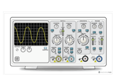
LAB 4: SEMICONDUCTOR DIODES I

UC Berkeley Donald A. Glaser Instrumentation Laboratory

GENERAL GUIDELINES

- **You should complete the prelab questions before beginning benchwork.** Information required to answer the prelab questions can be found in the background material at the beginning of the lab, from lecture, in the stated references, and on the web.
- **Please ask questions of the GSIs or professor(s)** at any time during the course!
- Wikipedia has many useful and informative articles. You are strongly encouraged to use Wikipedia and other online resources to better understand any material.
-  **Problems with this icon need to be checked off by GSIs.**
-  **Problems with this icon indicate that you should take a picture of the circuit built for the problem and include it in your report. Include your student ID, with name clearly visible, in the photo.**
-  **Problems with this icon indicate that you should take a screenshot of the WaveForms oscilloscope.**
- **Important Safety Habits**
  - Before stopping for the day or taking a break, make sure you power down all equipment.
  - When possible, use the switches for the power supplies to power down the circuit when changing the wiring. It is easy to accidentally short wires and damage your equipment or electronic components.
  - Never place food or drink next to any apparatus. Accidental spills can damage or destroy the equipment and your experiment.

Check-off	Instructor Name/Signature	Date
Pre-lab		

Table 1. Check-off Table

1. LEARNING GOALS

(1) Characterize and understand sermiconductor diodes.

(2) Use diodes for common applications, including voltage regulation (Zener diodes) and AC-DC conversion.

2. LAB BACKGROUND READING

2.1. **Important Safety Notes.** The semiconductor components used henceforth in this course can get quite hot... especially if they are hooked up incorrectly. They can easily burn you. If you must touch components while the power is on, touch them gingerly first to assess their temperature before grasping them with force. Better yet, turn the power off, and wait a few moments for the components to cool down.

Diodes will burn out if you connect them directly to a power supply without a current limiting resistor. Do not do this.

2.2. **Semiconductors and Diodes.** So far, we have only considered circuit elements for which the current and voltage are linearly related. However, non-linear elements such as diodes and transistors are required for the most interesting application in electronics, including creating DC voltages from AC signals, the creation of AC waveforms, amplifying small signals, and the construction of digital logic and memory. Not every non-linear circuit element is a semiconductor, but most of those that we will consider are. In this lab, we will explore applications of the most basic semiconductor element, the diode. Just like a resistor, the diode is a two-terminal device. However, the diode has a very non-linear relationship between voltage and current and an extreme asymmetry in the current for positive and negative voltages. We will begin with a very brief discussion of the physics of the diode and then explore some of its many applications. Chapter 3 of ***Sedra and Smith*** has a very good treatment of diode physics and diode circuits of this material, and you could start by reading that.

2.3. **P- and n-type semiconductors.** *Diodes* and *transistors* are made from semiconducting materials: most commonly, crystalline silicon, though occasionally germanium or composite materials like gallium-arsenide are also used. Semiconducting materials are materials which are not insulators at room temperature but have relatively low conductivity compared to good conductors like metals. In crystalline silicon, the four outer shell electrons of each atom inhabit low energy delocalized covalent bonding quantum states that attract the various atoms to each other. At absolute zero in a perfect crystal, all of the low energy "valence" states are filled, and thus there is no way for the electrons to move and produce current flow without being excited to a significantly higher energy conduction band (anti-bonding) quantum state. The highest energy of the valence band is below the lowest energy of the conduction band, where free electrons can propagate; this energy difference is the *bandgap energy* E_G.

At finite temperature it is possible for an electron to be randomly/thermally excited to a conduction band state, where it becomes free to move, leaving behind a positively charged stationary ion. An electron from a neighboring atom can move to the newly formed ion, leaving behind a new positively charged atom. In this way, a positively charged "hole" is also free to move in the crystal; holes behave almost exactly like positively charged electrons; they move, respond to electric fields, and appear to have a mass close to the electron mass. Thus, thermal ionization of an atom creates both a mobile electron and a mobile hole. Both electrons and holes act as charge carriers, and, on an individual basis, act with essentially equal efficacy. However, at room temperature, only about one in 10^9 atoms in pure silicon is ionized. Pure silicon at room temperature is very resistive.

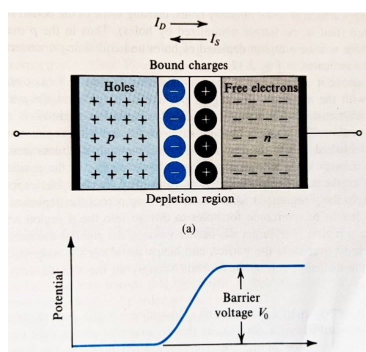


FIGURE 1. A p-n junction with no applied voltage (from Sedra and Smith, 4th edition, Figure 3.12). Outside of the depletion region, both the p- and n-type materials are charge neutral. The E field in the depletion region points from the (+) side to the (-) side (right to left in this diagram), creating the voltage difference ("barrier potential") shown below.

