Theo Shin

Problem Set 3

1. Let Q(x,y) denote x has been a contestant on quiz show y.
   a) "There exists a student who has been a contestant on a quiz show" or "There exists some student who has been a contestant on some quiz show"
   $\exists x \exists y\, Q(x, y)$
   b) "No student at your school" = "there does not exist a student" = $\neg\exists x$

   I had come up with $\neg\exists x \exists y\, Q(x, y)$ but came across the solution of $\forall x \forall y\, \neg Q(x, y)$ which is the NEGATION of my original statement (which basically means I couldn't be any more wrong!). However, I can read the statement as "Out of all the students and all the quiz shows, no student has been on a quiz show" so I understand the book's solution.

   c) "There exists a student at your school who has been a contestant on Jeopardy and Wheel of Fortune" or "A student has been on both jeopardy and wheel of fortune (from domain of all tv shows)"
   $\exists x \forall y\, (Q(x, \text{Jeopardy}) \wedge Q(x, \text{Wheel of Fortune}))$

   d) "Every television quiz show has had a student from your school as a contestant"
   All quiz shows have had a student be a contestant
   $\forall y \exists x\, Q(x, y)$
   e) "At least two students from your school have been contestants on Jeopardy"
   Q(x, Jeopardy)
   X=student 1; Y=student 2; Z=Quiz Show
   $\exists x \exists y$ ((X ≠ Y) ----)> (Q(x,Jeopardy) ΛQ(y, Jeopardy))
   I wasn't sure how to annotate AT LEAST 2, so the best I could come up with was showing "If two students who are not the same student x and student y have been on Jeopardy."

Following this week's quiz, I've identified I struggle the most with correctly going back and forth in translating nested quantifiers.

2. a) $\neg\forall x \forall y P(x, y)$

$\exists x \neg \forall y P(x, y)$

$\exists x \exists y \neg P(x, y)$

b) $\neg\forall y \exists x P(x, y)$

$\exists y \neg \exists x P(x, y)$

$\exists y \forall x \neg P(x, y)$

c) $\neg\forall y \forall x(P(x, y) \vee Q(x, y))$

$\exists y \neg \forall x(P(x, y) \vee Q(x, y))$

$\exists y \exists x \neg(P(x, y) \vee Q(x, y))$

$\exists y \exists x(\neg P(x, y) \wedge \neg Q(x,y))$ De Morgan's Law

d) $\neg(\exists x \exists y \neg P(x, y) \wedge \forall x \forall y Q(x, y))$

$\neg \exists x \exists y \neg P(x,y) \vee \neg \forall x \forall y Q(x, y))$ De Morgan's Law

$\forall x \neg \exists y P(x,y) \vee \exists x \neg \forall y Q(x, y))$

$\forall x \forall y \neg\neg P(x,y) \vee \exists x \exists y \neg Q(x, y))$

$\forall x \forall y P(x,y) \vee \exists x \exists y \neg Q(x, y))$ Double Negation

e) $\neg\forall x(\exists y \forall z P(x, y, z) \wedge \exists z \forall y P(x, y, z))$

$\exists x \neg(\exists y \forall z P(x, y, z) \wedge \exists z \forall y P(x, y, z))$

$\exists x(\neg \exists y \forall z P(x, y, z) \vee \neg \exists z \forall y P(x, y, z))$ De Morgan's Law

$\exists x(\forall y \neg \forall z P(x, y, z) \vee \forall z \neg \forall y P(x, y, z))$

$\exists x(\forall y \exists z \neg P(x, y, z) \vee \forall z \exists y \neg P(x, y, z))$

I thought working through this problem helped solidify my understanding of negating expressions. I made a comment in Piazza how it was important for me to negate each quantifier individually, otherwise I would incorrectly flip the quantifier. In problem d) I didn't get the question right at first because I had moved the negation operator without noticing I hadn't flipped the quantifier.

3.

N = {0,1,2,3…}

a. $\forall m\ \exists n\ (n > m)$

For every number m there exists a number n such that n is greater than m. **True** because whether you set m as 0, 1, 5, any number, there does exist a number n that will be greater than m.

Ex. m=0; n=1; (m+1) will always make the proposition true.

b. $\forall m\ \exists n\ (m > n)$

For every number m there exists a number n such that m is greater than n (or n is less than m). **False**. If m is 0 then n can only go to 0 making the proposition false.

