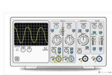
LAB 3: LINEAR CIRCUITS III

UC Berkeley Donald A. Glaser Instrumentation Laboratory

GENERAL GUIDELINES

- **You should complete the prelab questions before beginning benchwork.** Information required to answer the prelab questions can be found in the background material at the beginning of the lab, from lecture, in the stated references, and on the web.
- **Please ask questions of the GSIs or professor(s)** at any time during the course!
- Wikipedia has many useful and informative articles. You are strongly encouraged to use Wikipedia and other online resources to better understand any material.
-  **Problems with this icon need to be checked off by GSIs.**
-  **Problems with this icon indicate that you should take a picture of the circuit built for the problem and include it in your report. Include your student ID, with name clearly visible, in the photo.**
-  **Problems with this icon indicate that you should take a screenshot of the WaveForms oscilloscope.**
- **Important Safety Habits**
  - Before stopping for the day or taking a break, make sure you power down all equipment.
  - When possible, use the switches for the power supplies to power down the circuit when changing the wiring. It is easy to accidentally short wires and damage your equipment or electronic components.
  - Never place food or drink next to any apparatus. Accidental spills can damage or destroy the equipment and your experiment.

Check-off	**Instructor Name/Signature**	**Date**
Pre-lab		

Table 1. Check-off Table

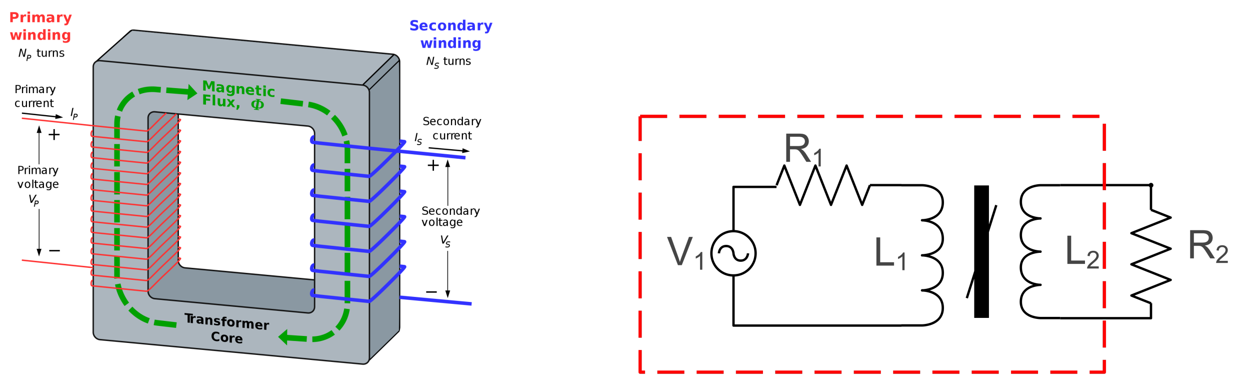
1. LEARNING GOALS

(1) Resonant circuits involving inductors and capacitors

(2) Transformers

2. LAB BACKGROUND READING

2.1. **Transformers.** Transformers are widely used in electronic circuits and the utility grid. By far their most common application is for AC voltage changes. Pacific Gas and Electric provides us with 60 Hz AC power at $V_{\text{RMS}} \sim 120\,\text{V}$, but for transport raises the voltage to $\mathcal{O}(100\,\text{kV})$, so the utility grid needs both "step-up" (boost the voltage) and "step-down" (reduce the voltage) transformers. For electronics, even AC voltages around 120 V at RMS is too large: most electronic circuitry works at far lower voltages: typically, $\pm 12\,\text{V}$ DC for analog circuits and $+5\,\text{V}$ (or $+3.3\,\text{V}$) for digital circuits. Thus, transformers are used to reduce the wall voltages down to levels suitable for electronics. Transformers are also used to match impedances between circuits, and to provide galvanic (DC voltage) isolation between circuits.



(a) Schematic of a transformer.

(b) A circuit involving a transformer. The red box encloses all the components that will be represented as an effective Thévenin voltage and output impedance.

FIGURE 1. Transformer internal structure and circuit.

As shown in Fig. 1, a transformer is a pair of inductors which are magnetically coupled through a mutual inductance. By this, we mean that a change in the current through inductor 1 (the primary circuit) produces a change in the magnetic flux through inductor 1, termed *self-inductance* L_1, and also a change in the magnetic flux through inductor 2, termed *mutual inductance* M_{21}, of the secondary circuit.

