# Modified Conjugate Beamforming for Cell-Free Massive MIMO

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***Abstract*—We present a modification of conjugate beamforming for the forward link of cell-free massive MIMO networks. This modification eliminates the self-interference and yields a performance that, without forward pilots, closely approaches what would be achieved with such pilots in place. The simplicity of conjugate beamforming is preserved, with no need for matrix inversions, at the expense of fading-rate coordination among the access points.**

***Index Terms*—Cell-free networks, massive MIMO, conjugate beamforming, power allocation.**

## I. Introduction

CELL-FREE massive MIMO can be regarded as a deconstruction of cellular massive MIMO: the many antennas that would be collocated at the cell sites are scattered over the network and the associations between users and cells are released. The result is a dense infrastructure of access points (APs), each featuring one or a few antennas, with every user potentially served from every AP via conjugate beamforming [1]–[5]. Capitalizing on extensive backhaul, cell-free networks offer several advantages over their cellular counterparts, including increased large-scale diversity and user proximity.

As in cellular massive MIMO, the total number of antennas is substantially larger than the number of users per time-frequency resource; this renders conjugate beamforming effective and ensures low multiuser interference. In contrast, the channel hardening observed in cellular massive MIMO does not carry over to cell-free networks because, in such networks, the channel gains are not IID; they are independent, but have very disparate strengths. As a result, substantial self-interference arises in forward-link transmissions devoid of pilots. This problem can be remedied at the user receivers, at the expense of incorporating precoded forward pilots for each user [6]. Alternatively, a partial recovery is possible through blind methods operating on data observations [7].

This letter proposes a modified conjugate beamforming technique that prevents self-interference completely, with no action required at the receivers.

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## II. Network and Channel Models

The networks under consideration feature $N$ APs, each equipped with $M$ antennas (where $M$ is small), and $K$ single-antenna users. Every AP can communicate with every user on each time-frequency resource. With time-division duplexing and perfect calibration of the transmit-receive chains [8], the forward and reverse channels are reciprocal. A share of the resources are reserved for pilot transmissions from the users, based on which the channels are estimated by the APs. The remaining resources, apportioned between the forward and reverse directions as desired, are available for data transmission.

### A. Large-Scale Modeling

Provided the AP locations are agnostic to the radio propagation, shadowing has been shown to make such locations seem Poisson-distributed from the vantage of any user [9]. This approximation sharpens as the shadowing strengthens, being highly precise for values of interest [9], [10]. Leveraging this result, we place the APs and users randomly over the network, such that their locations conform to respective (mutually independent) binomial point processes; as the network grows, these converge to Poisson point processes.

Each CDF in this letter corresponds to 1000 network snapshots in a wrapped-around universe with 200 APs, ensuring a 95% confidence interval of 0.3% in absolute terms.

Signals are subject to pathloss with exponent $\eta$, giving a large-scale channel gain $G_{n,k} = d_{n,k}^{-\eta}$ between the $n$th AP and the $k$th user, distanced by $d_{n,k}$. The forward- and reverse-link large-scale SNRs equal $\mathsf{SNR}_{n,k} = G_{n,k} P/\sigma^2$ and $\mathsf{SNR}^{\mathsf{r}} = G_{n,k} P^{\mathsf{r}}/\sigma^2$ with $P$ and $P^{\mathsf{r}}$ the maximum transmit powers at APs and users, respectively, measured at 1 m from their source so that no scaling constants are needed. In turn, $\sigma^2$ is the noise power.

Defining $\rho = P/P^{\mathsf{r}}$, we can relate the large-scale SNRs in both directions via

$$\mathsf{SNR}^{\mathsf{r}}_{n,k} = \frac{\mathsf{SNR}_{n,k}}{\rho}. \tag{1}$$

### B. Small-Scale Modeling

Besides $G_{n,k}$, the reverse-link channel between the $k$th user and the $n$th AP features the small-scale fading vector $\boldsymbol{h}_{n,k} \sim \mathcal{N}_{\mathbb{C}}(\mathbf{0}, \boldsymbol{I}_M)$, independent across users and APs. Owing to reciprocity, the forward-link fading between the $n$th AP and the $k$th user is $\boldsymbol{h}^*_{n,k}$.

