<u>Sets, Functions, and Algorithms Problem Set</u>

CSCI 2824

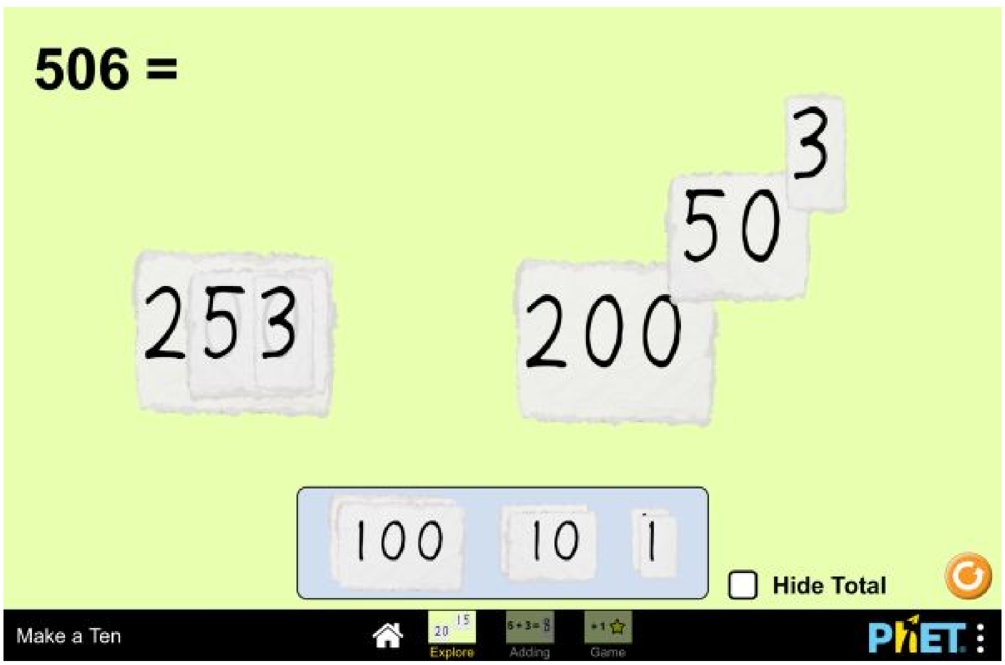
1. Prove the "other" DeMorgan's law following Chris' examples in the set video - slide 26 (similar to example 11 p. 131)

2. Make a Ten https://phet.colorado.edu/en/simulation/make-a-ten is an educational app developed by Beth Stade and the Phet team at the University of Colorado. Try out the app, all three sections, then answer the questions below.

    - The app creates a visual model of adding by focusing on a subset of the real numbers. Describe the set of numbers used in Make a Ten with set builder notation.

    - Which of the fundamental properties of algebra/numbers do these numbers follow (see appendix or week 1) ?

    - When developing this app, a key element was choosing which numbers would "add" when brought together. The big idea is that when 2 numbers added over 10 (or needed to carry), kids would need to break up the number first before adding. This strategy gets young children to use basic principles and not number trick to add (though they discover tricks as they go). The first algorithm describing when 2 numbers can add was almost a page long, but we ultimately reduced it to one line. Write an algorithm describing the conditions when two numbers should "add" in Make a Ten by watching the behavior of the numbers. (hint: our one line solution used "mod" - and yours can be longer than one line).



3. Prove the algebra rules for logs. Like exponents, logs are just an added definition, not a new set of rules.

We define the log base a of a number x as: "The power of a that give us x"

$$log_a(x) = y$$

We can use this **definition, the algebra properties, and the properties of exponents** to discover how log functions work - the log rules. Let's just consider log base 10 for simplicity.

For example:

- Why does $log(a\,b) = log_{10}(a) + log_{10}(b)$?

    - Let's say:

$$log_{10}(a) = C$$
$$log_{10}(b) = D$$

    - By definition of log we know:

$$10^C = a$$
$$10^D = b$$
$$a\,b = 10^C \cdot 10^D = 10^{(C+D)} \qquad \text{by substitution}$$

$$\begin{aligned} log_{10}(a\,b) &= log_{10}(10^C \cdot 10^D) && \text{by substitution} \\ &= log_{10}10^{(C+D)} && \text{exponent rules} \\ &= C + D && \text{def. of log.} \\ &= log_{10}(a) + log_{10}(b) \end{aligned}$$

Weird result - yet true! All because logs ARE exponents.

