Beamforming Based on Specular Component for Massive MIMO Systems in Ricean Fading

Dian-Wu Yue, *Member, IEEE*, Yan Zhang, and Yanan Jia

***Abstract*—This paper is concerned with massive MIMO systems over Ricean fading channels. To reduce the overhead for estimating channel state information (CSI), we investigate transmit/receive beamforming transmission only based on the specular component and treat the scattered component as interference. For the scenario without co-channel interference, we show that the ergodic achievable rate can remain unchanged when the transmit power is scaled down by the multiplication of the two numbers of antennas at the transmitter and the receiver, and can be equal to that of the corresponding beamforming based on perfect CSI when both of the numbers of antennas go without bound. Theoretical analysis is also extended to the scenario with co-channel interference. Simulation results further show that even though the numbers of antennas are not very large, the exact rate is quite close to its limit.**

***Index Terms*—Massive MIMO, large-scale antenna array, ricean fading, rician, beamforming, power scaling.**

I. Introduction

Massive MIMO is an emerging technology that employs hundreds of antennas at the base station (BS) to serve multiple users, can considerably improve the capacity of cellular systems and extend their coverage [1]. Furthermore, the benefits of massive MIMO can be reaped only by using simple signal processing techniques such as beamforming (BF) and zero-forcing (ZF) processing [2], [3]. So far there have appeared a lot theoretical contributions that provide various asymptotic performance with BF and ZF processing.

Massive MIMO systems substantially offer not only high data rates and increased link reliability but also a big reduction of transmit power. To quantify the power savings, the power scaling law with a very large antenna array has been deeply studied. In particular, [3] and [4] have addressed uplink massive MIMO systems employed BF and ZF detectors in Rayleigh and Ricean fading environments, and showed that when perfect channel state information (CSI) is available, the average (ergodic) achievable rate can converge to a constant when the number of antennas at the BS, M, grows large and the transmit power of each user is scaled down proportionally to $1/M$.

Obviously, we must carry on channel estimation to obtain the needed CSI. Unfortunately, the channel estimation in multi-cells will result in pilot contamination, which limits the system performance and becomes a fundamental challenge in massive-MIMO systems [1]. In Ricean fading environments, however, it is much easier to estimate only channel information corresponding to the specular component and ignore the scattered component. Therefore, if we can perform BF or ZF processing only based on the specular component, the overhead for acquiring CSI will be significantly reduced and the pilot contamination will be completely avoided.

So far, most of research in massive MIMO assumes that channels are with Rayleigh fading. However, there is only limited work for Ricean fading channels despite its practical significance [4], [5]. In addition, BF processing may be preferable to ZF processing due to its greater robustness [2]. Motivated by these mentioned-above reasons, in this letter we will investigate BF only based on the specular component for single-user massive MIMO systems without and with co-channel interference. We will assume that both the transmitter and the receiver are equipped with a large antenna array, and focus on the power scaling property when the number of antennas at the transmitter or at the receiver grows without bound.

II. System Model

We consider a single-user MIMO system with N transmit antennas and M receive antennas. Then $M \times 1$ received signal vector can be represented as

$$\mathbf{y} = \sqrt{p}\mathbf{H}\mathbf{x} + \mathbf{z} \tag{1}$$

where p is the short-term average transmitted power of the user, $\mathbf{H} = [h_{mn}]_{m,n=1}^{M,N}$ is the $M \times N$ normalized channel matrix with h_{mn} being the channel gain between the m-th receive antenna and the n-th transmit antenna; $\mathbf{x}$ is the normalized signal vector transmitted by the user; and $\mathbf{z}$ is the additive white Gaussian noise (AWGN) vector with zero-mean and covariance matrix $\sigma^2\mathbf{I}_M$, where $\mathbf{I}_M$ denotes the $M \times M$ identity matrix.