Likewise, changes in the current through the secondary circuit produces a change in the magnetic flux through the secondary inductor (L_2) and a change in the flux through inductor 1, M_{12}. The 2 mutual inductances M_{12} and M_{21} are precisely equal and thus we will just write them as M. Changes in flux due to a changes in either the primary or secondary circuit will produce an EMF in both circuits.

The mutual inductance of the transformer is defined by $M = k\sqrt{L_1 L_2}$, where $-1 < k \leq 1$ defines the coupling between the inductors.

Following standard complex circuit analysis, using the definition of mutual inductance between the primary and secondary inductors, paying attention to the inductor polarity for mutual induction, and noting the defined current directions, Kirchoff's voltage law equations for the primary and secondary loops are

$$\begin{aligned} V_1(t) - I_1 R_1 - L_1 \frac{dI_1}{dt} + M\frac{dI_2}{dt} &= 0 \\ -I_2 R_2 - L_2 \frac{dI_2}{dt} + M\frac{dI_1}{dt} &= 0 \end{aligned} \tag{1}$$

After Fourier transforming these equations and splitting the Fourier components, these coupled linear differential equations become:

$$\begin{bmatrix} R_1 + j\omega L_1 & -j\omega M \\ -j\omega M & R_2 + j\omega L_2 \end{bmatrix} \begin{bmatrix} \bar{I}_1(\omega) \\ \bar{I}_2(\omega) \end{bmatrix} = \begin{bmatrix} \bar{V}_1(\omega) \\ 0 \end{bmatrix}. \tag{2}$$

(Note: rather than starting with the coupled ODEs (Eq. 1), one could have immediately written down Eq. 2 using complex impedances for inductors.) To find $\bar{I}_1$ and $\bar{I}_2$, we invert the matrix:

$$\begin{bmatrix} \bar{I}_1(\omega) \\ \bar{I}_2(\omega) \end{bmatrix} = \frac{1}{(R_1 + j\omega L_1)(R_2 + j\omega L_2) - (j\omega M)^2} \begin{bmatrix} R_2 + j\omega L_2 & j\omega M \\ j\omega M & R_1 + j\omega L_1 \end{bmatrix} \begin{bmatrix} \bar{V}_1(\omega) \\ 0 \end{bmatrix}. \tag{3}$$

Now that we have solved for $\bar{I}_1$ and $\bar{I}_2$ for a given voltage excitation, $\bar{V}_1$, we can just plug in various different load resistor values (R_2) to figure out the Thévenin equivalent circuit for the dotted red black box in Fig. 1. Specifically, V_{th} can be found by calculating the open circuit voltage when one takes $R_2 \to \infty$:

$$\begin{aligned} V_{\text{th}} &= \lim_{R_2 \to \infty} \bar{V}_2 \\ &= \lim_{R_2 \to \infty} R_2 \bar{I}_2 \\ &= \frac{j\omega M}{R_1 + j\omega L_1} \bar{V}_1 \\ &= \frac{j\omega k \sqrt{L_1 L_2}}{R_1 + j\omega L_1} \bar{V}_1 \end{aligned} \tag{4}$$

An ideal transformer perfectly and efficiently transfers power from the source to the load (i.e. minimal losses in R_1). To do this, two conditions must be satisfied:

- The magnetic coupling coefficient k must approach 1. As you will measure, this is pretty close to true for low resistance ferrite core transformers since nearly all the B-fields remain in the ferrite.
- $R_1 \ll N_{12}^2 R_2 \ll |Z_{L_1}| = \omega L_1$ or equivalently $N_{21}^2 R_1 \ll R_2 \ll |Z_{L_2}| = \omega L_2$ (you will derive this in P3.3)

Let's simplify V_{th} with these approximations in mind. First, let's choose either ω_0 and/or L_1 such that $\omega_0 L_1 \gg R_1$. In this limit, V_{th} becomes

$$V_{\text{th}} = k\sqrt{\frac{L_2}{L_1}} \bar{V}_1$$

Since the inductance of an inductor scales as the number of turns $L \propto N^2$, so if L_1 and L_2 have N_1 and N_2 turns, then $N_{21} = N_2/N_1 = \sqrt{L_2/L_1}$ and this becomes

$$V_{\text{th}} = kN_{21}\bar{V}_1 \tag{5}$$

For an ideal transformer which has perfect power transfer ($R_1 \ll \omega L_1$) and perfect magnetic coupling ($k = 1$), $V_{\text{th}} = N_{21}\bar{V}_1$. Thus, a *step-down* transformer (which reduces the voltage amplitude) has more turns in the primary than the secondary: $N_{21} < 1$. By contrast, a *step-up* transformer (which boosts the voltage) has fewer turns in the primary than the secondary: $N_{21} > 1$.