Ex. m=0; n=0; 0 is not less than 0 and n cannot have a negative value, therefore it's false.

c. $\exists n\ \forall m\ ((m =/ n) \rightarrow (m > n))$

There exists a number n such that for all numbers m if m and n are not the same then m is greater than n (or n is less than m). By qualifying m and n cannot be the same (hence both cannot be 0) then the proposition is **true**. This is similar to the last problem (.b) except m and n cannot both be 0.

Ex. m=1; n=0; since the lowest value for m can be 1 then the proposition is true.

d. $\forall m\ \forall n\ ((m > 0) \land (n > 0)) \rightarrow ((mn > m) \lor (mn > n)))$

For all numbers m and all numbers n IF m is greater than 0 AND n is greater than 0 (if both numbers are positive which is already the domain) then m times n is greater than m OR m times n is greater than n. **FALSE**. If the proposition stated m≠n then the overall proposition would be true.

Ex. m=1, n=1; 1(m) * 1(n) is not greater than 1(m) OR 1(m)*1(n) is not greater than 1(n)

4. Doug knows how to write programs in JAVA AND is a student in the class. Everyone (universal quantifier) who can write programs in JAVA can get a high paying job (If $\Rightarrow$ then). Therefore ($\therefore$) someone in the class can get a high paying job.

   #I sometimes have difficulties differentiating between "if $\Rightarrow$ then" statements versus "And" statements unless the sentence makes it very clear or if the "if" portion in the sentence comes after the "then" portion (not in example).

   a) "Doug, a student in this class, knows how to write programs in JAVA. Everyone who knows how to write programs in JAVA can get a high-paying job. Therefore, someone in this class can get a high-paying job."

   Predicates:

   C(x) denote x is a student in this class

   J(X) denote x can write programs in JAVA

   P(x) denote x can get high-paying job

   Premises:

   $\exists x\ (C(a) \land J(a))$

   $\forall x\ (J(x) \Rightarrow P(x))$

   $\therefore \exists x\ (C(x) \land P(x))$

1.	$\exists x\ (C(x) \land J(x))$	Premise
2.	$(C(a) \land J(a))$	Existential Instantiation (1)
3.	$J(a)$	Simplification (2)
4.	$\forall x\ (J(x) \Rightarrow P(x))$	Premise
5.	$(J(a) \Rightarrow P(a)$	Universal Instantiation (4)
6.	$P(a)$	Modus Ponens using (3) and (5)
7.	$C(a)$	Simplification (2)
8.	$P(a) \land C(a)$	Conjunction using (6) and (7)
9.	$\exists x\ (C(x) \land P(x)$	Existential Generalization

   # The tricky part for me is establishing the correct premises. I initially set up the conclusion as $\therefore \exists x\ (C(x) \Rightarrow P(x))$ instead of $\therefore \exists x\ (C(x) \land P(x))$ and got stuck.

b) “Somebody in this class enjoys whale watching. Every person who enjoys whale watching cares about ocean pollution. Therefore, there is a person in this class who cares about ocean pollution”

Predicates:

$C(x)$ denote x is a student in the class

$W(x)$ denote x enjoys whale watching

$O(x)$ denote x cares about ocean pollution

Premises:

$\exists x\ (C(x) \land W(x))$

$\forall x\ (W(x) \Rightarrow O(x))$

$\therefore\ \exists x\ (C(x) \land O(x))$

1.	$\exists x\ (C(x) \land W(x))$	Premise
2.	$C(a) \land W(a)$	Existential Instantiation (1)
3.	$W(a)$	Simplification (2)
4.	$\forall x\ (W(x) \Rightarrow O(x))$	Premise
5.	$W(a) \Rightarrow O(a)$	Universal Instantiation (4)
6.	$O(a)$	Modus Ponens using (3) and (5)
7.	$C(a)$	Simplification (2)
8.	$O(a) \land C(a)$	Conjunction from (6) and (7)
9.	$\exists x\ (C(x) \land O(x))$	Existential Generalization (8)

This problem was a lot smoother than trying (a) because I had setup the predicates and premises correctly. I think if either a predicate or premise happens to be written incorrectly, I’ll hit an obvious snag in trying to logically prove the statements. Correctly translating the sentence appears to be half the battle.

c) “Each of the 93 students in this class owns a personal computer. Everyone who owns a personal computer can use a word processing program. Therefore, Zeke, a student in this class, can use a word processing program.”