For a Ricean fading channel, the channel matrix, $\mathbf{H}$, can be decomposed into a sum of a specular matrix and a scattered matrix as

$$\mathbf{H} = \sqrt{\bar{\kappa}}\overline{\mathbf{H}} + \sqrt{\tilde{\kappa}}\widetilde{\mathbf{H}} \tag{2}$$

where $\overline{\mathbf{H}}$ is the specular matrix, and $\widetilde{\mathbf{H}}$ is the scattered matrix, $\bar{\kappa} = \frac{\kappa}{1+\kappa}$, $\tilde{\kappa} = \frac{1}{1+\kappa}$, and $\kappa > 0$ is the Ricean K-factor that represents the power ratio of specular and scattered components. From [6], the specular matrix $\overline{\mathbf{H}}$ in (2) can be expressed as

$$\overline{\mathbf{H}} = \mathbf{r}\mathbf{t}^{\mathrm{T}} \tag{3}$$

where $\mathbf{r}$ and $\mathbf{t}$ are the specular array responses at the receiver and the transmitter, respectively, and can be written as

$$\mathbf{r} = \left[1, e^{j2\pi d_r \sin(\theta)}, \ldots, e^{j2\pi(M-1)d_r \sin(\theta)}\right]^{\mathrm{T}} \tag{4}$$

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D.-W. Yue, Y. Zhang, and Y. Jia are with the College of Information Science and Technology, Dalian Maritime University, Dalian 116026, China (e-mail: dwyue@dlmu.edu.cn).

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and

$$\mathbf{t} = \left[1, e^{j2\pi d_t \sin(\phi)}, \ldots, e^{j2\pi(N-1)d_t \sin(\phi)}\right]^{\mathrm{T}} \quad (5)$$

where θ and ϕ are the angles of arrival and departure of the specular component, and d_r and d_t are the antenna spacings normalized by wavelength at the receiver and the transmitter, respectively. The entries in the scattering matrix $\widetilde{\mathbf{H}}$ are independent and identically distributed (i.i.d) circular complex Gaussian random variables with zero mean unit variance, i.e., $[\widetilde{H}]_{mn} \sim \mathtt{CN}(0,1)$.

For a system with a very large number of antennas, the scattered component is relatively hard to estimate and hence is assumed to be unknown at the transmitter and the receiver. The specular component depends only on the angles of arrival and departure and the spacings between antennas and is very easy to estimate. Thus it is assumed to be available at the transmitter and the receiver for transmit/receive BF transmission.

III. BF based on the Specular Component

Transmit/receive BF system transmits one symbol at a time. Denote s as the information symbol and let $\mathbb{E}(s^H s) = 1$ where $\mathbb{E}(\cdot)$ is the expectation operator. The information symbol s is first weighted by the transmit beamformer $\mathbf{w}_t$ with $\mathbb{E}(\mathbf{w}_t^{\mathrm{H}}\mathbf{w}_t) = 1$ before feeded into the N transmit antennas. Then the transmitted symbol vector will be $\mathbf{x} = \mathbf{w}_t s$, and the received vector in (1) can be rewritten as

$$\begin{aligned}\mathbf{y} &= \sqrt{p\bar{\kappa}}\bar{\mathbf{h}}s + \sqrt{p\tilde{\kappa}}\tilde{\mathbf{h}}s + \mathbf{z} \\ &= \sqrt{p\bar{\kappa}}\bar{\mathbf{h}}s + \bar{\mathbf{z}} \end{aligned} \quad (6)$$

where $\bar{\mathbf{h}} = \bar{\mathbf{H}}\mathbf{w}_t$, $\tilde{\mathbf{h}} = \tilde{\mathbf{H}}\mathbf{w}_t$, and $\bar{\mathbf{z}} = \sqrt{p\tilde{\kappa}}\tilde{\mathbf{h}}s + \mathbf{z}$. At the receiver, the received vector $\mathbf{y}$ is processed with a weighting vector $\mathbf{w}_r$ to form a single decision variable, that is,

$$\mathbf{w}_r^H \mathbf{y} = \sqrt{p\bar{\kappa}}\mathbf{w}_r^H \bar{\mathbf{h}}s + \mathbf{w}_r^H \bar{\mathbf{z}}. \quad (7)$$