## III. Hardening-Based Conjugate Beamforming

### A. Channel Hardening

The effectiveness of conjugate beamforming descends from the law of large numbers. Let $\boldsymbol{h}^*_k$ and $\boldsymbol{h}^*_{\mathrm{k}}$ be the $N$-dimensional fading vectors from all antennas to users $k$ and k. In cellular massive MIMO, where the antennas are collocated, these

$$y_k = \underbrace{\sum_{n=1}^{N} \sqrt{\frac{M\mathsf{SNR}^{\mathsf{r}}_{n,k}}{1+\mathsf{SNR}^{\mathsf{r}}_{n,k}}} \sqrt{G_{n,k}p_{n,k}P}\, s_k}_{\text{Desired Signal: } S_k} + \underbrace{\sum_{n=1}^{N}\left(\sqrt{\frac{1+\mathsf{SNR}^{\mathsf{r}}_{n,k}}{M\mathsf{SNR}^{\mathsf{r}}_{n,k}}}\boldsymbol{h}^*_{n,k}\hat{\boldsymbol{h}}_{n,k} - \sqrt{\frac{M\mathsf{SNR}^{\mathsf{r}}_{n,k}}{1+\mathsf{SNR}^{\mathsf{r}}_{n,k}}}\right)\sqrt{G_{n,k}p_{n,k}P}\, s_k}_{\text{Self-Interference: } E_k}$$
$$+ \underbrace{\sum_{n=1}^{N}\sqrt{G_{n,k}P}\sum_{\mathrm{k}\neq k}\sqrt{\frac{1+\mathsf{SNR}^{\mathsf{r}}_{n,\mathrm{k}}}{M\mathsf{SNR}^{\mathsf{r}}_{n,\mathrm{k}}}}\boldsymbol{h}^*_{n,k}\hat{\boldsymbol{h}}_{n,\mathrm{k}}\sqrt{p_{n,\mathrm{k}}}\, s_{\mathrm{k}}}_{\text{Multiuser Interference: } I_k} + v_k \tag{2}$$

vectors have IID entries and, for $\frac{N}{K} \to \infty$,

$$\frac{1}{N}\boldsymbol{h}^*_k\boldsymbol{h}_{\mathrm{k}} \xrightarrow{\text{a.s.}} \mathbb{E}\left[h^*_{n,k}h_{n,\mathrm{k}}\right] = \begin{cases}1 & k=\mathrm{k}\\ 0 & k\neq \mathrm{k}.\end{cases} \tag{3}$$

Thus, for $N \gg K$, a precoder $\boldsymbol{f}_k \propto \boldsymbol{h}_k$ leads to (*i*) a hardened (over the fading) precoded channel at user *k*, whereby forward pilots are not needed and the decoding can rely on large-scale quantities, and (*ii*) minimal interference onto users $\mathrm{k} \neq k$.

In cell-free networks, the channel gains connecting the *N* APs with a user are no longer IID and hence (*i*) is not upheld.

### B. Reverse-Link Channel Estimation

Let $\mathcal{P}_k$ be the set of users (including user *k*) that share the pilot of user *k*. The simultaneous transmission from the users in this set of a pilot of power $P^{\mathsf{r}}$ is observed at the *n*th AP as

$$\boldsymbol{y}_n = \sum_{\mathrm{k}\in\mathcal{P}_k}\sqrt{G_{n,\mathrm{k}}}\boldsymbol{h}_{n,\mathrm{k}}\sqrt{P^{\mathsf{r}}} + \boldsymbol{v}_n \tag{4}$$

where $\boldsymbol{v}_n \sim \mathcal{N}_{\mathbb{C}}(\boldsymbol{0}, \sigma^2\boldsymbol{I}_M)$. From $\boldsymbol{y}_n$, the *n*th AP produces the LMMSE channel estimate $\hat{\boldsymbol{h}}_{n,k}$ satisfying $\boldsymbol{h}_{n,k} = \hat{\boldsymbol{h}}_{n,k}+\tilde{\boldsymbol{h}}_{n,k}$ where $\tilde{\boldsymbol{h}}_{n,k} \sim \mathcal{N}_{\mathbb{C}}(\boldsymbol{0}, \mathsf{MMSE}_{n,k}\boldsymbol{I})$ is uncorrelated error and

$$\mathsf{MMSE}_{n,k} = \frac{1+\sum_{\mathrm{k}\in\mathcal{P}_k,\mathrm{k}\neq k}\mathsf{SNR}^{\mathsf{r}}_{n,\mathrm{k}}}{1+\sum_{\mathrm{k}\in\mathcal{P}_k}\mathsf{SNR}^{\mathsf{r}}_{n,\mathrm{k}}}. \tag{5}$$