Predicates:

$C(x)$ denote x is in this class

$P(x)$ denote x owns a personal computer

$W(x)$ denote x can use a word processing program

Premises:

$C(z)$

$\forall x\ (C(x) \Rightarrow (P(x))$

$\forall x\ (P(x) \Rightarrow W(x))$

$\therefore\ W(z)$

1.	$\forall x\ (C(x) \Rightarrow (P(x))$	Premise
2.	$\forall x\ (P(x) \Rightarrow W(x))$	Premise
3.	$C(z) \Rightarrow (P(z)$	Universal Instantiation (1)
4.	$C(z)$	Premise
5.	$P(z)$	Modus Ponens (3) and (4)
6.	$P(z) \Rightarrow W(z)$	Universal Instantiation (2)
7.	$C(z) \Rightarrow W(z)$	Hypothetical Syllogism (3) and (6)
8.	$W(z)$	Modus Ponens (3) and (7)

I went about this problem slightly differently than the book’s solution. Instead of using modus ponens twice (I do understand that route), but decided to go with hypothetical syllogism in line 7.

d) “Everyone in New Jersey lives within 50 miles of the ocean. Someone in New Jersey has never seen the ocean. Therefore, someone who lives within 50 miles of the ocean has never seen the ocean.”

Predicates:

N(x) denote x lives in New Jersey

O(x) denote x lives within 50 miles of the ocean

S(X) denote x has seen the ocean

Premises:

$\forall x\ (N(x) \Rightarrow O(x))$

$\exists x\ (N(x) \wedge \neg S(x))$

$\therefore\ \exists x\ (O(x) \wedge \neg S(x))$

1. $\forall x\ (N(x) \Rightarrow O(x))$ Premise
2. $N(a) \Rightarrow O(a)$ Universal Instantiation (1)
3. $\exists x\ (N(x) \wedge \neg S(x))$ Premise
4. $N(a) \wedge \neg S(a)$ Existential Instantiation (3)
5. $N(a)$ Simplification (4)
6. $O(a)$ Modus Ponens (2) and (5)
7. $\neg S(a)$ Simplification (4)
8. $O(a) \wedge \neg S(a)$ Conjunction (6) and (7)
9. $\therefore\ \exists x\ (O(x) \wedge \neg S(x)$ Existential Generalization (8)

I fortunately set up the premises and predicates similar to the book so my answer and work were pretty much the same.

5. Using 13(a) in repeating proof with premises listed first:
   $\therefore\ \exists x\ (C(x) \wedge P(x))$

   1. $\exists x\ (C(x) \wedge J(x))$ Premise
   2. $\forall x\ (J(x) \Rightarrow P(x))$ Premise
   3. $C(a) \wedge J(a)$ Existential Instantiation (2)
   4. $J(a) \Rightarrow P(a)$ Universal Instantiation (1)
   5. $C(a)$ Simplification (3)
   6. $J(a)$ Simplification (3)
   7. $P(a)$ Modus Ponens (4) and (6)
   8. $C(a) \wedge P(a)$ Conjunction (5) an d (7)
   9. $\exists x\ (C(x) \wedge P(x))$ Existential Generalization (8)

   # I’m not sure if the intent of stating premises first is lost on me, but the problem was solvable either way, just the order of executing the proof differed.

6. By not "isolating" the J(a) where a is a particular member of the domain first, then we cannot use modus ponens for some arbitrary (a) in the domain.

7. $\forall x(P(x) \rightarrow (Q(x) \land S(x))$

   $\forall x(P(x) \land R(x))$

   $\therefore \forall x(R(x) \land S(x))$

   1. $\forall x(P(x) \rightarrow (Q(x) \land S(x))$ *Hypothesis*
   2. $P(a) \Rightarrow Q(a) \land S(a)$ Universal Instantiation (1)
   3. $\forall x(P(x) \land R(x))$ *Hypothesis*
   4. $P(a) \land R(a)$ Universal Instantiation (3)
   5. $P(a)$ Simplification (4)
   6. $Q(a) \land S(a)$ Modus Ponens (2) and (5)
   7. $R(a)$ Simplification (4)
   8. $S(a)$ Simplification (6)
   9. $R(a) \land S(a)$ Conjunction (7) and (8)
   10. $\forall x(R(x) \land S(x))$ *Universal Generalization (9)*

   # Never considered Universal modus ponens, but glad to add to the toolbag. So far, I think the general formula is using instantiation (universal or existential) then simplifying if it's a conjunction or looking for disjunctive syllogism with disjunctions and usually proving the conclusion by conjunction or modus ponens.