- Now you discover/show: $log_{10}(a^b) = b\,log_{10}a$

- Is $(log_{10}(a))^b = b\,log_{10}a$? Prove or disprove

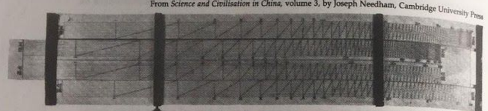
4. Do the following slide rule activity from Jacobs page 240 -241, #1 only. I have included additional chapter pages in Moodle for you to do and review as needed to prepare. Post a photo of your result.

Lesson 5

1. Experiment: *A Slide Rule*

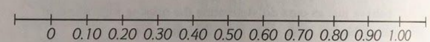
The slide rule was invented by the English mathematician William Oughtred in 1622, soon after the invention of logarithms. In the following experiment, you will construct a simple slide rule and discover how it worked.

From *Science and Civilisation in China*, volume 3, by Joseph Needham, Cambridge University Press



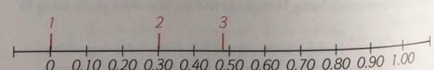
A Chinese slide rule dated at 1660

First, draw a scale 6 inches long on graph paper that has 10 squares to the inch. Divide the scale into 0.5-inch lengths and label it with numbers as shown in the figure below.

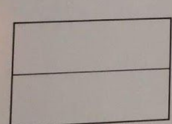


Number	1	2	3	4	5	6	7	8	9	10
Log	0	0.30	0.48	0.60	0.70	0.78	0.85	0.90	0.95	1.00

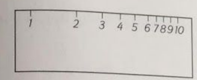
Next, use the table of logarithms above to add a logarithmic scale above the scale you have just made. Do this by locating each of the numbers from 1 through 10 directly above its logarithm. The first three numbers are shown in the figure below.



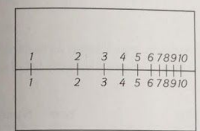
Next, draw a line on a 4-by-6-inch card to divide it in half as shown at the left, and cut it into two pieces along the line.



Take the lower piece of your card and place it on your graph paper so that the upper edge falls along the scale. Carefully make marks on the upper edge of the card corresponding to the numbers from 1 through 10. Your finished card should look like the one shown in the figure at the top of the next page.



Now put it below the other half of the card and carefully copy the scale and the numbers from the finished card onto the lower edge of the other card, as shown in the figure below. When you have done this, your slide rule is ready to use.



a. Using a simple example like 2×3, explain how the slide rule can be used to multiply.
b. Explain how the slide rule can be used to divide.

2. The largest number that can be written using only two digits is

$$9^9$$

Its approximate value in decimal form can be found by using logarithms as shown here.

$$\begin{array}{rcl} 9 & \longrightarrow & 0.954 \\ \times 9 & \longrightarrow & +0.954 \\ \times 9 & \longrightarrow & +0.954 \\ \times 9 & \longrightarrow & +0.954 \\ \times 9 & \longrightarrow & +0.954 \\ \times 9 & \longrightarrow & +0.954 \\ \times 9 & \longrightarrow & +0.954 \\ \times 9 & \longrightarrow & +0.954 \\ \underline{\times 9} & \longrightarrow & \underline{+0.954} \\ & & 9(0.954) = \\ 3.85 \times 10^8 & \longleftarrow & 8.586 \end{array}$$

So $9^9 \approx 385{,}000{,}000$, a number nine digits long.

The largest number that can be written using three digits is

$$9^{9^9}$$

9

5.

A quick exploration:

- Open up Desmos.com
- Graph:
  - f(x) = x
  - g(x) = 1000 log (x)

  From looking at how these two graphs grow, as x gets larger and larger, which function will "win" as x gets really large? That is, can we say that g(x) = 1000 log(x) is always going to be "bigger" than y = x? Go out on the x axis to find a place where f(x) "wins."

- Experiment with other values of the log function:
  - p(x) = 10,000 log (x)
  - t(x) = 100,000 log(x)

  Can we ever get a log function to "beat" y = x as x gets large? Why or why not? (not proof, but what are you seeing? This is a HUGE and IMPORTANT computer science concept. Know and remember).