Heuristic Analysis

I implemented the following position evaluation heuristic functions:

I. The square of the distance from the center of the board to the position of the player minus the square of the distance from the center of the board to the opponent's position.

The idea is that if just the distance to the players' position works as an evaluation function then also taking into consideration the opponent's position could work even better.

II. The squared number of possible moves for the player, divided by the squared number of possible moves for the opponent.

Both squaring the numbers and dividing, instead of subtracting the opponent's number of moves makes the agent prioritize positions where the opponent has the lowest possible number of moves to keep him out of options and promote aggressive plays. For example, consider two possible scenarios:

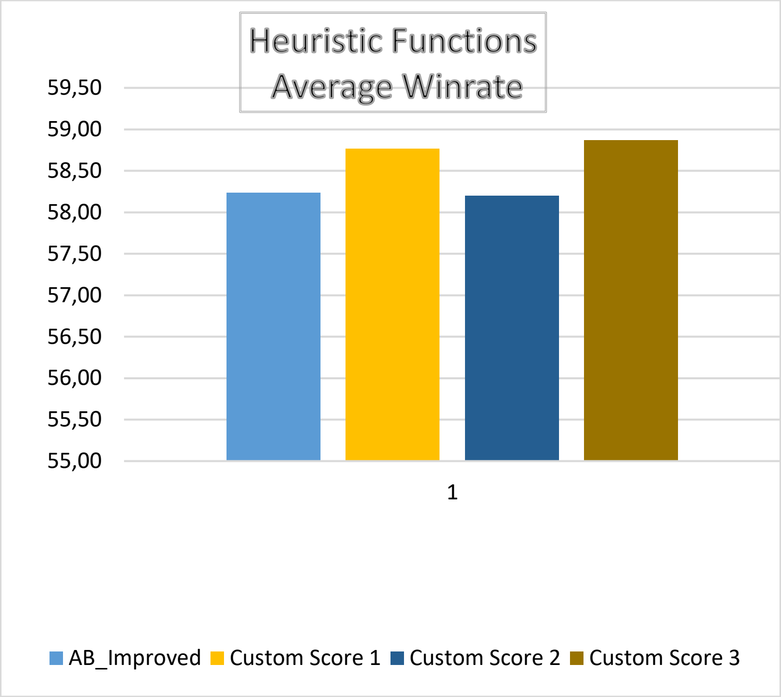
1. Player moves : 8, Opponent moves: 6
2. Player moves : 3, Opponent moves: 1

Although the second scenario appears better, the improved_score(difference of number of moves) would return score 2 for both, while this function would return a better score for the second scenario.

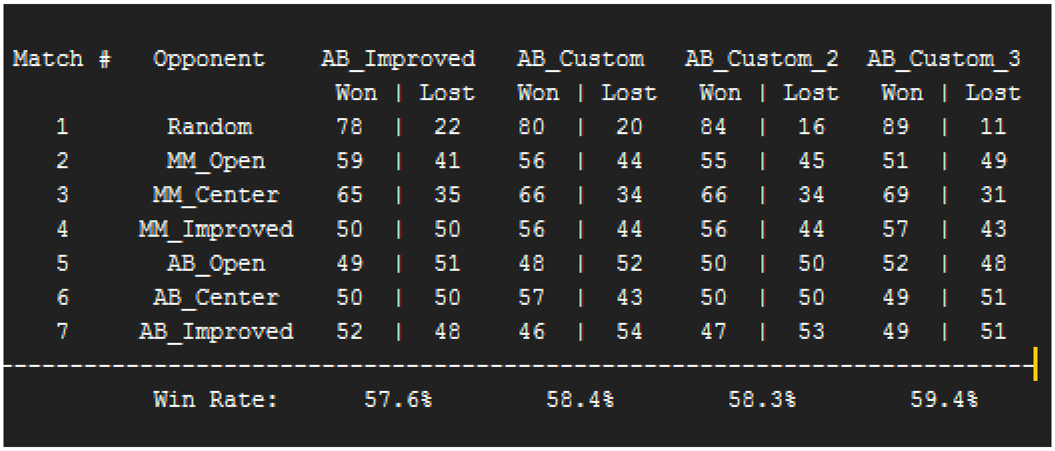
III. For the third function, I tried combining the previous two functions since they worked fairly well, so the combination could possibly get even better results. The third function is the result of the first one plus the result of the second one, weighted by five. The constant "five" was chosen by testing various options and looking at the results.

This is a short summary of the results from running three tournaments:

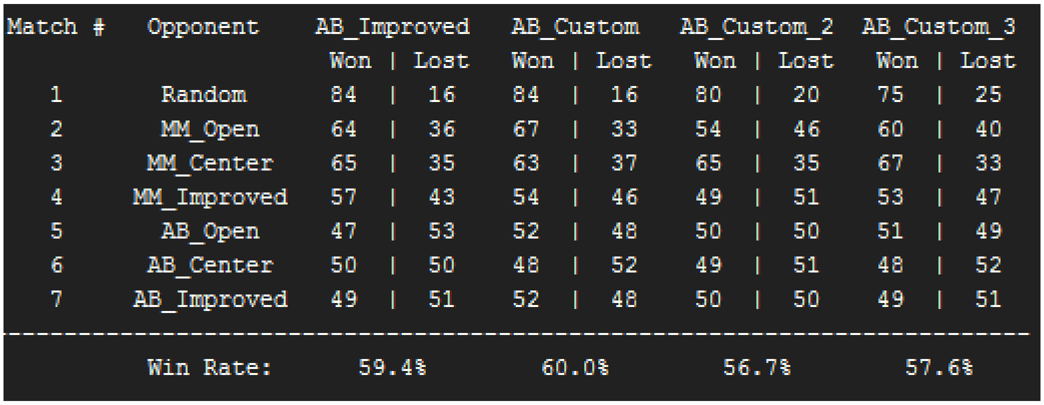
Heuristic	**AB_Improved**			**Custom Score 1**			**Custom Score 2**			**Custom Score 3**		
Win rates	57,6	59,4	57,7	58,4	60	57,9	58,3	56,7	59,6	59,4	57,6	59,6
Average Win rate	**58,23**			**58,77**			**58,20**			**58,87**		



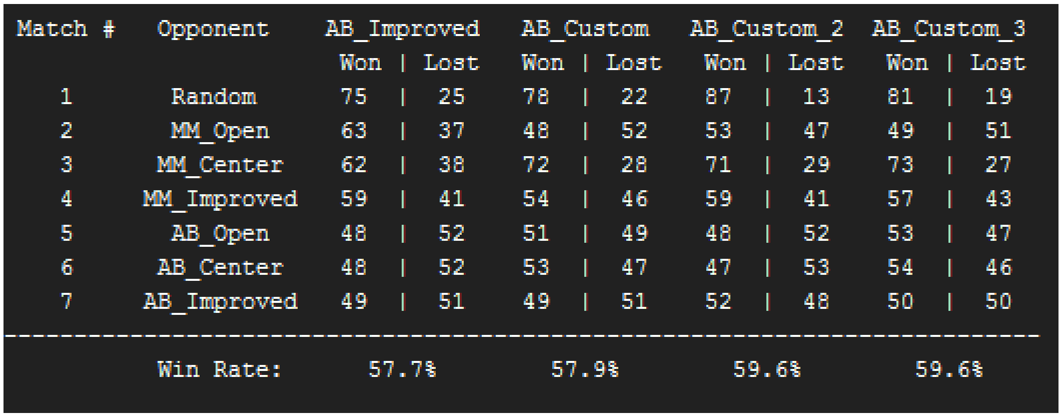
In addition, these are the actual results:



```
Match #    Opponent    AB_Improved   AB_Custom   AB_Custom_2  AB_Custom_3
                        Won | Lost   Won | Lost  Won | Lost   Won | Lost
    1       Random      78  |  22    80  |  20   84  |  16    89  |  11
    2       MM_Open     59  |  41    56  |  44   55  |  45    51  |  49
    3      MM_Center    65  |  35    66  |  34   66  |  34    69  |  31
    4     MM_Improved   50  |  50    56  |  44   56  |  44    57  |  43
    5       AB_Open     49  |  51    48  |  52   50  |  50    52  |  48
    6      AB_Center    50  |  50    57  |  43   50  |  50    49  |  51
    7     AB_Improved   52  |  48    46  |  54   47  |  53    49  |  51
--------------------------------------------------------------------------
           Win Rate:      57.6%        58.4%        58.3%        59.4%
```



```
Match #    Opponent     AB_Improved   AB_Custom    AB_Custom_2  AB_Custom_3
                         Won | Lost   Won | Lost   Won | Lost   Won | Lost
   1         Random      84  |  16    84  |  16    80  |  20    75  |  25
   2         MM_Open     64  |  36    67  |  33    54  |  46    60  |  40
   3        MM_Center    65  |  35    63  |  37    65  |  35    67  |  33
   4       MM_Improved   57  |  43    54  |  46    49  |  51    53  |  47
   5         AB_Open     47  |  53    52  |  48    50  |  50    51  |  49
   6        AB_Center    50  |  50    48  |  52    49  |  51    48  |  52
   7       AB_Improved   49  |  51    52  |  48    50  |  50    49  |  51
--------------------------------------------------------------------------
           Win Rate:      59.4%        60.0%        56.7%        57.6%
```



```
Match #    Opponent     AB_Improved   AB_Custom    AB_Custom_2  AB_Custom_3
                         Won | Lost   Won | Lost   Won | Lost   Won | Lost
   1         Random      75  |  25    78  |  22    87  |  13    81  |  19
   2         MM_Open     63  |  37    48  |  52    53  |  47    49  |  51
   3        MM_Center    62  |  38    72  |  28    71  |  29    73  |  27
   4       MM_Improved   59  |  41    54  |  46    59  |  41    57  |  43
   5         AB_Open     48  |  52    51  |  49    48  |  52    53  |  47
   6        AB_Center    48  |  52    53  |  47    47  |  53    54  |  46
   7       AB_Improved   49  |  51    49  |  51    52  |  48    50  |  50
--------------------------------------------------------------------------
           Win Rate:      57.7%        57.9%        59.6%        59.6%
```

The three evaluation functions performed very close both to one another and to the AB_Improved heuristic. I choose the third one, because:

- It performed slightly better than the others did.
- It takes into account all the variables the other two do.
- The complexity of all three functions is O(n), so although the third function has a bit more calculations, this should have minimal effect on its performance.