The Thévenin output impedance can also be estimated by calculating the short circuit current:

$$\begin{aligned} Z_{\text{out}} &= \frac{V_{\text{th}}}{\lim_{R_2 \to 0} \bar{I}_2} \\ &= \frac{(R_1 + j\omega L_1) j\omega L_2 - (j\omega M)^2)}{R_1 + j\omega L_1} \\ &= \frac{j\omega L_2 R_1 + (1 - k^2)(j\omega)^2 L_1 L_2}{R_1 + j\omega L_1} \end{aligned} \tag{6}$$

Let's apply both ideal transform conditions separately (since both the limits are individually useful) and then together. In the limit that $k \to 1$,

$$\begin{aligned}\lim_{k\to 1} Z_{\text{out}} &= \frac{j\omega L_2 R_1}{R_1 + j\omega L_1} \\ &= \frac{L_2}{L_1}\frac{j\omega L_1 R_1}{R_1 + j\omega L_1} \\ &= N_{21}^2 Z_{R_1||L_1}\end{aligned} \tag{7}$$

Thus, if we need to decrease the output impedance of high impedance circuit to couple it to low impedance readout, we can use a step down transformer: if $N_{21} = 10^{-2}$, R_{out} will be a factor of 10^4 less than R_1. To be more concrete, in my lab we've considered using an $N_{21} \ll 1$ superconducting toroidal inductor to measure an extremely high impedance ionization detector with a SQUID, a very low impedance superconducting current sensor. In more mundane (though way more useful) applications, the transformation of the load impedance is important for efficient power transfer between devices with different impedances.

For our radio later in this course, we will also use a low-k transformer to excite a high Q resonant circuit for calibration. In the limit when k is arbitrary but $\omega L_1 \gg R_1$ we can Taylor expand Eq. 6 to find

$$\lim_{\omega L_1 \gg R_1} Z_{\text{out}} = j\omega L_2(1-k^2) + k^2\frac{L_2}{L_1}R_1 + ... \tag{8}$$

For the ideal transformer, $k \to 1$ and $j\omega L_1 \gg R_1$ and R_{out} becomes

$$Z_{\text{out}} = R_1\frac{L_2}{L_1} = R_1 N_{21}^2 \tag{9}$$

In summary, in the ideal transformer limit, the action of the transformer is to scale the voltage between the primary and secondary and simultaneously transform the output impedance. As mentioned above, the voltage scaling is convenient for stepping down or stepping up an input voltage to a desired output voltage.

2.2. **Resonant LC Circuits.** The impedance of an inductor and capacitor are both imaginary but have opposite signs: $Z_L = j\omega L$ and $Z_C = 1/j\omega C = -j/\omega C$. The impedance of this parallel combination (Fig. 2) is given by

$$Z_{L||C} = Z_L||Z_C = \frac{Z_L Z_C}{Z_L + Z_C} = \frac{-j\omega}{C\left(\omega^2 - \frac{1}{LC}\right)}$$

When driven with a sine wave at the resonant frequency $\omega_0 = 1/\sqrt{LC}$, the impedance of this circuit is infinite. Although no current will pass through the parallel combination at resonance, the current through the inductor and capacitor individually will be oscillating at ω_0. LC parallel resonant circuits, frequently referred to as *tank* circuits, are commonly used in physics and electronics. They have two primary applications: detecting signals oscillating at a known frequency and generating signals at a specific frequency. The first application is exemplified by radio tuners which pick out a specific radio station (say at 1 MHz) from the enormous selection of available radio stations, and the second is exemplified by the radio station transmitter itself, which must broadcast at a precisely controlled frequency[1]. Tank circuits are endlessly analyzed in the course textbooks. Here we will review only a few points:

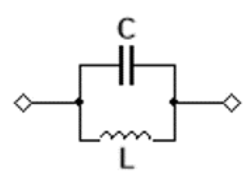


FIGURE 2. LC parallel combination.