Based on the structure of $\bar{\mathbf{H}}$ in (3), a double of weighting vectors $\mathbf{w}_t$ and $\mathbf{w}_r$ can be chosen as

$$\mathbf{w}_t = \frac{\mathbf{t}}{\sqrt{N}},\ \mathbf{w}_r = \frac{\mathbf{r}}{\sqrt{M}}. \quad (8)$$

Since the scattered component is not available, the effective output signal-to-interference-plus-noise ratio (SINR) should be given by

$$\gamma = \frac{p\bar{\kappa}\left|\mathbf{w}_r^H \bar{\mathbf{h}}\right|^2}{\mathbb{E}\left|\mathbf{w}_r^H \bar{\mathbf{z}}\right|^2} = \frac{p\bar{\kappa}MN}{p\tilde{\kappa}|\mathbf{w}_r^H \tilde{\mathbf{h}}|^2 + \mathbb{E}\left|\mathbf{w}_r^H \mathbf{z}\right|^2}. \quad (9)$$

We denote the ergodic achievable rate by $R = \mathbb{E}\log_2(1+\gamma)$. Through the derivation given in Appendix A, we have the following results with respect to R.

Lemma 1:

$$\log_2\left(1 + \frac{p\bar{\kappa}MN}{p\tilde{\kappa}+\sigma^2}\right) \le R \le \log_2\left(1 + \frac{p\bar{\kappa}MN}{\sigma^2}\right) \quad (10)$$

Proposition 1: Let $E = MNp$ be fixed when $MN \to \infty$. Then

$$\lim_{MN\to\infty} R = \log_2(1 + E\bar{\kappa}/\sigma^2). \quad (11)$$

Proposition 1 reveals such a power scaling law that without degradation in the rate performance, the transmit power can be reduced by a factor of $\frac{1}{MN}$ as MN grows large.

IV. Comparison with BF Based on Perfect CSI

In the previous section, we have presented the simple transmit/receive BF scheme, where the transmitter and the receiver only need to know partial information about the specular component, $\mathbf{t}$ and $\mathbf{r}$, respectively. Now we consider to compare this scheme with the corresponding BF scheme that can employ perfect CSI.

Suppose that both the specular component and the scattered component are available at the transmitter and the receiver. Then we should chose the weighting vectors $\mathbf{w}_t$ and $\mathbf{w}_r$ in such a way that the output signal-to-noise ratio (SNR) is maximized. And the resulting output SNR is given by (see [7] and [8])

$$\gamma_P = \frac{p}{\sigma^2}\lambda_{\max}(\mathbf{H}^H\mathbf{H}) \quad (12)$$

where $\lambda_{\max}(\mathbf{H}^H\mathbf{H})$ is the largest eigenvalue of matrix $\mathbf{H}^H\mathbf{H}$. Through the derivation given in Appendix B, we obtain the following results.

Lemma 2: Suppose that $N/M \to \mu$ when $M \to \infty$. Then

$$\bar{\kappa} \le \lim_{M\to\infty} \lambda_{\max}\left(\frac{1}{MN}\mathbf{H}^H\mathbf{H}\right) \le \bar{\kappa} + \tilde{\kappa}(1+\sqrt{\mu})^2/N. \quad (13)$$

Proposition 2: Let $E = MNp$ be fixed and $N/M \to \mu$ when $M \to \infty$ and $N \to \infty$. Then

$$\lim_{M,N\to\infty} R_P = \lim_{M,N\to\infty} R = \log_2(1 + E\bar{\kappa}/\sigma^2) \quad (14)$$

where R_P represents the ergodic achievable rate for the scheme with perfect CSI.

