Partial Differential Equations

(Semester II; Academic Year 2024-25)

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Assignment - 2

Given Date: January 15, 2025 Submission Date: January 24, 2025
Number of questions: 4 Maximum Marks: 65

1. Consider the PDE $Lu := x(u_x)^2 + yu_y - u = 0$. (15)
   (a) Find the equation of the Monge's cone at $(1, 1, -1)$.
   (b) Is it possible to find Monge's cone at $(0, 0, 0)$? Conclude the solvability of the PDE with initial data given in a curve containing origin.
   (c) Find the integral surface passing through the line $y = 1,\ x + z = 0$.

2. Solve the following IVP: (30)
   (a) $u_y = u_x^3,\ u(x, 0) = 2x^{3/2}$.
   (b) $u_x^2 + u_y^2 = 1, u(x, y) = 0$ on the line $x + y = 1$.
   (c) $xp^2 + yq^2 = z, y = 1$ on the line $x + z = 0$.
   (d) $u_t + (x \cos t)u_x = 0,\ u(x, 0) = \frac{1}{1+x^2},\ x \in \mathbb{R},\ t > 0$.
   (e) $u_t + (x + t)u_x + t(x + 1)u = 0,\ u(x, 0) = \phi(x),\ x \in \mathbb{R},\ t > 0$.
   (f) $u_t + u^2 u_x = 0,\ u(x, 0) = x,\ x \in \mathbb{R},\ t > 0$.

3. Solve: (10)
   (a) $u_t - \sqrt{u_{x_1}^2 + u_{x_2}^2} = 0,\ u(x_1, x_2, t_0) = \psi(x_1^2 + x_2^2),\ \psi' > 0;\ (x_1, x_2) \in \mathbb{R}^2,\ t > t_0$
   (b) $u_t + u(u_x + u_y) = 0,\ x, y \in \mathbb{R},\ t > 0, u(x, y, 0) = x + y,\ x, y \in \mathbb{R}$.

4. Solve: (10)
$$u_t + \frac{1}{1 + |x|}u_x = 0,\ u(x, 0) = \phi(x),\ x \in \mathbb{R},\ t > 0.$$