Since the problem we address is not directly related to pilot contamination, and there are ways of keeping such contamination at bay [1], [11], we disregard it to avoid distractions and the need to posit specific pilot assignments. This amounts to $\mathcal{P}_k$ containing only user *k*, from which

$$\mathbb{E}\left[\|\hat{\boldsymbol{h}}_{n,k}\|^2\right] = \frac{M\mathsf{SNR}^{\mathsf{r}}_{n,k}}{1+\mathsf{SNR}^{\mathsf{r}}_{n,k}},$$
$$\mathbb{E}\left[\|\tilde{\boldsymbol{h}}_{n,k}\|^2\right] = \frac{M}{1+\mathsf{SNR}^{\mathsf{r}}_{n,k}}. \tag{6}$$

### C. Forward-Link Data Transmission

The precoder applied by the *n*th AP to beamform to user *k* is $\boldsymbol{f}^{\mathrm{cb}}_{n,k} \propto \hat{\boldsymbol{h}}_{n,k}$. Altogether, the *n*th AP generates the signal

$$\boldsymbol{x}_n = \sum_{k=1}^{K}\boldsymbol{f}^{\mathrm{cb}}_{n,k}s_k \tag{7}$$

with $s_k$ the unit-variance symbol intended for user *k* and

$$\boldsymbol{f}^{\mathrm{cb}}_{n,k} = \sqrt{\frac{p_{n,k}P}{\mathbb{E}\left[\|\hat{\boldsymbol{h}}_{n,k}\|^2\right]}}\hat{\boldsymbol{h}}_{n,k} \tag{8}$$

where, by virtue of the normalization by $\mathbb{E}[\|\hat{\boldsymbol{h}}_{n,k}\|^2]$, the share of power that the *n*th AP devotes to user *k* is $p_{n,k}$ with $\sum_{k=1}^{K}p_{n,k} \leq 1$. Within the class of large-scale-based power allocations, the most natural are:

- Maximal-ratio, $p_{n,k} = \frac{G_{n,k}}{\sum_{\mathrm{k}=1}^{K}G_{n,\mathrm{k}}}$.
- Max-min (a quasi-convex optimization solved iteratively), which equalizes the SINRs and maximizes fairness [1].

User $k$ observes

$$y_k = \sum_{n=1}^{N}\sqrt{G_{n,k}}\boldsymbol{h}^*_{n,k}\boldsymbol{x}_n + v_k \tag{9}$$

where $v_k \sim \mathcal{N}_{\mathbb{C}}(0,\sigma^2)$. With hardening-based reception (no forward pilots), user *k* recovers the projection of $y_k$ onto [1]

$$\sqrt{G_{n,k}}\mathbb{E}\left[\boldsymbol{h}^*_{n,k}\boldsymbol{f}^{\mathrm{cb}}_{n,k}\right] = \sqrt{\frac{M\mathsf{SNR}^{\mathsf{r}}_{n,k}}{1+\mathsf{SNR}^{\mathsf{r}}_{n,k}}}\sqrt{G_{n,k}p_{n,k}P} \tag{10}$$

where we have invoked (6). In turn, the projection of $y_k$ on $\sqrt{G_{n,k}}\boldsymbol{h}^*_{n,k}\boldsymbol{f}^{\mathrm{cb}}_{n,k} - \sqrt{G_{n,k}}\mathbb{E}[\boldsymbol{h}^*_{n,k}\boldsymbol{f}^{\mathrm{cb}}_{n,k}]$ is self-interference. Combining (6)–(10), the observation at user *k* can be written as (2) at top of this page, from which the SINR emerges as