The lifetime of electrons and holes created by thermal ionization is not infinite.[1] Instead, electrons and holes occasionally come close to each other and recombine (sometimes called annihilation). This changes the ion temporarily associated with the hole back into a neutral atom. In a real material, an equilibrium is very quickly set up between thermal ionization, which creates free electrons and holes, and recombination, which destroys them. If n and p are the densities of free electrons and holes in units of cm^{-3}, then, for silicon at room temperature, the equilibrium condition is $np = 10^{20}\,\mathrm{cm}^{-6}$ where the numerical constant is an exponentially increasing function of the temperature.[2]

To increase and control its conductivity, the silicon is normally *doped*, i.e. deliberately contaminated, with other elements. Some dopants, like phosphorous, arsenic, and antimony, have 5 valence electrons; after 4 electrons form strong covalent bonds, the atom is left with one very weakly bound electron. These weakly bound electrons are easily thermally ionized and donate mobile electrons to the material, hence these dopants are called *donor* dopants. These donated electrons are free to move about the crystal, and its conductivity increases dramatically. Only a small density of dopants is needed; one dopant atom per 10^8 silicon atoms will increase the density of the of the free electrons to about $n = 10^{15}\,\mathrm{cm}^{-3}$ and increase the conductivity by a factor of approximately 10^5 compared to pure silicon. Because the np equilibrium condition still holds, the density of holes will decrease to about $p = 10^5\,\mathrm{cm}^{-3}$.

When dopant atoms give up an electron and become positively charged, the net charge of the material remains zero. Because the electrons that the dopant atoms give up were only weakly bound, the dopant atoms do not have the wherewithal to steal electrons from one of their neighbors. Consequently, the positive charges remain on the dopant atoms, and holes are not created by donor dopants.

Dopants with three valence electrons like boron, indium and aluminum, are missing an electron and steal an electron from a neighboring silicon atom to create a missing electron (a hole). These dopants are called *acceptor* dopants. This now-positive silicon ion tries to regain its neutrality by stealing electrons from their neighbors. This results in positively charged holes that are free to move through the crystal. The four covalent bonds around the acceptor dopants are very strong. The negative charges remain on the acceptor atoms, and free electrons are not created by acceptor dopants.

A donor-doped semiconductor with more free electrons than holes is called an n-type *semiconductor; conversely, an acceptor-doped semiconductor with more holes than free electrons is called a* p-type *semiconductor.*

2.4. **P-N Junctions.** If doping's only effect was to increase semiconductor conductivity, semiconductors would be obscure, little-used materials. The utility of semiconductors comes from the remarkable effects of placing p- and n-type materials in contact. Such juxtapositions are called *p-n junctions*. An isolated p-n junction makes a semiconductor diode. Other semiconductor components are made from more complicated arrangements; bipolar NPN transistors, for example, are made by sandwiching a p-type layer in between two n-type layers. When a p-type material and an n-type material are put in contact, free holes and electrons diffuse across the barrier where they recombine on the other side of the junction. This leaves behind a *depletion* region where the n-type material will have an excess of positive stationary charge and the p-type material will have an excess of negative stationary charge. The separation of charge results in a barrier potential across the junction, as is shown in Fig. 1. The barrier potential halts the further diffusion of charge across the junction and no current flows in the device. However, if a positive potential is applied on the anode (p) relative to the cathode (n) of the junction, then this barrier can be overcome, and large currents can be made to flow. Applying a voltage in the opposite direction only increases the barrier to charge flow and, under normal conditions, only a small leakage current will flow.

2.5. **I-V Characteristics.** As shown in Fig. 2, the relationship between current and voltage for a diode, called its I-V characteristic, is highly non-linear. In contrast, the curve in Fig. 2 for a resistor would simply be a diagonal line. This non-linear and asymmetric dependence of current and voltage is what makes the diode so useful.

The diode I-V characteristic curve has three main regions. On the right-hand side of the plot, for $V_\mathrm{D} > 0$, we have the **forward-bias** region. On the left-hand side of the plot, for $V_\mathrm{D} < 0$, we have the **reverse-bias** region. Finally, on the far left of the plot, we have the **"breakdown"** region.

[1] If it was infinite, eventually all the atoms would be ionized.

[2] This equilibrium is very similar to the equilibrium between free hydrogen and hydroxyl ions in water. It follows from the same physics and is the basis of the pH scale.

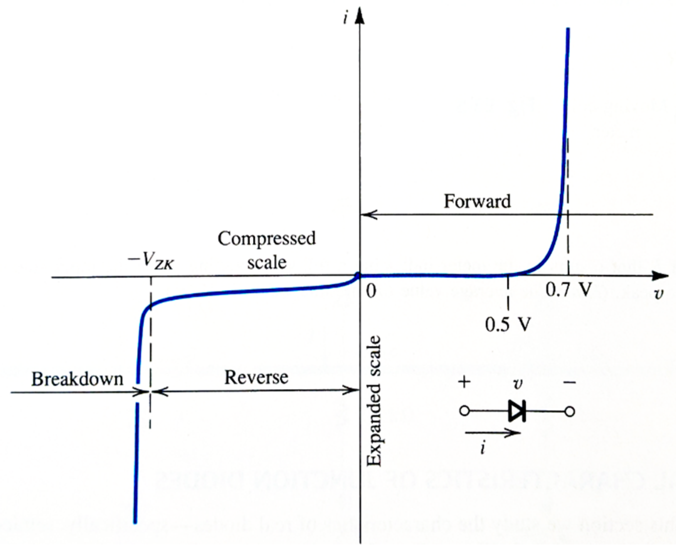


FIGURE 2. Diode characteristic I-V curve (from Sedra and Smith, 4th edition, Fig. 3.8). The kink near 0 V is an artifact of the compressed scale used for negative voltages; in reality, the curve is smooth around zero.