- Understand the energy flow: the energy oscillates back and forth (at the resonant frequency) between the capacitor (maximum capacitor voltage, no current) and the inductor (maximum current, no voltage across the capacitor)

[1]For instance, KFOG-FM's carrier frequency is allowed to drift only 2 kHz (0.002%) from its assigned frequency of 104.5 MHz.

- The oscillation amplitude of a resonant circuit will be much higher when driven with a frequency near resonance than when driven off resonance. Thus, resonators will preferentially respond to near resonant signals, and can be used to detect such signals even when these signals are masked by signals at other frequencies.
- By driving this parallel combination through a large series resistor, we can create a band pass filter.

Inductors and capacitors are also frequently connected in series as shown in Fig. 3. The impedance of this combination is

$$Z_{L+C} = Z_L + Z_C = \frac{jL}{\omega}\left(\omega^2 - \frac{1}{LC}\right)$$

At the resonant frequency, $\omega_0 = 1/\sqrt{LC}$, the impedance is zero. Remember that the impedance of an inductor is proportional to j while the impedance of a capacitor is proportional to $1/j = -j$. Both inductors and capacitors induce 90° phase shifts, but in opposite directions. Consequently, their phase shifts are 180° apart from each other. At resonance the magnitudes of the impedances are equal. Add them together and you get zero! By driving this combination through a resistor, we can match a notch filter. A *notch* filter is the opposite of a bandpass and selectively attenuates a narrow band of frequency about ω_0.

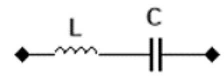


FIGURE 3. LC series combination.

2.3. **Resonator Quality Factor.** The *quality factor* Q of a resonator is defined as the ratio of the resonant frequency to the 3 dB bandwidth of the resonance. The 3 dB bandwidth (Fig. 5) is defined as the width where the resonator response falls from its peak gain or attenuation (notch filter) by 3 dB, which is a factor of 2 in power or $\sqrt{2}$ in gain. The bandwidth is defined as the full width BW $= 2\Delta\omega$, which is twice the frequency detuning $\Delta\omega = \omega - \omega_0$ needed to decrease the gain by $\sqrt{2}$.

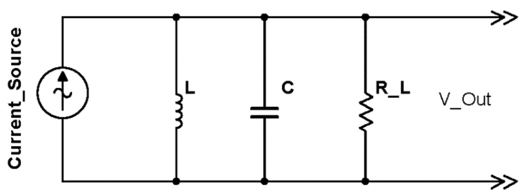


FIGURE 4. An LCR resonator.

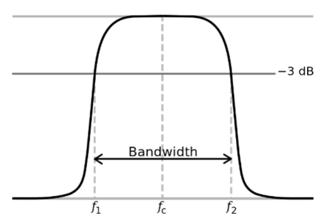


FIGURE 5. Illustration of Q. (credit: Wikipedia)

In Fig. 4, we show a parallel LCR combination being driven by an AC current source. An ideal current source is simply a source of current with an infinite impedance. As we will see in this lab, we can approximate a current source (Fig. 6) by a voltage source and a very large impedance R_S. So, as long as $R_S \gg R$, this will be a good approximation.

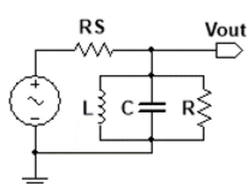


FIGURE 6. A parallel band-pass LCR filter.

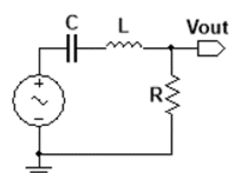


FIGURE 7. A series bandpass LCR filter.

At $\omega_0 = 1/\sqrt{LC}$, the parallel LCR has an impedance of $R_{||}$ and a maximum voltage of $V = IR$. As you will demonstrate in P3.4, solving for the Q of this circuit gives:

$$Q = R_{||}\sqrt{\frac{C}{L}} = \frac{R_{||}}{\omega_0 L} = \omega_0 R_{||} C$$

A larger parallel resistance $R_{||}$ gives larger Q.