8. $\forall x(P(x) \vee Q(x))$

$\forall x(\neg Q(x) \vee S(x))$

$\forall x(R(x) \rightarrow \neg S(x))$

$\exists x \neg P(x)$

$\therefore \exists x \neg R(x)$

1.	$\exists x \neg P(x)$	*Hypothesis*
2.	$\neg P(c)$	*Existential Instantiation (1)*
3.	$\forall x(P(x) \vee Q(x))$	*Hypothesis*
4.	$(P(c) \vee Q(c)$	*Universal Instantiation (3)*
5.	$Q(c)$	*Disjunctive Syllogism (2) and (4)*
6.	$\forall x(\neg Q(x) \vee S(x))$	*Hypothesis*
7.	$\neg Q(c) \vee S(c)$	*Universal Instantiation (6)*
8.	S(c)	Disjunctive Syllogism (5) and (7)
9.	$\forall x(R(x) \rightarrow \neg S(x))$	*Hypothesis*
10.	$R(c) \rightarrow \neg S(c)$	*Universal Instantiation (9)*
11.	$\neg R(c) \vee \neg S(c)$	*Reverse RBI (10)*
12.	$\neg S(c) \vee \neg R(c)$	*Commutativity (11)*
13.	$\neg R(c)$	*Disjunctive Syllogism (8) and (12)*
14.	$\exists x \neg R(x)$	*Existential generalization (13)*

 # My work through of this problem differed slightly from the book. Mainly I stuck with using disjunctive syllogism after using reason by induction to try and isolate $\neg R(c)$.

9. a) $\forall x(A(x) \vee E(x))$

$\forall x(E(x) \rightarrow T(x))$

$\therefore \forall x(\neg T(x) \rightarrow A(x))$

1.	$\forall x(E(x) \rightarrow T(x))$	hypothesis
2.	$E(c) \rightarrow T(c)$	universal instantiation (1)
3.	$\neg E(c) \vee T(c)$	RBI (2)
4.	$\forall x(A(x) \vee E(x))$	hypothesis
5.	$A(c) \vee E(c)$	universal instantiation (5)
6.	$E(c) \vee A(c)$	*commutative (5)*
7.	$T(c) \vee A(c)$	Resolution from (3) and (6)
8.	$\neg\neg T(c) \vee A(c)$	Double Negation (7)
9.	$\neg T(c) \Rightarrow A(c)$	Reverse RBI (8)
10.	$\forall x(\neg T(x) \rightarrow A(x))$	Universal Generalization (9)

b) (Ax) denotes x is a member of the College of Arts and Sciences

E(x) denotes x is a member of the College of Engineering

T(x) denotes x is required to take Ethics and Technology before graduating

All students either belong to the College of Arts and Sciences or the College of Engineering. Students who belong to the College of Engineering are required to take Ethics and Technology before graduating ("If the student belongs to College of Engineering then they are required to take Ethics and Technology"). Therefore, if they haven't taken Ethics and Technology before graduating then they are a member of the College of Arts and Sciences.

c) Could a student potentially be a member of both colleges?

First, I can write out this statement as:

$\exists x\ (E(x) \wedge (A(x))$ "There exists some student who is in both college of engineering and arts and sciences"

Negation:

$\forall x\ \neg(E(x) \wedge A(x))$

$\forall x\ (\neg E(x) \vee \neg A(x))$ "For all students, they are either not in the college of engineering or not in the college of arts and sciences."

This means $\neg\exists x\ (E(x) \wedge (A(x))$ is logically equivalent to $\forall x\ (\neg E(x) \vee \neg A(x))$ and $\neg\exists x\ (E(x) \wedge (A(x))$

knowing $\neg\exists x\ (E(x) \wedge (A(x))$ is true if and only if $\exists x\ (E(x) \wedge (A(x))$ is false. My conclusion is because the premise $\forall x(A(x) \vee E(x))$ is not an exclusive or ($\oplus$), a student could be a student of both engineering and arts and sciences.