For voltages more positive than the breakdown voltage, the current through an ideal p-n junction as a function of applied voltage is well described by the following equation,

$$I_D(V_D) = I_S\left[e^{\frac{eV_D}{nkT}} - 1\right] \tag{1}$$

where V_D is the voltage drop across the junction; I_S is a constant called the *saturation current* that depends on the temperature, the particular geometry of the junction, and on the material of the junction; $e = 1.6 \times 10^{-19}$ C is the charge of an electron (also called the elementary charge), $k = 1.38 \times 10^{-23}\,\mathrm{J\,K^{-1}}$ is Boltzmann's constant, and T is the temperature in Kelvin. We can express the argument of the exponential as a ratio of the applied voltage over the *thermal voltage* V/nV_T with $V_T = kT/e$. At room temperature, $V_T \approx 25.7$ mV, which is often crudely approximated as $kT/e \approx 1/40$V . The constant n is called the *ideality factor* and varies between 1 and 2 depending on the particular diode but is typically very close to $n = 2$ for discrete silicon diodes. The derivation of this equation involves some interesting physics and you are encouraged to read more about it in the references.

While Eq. 1 properly describes the diode voltage dependence at a fixed temperature, the value of I_S is a steep function of temperature,

$$I_S \propto T^3 e^{\frac{-E_G}{nkT}} \tag{2}$$

where E_G is the "bandgap" of the intrinsic semiconductor and n is the same ideality factor in Eq. 1. The bandgap is 1.14 eV for silicon, and 0.67 eV for germanium. At room temperature, the saturation current of a silicon diode with ideality factor $n = 2$ doubles for every increase in temperature of approximately 9 °C (the engineering rule of thumb is that it doubles for every increase of 10 °C). For the 1N4448 diodes used in this lab, the room temperature saturation current is $I_S \sim 5$ nA. We can use the dependence of saturation current on temperature as a thermometer or to measure the semiconductor bandgap energy. Putting Eq. 1 and Eq. 2 together, we have

$$I_D \propto T^3 e^{\frac{-E_G}{nkT}}\left[e^{\frac{eV_D}{nkT}} - 1\right] \tag{3}$$

Notice from Eq. 1 that the diode's response is directional and highly nonlinear. When *forward-biased* (V_D positive), large currents can flow through the diode with a modest voltage across the diode (small effective resistance) because of the exponential dependence of I on V_D. When the diode is *reverse-biased* (V_D negative), the current then approaches $-I_S$. Since I_S is typically very small (as small as picoamps or even femtoamps for some diodes), very little current flows. Thus, the diode acts like a one-way valve; significant current can only flow in one direction. It is this particular behavior of diodes that makes them useful in applications.

When forward-biased, ***the positive end of the diode is called the anode***, and ***the negative end is called the cathode.*** The terms anode and cathode date from the days of vacuum tube diodes. The symbol for a diode is shown in Fig. 3. The direction of the "arrow" indicates the direction of positive current flow. On an actual diode, the cathode is normally marked with a painted band as also shown at right. This band can be hard to see on some diodes.

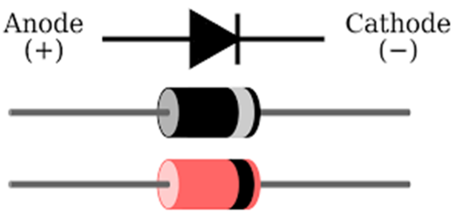


FIGURE 3. Diode symbol from Wikipedia.

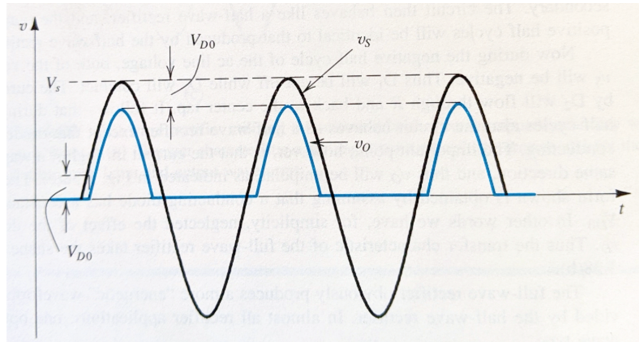


FIGURE 4. Input (blue) and output (black) voltages of the diode rectifier circuit shown in Fig. 5, taken from Sedra and Smith.