We can carry out a similar analysis for a series LCR combination as shown in Fig. 7 (this was done explicitly in lecture 3). In this case, we are driving the resonator with a voltage source. At a frequency $\omega_0 = 1/\sqrt{LC}$, the resistor sees the full applied voltage and the current has a maximum value of $i = V/R_+$. A similar treatment to that of the parallel resonator gives:

$$Q = \frac{1}{R_+}\sqrt{\frac{L}{C}} = \frac{\omega_0 L}{R_+} = \frac{1}{\omega_0 R_+ C}$$

A smaller series resistance R_+ gives larger Q.

In addition to these bandpass filters, it is also possible to construct high-Q LCR notch filters that attenuate signals in a narrow BW about ω_0. These are useful when making sensitive measurements in the presence of strong interference at specific frequencies.

Resonating circuits always have some dissipation, either from a deliberately added resistor or from imperfections in the circuit components; typical imperfections include resistance in the inductor windings and dissipation in the inductor core and the capacitor dielectric. The Q of a resonator is a measure of the "quality" of the resonator; the lower the dissipation, the higher the Q. Mechanical resonators also have Q values that can be defined similarly. In addition to the precise definition of Q for LCR circuits given above, there several other ways to estimate the Q of a resonator including:

- The number of oscillation cycles the resonator will complete after it has experienced an impulse excitation before the amplitude drops to $e^{-1/2}$.
- The number of oscillation cycles for the energy to drop by $1/e$.

A common misconception is that the highest possible Q circuit is always best. Very high values of Q are desirable for sources like fixed frequency transmitters or sine wave sources, where very pure frequencies are needed. However, very high Q is not necessarily desirable for transmitters or receivers broadcasting information; information cannot be conveyed by a pure, single frequency wave. The signal has to have a bandwidth large enough to encode the highest frequency component of the signal to be conveyed. Thus, in such applications, circuitry will be designed with resonators with $Q \approx \omega_0/\mathrm{BW}$. For example, radio stations broadcast information around a central base frequency called the carrier frequency, $\omega_0 = 2\pi f_0$. But since they have to transmit information, they actually broadcast using a band of frequencies surrounding their carrier frequency. The station KFOG-FM, for instance, uses a carrier frequency of f_0 =104.5 MHz, but it actually broadcasts between about 104.450 MHz and 104.550 MHz for a full-width bandwidth of $\Delta f_{\mathrm{BW}} = 2$ (50 kHz) = 100 kHz, and a quality factor of about $Q \approx f_0/\Delta f_{\mathrm{BW}}$ = (104.500 MHz)/(100 kHz) = 1045.

3. PRE-LAB QUESTIONS

P3.1) **Transformers: Understanding current flow in the primary coil:** In Sec. 2.1, after deriving the dynamical solution for the current in both windings (Eq. 3), we focused on I_2 and V_2 to calculate the Thévenin output equivalent circuit. Let's try to also understand what's happening to $\bar{I}_1$ to improve qualitative understanding.

(a) When the secondary winding is in the open circuit limit ($R_2 \to \infty$) what is I_1? What is the effective circuit that this looks like?

(b) When the secondary winding is in the short circuit limit ($R_2 \to 0$) what is I_1? Use the fact that $M = k\sqrt{L_1 L_2}$ to simplify your work. What is the effective circuit that this looks like? What is the effective inductance of this circuit? Can you qualitatively understand this in terms of magnetic field energy within the transformer?

(c) How can you use the above limits to estimate k?

P3.2) **Transformers: Thévenin Input Impedance:** In Sec. 2.1 and Fig. 1, we put the transformer in the black box with the primary voltage source (V_1, R_1) and calculated the Thévenin equivalent source parameters. Now, as shown in Fig. 8, put the transformer in a black box with the load resistance, R_2, and calculate the effective load impedance in the limit that $k = 1$ (an important case that you will need for this lab). Note that $R_1 = 0$ as well to make the calculation easier (though I note that this choice doesn't change the answer, just the complexity of the math). Because of the fundamental symmetry of the transformer equations, it's also possible to just take Eq. 7 and do some variable swaps!

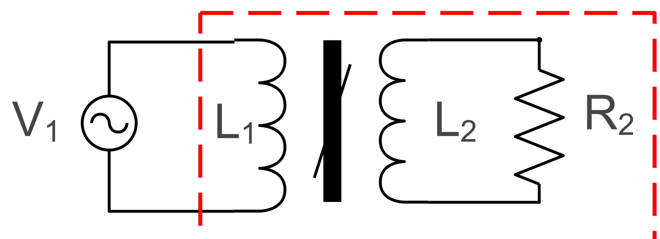


FIGURE 8. Thévenin equivalent black box (or in this case red box) to calculate an effective Thévenin input impedance.