V. Extension to the Scenario with Interference

Now we consider to extend the mentioned-above analysis to the scenario in the presence of co-channel interference. For that, we necessarily modify the MIMO system model in Section II. Assuming that there are a total of L interfering users each equipped with N_i transmit antennas, $i = 1, 2, \ldots, L$, the $M \times 1$ received vector of the desired user is rewritten as [5], [7], [8]

$$\mathbf{y} = \sqrt{p}\mathbf{H}\mathbf{x} + \sum_{i=1}^{L}\sqrt{p_i}\mathbf{H}_i\mathbf{x}_i + \mathbf{z} \quad (15)$$

where p_i, $\mathbf{H}_i$ and $\mathbf{x}_i$ are the short-term average received power, the normalized channel matrix, and the normalized transmitted vectors for the i-th co-channel interferer, respectively. Moreover, p_i depends on the transmit power, path-loss, and shadowing between the transmitter of the i-th interferer and the receiver of the desired user; the entries of $\mathbf{H}_i$ are i.i.d. circular complex Gaussian random variables with zero-mean and unit variance; the covariance matrix of $\mathbf{x}_i$ can be any valid, but is normalized, i.e., $\mathrm{tr}(\mathbb{E}\mathbf{x}_i\mathbf{x}_i^H) = 1$, where $\mathrm{tr}(\cdot)$ denotes the trace function.

We assume that neither the receiver nor the transmitter of the desired user knows any information about $\{\mathbf{H}_i\}$. By the

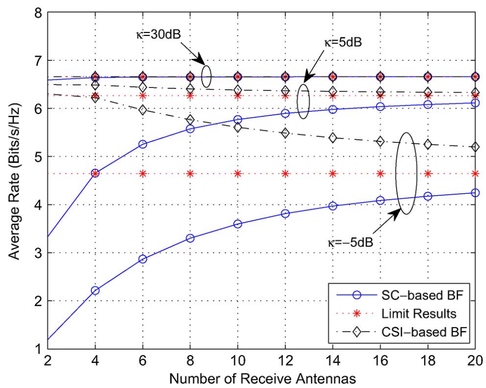

Fig. 1. Average rate versus the number of receive antennas for making a comparison between the SC- and CSI-based schemes when several values of κ are used.

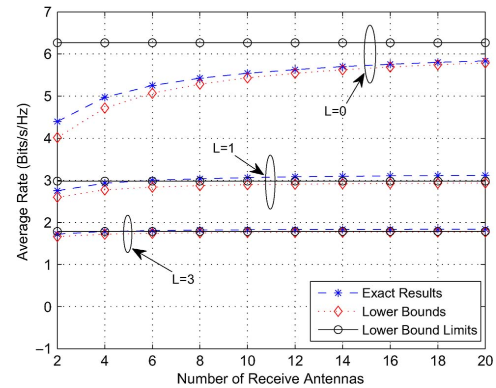

Fig. 2. Average rate versus the number of receive antennas for making a comparison among the exact rate, its lower bound, and the limit of lower bound when several values of L are used.

derivation given in Appendix C, the ergodic achievable rate with interference, denoted as R_I, has the following lower bounds.

Lemma 3: Let $\sigma_I^2 = \sum_{i=1}^{L} p_i + \sigma^2$. Then

$$R_I \geq \log_2\left(1+\frac{p\bar{\kappa}MN}{p\tilde{\kappa}+\sigma_I^2}\right) \tag{16}$$

Proposition 3: Let $E = MNp$ be fixed when $MN \to \infty$. Then

$$\lim_{MN\to\infty} R_I \geq \log_2(1+E\bar{\kappa}/\sigma_I^2). \tag{17}$$

VI. Numerical Results

In this section, we present analytical results and simulation results for a MIMO system operating over Ricean fading channels. For simplicity, all of these spacings between adjacent antennas at the transmitter and the receiver are assumed to be 0.5. Moreover, we set the parameters $E = 20$ dB and $\sigma^2 = 1$.