2.6. **Most Simplistic Diode Model: Constant Voltage Drop.** For many switching and rectification applications, it is sufficient to think of the diode as an ideal switch that turns on with a negligible dynamic impedance when forward-biased above a certain critical voltage,$V_D \sim 0.7V$ for Si, while below the critical voltage it has no current flow (and thus infinite impedance). Let's use this model to qualitatively understand the performance of the rectifier[3] circuit shown in Fig. 5 where the resistance of the load is much greater than the forward biased dynamic impedance of the diode, but much less than the reverse biased dynamic impedance:

- For $V_{in} - V_{out} < V_D$, the diode is reversed biased and allows negligible current flow. Thus, it can be approximated as a very high impedance resistor. This means that the circuit looks like a voltage divider where the entirety of the voltage is across diode's high resistance and consequently $V_{out} = 0V$.
- If $V_{in} > V_D$, then the diode is forward biased and in this approximation has zero impedance. Thus no matter how much current flows, the $V_{in} - V_{out} = V_D$ and thus $V_{out} = V_{in} - V_D$.

Therefore, the output looks like that shown in blue in Fig. 4.

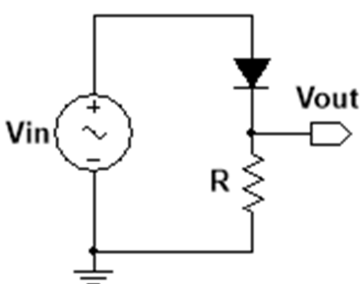


FIGURE 5. A rectifier circuit.

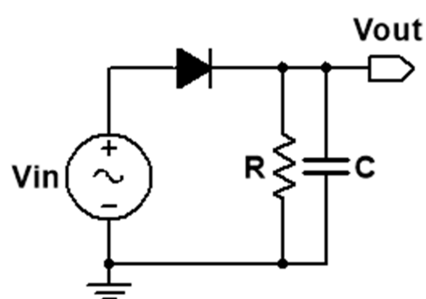


FIGURE 6. A filtered rectifier.

The output of a rectifier is usually filtered to reduce the variations in voltage and more closely approximate a DC voltage. Fig. 6 shows a rectifier with a filtering capacitor. You can model the ripple as a capacitor being discharged from its peak voltage across a resistor during the time $T = 1/f$ between peaks in the rectified voltage. To achieve significant filtering, we want to choose $C \gg 1/(2\pi f R)$. Smaller load resistors, and corresponding greater current draw, will require larger filtering capacitors.

[3]A *rectifier* is another name for a diode and is commonly used for this and similar applications.

2.7. **Forward-bias region.** When the diode is forward-biased ($V_{\mathrm{D}} \gg nV_T$), Eq. 1 can be approximated as

$$\lim_{V_{\mathrm{D}} \gg nV_t} I_{\mathrm{D}}(V) = I_{\mathrm{S}} e^{\frac{eV_{\mathrm{D}}}{nkT}} = I_{\mathrm{S}} e^{\frac{V_{\mathrm{D}}}{nV_T}}. \tag{4}$$

If we take the logarithm of the current, the equation becomes,

$$\ln(I_{\mathrm{D}}) = \ln(I_{\mathrm{S}}) + \frac{1}{nV_T} V_{\mathrm{D}}. \tag{5}$$

Consequently the plot of $\ln(I_{\mathrm{D}})$ vs V_{D} allows one to directly estimate both n and I_{S} from the slope and intercept.

We can also invert Eq. 4 to express V_{D} in terms of I_{D}:

$$V_{\mathrm{D}} = nV_T \ln\left(\frac{I_{\mathrm{D}}}{I_{\mathrm{S}}}\right) \tag{6}$$

to find both the large signal non-linear resistance that decreases with increasing current through the diode,

$$R_{\mathrm{D}} = \frac{V_{\mathrm{D}}}{I_{\mathrm{D}}} = \frac{nV_T}{I_{\mathrm{D}}} \ln\left(\frac{I_{\mathrm{D}}}{I_{\mathrm{S}}}\right)$$

and the *small-signal dynamic impedance*, r_{d}, for a small signal excitation around an equilibrium current, I_{D0},

$$r_{\mathrm{d}} = \frac{\partial V_{\mathrm{D}}}{\partial I_{\mathrm{D}}}(I_{\mathrm{D}} = I_{\mathrm{D0}}) = \frac{nV_T}{I_{\mathrm{D0}}}. \tag{7}$$

This diode small-signal dynamic impedance is then inversely proportional to the equilibrium current and for large currents can be quite low. For example, at a current of 10 mA, the static impedance for a $n = 2$ diode is $R_{\mathrm{D}} \sim 70\,\Omega$ and the dynamic impedance is $r_{\mathrm{d}} \sim 5\,\Omega$.

2.8. **Temperature dependence in the forward-bias region.** When the diode is forward-biased ($E_g/e \geq V_{\mathrm{D}} \gg nV_T$), Eq. 3 can be approximated as

$$\lim_{V_{\mathrm{D}} \gg nV_T} I_{\mathrm{D}} \propto T^3 e^{\frac{eV_{\mathrm{D}} - E_{\mathrm{G}}}{nkT}} \tag{8}$$

If the diode is biased with a fixed current, then

$$\frac{E_{\mathrm{G}} - eV_{\mathrm{D}}}{T} \simeq \text{constant} \tag{9}$$

because the exponential dependence on the inverse temperature will dominate the T^3 dependence. Thus when the diode temperature goes up, the numerator ($E_g - eV_D$) must increase to compensate for the increase in the temperature as well, and thus V_{D} will drop for a fixed I_{D}. On a microphysics level, $E_g - eV_D$ is related to the potential barrier in the depletion region.

