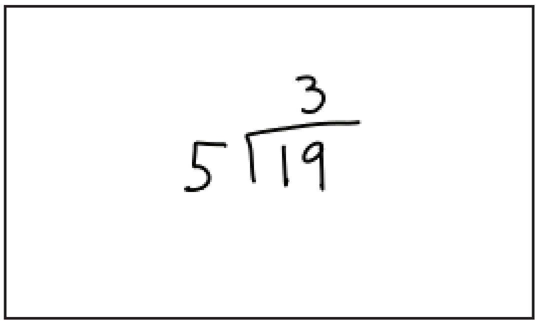


Fig 22 Setting up 19 divided by 5



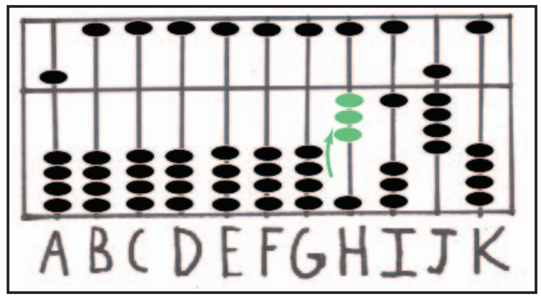
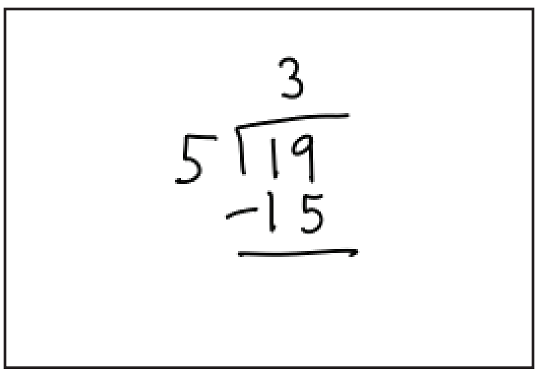


Fig 23 Determing the number of 5's in 19



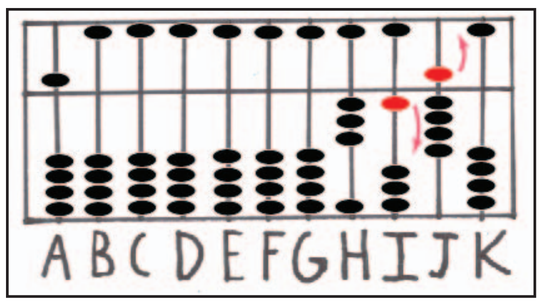
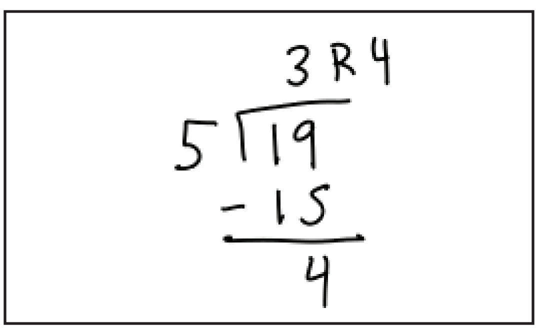


Fig 24 Multiplying divisor by quotient and subtracting from dividend



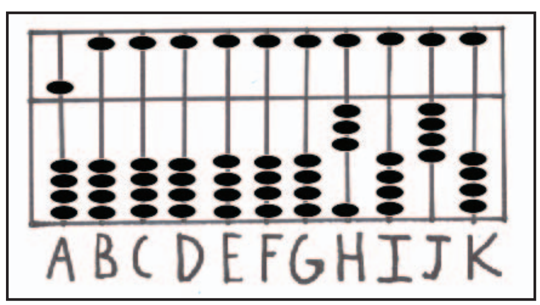


Fig 25 Final solution, $19 \div 5 = 3R4$

Summary

The abacus has played a vital role in mathematics that can still be seen today. Its portability made it a popular tool used by merchants and allowed it to be introduced to all parts of the world. Its flexibility allowed the abacus to evolve into a tool that could be used across cultures. Its mechanical operation allows addition, subtraction, multiplication, and division to be done accurately and efficiently without pencil and paper, rivaling that of a four-function calculator. The abacus is window into the past which allows users to process operations in the same fashion of that done for thousands of years. Moreover, the device provides students in today's classrooms with alternatives to paper-and-pencil procedures that let them explore calculations in a more hands-on manner.

References

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Sources of Images

Figure 1: http://homepage.mac.com/shelleywalsh/MathArt/CBAddition.html
Figure 2: http://encyclopedia2.thefreedictionary.com/Bead+frame
Figure 3: http://www.historyofinformation.com/index.php?id=1664
Figure 4: http://www.mlahanas.de/Greeks/Counting.htm
Figure 5: http://www.ucmas.ca/our-programs/how-does-it-work/what-is-an-abacus/
Figure 6: http://webhome.idirect.com/~totton/abacus/
Figures 7-25: Drawn by the author using Adobe Illustrator