In the case where the numbers of antennas at the transmitter and the receiver grow large with a fixed ratio, we consider to validate Propositions 1 and 2 by comparing the ergdic achievable rate of the proposed scheme based on the specular component (SC) with that of the perfect CSI-based scheme. For the Ricean factor $\kappa = -5, 5, 30$ dB, Fig. 1 plots the two average rates of R and R_P as M increases from 2 to 20 in the case with $\mu = 1/2$. For comparison, Fig. 1 includes three curves, which corresponds to the rate limit $\log_2(1+E\bar{\kappa}/\sigma^2)$. From Fig. 1 it can be observed that both of the average rates for the SC- and CSI-based schemes tend to the corresponding limit results for the three different values of κ. For $\kappa \geq 5$ dB, the exact rates can be close to the rate limits even though M is small. For $\kappa < 5$ dB, however, the speeds of rate convergence become slower and slower as κ decreases. In particular, when $M = 20$, the exact rates are not yet close to the corresponding limit results. Nevertheless, it can be shown by simulation that when $M = 100$, the exact rates with $\kappa = -5$ dB can be still quite close to their limit values.

Finally, we verify Proposition 3 for the scenario in the presence of co-channel interference when $N_i = N = 3$. For the number of interferers $L = 0, 1, 3$, Fig. 2 plots the exact average rate R_I, its lower bound $\log_2(1+E\bar{\kappa}/(p\tilde{\kappa}+\sigma_I^2))$, and the limit of the lower bound $\log_2(1+E\bar{\kappa}/\sigma_I^2)$ as M increases from 2 to 20. In Fig. 2, we set $\kappa = 5$ dB, $p_i = 10$ dB, and $\mathbb{E}\mathbf{x}_i\mathbf{x}_i^H = \mathbf{I}_N/N$, $i = 1, 2, \ldots, L$. It can be seen from this figure that in the presence of co-channel interference the lower bound and its limit for the exact average rate are tight for any M. In the case without interference $(L = 0)$, the lower bound becomes quite tight when $M \geq 10$ while its limit is not tight when $M \leq 20$. On the other hand, as the number of interferers grows large the lower bound and its limit become much tighter.

VII. Conclusion

In this letter, we have investigated the BF transmission scheme based on the specular component for single-user massive-MIMO systems over Ricean fading channels, derived expressions of the achievable rate, and presented several power scaling properties for different cases. By comparison with pure Rayleigh fading environments, massive MIMO systems are more suitable to be deployed in Ricean fading environments, especially when the specular component is strong.

Appendix

A. Proof of Lemma 1 and Proposition 1

Proof: By using Jensen's inequality, we obtain the following lower bound on the ergodic achievable rate

$$\begin{aligned} R &\geq \log_2\left(1+\frac{1}{\mathbb{E}(1/\gamma)}\right) \\ &= \log_2\left(1+\frac{p\bar{\kappa}MN}{p\tilde{\kappa}\mathbb{E}|\mathbf{w}_r^H\tilde{\mathbf{h}}|^2+\mathbb{E}|\mathbf{w}_r^H\mathbf{z}|^2}\right). \end{aligned} \tag{18}$$

Since $\mathbb{E}|\mathbf{w}_r^H\mathbf{z}|^2 = \sigma^2$ and $\mathbb{E}|\mathbf{w}_r^H\tilde{\mathbf{h}}|^2 = 1$, we have further

$$R \geq \log_2\left(1+\frac{p\bar{\kappa}MN}{p\tilde{\kappa}+\sigma^2}\right). \tag{19}$$

On the other hand, it follows from (9) that

$$\begin{aligned} R &= \mathbb{E}\log_2\left(\frac{p\bar{\kappa}MN}{p\tilde{\kappa}|\mathbf{w}_r^H\tilde{\mathbf{h}}|^2+\sigma^2}\right) \\ &\leq \mathbb{E}\log_2\left(1+\frac{p\bar{\kappa}MN}{\sigma^2}\right) = \log_2\left(1+\frac{p\bar{\kappa}MN}{\sigma^2}\right). \end{aligned} \tag{20}$$

If $E = MNp$ is fixed when $MN \to \infty$, by using the upper and lower bounds we easily obtain the desired result (11).