With measurements of the voltage across a current biased diode at two (or more) temperatures, it is possible to solve for the bandgap energy E_g of the semiconductor.

Differentiating Eq. 9 and doing some algebra, we find

$$\frac{\partial V_{\mathrm{D}}}{\partial T} = \frac{V_{\mathrm{D}} - E_{\mathrm{G}}/e}{T}$$

The bandgap for silicon is $E_{\mathrm{G}} = 1.11\,\mathrm{eV}$, and with a current of $I_{\mathrm{D}} = 10\,\mu\mathrm{A}$, $V_{\mathrm{D}} \sim 0.4\,\mathrm{V}$, we find that $\frac{\partial V_{\mathrm{D}}}{\partial T} \sim -2.4\,\frac{\mathrm{mV}}{^\circ\mathrm{C}}$. Diodes used in this way are referred to as bandgap temperature sensors.

2.9. **Reverse-bias region.** When the voltage across the diode becomes very negative ($V_D \ll -nV_T$), the junction is reverse biased and following Eq. 1 the current across the junction becomes

$$I_{\mathrm{D}} = -I_{\mathrm{S}}$$

In practice, the actual measured reversed current is much larger than $-I_{\mathrm{S}}$ and increases slowly with increasing voltage. Therefore, the ideal diode equation is not very useful for describing this region. If the reverse current matters in your design, you can find it on the data sheet for the diode. For the 1N4448 it is listed as 25 nA at 20 °C. However, it climbs rapidly with temperature and reaches nearly 3 µA when the junction temperature reaches 100 °C. So, beware that leakage currents can be large in applications where the forward current is intermittently high and the junction becomes heated.

2.10. **Breakdown region and Zener diodes.** When the magnitude of the reverse bias voltage reaches the breakdown voltage for the device V_{ZK}, the reverse diode current climbs very steeply. This can be seen on the far left of Fig. 2. In normal operation, a signal diode is not subjected to voltages large enough to cause breakdown (ZK stands for "Zener knee"). However, there is a special class of diodes known as *Zener diodes* that are engineered to have a low and precisely specified reverse breakdown voltage. The symbol used for a Zener diode in a circuit diagram is shown in Fig. 7. The anode, cathode, and definitions for forward and reverse bias are identical; however, the device is normally operated with a reverse bias. Heavily-doped materials are used to create a very thin p-n junction layer.

The details of doping and construction can be used to tune the breakdown voltage over wide range of voltage from 3 V to 300 V. When biased in the forward direction, Zener diodes behave like normal diodes. For diodes with $V_{\mathrm{ZK}} \lesssim 5\,\mathrm{V}$, a significant number of holes and electrons begin tunneling through the thin depletion layer and creating a significant reverse current as the reverse bias voltage approaches V_{ZK}. This results in a comparatively gradual increase in reverse current at V_{ZK}. However, this I-V curve is still much steeper than the increase in current for a forward-biased diode. Zener diodes with $V_{\mathrm{ZK}} \gtrsim 5\,\mathrm{V}$ make use of a wider depletion layer. As the reverse voltage approaches V_{ZK}, electrons and holes entering the depletion layer are accelerated by the electric field in the depletion layer. In this accelerated state, they collide with other atoms and knock electrons from their atomic bonds which go on to create more electron/hole pairs leading to a cascade or *avalanche* of charge carriers.[4] In devices with this type of avalanche breakdown, there is a sudden and extreme increase in reverse current at V_{ZK}. The extremely steep I-V curve in the breakdown region results in extremely low values of dynamic impedance $r_{\mathrm{z}} = \partial V/\partial I$. This makes these devices particularly useful for constructing voltage references and regulators that maintain a precise voltage despite supplying a changing current to a load.

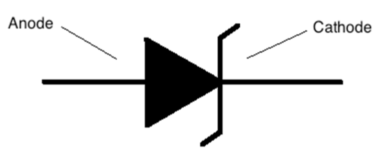


FIGURE 7. Zener diode symbol.

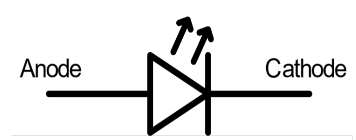


FIGURE 8. LED symbol.

2.11. **Light Emitting Diodes.** The symbol for the *light-emitting diode (LED)* shown in Fig. 8 represents the LED's behavior as a diode, and the ability of the LED to emit light. An LED is a p-n junction that has been constructed so as to allow energy released as photons when electrons and holes that cross the junction in the diffusion current recombine. Each recombination releases energy equal to the semiconductor bandgap of the semiconductor material. LEDs are constructed from materials in which the bandgap has been tuned to produce photons of particular energy and wavelength (i.e., color). In a normal diode, this energy from recombination is just dissipated as heat in the diode. However, in an LED, the recombination occurs in a thin highly doped semiconductor layer near a transparent window, and the resulting photons are reflected out of the junction and directed into a broad beam.

[4] The key here is that the electrons are accelerated enough by the electric field that they can knock out orbital electrons, a phenomenon called *impact ionization*.

The basic physical structure of a discrete packaged LED is shown in Fig. 9. As shown in Fig. 9, most discrete light emitting diodes have their cathode (–) terminal identified by either a notch or flat spot on the body or by the cathode lead being shorter than the anode (+) lead.