$$\mathsf{SINR}^{\mathrm{cb}}_k = \frac{\mathbb{E}\left[|S_k|^2\right]}{\sigma^2+\mathbb{E}[|E_k|^2]+\mathbb{E}[|I_k|^2]} \tag{11}$$
$$= M\frac{\left(\sum_{n=1}^{N}\sqrt{\frac{\mathsf{SNR}^2_{n,k}p_{n,k}}{\rho+\mathsf{SNR}_{n,k}}}\right)^2}{1+\sum_{n=1}^{N}\mathsf{SNR}_{n,k}\sum_{\mathrm{k}=1}^{K}p_{n,\mathrm{k}}}, \tag{12}$$

expressed as function of only the forward-link SNRs via (1). In interference-limited conditions, the above reduces to

$$\mathsf{SIR}^{\mathrm{cb}}_k = M\frac{\left(\sum_{n=1}^{N}\sqrt{G_{n,k}p_{n,k}}\right)^2}{\sum_{n=1}^{N}G_{n,k}\sum_{\mathrm{k}=1}^{K}p_{n,\mathrm{k}}}. \tag{13}$$

## IV. Hardening Shortfall in Cell-Free Networks

The fluctuations of the precoded channels over their expected values have two detrimental effects, opposite sides of the same coin, on hardening-based receivers: they subtract signal power, turning it into self-interference. These effects are gauged in Fig. 1, which depicts, as a function of *M*,

$$\frac{\mathbb{E}\left[|E_k|^2\right]}{\mathbb{E}[|S_k|^2]} \quad \text{and} \quad \frac{\mathbb{E}\left[|E_k|^2\right]}{\mathbb{E}[|E_k|^2]+\mathbb{E}[|I_k|^2]} \tag{14}$$

further averaged over the user locations, for maximal-ratio power allocation, $\eta = 4$, and *N*/*K* = 4 and 10. These ratios quantify the self-interference, respectively as a share of the desired signal and of the total interference. For $M = 1$, self-interference steals about a third of the desired signal and it represents about two thirds of the interference, and only for substantial *M* is this largely corrected. For cell-free networks with small *M*, therefore, self-interference is a major issue.

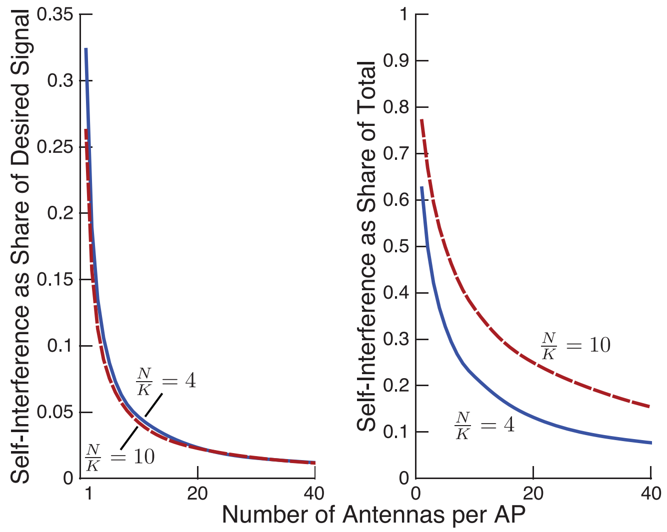


Fig. 1. Average (over all locations) ratios of self-interference to desired signal and of self-interference to total interference. Both ratios are for maximal-ratio power allocation, $\eta = 4$, and *N*/*K* = 4,10, as a function of *M*.

## V. Genie-Aided Upper Bound

To calibrate the loss in SIR caused by self-interference when $M = 1$, we can contrast (13) with a genie-aided counterpart where users have perfect knowledge of their precoded channels, i.e., user $k$ knows $\boldsymbol{h}^*_{n,k}\boldsymbol{f}_{n,k}$ or equivalently $\boldsymbol{h}^*_{n,k}\hat{\boldsymbol{h}}_{n,k}$. Then, the projection onto this quantity becomes the desired signal, allowing us to rewrite (2) as