B. Proof of Lemma 2 and Proposition 2

Proof: Based on (2) it follows that

$$\frac{1}{M}\mathbf{H}^H\mathbf{H} = \frac{1}{M}\left(\sqrt{\bar{\kappa}}\overline{\mathbf{H}}+\sqrt{\tilde{\kappa}}\widetilde{\mathbf{H}}\right)^H\left(\sqrt{\bar{\kappa}}\overline{\mathbf{H}}+\sqrt{\tilde{\kappa}}\widetilde{\mathbf{H}}\right)$$
$$= \frac{\bar{\kappa}}{M}\overline{\mathbf{H}}^H\overline{\mathbf{H}}+\frac{\tilde{\kappa}}{M}\widetilde{\mathbf{H}}^H\widetilde{\mathbf{H}}$$
$$+\frac{\sqrt{\bar{\kappa}\tilde{\kappa}}}{M}\left(\overline{\mathbf{H}}^H\widetilde{\mathbf{H}}+\widetilde{\mathbf{H}}^H\overline{\mathbf{H}}\right). \tag{21}$$

Now define $\mathbf{G} = \frac{1}{M}\overline{\mathbf{H}}^H\widetilde{\mathbf{H}} = [g_{uv}]_{u,v=1}^{N,N}$. Then we know from (3)

$$g_{uv} = \frac{1}{M}e^{-j2\pi(u-1)d_t\sin(\phi)}\sum_{k=1}^{M}e^{-j2\pi(k-1)d_r\sin(\theta)}[\widetilde{\mathbf{H}}]_{kv}. \tag{22}$$

Note that $[\widetilde{\mathbf{H}}]_{kv}\sim \mathtt{CN}(0,1)$ for $1\le k\le M$, and they are independent each other. Thus $g_{uv}\sim\mathtt{CN}(0,1/M)$. So $\mathbf{G}$ tends to an all zero matrix when $M\to\infty$. Similarly, $\mathbf{G}^H=\frac{1}{M}\widetilde{\mathbf{H}}^H\overline{\mathbf{H}}$ also tends to an all zero matrix when $M\to\infty$. In the case where M is large enough, we thus obtain that

$$\lambda_{\max}\left(\frac{1}{M}\mathbf{H}^H\mathbf{H}\right)=\lambda_{\max}\left(\frac{\bar{\kappa}}{M}\overline{\mathbf{H}}^H\overline{\mathbf{H}}+\frac{\tilde{\kappa}}{M}\widetilde{\mathbf{H}}^H\widetilde{\mathbf{H}}\right)$$
$$\le\lambda_{\max}\left(\frac{\bar{\kappa}}{M}\overline{\mathbf{H}}^H\overline{\mathbf{H}}\right)+\lambda_{\max}\left(\frac{\tilde{\kappa}}{M}\widetilde{\mathbf{H}}^H\widetilde{\mathbf{H}}\right)$$
$$=\bar{\kappa}\lambda_{\max}\left(\frac{1}{M}\overline{\mathbf{H}}^H\overline{\mathbf{H}}\right)+\tilde{\kappa}\lambda_{\max}\left(\frac{1}{M}\widetilde{\mathbf{H}}^H\widetilde{\mathbf{H}}\right). \tag{23}$$

If $N/M\to\mu$ when $M\to\infty$, it follows further from [9, Theorem 2.37] that

$$\lambda_{\max}\left(\frac{1}{M}\widetilde{\mathbf{H}}^H\widetilde{\mathbf{H}}\right)\to(1+\sqrt{\mu})^2 \tag{24}$$

Moreover, $\lambda_{\max}(\frac{1}{M}\overline{\mathbf{H}}^H\overline{\mathbf{H}})=N$. So we finally have

$$\lambda_{\max}\left(\frac{1}{MN}\mathbf{H}^H\mathbf{H}\right)\le\bar{\kappa}+\tilde{\kappa}(1+\sqrt{\mu})^2/N. \tag{25}$$

On the other hand, we also have that

$$\lambda_{\max}\left(\frac{1}{MN}\mathbf{H}^H\mathbf{H}\right)\ge\bar{\kappa}\lambda_{\max}\left(\frac{1}{MN}\overline{\mathbf{H}}^H\overline{\mathbf{H}}\right)=\bar{\kappa} \tag{26}$$