In some devices, the p-n junction is placed in a reflecting cavity and reflected light leads to the stimulated emission of photons with identical direction and wavelength. These are known as *laser diodes* and are used to efficiently produce highly directed coherent light sources for a wide range of applications ranging from medical technology to laser pointers.

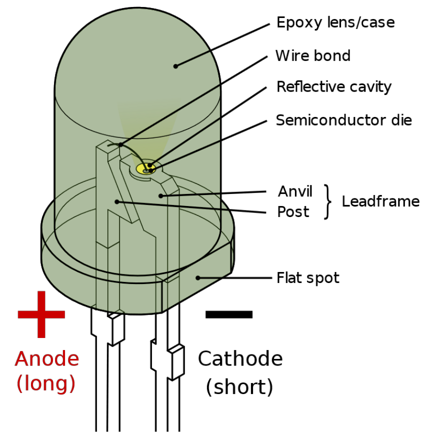


FIGURE 9. LED schematic.

The electrical operation of an LED is very similar to that of a signal diode with the exception that they typically operate with much higher forward voltages. Just like a signal diode, A LED will pass current in its forward direction but block the flow of current in the reverse direction. The wavelength of the light created by a LED is directly related to the bandgap of the semiconductor by $E_G = hc/\lambda$. Different semiconductor materials are utilized to create LEDs with different colors. For example, a red LED constructed with gallium arsenide phosphide (GaAsP) as the semiconductor material has a bandgap energy $E_G = 1.91\,\text{eV}$ and produces light with wavelength of 650 nm. It is a common misconception that the voltage drop across an LED is a direct measurement of the bandgap energy. Certainly, the forward voltage when the LED is operating (reaches a significant current) is related to the bandgap, and larger bandgap energies will require larger forward operating voltages. However, in order to precisely measure the bandgap, you need to measure the change in the voltage or current with temperature. Because LEDs directly convert recombining holes and electrons into photons, they are able to achieve high efficiencies in the conversion of electrical energy to light. For some devices this efficiency is as high as 70%, which can be compared with the efficiency of an incandescent bulb in converting electrical energy into light of $\sim 5\%$. Because LEDs are solid-state devices, they can be small, durable, and provide much longer life than other light sources.

2.12. **Nonlinear circuit equilibrium.** Unlike purely linear circuits, circuits containing nonlinear elements like diodes cannot be reduced to systems of linear differential equations. Consequently, the equilibrium voltages and currents in nonlinear circuits are more difficult to determine. Solutions to these equations can be found by using a numerical root finder or by building the circuit in MultiSim (or some other SPICE-based program) and using it to find the solution. Historically, these types of problems were solved by graphical analysis or numerical iteration. Both of these approximate methods provide insight into the solution of the diode equilibrium conditions and can be used to get quick results. Below we will discuss each of the three methods: *graphical analysis* in Sec. 2.12.1, *iterative analysis* in Sec. 2.12.2, and *computation* in Sec. 2.12.3.

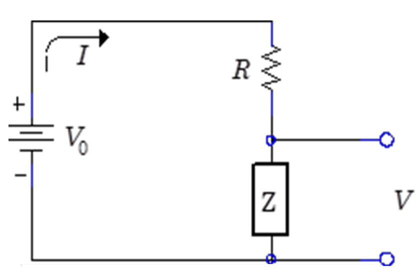


FIGURE 10. Circuit for load line analysis.

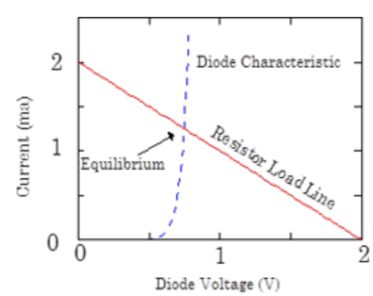


FIGURE 11. Diode equilibrium solution via load line analysis.

Graphical analysis: load lines. Consider the simple circuit in shown in Fig. 10, which contains a voltage source V_0, a resistor R and a generic nonlinear component that does not contain a voltage source and has a voltage-dependent impedance $Z(V)$, where V is the voltage across the component. The voltage of the source must be equal to the sum of the voltage across the resistor and non-linear component: $V_0 = V + IR$. We can then solve for the current as a function of the voltage across the non-linear component:

$$I(V) = \frac{V_0 - V}{R} \tag{10}$$

If the voltage across the non-linear element (V) is large, there must be a smaller voltage across the resistor and thus a smaller current. If the voltage across the non-linear is small, then there is a larger voltage drop across the resistor and thus larger current flow.

The line determined by Eq. 10 is called the *load line* because it is determined solely by the load resistor (and the voltage source), not by the nonlinear component. The nonlinear component obeys its own equation, or *characteristic curve* $I_Z(V)$. In equilibrium, both the load line and the characteristic curve must be satisfied simultaneously: $(V_0 - V)/R = I_Z(V)$. Consequently, the equilibrium current and voltage for the circuit are given by the intersection of the load line and the characteristic equation $I_Z(V)$. For example, assume that the nonlinear component is a room temperature diode ($I_S = 4 \times 10^{-10}\,\mathrm{A}, n = 2$) driven by a $V_0 = 2\,\mathrm{V}$ battery through a $1\,\mathrm{k\Omega}$ resistor as shown in Fig. 12. The load line and diode characteristic for this circuit intersect, as shown in Fig. 11, at equilibrium voltage $V = 0.75\,\mathrm{V}$ and current $I = 1.25\,\mathrm{mA}$. The equilibrium voltage and current across the diode is sometimes called the *operating point* of the circuit. Frequently, we are interested in small deviations about the equilibrium values and determining them is the first step in the analysis.

