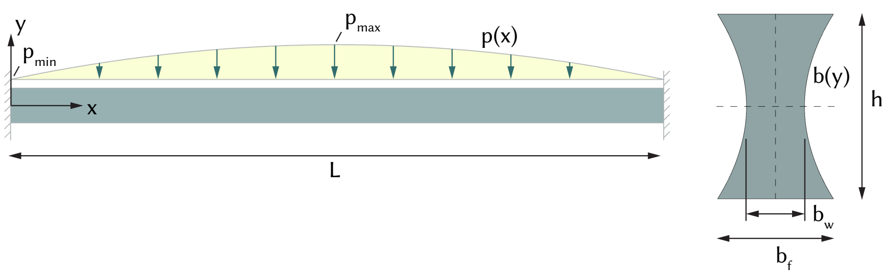
MECE5397/6397: Assignment 1 - design of a beam

Due date: 5:30pm on Feb. 16

In this assignment, your task is to optimize the cross sectional geometry of a doubly clamped beam. The cross section is constructed with two parabolic curves in the form of $b(y)$ that is dependent on b_{f}, b_{w} and h. The distributed load applied to the beam is also in a parabolic form $p(x)$ that is dependent on $p_{\max}$, $p_{\min}$ and L. The Young's modulus of the material is E, the yield strength is σ_{y}, and the deflection is $u(x)$. Note that all the question are to be completed in a Jupyter notebook and submitted on blackboard.



In this problem b_{w} and h are the optimization variables.

Task 1

State the cross sectional equations for beam width - $b(y)$, area - $A(y)$, first moment of area - $Q(y)$, and second moment of area - $I(y)$. Note that the first moment of area is typically integrated from y to $h/2$, where as both A and I are integrated from $-h/2$ to $h/2$.
Hint:

$$A(y) = \int dA$$

$$Q(y) = \int y dA$$

$$I(y) = \int y^2 dA$$

Hint: first derive $b(y)$, then substitute dA by $b(y)dy$.

Task 2

Write the governing ODE equation. Note that $p(x)$ is not a hint, but rather, a function of x.

Task 3

Derive the general solution to the governing ODE.

Task 4

State the boundary conditions.

Task 5

Calculate the integration constants.

Task 6

Derive moment, shear, and square of the von mises stress using the following equations.

$$M(x) = EI\frac{d^2u(x)}{dx^2}$$
$$V(x) = EI\frac{d^3u(x)}{dx^3}$$
$$\sigma_{vm}(x,y)^2 = \sigma_{xx}^2 + 3\sigma_{xy}^2$$

where:

$$\sigma_{xx} = \frac{M(x)y}{I(y)}$$
$$\sigma_{xy} = \frac{V(x)Q(y)}{b(y)I(y)}$$

Task 7

Here is the optimization problem. Please explain in words what the following means

$$\begin{aligned} \min_{h,b_{\mathrm{w}}} \quad & \mathrm{A(y)L}\rho \\ \text{s.t.} \quad & \sigma_{\mathrm{vm}}^2 - \sigma_{\mathrm{y}}^2 < 0 \\ & u_{\max} - u_{\mathrm{limit}} < 0 \\ & b_{\mathrm{w}} - b_{\min} < 0 \\ & h - h_{\min} < 0 \end{aligned} \tag{1}$$

Task 8

Substitute constants into the above equations, so that in the end, the objective and each constraint can be plotted.
Hint:
Think about at what x coordinates does $u_{\max}$ occur.
Think about at what (x,y) coordinates does $\sigma_{\max}$ occur. For this assignment, test four points $(x,y) = (0, \frac{h}{2})$, $(x,y) = (\frac{L}{2}, -\frac{h}{2})$, $(x,y) = (0,0)$, $(x,y) = (\frac{L}{4}, \frac{h}{4})$

Use constants:

$$p_{\min} = 0\mathrm{N/m}$$
$$p_{\max} = -20\mathrm{kN/m}$$
$$b_{\mathrm{f}} = 0.25\mathrm{m}$$
$$L = 15\mathrm{m}$$
$$\sigma_{\mathrm{y}} = 180\mathrm{MPa}$$
$$u_{\max} = \frac{L}{250}$$
$$b_{\min} = 0.01\mathrm{m}$$
$$b_{\max} = b_{\mathrm{f}}$$

$$h_{\min} = 0.01\text{m}$$
$$h_{\max} = 0.5\text{m}$$
$$\rho = 7800\text{kg/m}^3$$
$$E = 200\text{GPa}$$

Substitute constants into the above equations. Note you need a separate sigma function for each of the four test points above.

Task 9

Lambdify the expressions into Python function definitions, with h and h_{w} as input variables.

Task 10

Use meshgrid to generate a grid of test points for both of the optimization variables h and b_{w}, and generate values for the mass, the displacement constraint, and the stress constraints.

Task 11

Plot the optimization objective a filled contour plot.
Draw the constraints as not-filled contour plot.
Find the optimal h and b_{w} by looking at this contour plot.