When $N\to\infty$ and $M\to\infty$, we get further by (25) and (26)

$$\lambda_{\max}\left(\frac{1}{MN}\mathbf{H}^H\mathbf{H}\right)\to\bar{\kappa} \tag{27}$$

Note that

$$R_p = \mathbb{E}\log_2\left(1+\frac{p}{\sigma^2}\lambda_{\max}(\mathbf{H}^H\mathbf{H})\right)$$
$$=\mathbb{E}\log_2\left(1+\frac{pMN}{\sigma^2}\lambda_{\max}\left(\frac{1}{MN}\mathbf{H}^H\mathbf{H}\right)\right). \tag{28}$$

Therefore, if $E=pMN$ is fixed when $M\to\infty$ and $N\to\infty$, we can have by (28)

$$\lim_{M,N\to\infty}R_p=\log_2(1+E\bar{\kappa}/\sigma^2)=\lim_{M,N\to\infty}R \tag{29}$$

C. Proof of Lemma 3 and Proposition 3

For the scenario in the presence of co-channel interference, we let $\Omega_i=\mathbb{E}\mathbf{x}_i\mathbf{x}_i^H, i=1,2,\ldots,L$. Then the effective output SINR can be given by

$$\gamma_I = \frac{p\bar{\kappa}MN}{p\tilde{\kappa}\left|\mathbf{w}_r^H\tilde{\mathbf{h}}\right|^2+\sum_{i=1}^L p_i\left\|\mathbf{w}_r^H\mathbf{H}_i\Omega_i^{1/2}\right\|^2+\mathbb{E}\left|\mathbf{w}_r^H\mathbf{z}\right|^2}$$
$$=\frac{p\bar{\kappa}MN}{p\tilde{\kappa}\left|\mathbf{w}_r^H\tilde{\mathbf{h}}\right|^2+\sum_{i=1}^L p_i\left|\mathbf{w}_r^H\mathbf{H}_i\Omega_i\mathbf{H}_i^H\mathbf{w}_r\right|+\sigma^2}. \tag{30}$$

Due to the fact that Ω_i is positive semi-definite, we can write $\Omega_i=\mathbf{U}_i\Lambda_i\mathbf{U}_i^H$, where $\mathbf{U}_i^H$ is the unitary matrix consisting of eigenvectors of Ω_i and Λ_i is the diagonal matrix composed of the corresponding non-negative eigenvalues. Since $\mathbf{H}_i\mathbf{U}_i$ has the same distribution as $\mathbf{H}_i$, we can have that

$$\mathbb{E}\left|\mathbf{w}_r^H\mathbf{H}_i\Omega_i\mathbf{H}_i^H\mathbf{w}_r\right|=\mathbb{E}\left|\mathbf{w}_r^H\mathbf{H}_i\Lambda_i\mathbf{H}_i^H\mathbf{w}_r\right|=\mathrm{tr}(\Lambda_i)=1. \tag{31}$$

With the help of Jensen's inequality, (30) and (31), the ergodic achievable rate with interference is given by

$$R_I\ge\log_2\left(1+\frac{1}{\mathbb{E}(1/\gamma_I)}\right)$$
$$=\log_2\left(1+\frac{p\bar{\kappa}MN}{p\tilde{\kappa}+\sum_{i=1}^L p_i\mathbb{E}\left|\mathbf{w}_r^H\mathbf{H}_i\Omega_i\mathbf{H}_i^H\mathbf{w}_r\right|+\sigma^2}\right)$$
$$=\log_2\left(1+\frac{p\bar{\kappa}MN}{p\tilde{\kappa}+\sigma_I^2}\right). \tag{32}$$

Furthermore, if let $E=MNp$ be fixed when $MN\to\infty$, it is obvious that

$$\lim_{MN\to\infty}R_I\ge\log_2\left(1+E\bar{\kappa}/\sigma_I^2\right). \tag{33}$$

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