Iterative analysis. Nonlinear equilibria can also be found iteratively: by guessing an initial solution, determining the consequences of the guess, and then iteratively refining the guess. For many diode applications, this simple process rapidly converges to a highly accurate solution.

This method is best explained with an example: Using the diode circuit in Fig. 12, guess a current. For the purpose of demonstration, we make a crude guess using the maximum possible current $I = V_0/R = 2\,\mathrm{mA}$ and solve for the resulting diode voltage,

$$V_D(I) = \left(\frac{nkT}{e}\right) \ln\left(1 + \frac{I_D}{I_S}\right) \tag{11}$$

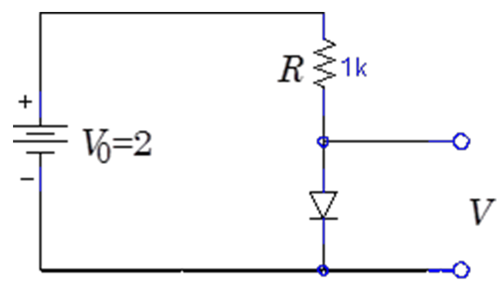


FIGURE 12. Iterative analysis example circuit.

With $I_S = 4 \times 10^{-10}\,\mathrm{A}$, we use Eq. 11 solve for the voltage across the diode that would produce this current, $V = 0.771\,247\,\mathrm{V}$. Next, use this voltage in Eq. 10 to determine a new estimate for the current $I = 1.228\,75\,\mathrm{mA}$. Repeat and continue iterating until the numbers converge. This kind of iteration is particularly easy to implement in a spreadsheet program like Excel to produce output like Table 2. The results have converged to five decimal places by the third iteration and six decimal places by the fourth iteration.

Iteration	Current (mA)	Diode Voltage (V)	Resistor Voltage (V)
Initial Guess	2.00000	0.771247	1.22875
1	1.22875	0.746890	1,25311
2	1.25311	0.747871	1.25213
3	1.25217	0.747832	1.25217
4	1.25217	0.747834	1.25217
5	1.25217	0.747834	1.2521

TABLE 2. Diode equilibrium values computed via iteration.

The logarithmic variation of the diode voltage with current means that for percent-level accuracy, you can often just assume that the voltage across a forward-biased silicon signal diode $V_\mathrm{D} \sim 0.7\,\mathrm{V}$ and proceed without iteration. The apparent precision of the iterative method is deceptive, and practically unnecessary as it relies on exact knowledge of I_S and n, which you won't know precisely unless you have measured them for the diode you are using. Don't worry; answers with accuracy better than a few percent are rarely required in electronics.

This method converges rapidly because the diode voltage depends only logarithmically on the current and the current is largely set by the series resistor. However, be careful: iterative methods do not always converge. In fact, running the described sequence backwards (guess the diode voltage, calculate the diode current, find the resistor voltage drop, and subtract from the battery voltage to refine the diode voltage guess) does not converge. Try it yourself![5]

Computational methods. Numerical minimization routines can be used to quickly find solutions to transcendental equations like that describing diode equilibrium. Essentially, this is just a more efficient automated version of the iteration method used above. To use these methods, you start by writing the transcendental equation describing the load line:

$$\begin{aligned} V_0 - V_\mathrm{D} - RI_\mathrm{D} &= 0 \\ V_0 - V_\mathrm{D} - RI_\mathrm{S}\left[e^{\frac{V_\mathrm{D}}{nV_T}} - 1\right] &= 0 \end{aligned} \tag{12}$$

Putting in the numerical values for this example ($V_0 = 2\,\mathrm{V}$, $R = 1\,\mathrm{k\Omega}$, $n = 2$, $I_\mathrm{S} = 4\times10^{-10}\,\mathrm{A}$, $V_T = 25.7\,\mathrm{mV}$), we have

$$2 - V_\mathrm{D} - 4\times10^{-7}\left[\exp\left(\frac{V_\mathrm{D}}{0.0514}\right) - 1\right] = 0$$

Python: In Python, you can use the `fsolve` equation solver from the `scipy` package. You need to define the equation $f(x)$ which will be zero when solved, and provide the algorithm with an initial guess. In the below code, x represents the diode voltage.

```python
from scipy import optimize
# Define the function:
def f(x):
    return 2-x-1e3*4e-10*(np.exp(x/0.0514)) -1)
# Solve for diode voltage with an initial guess of 0.7
print(optimize.fsolve(f, 0.7))
```

Wolfram Alpha: Go to https://www.wolframalpha.com/input/ and copy-paste the following text into the input box:

```
using Newton's method solve 2-x-(1*10^3*4*10^{-10})*(exp(x/0.0514)-1)=0 start V=0.7
```

Either of these will instantly output a precise value of the diode voltage. Putting the diode voltage into Eq. 1 will give the current. Of course, you could also use a SPICE based circuit solver like MultiSim to solve for the diode operating point. However, if you want results for specific diode parameters such as n or I_S, you will need to modify the diode model.