P3.3) **Ideal Transformer:** An "ideal" transformer perfectly transfers all power produced in the source to the load without significant absorption in R_1 (see Fig. 1). This is equivalent to requiring that $R_1 \ll N_{12}^2 R_2 \ll \omega L_1$ or $N_{21}^2 R_1 \ll R_2 \ll \omega L_2$ for a transformer with $k = 1$. Derive this constraint by thinking about the effective voltage divider between R_1 and the Thévenin equivalent input impedance that you derived in P3.2.

P3.4) Consider the RLC tank circuit shown in Fig. 9 being driven by an AC current source.

(a) Calculate $H(\omega) = \bar{V}_{\text{out}}/\bar{I}(\omega)$. Note that in this case the transfer function $H(\omega)$ has units of ohms because the output is a voltage while the input is a current.

(b) Explicitly find both the magnitude and phase of $H(\omega)$ for the cases of $\omega \to 0$, $\omega \to \infty$, and $\omega = \omega_0$, where ω_0 is the resonant frequency $1/\sqrt{LC}$. Finding $|H(\omega)|$ is quite simple for all three cases, but the phase takes a bit more effort.

(c) Qualitatively graph $|H(\omega)|$ and $\phi(\omega)$.

(d) Find the complex *poles* ω_p of $H(\omega)$. These are the complex values of ω where the denominator of $H(\omega)$ is equal to zero. As discussed in lecture 3, these are the dynamical eigenvalues of the homogenous differential equation for the circuit. The real part of the pole is the resonant frequency, and the imaginary part of the pole is the inverse of the characteristic time constant for the circuit. Show that if $LC \ll (RC)^2$, the resonant frequency is $\frac{1}{\sqrt{LC}}$.

(e) The quality factor Q of the circuit is $\frac{\mathrm{Re}(\omega_p)}{2\,\mathrm{Im}(\omega_p)}$. Note that the demoniator's factor of 2 comes from the fact that the energy decays away twice as fast as the amplitude of the signal. Calculate Q in the small damping limit $LC \ll (RC)^2$.

(f) Calculate the detuning $\Delta\omega$ required to reduce $G(\omega)$ by a factor of $\sqrt{2}$, which is the 3 dB point of the resonator. The detuning is the frequency shift away from the resonant frequency. Use this to determine the bandwidth and the quality factor Q of the filter. Express your results in terms of R_{L}, L, and C.

There are lots of ways to Taylor expand, but many of these ways lead you to enormous amounts of algebra. Here is a simple way, using the fact that it's easier to take derivatives of $1/H(\omega)$ than of $H(\omega)$:

- Taylor expand the function $\frac{1}{H(\omega)}$ to first order about ω_0,
- Find the $\delta\omega$ where $|\frac{1}{H(\omega)}| = \sqrt{2}|\frac{1}{H(\omega_0)}|$ which is equivalent to $|H(\omega)| = \frac{1}{\sqrt{2}}|H(\omega_0)|$.

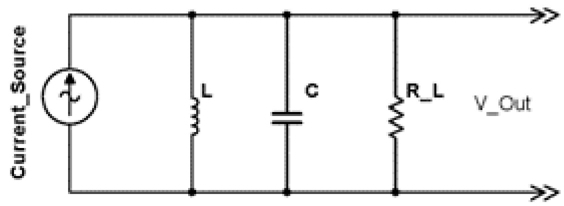


FIGURE 9. RLC resonator for P3.4

4. LAB EXERCISES

Problem L3.1 - Transformer Construction. In this exercise, we turn our inductor that you made in the last problem of Lab 2 into a transformer. Construct a 25:5 transformer by winding five additional turns of a second wire around your toroid. You will need to start with about 15" of wire. As before, leave about 3-4" of wire on each end for the leads. Your transformer should look like Fig. 10.

FIGURE 10. Transformer with turns ratio 25:5.

(a) Estimate the value of L_2 from the measurement of L_1 in Lab 2.

(b) Use the Impedance instrument to **measure the inductance of this new set of windings**. The procedure is the same as that from the previous lab. However, because the inductance of this coil is much lower than the 25-turn coil, you should use a different R in your RL circuit. What R should you choose? What is the measured L_2? (Don't forget to change the resistor setting in the Impedance instrument.)

