Theo Shin

HW - Week #2

1. Consider the paperclip and penny game posted in Piazza

- Give an inductive argument for the winning strategy.

I came to the conclusion placing pennies on the same colored squares on the 4 by 4 game board prohibited player B from successfully laying all 7 paperclips on the board. In order for player B to win, player B needs all remaining 14 spaces (2 spaces occupied by pennies). Since the same colored squares align diagonally like a checkers board the max number of spaces player B can use after the pennies are placed on the same colored squares (note: aligned diagonally) is 13 (since each paperclip needs one of each colored square). Placing the pennies on the same colored square effectively blocks off one square. In contrast, if the pennies are placed on different colored squares, all remaining squares can be grouped into adjacent blocks for the paperclips to fit nicely. Conveniently, if the paperclips could be placed diagonally, player B would always have a chance to win.

- Write a deductive argument that "proves" your strategy works.

There are 7 white and 7 red squares for a total of 14 squares on the 4 by 4 game board. My inductive reasoning was because a paperclip has to occupy 1 white and 1 red square, placing both pennies on the same color square (doesn't matter if it's red or white), leaves 8 of one color and 6 of the other. This means player B can only use up the amount of paperclips equal to the number of squares remaining after the pennies have occupied 2 of the same colored squares, in this case 6. In order for player B to win, player A has to place 1 penny on the red and 1 penny on the white, so that an equal number of red and white squares remain (7 red; 7 white; 7 paperclips).

- Write an algorithm or list the steps of how to win at this game

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# Number of remaining red squares after pennies placed

r = input("number of red squares:" )

# Number of remaining white squares after pennies placed

w = input("number of white squares:" )

#Dividing number of red and white squares by 2 to determine if placed on same color

r_int = int(r)

w_int = int(w)

r_even = (r_int/2)

w_even = (w_int/2)
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# int to round down if pennies were placed on different colors: 7 / 2  = 3.5; int(3.5) = 3

r_even_int = int(r_even)

w_even_int = int(w_even)

# total will equal either 7 or 6

# 8 / 2 + 6 / 2 = 7, then player A must win

# int(7/2) + int (7/2) = 6 (3.5 rounds down to 3); player B wins

total = r_even_int + w_even_int

if total >= 7:

	print("Congrats Player A wins")

else:

	print("Congrats Player B wins")

# Player A can win when inputting 8 and 6 into the input statements

# Player B can win when inputting 7 and 7 into the input statements
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2. (a) $p \to q \equiv \neg p \vee q$

p	q	¬p	p→q	¬p∨q
T	T	F	T	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

(b) $p \to q \equiv \neg q \to \neg p$

$p \to q \equiv \neg p \vee q$ RBI

$\equiv q \vee \neg p$ Commutativity

$\equiv \neg\neg q \vee \neg p$ Double Negation

$\equiv \neg q \to \neg p$ Reverse RBI

(c) $p \vee q \equiv \neg p \to q$

$\neg(\neg p) \vee q$ RBI

$\neg p \to q \equiv p \vee q$ Double Negation

(d) $p \wedge q \equiv \neg(p \to \neg q)$

$\neg(p \to \neg q) \equiv \neg(\neg p \vee \neg q)$ RBI

$\equiv \neg(\neg(p \wedge q))$ De Morgan's Law

$\equiv p \wedge q$ Double Negation

(e) $\neg(p \to q) \equiv p \wedge \neg q$

$\neg(p \to q) \equiv \neg(\neg p \vee q)$ RBI

$\equiv \neg\neg p \wedge \neg q$ De Morgan's Law

$\equiv p \wedge \neg q$ Double Negation

3. (a) Truth Table

p	q	$\neg p$	$\neg q$	$p \leftrightarrow q$	$p \wedge q$	$\neg p \wedge \neg q$	$(p \wedge q) \vee (\neg p \wedge \neg q)$
T	T	F	F	T	T	F	T
T	F	F	T	F	F	F	F
F	T	T	F	F	F	F	F
F	F	T	T	T	F	T	T

same truth values

(b) $p \leftrightarrow q \equiv (p \wedge q) \vee (\neg p \wedge \neg q)$

$(p \wedge q) \vee (\neg p \wedge \neg q) \equiv [(p \wedge q) \vee \neg p] \wedge [(p \wedge q) \vee \neg q]$ Distribution

$\equiv [\neg p \vee (p \wedge q)] \wedge [\neg q \vee (p \wedge q)]$ Commutativity

$\equiv [(\neg p \vee p) \wedge (\neg p \vee q)] \wedge [(\neg q \vee p) \wedge (\neg q \wedge q)]$ Dist.

$\equiv [(p \to p) \wedge (p \to q)] \wedge [(q \to p) \wedge (q \to q)]$ RBI

$\equiv [(T) \wedge (p \to q)] \wedge [(q \to p) \wedge (T)]$

Tautology

$p \leftrightarrow q \quad \equiv [(p \to q)] \wedge [(q \to p)]$ Identity

4.

#8 (Rosen p.53)

R (x) is "x is a rabbit"

H (x) is "x hops"

Doman: all animals

a) For all animals, if it is a rabbit then it hops or simply, "all rabbits hop"

b) Every animal is a rabbit and hops

c) There exists an animal such that if it is a rabbit, then it hops.

d) Some animal or there exists an animal that is a rabbit and hops.

#10 (Rosen p.53)

C(x) = "x has a cat"

D(x) = "x has a dog"

F(x) = "x has a ferret"

Domain = all students in my class

a) A student in your class has a cat, a dog, and a ferret.

$\exists x\, (C(x) \land D(x) \land F(x))$

b) All students in your class have a cat, a dog, or a ferret.

$\forall x\, (C(x) \lor D(x) \lor F(x))$

c) Some student in your class has a cat and a ferret, but not a dog.

$\exists x\, (C(x) \land F(x) \land \neg D(x))$

d) No student in your class has a cat, a dog, and a ferret.

$\neg \exists x\, (C(x) \land D(x) \land F(x))$

e) For each of the three animals, cats, dogs, and ferrets, there is a student in your class who has this animal as a pet.

$\exists x\, (C(x)) \land \exists x\, (D(x)) \land \exists x\, (F(x))$

Comments for problems #8, #10: I had an easier time translating the statements to English (#8) compared to expressing the statements in terms of quantifiers and logical connectives. I guess interpreting the logical statements into English allows for more open interpretation and is generally easier to formulate in the mind, whereas, it can get confusing discerning which logical connectives applies to a given situation.