10.

$\forall\ a, b, c,\ d\ \in real\ numbers\ \neq 0,$ $$\frac{\frac{a}{b}}{\frac{c}{d}} = \frac{a}{b} * \frac{d}{c}$$

In addition to the rules in the appendix, you may use:

- $\forall x,\ y\ \in\ reals\ \neq 0,\ \ the\ inverse\ of\ \ x/y\ \ is\ y/x$
- $\forall x,\ y\ \in\ reals\ \neq 0,\ \ x/x\ =\ 1$
- $\forall\ w, x,\ y,\ z\ \in\ reals\ \neq 0,\ \ \frac{w}{y} * \frac{z}{x} = \frac{wz}{yx}$

Hint: Just start with the left hand side and find a way to make the denominator equal to one. You do not need to use the rules of inference.

Proof: From the x/x =1 rule we can multiply both sides by (d/c)/(d/c). From the (w/y) * (z/x) = (wz)/(yx) rule we can join values of the separate fractions into one larger fraction. From x/x=1 rule again, we can remove the (d/c)/(d/c) from the right hand side of the equation. From the commutative law we are left with (c*d)/(c*d) which we know equals 1 because a number divided by itself is always equal to 1.

- ((a/b) * (d/c))/(d/c)/(d/c) = (a/b)/(c/d) * (d/c)/(d/c) Multiply both sides by (d/c)/(d/c)

Based on ∀*x*, *y* ∈ *reals* =/ 0, *x*/*x* = 1

- ((a*d)/(b*c))/((c*d)/(d*c)) = (a*d)/(b*c)* (d/c)/(d/c) Joined values a,b,c,d on both sides

Based on ∀*w,x,y,z* ∈ reals =/ 0, (w/y) * (z/x) = (w*z)/(y*x)

- ((a*d)/(b*c))/((c*d)/(d*c)) = (a*d)/(b*c) Removed (d/c)/(d/c)

Based on ∀*x*, *y* ∈ *reals* =/ 0, *x*/*x* = 1

(c*d)/(d*c) = (c*d)/(c*d) due to commutative law then we know (c*d)/(c*d) = 1 because x/x =1. We are left with:

- ((a*d)/(b*c)) = (((a*d)/(b*c))

Ungraded Extra Challenge:

$\forall x(P(x) \rightarrow N(x))$

$\forall x(N(x) \rightarrow (E(x) \vee O(x)))$

$\neg\forall x(P(x) \rightarrow O(x))$ logically equivalent to $\exists x\neg(P(x) \Rightarrow O(x))$

$\exists x(P(x) \wedge E(x))$

1.	$\forall x(P(x) \Rightarrow N(x))$	Premise
2.	$P(c) \Rightarrow N(c)$	Universal Instantiation (1)
3.	$\neg P(c) \vee N(c)$	RBI (2)
4.	$\exists x\neg(P(x) \Rightarrow O(x))$	Premise(pushed negation through from above)
5.	$\neg(P(c) \Rightarrow O(c))$	Existential Instantiation (4)
6.	$\neg(\neg P(c) \vee O(c))$	RBI (5)
7.	$P(c) \wedge \neg O(c)$	Double negation (7) and de morgan's (7)
8.	$P(c)$	Simplification (7)
9.	$\forall x(N(x) \Rightarrow (E(x) \vee (O(x))$	Premise
10.	$N(c) \Rightarrow E(c) \vee O(c)$	Universal Instantiation (9)
11.	$P(c) \Rightarrow E(c) \vee O(c)$	Hypothetical Syllogism (2) and (10)
12.	$\neg P(c) \vee ((E(c) \vee O(c))$	RBI (11)
13.	$E(c) \vee O(c)$	Disjunctive Syllogism (8) and (12)
14.	$\neg O(c)$	Simplification (7)
15.	$E(c)$	Disjunctive syllogism (13) and (14) and commutative (13)
16.	$P(c) \wedge E(c)$	Conjunction (8) and (15)
17.	$\exists x(P(x) \wedge E(x))$	Existential Generalization (16)