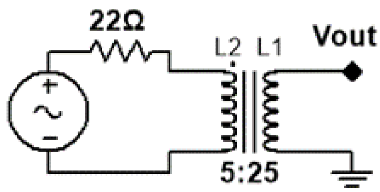
(c) Does the ratio of the two inductances L_1/L_2 agree with theoretical expectations?

Problem L3.2 - Transformer. In this exercise, we will be testing the transformer built in L3.1. The inductance, and resulting impedance, of the 5-turn winding is relatively low. We will need to operate with low load impedances and/or at high frequencies to be in the ideal transformer regime. A $22\,\Omega$ resistor in series with the input coil (primary) will be used to measure the current through the primary. We will operate the transformer in both step-down and step-up modes, exchanging the roles of the 5- and 25-turn coils.

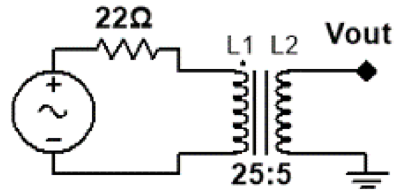
(a) Make a step-up transformer by driving the 5-turn winding with a 1 V amplitude 100 kHz signal through a $22\,\Omega$ resistor as shown in Fig. 11a. Calculate the impedance of the 5 turn and 25 turn inductors at 100 kHz. At 100 kHz, does the $22\,\Omega$ (R_2) significantly effect the transformer response?

(b) Configure both scopes so that Channel 1 shows the input voltage across the 5-turn primary and Channel 2 shows the output voltage across the 25-turn secondary. Set the scope Time Base to 5 µs/div and the vertical scale so that you can clearly see both traces. To reduce noise, set the scope to trigger on Wavegen output and average multiple traces. To do this, go into the menu at the top of the Wavegen Instrument and set the window that says No synchronization to Auto synchronization. Then click the green arrow that says Run All. This will produce an input waveform with a synchronized trigger for the scope. In the Scope window, set the trigger Source to Wavegen 1. Click the green down arrow in the Time box to display the full menu. Set Average to 100; this will produce an average of 100 scans with significantly reduced the noise. Use the Measurements function to **measure the signal Amplitude for both Scope channels**. Report these values. Does the step-up voltage ratio $V_{\text{out}}/V_{\text{in}}$ agree with the predicted value (V_{in} being the voltage across the primary)?

(c) Now **reverse** the transformer as in Fig. 11b and drive the 25-turn primary winding with the 1 V amplitude 100 kHz sine wave through the $22\,\Omega$ resistor and measure the output on the 5-turn secondary winding. Use the same measurement method as for part (b). What is the **voltage ratio** between the input and output windings? Does this step-down ratio $V_{\text{out}}/V_{\text{in}}$ agree with its predicted value?



(a) A step-up transformer.



(b) A step-down transformer.

FIGURE 11. The two types of transformers.

(d) If the voltage ratios measured in (b) and (c) do not agree with expectations, it could be because the coupling constant between the coils k is less than 1, or the ratio of turns $N_1/N_2 = N_{12} \neq 5$ exactly. Use your 2 measurements from parts (b) and (c) to **determine exact values for k and N_{12}**.[2]

(e) Attach Scope Channel 1 across the $22\,\Omega$ resistor and use this to determine the amplitude of the current I through the resistor and the 25-turn primary of the transformer. Attach Scope Channel 2 across the primary of the transformer and measure the voltage amplitude V. Use your measurement of the voltage and current to **determine the magnitude of the input coil impedance** $Z_{\text{primary}} = V/I$. Does this value agree with expectations?

(f) For your measured secondary inductance L_2 and a signal frequency of 100 kHz, what condition must the load impedance Z_{load} satisfy so that the transformer operates in the ideal regime? (Z_{load} is the impedance of any load attached across the secondary, for instance, the $4.7\,\Omega$ resistor in Fig. 12.)

(g) Use a $4.7\,\Omega$ resistor to load the 5-turn transformer output winding as in Fig. 12. **Remeasure** the voltage step-down ratio. Is it different than in the unloaded case?

(h) **Repeat** the voltage and current measurements of the input coil of the transformer from part (e) to determine the transformer input impedance $Z_{\text{primary}} = V/I$. Does this measured value agree with expectations? Why is it different from the result of (e)?

