Problem Set for Module 4

Problem 1 (30 points).

In lecture, we discussed Robert Axelrod's Prisoner's Dilemma Tournament, employing the following payoff matrix:

	Cooperate	Defect
Cooperate	3, 3	0, 5
Defect	5, 0	1, 1

As we mentioned, the TIT-FOR-TAT strategy (cooperate on the very first round, then on each subsequent round do whatever the other player did on the previous round) works well in this setting.

1a. (5 point) Show the first few rounds of a tournament game between two (identical) TIT-FOR-TAT strategies. How many points, on average, does each player earn per round?

1b. (10 points) Sketch a proof (not a formal proof, necessarily, but a reasonable argument) that shows that TIT-FOR-TAT is incapable of winning a game against another player–that is, when TIT-FOR-TAT plays a series of rounds against any other strategy whatever, it can only draw or lose.

1c. (5 points) Consider the strategy SKEPTICAL-TIT-FOR-TAT (or STTT, for short): this strategy is just like TIT-FOR-TAT, except that it defects on the very first round, after which it imitates the other player's previous action. Show the first few rounds of a tournament game between two (identical) STTT players. How many points, on average, does each player earn per round?

1d. (10 points) Show the first six rounds of a series between STTT and TIT-FOR-TAT. How many points, on average, does each player earn per round? Explain why this result is relevant to the fact that a "true" Prisoner's Dilemma matrix must have the property that

CC > (DC + CD)/2

or, in other words, that the reward for mutual cooperation is greater than the average reward for two rounds in which players "trade" cooperation and defection.

Problem 2. (15 points)

Let's consider another game theory scenario. Take a look at the matrix below:

	Cooperate	Defect
Cooperate	3, 3	2, 4
Defect	4, 2	0, 0

(a) Suppose you're the red player. If blue cooperates, what is your preferred strategy? How about if blue defects? Is this a prisoner's dilemma game?

(b) Suppose blue could somehow convince you that they are going to defect. (For instance, blue says to you, "Before you make your choice, I just want you to know that I have already defected, and am now going home. See you later." Rationally, what you should you do? Would blue's pre-emptive announcement work the same way in a prisoner's dilemma game? (By the way, if you want to see more about this situation, it is called a *snowdrift game*, for reasons I will leave you to discover.)

Problem 3. (15 points)

In lecture, we discussed Schelling's simple model of neighborhood formation, and mentioned that there might be variations of Schelling's model that could avoid (if desired) the emergence of large homogeneous neighborhoods. For this problem, suggest at least two changes that you might attempt to make to Schelling's model–changes that you think might plausibly affect the results of his original simulation, and predict (or really, hypothesize) about the sort of new behavior you might expect.

Problem 4. (20 points)

In lecture, we discussed a variety of scenarios for "large-scale" games in which the decisions of an individual are influenced by the (potential or anticipated) decisions of a crowd. Describe–from your own experience–examples of the following game scenarios:

a. I want to do "X" only if few or no others are doing "X".
b. I want to do "X" only if many (nearly all? All?) others are doing "X".
c. I would like to choose strategy "X", but only if I can do so without my decision being made public.

Problem 5. (20 points)

The following is a description of a well-known experimental, or laboratory, game called "The Ultimatum Game". The game is played by two players, A and B. Here is the scenario:

• A and B are sitting across a table from each other. The experimenter places a large sum of money in front of Player A (let's say, $1000).

• Player A now has one choice to make: namely, how much of the $1000 to offer to Player B. For example, Player A might decide to offer $500; or $5; or nothing at all; or indeed, if A wishes, the entire $1000.

• Now Player B has one choice to make: either *accept* A's offer, in which case both players go home with the amount of money in front of them, or *reject* A's offer. If B rejects the offer, the experimenter takes back the entire pot of money and *both* players go home with nothing.

Now, consider this game in terms of "classical" game theory for a moment. Suppose you are Player B. You watch Player A receive $1000 at the other end of the table. Player A now offers you one dollar (which means A gets $999). Would you take this offer? According to a certain interpretation of game theory, you *should* take the offer: after all, you came into the room with nothing, and now you get to leave the room with one dollar. Many people, however, would choose to reject the offer in this case, so that both Player A and B wind up with nothing.

Write a short discussion of this scenario: what does the ultimatum game say about the potential limitations of "classical" game theory, in which the measure of reward can be expressed clearly (often in monetary terms)? If you were Player A, how much do you think you would realistically offer in this game? If you were Player B, how much you would realistically accept? (If you want to read more about the Ultimatum Game, there is a good discussion of the scenario at Wikipedia, among other sites.)