$$y_k = \underbrace{\sum_{n=1}^{N}\sqrt{\frac{1+\mathsf{SNR}^{\mathsf{r}}_{n,k}}{M\mathsf{SNR}^{\mathsf{r}}_{n,k}}}\boldsymbol{h}^*_{n,k}\hat{\boldsymbol{h}}_{n,k}\sqrt{G_{n,k}p_{n,k}P}\,s_k}_{\text{Desired Signal: } S_k} + \underbrace{\sum_{n=1}^{N}\sqrt{G_{n,k}P}\sum_{\mathrm{k}\neq k}\sqrt{\frac{1+\mathsf{SNR}^{\mathsf{r}}_{n,\mathrm{k}}}{M\mathsf{SNR}^{\mathsf{r}}_{n,\mathrm{k}}}}\boldsymbol{h}^*_{n,k}\hat{\boldsymbol{h}}_{n,\mathrm{k}}\sqrt{p_{n,\mathrm{k}}}\,s_{\mathrm{k}} + v_k}_{\text{Multiuser Interference: } I_k} \tag{15}$$

free of self-interference. From (15), as a function of the realization of $\boldsymbol{h}^*_{n,k}\hat{\boldsymbol{h}}_{n,k}$,

$$\begin{aligned}\mathsf{SINR}^{\mathsf{genie}}_k &= \frac{\mathbb{E}\left[|S_k|^2 | \boldsymbol{h}^*_{n,k}\hat{\boldsymbol{h}}_{n,k}\right]}{\sigma^2 + \mathbb{E}\left[|I_k|^2 | \boldsymbol{h}^*_{n,k}\hat{\boldsymbol{h}}_{n,k}\right]} \\ &= \frac{\frac{1}{M}\left|\sum_{n=1}^{N}\sqrt{\rho+\mathsf{SNR}_{n,k}}\boldsymbol{h}^*_{n,k}\hat{\boldsymbol{h}}_{n,k}\sqrt{p_{n,k}}\right|^2}{1+\frac{1}{M}\sum_{n=1}^{N}\mathsf{SNR}_{n,k}\mathbb{E}\left[\|\boldsymbol{h}_{n,k}\|^2 | \boldsymbol{h}^*_{n,k}\boldsymbol{f}_{n,k}\right]\sum_{\mathrm{k}\neq k}p_{n,\mathrm{k}}}.\end{aligned} \tag{16}$$

In interference-limited conditions, the above reduces to

$$\mathsf{SIR}^{\mathsf{genie}}_k = \frac{\left(\sum_{n=1}^{N}\sqrt{G_{n,k}}\|\boldsymbol{h}_{n,k}\|^2\sqrt{p_{n,k}}\right)^2}{\sum_{n=1}^{N}G_{n,k}\|\boldsymbol{h}_{n,k}\|^2\sum_{\mathrm{k}\neq k}p_{n,\mathrm{k}}}. \tag{17}$$

Figs. 2–4 present SIR distributions over all locations for $M = 1$, confirming the deficiency of (13) relative to (17) when $M$ is small.

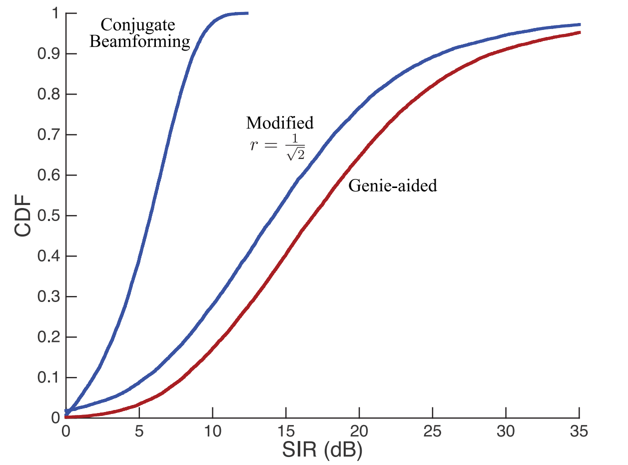


Fig. 2. CDF of SIR (averaged over the fading) for maximal-ratio power allocation, $\eta = 4$, $N/K = 10$ and $M = 1$.

## VI. Modified Conjugate Beamforming

The technique we propose exploits that the APs have more information than the users, hence they can (with only the uncertainty of channel estimation errors) compensate for the precoded channel fluctuations, tightening the overall gain around a target value. To make room for upward–downward corrections, such target needs to be below the hardening-based level in (10); we set the target to a portion $r \in [0, 1]$ of (10).