(i) Qualitatively **measure** the phase relationship between the voltage across and the current through the 25-turn primary. **Remove** the $4.7\,\Omega$ resistor, leaving the 5-turn output coil leads floating, and **remeasure** the phase. Finally, replace the $4.7\,\Omega$ resistor in Fig. 12 with a $0.33\,\mu\text{F}$ capacitor, and **remeasure** the phase. Qualitatively explain your results.

(j) **Restore** the $4.7\,\Omega$ resistor in Fig. 12. **Measure** the voltage across the $4.7\,\Omega$ load resistor and **calculate** the current through the 5-turn winding. What is the **ratio** of the output current through the 5-turn secondary measured here to the input current across the 25-turn primary that you measured in part (h)? Does this current ratio agree with the predicted value?

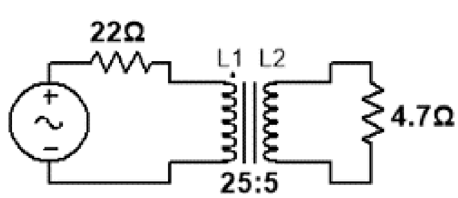


FIGURE 12. A step-down transformer with load.

[2]Hint: you have 2 measurements and 2 unknowns! Unless the measurements are degenerate, you can use these 2 measurements to separate and independently measure k and N_{12}. Thus, try to think of a way to combine the measurements from your step-up and step-down transformers to solve for both k and N_{12}, without making any assumptions about their values.

Problem L3.3 - RLC Circuit Resonator. Construct the RLC resonator circuit shown in Fig. 13. Use the 25-turn inductor, and one of separately-supplied (not in your main capacitor kit) 0.1 µF capacitors—these capacitors have a yellow head (like those in your kit) but have extra-long leads. In this circuit, the series resistance of $R_S = 220\,\mathrm{k\Omega}$ is selected to be much larger than the impedance of the tank circuit and you can treat the tank circuit as being driven by a current source.

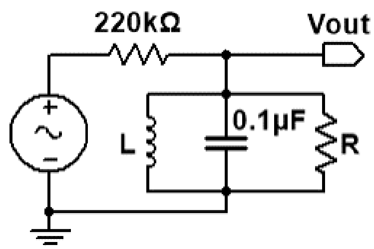


FIGURE 13. Parallel resonator.

(a) Using your measured value for the inductance L, **calculate** the resonant frequency f_0 of this circuit. Reminder: ignore the $220\,\mathrm{k\Omega}$ resistor, i.e. just consider the resistor and the voltage source to constitute a current source.

(b) With $R = 10\,\mathrm{k\Omega}$, take a Network Analysis from 1 kHz to 100 kHz. Decrease the frequency span of the Network Analysis about the resonance to accurately **measure the frequency of the resonance**. Does it agree with your expectations?

(c) Use the cursor to find the 3 dB points of the resonance from the Network Analysis. Use this and the central frequency to **determine the value of** Q for the resonator.

(d) With $R = 3\,\mathrm{k\Omega}$ and $1\,\mathrm{k\Omega}$, take a Network Analysis and, using the method of (c), **measure the** Q of the resonator.

(e) **Compare** the values of Q found for $R = 1\,\mathrm{k\Omega}$, $3\,\mathrm{k\Omega}$ and $10\,\mathrm{k\Omega}$ with the theoretical expectations.

(f) Losses in capacitor dielectrics and inductor cores can be modeled as an additional resistance R_p in parallel with R. So, rather than the impedance of the LC parallel combination becoming infinite on resonance, it becomes R_P. With R removed, use the measured peak of the network analysis to **determine** R_P.[3] The reason you built this circuit with the separately-supplied 0.1 µF capacitor is that it has lower loss (and resulting higher effective parallel R_P) than the capacitors in your capacitor kit.

(g) With R removed, **measure the** Q **of the resonator**. Does it agree with the theoretical expectation for Q assuming R_P from part (f) as the load?

(h) Treating the load as the parallel combination of R and R_P, **recompute** the theoretical expectations for Q with $R = 1\,\mathrm{k\Omega}$, $3\,\mathrm{k\Omega}$ and $10\,\mathrm{k\Omega}$. **Compare** these predictions with your measured values.

[3]Hint: it makes a voltage divider with $R_S = 220\,\mathrm{k\Omega}$.