Partial Differential Equations

(Semester II; Academic Year 2024-25)

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Assignment - 1

Given Date: January 7, 2025 Submission Date: January 7, 2025
Number of questions: 4 Maximum Marks: 50

1. Find and sketch some sample characteristic curves of the PDE: (5)

$$(x+2)u_x + 2yu_y = 2u$$

in the $x-y$ plane. Write the ODE for u along a characteristic curve with x as the parameter and then solve the PDE with the initial condition $u(-1,y) = \sqrt{|y|}, y > 0$.

2. Consider the PDE: (15)

$$xu_x + yu_y = 2u, \quad x > 0,\ y > 0.$$

Plot the characteristic curves and solve the equation with the following initial conditions in the domain given above:

   (a) $u = 1$ on the hyperbola $xy = 1$.

   (b) $u = 1$ on the circle $x^2 + y^2 = 1$.

   (c) Can you solve the equation, in general, if certain initial data is prescribed on the initial curve $y = e^x$? Justify with reasons.

3. Sketch the characteristic curve, the initial curve, and solve the following problems: (20)

   (a) $xu_x + yu_y = ku,\ x \in \mathbb{R},\ y \geq \alpha > 0;\ u(x,\alpha) = F(x)$, where k, α are fixed and F is a given smooth function.

   (b) $yu_x - xu_y = 0;\ u(x,0) = x^2$.

   (c) $x^2u_x - y^2u_y = 0;\ u(1,y) = F(y)$.

   (d) $yu_x + xu_y = 0;\ u(0,y) = e^{-y^2}$.

4. Solve the quasi-linear problem and verify transversality conditions: (10)

   (a) $uu_x + u_y = 0,\ u(x,0) = x$.

   (b) $uu_x + u_y = 1,\ u(x,x) = x/2,\ x \in (0,1]$.