Start with conjugate beamforming as per (8). If the overall gain from all APs to user $k$ exceeds the target, i.e., if

$$\sum_n \sqrt{G_{n,k}}\hat{\boldsymbol{h}}^*_{n,k}\boldsymbol{f}^{\mathrm{cb}}_{n,k} > r\mathbb{E}\left[\sum_n \sqrt{G_{n,k}}\hat{\boldsymbol{h}}^*_{n,k}\boldsymbol{f}^{\mathrm{cb}}_{n,k}\right], \tag{18}$$

then we declare an upward fluctuation and all the precoders intended for user $k$ are scaled down to

$$\boldsymbol{f}^{\mathrm{mod}}_{n,k} = \frac{r\ \mathbb{E}\left[\sum_n \sqrt{G_{n,k}}\hat{\boldsymbol{h}}^*_{n,k}\boldsymbol{f}^{\mathrm{cb}}_{n,k}\right]}{\sum_n \sqrt{G_{n,k}}\hat{\boldsymbol{h}}^*_{n,k}\boldsymbol{f}^{\mathrm{cb}}_{n,k}}\boldsymbol{f}^{\mathrm{cb}}_{n,k} \qquad \forall n \tag{19}$$

such that the overall gain is pushed back to the target.

If (18) is reversed, the fluctuation is downwards. To correct it with the minimal amount of interference to other users, we identify as $n_{\max}$ the AP having the strongest large-scale gain to user $k$ and adjust upwards only $\boldsymbol{f}_{n_{\max},k}$, setting it to

$$\begin{aligned}&\boldsymbol{f}^{\mathrm{mod}}_{n_{\max},k} \\ &= \frac{r\ \mathbb{E}\left[\sum_n \sqrt{G_{n,k}}\hat{\boldsymbol{h}}^*_{n,k}\boldsymbol{f}^{\mathrm{cb}}_{n,k}\right] - \sum_{n\neq n_{\max}}\sqrt{G_{n,k}}\hat{\boldsymbol{h}}^*_{n,k}\boldsymbol{f}^{\mathrm{cb}}_{n,k}}{\sqrt{G_{n_{\max},k}}\|\hat{\boldsymbol{h}}_{n_{\max},k}\|^2}\hat{\boldsymbol{h}}_{n_{\max},k}\end{aligned} \tag{20}$$

where by

$$\begin{aligned}&\sqrt{G_{n_{\max},k}}\hat{\boldsymbol{h}}^*_{n_{\max},k}\boldsymbol{f}^{\mathrm{mod}}_{n_{\max},k} \\ &= r\ \mathbb{E}\left[\sum_n \sqrt{G_{n,k}}\hat{\boldsymbol{h}}^*_{n,k}\boldsymbol{f}^{\mathrm{cb}}_{n,k}\right] - \sum_{n\neq n_{\max}}\sqrt{G_{n,k}}\hat{\boldsymbol{h}}^*_{n,k}\boldsymbol{f}^{\mathrm{cb}}_{n,k}\end{aligned} \tag{21}$$

and the target is met again. Altogether, every user experiences a stable overall gain, disturbed only by channel estimation

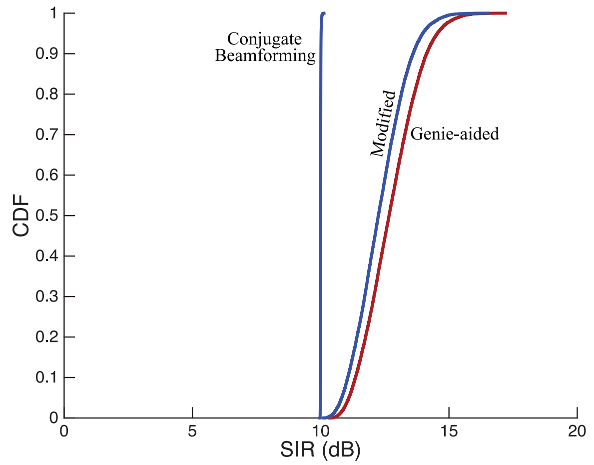


Fig. 3. CDF of SIR (averaged over the fading) for max-min power allocation, $\eta = 4$, $N/K = 10$ and $M = 1$.

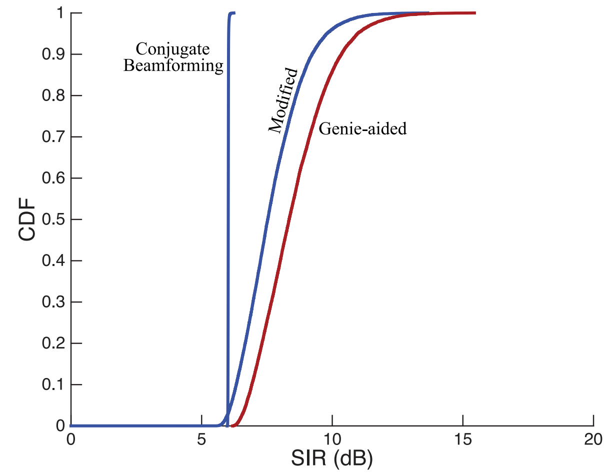


Fig. 4. CDF of SIR (averaged over the fading) for max-min power allocation, $\eta = 4$, $N/K = 4$ and $M = 1$.

errors, and the multiuser interference is curbed. (Since $r<1$, the per-AP transmit powers are lowered and a rescaling of all precoders is advisable if noise is significant; in interference-limited conditions, this is immaterial.) User $k$ observes

$$y_k = \underbrace{r\,\mathbb{E}\left[\sum_n \sqrt{G_{n,k}}\hat{\boldsymbol{h}}^*_{n,k}\boldsymbol{f}^{\text{cb}}_{n,k}\right] s_k}_{\text{Desired Signal: } S_k} + \underbrace{\left(\sum_n \sqrt{G_{n,k}}\boldsymbol{h}^*_{n,k}\boldsymbol{f}^{\text{mod}}_{n,k} - r\,\mathbb{E}\left[\sum_n \sqrt{G_{n,k}}\hat{\boldsymbol{h}}^*_{n,k}\boldsymbol{f}^{\text{cb}}_{n,k}\right]\right) s_k}_{\text{Self-interference: } E_k} + \underbrace{\sum_n \sum_{\text{k}\neq k} \sqrt{G_{n,k}}\boldsymbol{h}^*_{n,k}\boldsymbol{f}^{\text{mod}}_{n,\text{k}} s_{\text{k}}}_{\text{Mutual Interference: } I_k} + v_k \tag{22}$$

where, if $\hat{\boldsymbol{h}}_{n,k} = \boldsymbol{h}_{n,k}$ (interference-limited conditions without pilot contamination), $E_k$ indeed vanishes giving

$$\text{SIR}^{\text{mod}}_k = \frac{r^2 M \left(\sum_n \sqrt{G_{n,k} p_{n,k}}\right)^2}{\sum_n \sum_{\text{k}\neq k} G_{n,k}\mathbb{E}\left[\|\boldsymbol{f}^{\text{mod}}_{n,\text{k}}\|^2\right]} \tag{23}$$

with the denominator following from the independence of $\boldsymbol{f}^{\text{mod}}_{n,\text{k}}$ and $\boldsymbol{h}_{n,k}$. The portion $r$, which must be known by the users, can be optimized over; note that, in (23), $r$ affects the numerator and, through $\boldsymbol{f}^{\text{mod}}_{n,\text{k}}$, also the denominator.

## VII. Examples

Figs. 2–3 exemplify how, with $N/K = 10$ and $r = 1/\sqrt{2}$ for both maximal-ratio and max-min power allocations, the modifications push the SIR close to the (unachievable) genie-aided upper bound, erasing most of the deficit of conjugate beamforming. For shrinking $N/K$, self-interference is progressively overcome by multiuser interference, yet the modified beamformer continues to perform satisfactorily close to the genie-aided bound. This is illustrated in Fig. 4, which is the counterpart to Fig. 3 for $N/K = 4$.

In terms of $\eta$, its increase weakens the hardening and renders the modified beamforming even more effective. Conversely, a decrease in $\eta$ reduces the advantage.

## VIII. Discussion

The proposed modifications preserve the utter simplicity of conjugate beamforming, free of matrix inversions, at the expense of fading-rate coordination—needed anyway to combine and decode the reverse-link transmissions—among the APs. By translating these modifications from $\boldsymbol{f}_{n,k}$ to $p_{n,k}$, they can be construed as a fading-based power allocation, which can be overlaid onto any existing large-scale-based allocation.

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