2.13. **Simplified diode model: Linearized response.** In many circuit applications, we want to consider small perturbations about the equilibrium operating point of a diode. For small changes in operating parameters, we can construct a linear approximation to the diode voltage-current relationship. Suppose we have a diode in equilibrium at a voltage $V_{\mathrm{D}0}$ and current $I_{\mathrm{D}0}$. The current voltage relationship in the forward-biased region follows Eq. 1. Now

[5] The study of the convergence of these methods is called *iterated map theory*, and surprisingly, forms the basis for much of chaos theory.

suppose that the voltage is perturbed by a small voltage excitation v_{d} such that $V_{\mathrm{D}} = V_{\mathrm{D0}} + v_{\mathrm{d}}$.[6] Taylor expanding the resultant current response gives us

$$\begin{aligned} I_{\mathrm{D}} &= I_{\mathrm{D0}} + \frac{\partial I_{\mathrm{D}}}{\partial V_{\mathrm{D}}}(V_{\mathrm{D0}})v_{\mathrm{d}} + \frac{1}{2}\frac{\partial^2 I_{\mathrm{D}}}{\partial V_{D^2}}(V_{\mathrm{D0}})v_{\mathrm{d}}^2 + \cdots \\ &= I_{\mathrm{D0}}\left(1 + \frac{1}{nV_T}v_{\mathrm{d}} + \frac{1}{2}\frac{1}{(nV_T)^2}v_{\mathrm{d}}^2 + \cdots\right) \end{aligned} \tag{13}$$

where we can see that the current-voltage relationship is linear so long as

$$\left|\frac{ev_{\mathrm{d}}}{nkT}\right| = \left|\frac{v_{\mathrm{d}}}{nV_T}\right| \ll 1,$$

or for $V_T = kT/e = 25.7\,\mathrm{mV}$, $n = 2$ and $T = 300\,\mathrm{K}$, the linear approximation requires $|v_{\mathrm{d}}| \ll nV_T = 51.4\,\mathrm{mV}$. Thus, as long as changes in diode voltage are $|v_{\mathrm{d}}| \lesssim 10\,\mathrm{mV}$, the linear approximation should be adequate for most applications and

$$I_{\mathrm{D}} = I_{\mathrm{D0}} + \frac{I_{\mathrm{D0}}}{nV_T}v_{\mathrm{d}} + \cdots \tag{14}$$

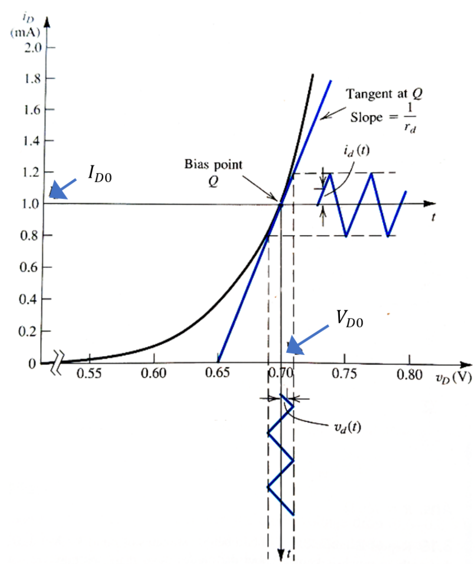


FIGURE 13. Graph of diode small signal response from Sedra & Smith.

The resulting small signal change in current i_{d} can be expressed as

$$i_{\mathrm{d}} = I_{\mathrm{D}} - I_{\mathrm{D0}} = I_{\mathrm{D0}}\left(\frac{ev_{\mathrm{d}}}{nV_T}\right) = v_{\mathrm{d}}\left(\frac{I_{\mathrm{D0}}}{nV_T}\right).$$

The small-signal impedance r_{d} is just the change in voltage for a small change in current,

$$r_{\mathrm{d}} = \frac{\partial V_{\mathrm{D}}}{\partial I_{\mathrm{D}}} = \frac{v_{\mathrm{d}}}{i_{\mathrm{d}}} = \frac{nV_T}{I_{\mathrm{D0}}}$$

For example, consider the case shown in Fig. 13, where a small triangle wave voltage perturbation is added to the equilibrium voltage $V_{\mathrm{D}} = V_{\mathrm{D0}} + v_{\mathrm{d}} = V_{\mathrm{D0}} + i_{\mathrm{d}}r_{\mathrm{d}}$ of the diode, resulting in a diode current $I_{\mathrm{D}} = I_{\mathrm{D0}} + i_{\mathrm{d}} = I_{\mathrm{D0}} + v_{\mathrm{d}}/r_{\mathrm{d}}$.

[6] This convention is different than that in Sedra & Smith.

We can apply this small-signal model for the diode to the circuit shown in Fig. 14, where the input voltage has a small AC component v_{in}. The total input voltage is equal to the sum of the voltage across the resistor and diode:

$$V_{\text{in}} = I_{\text{D}}R + V_{\text{D}} = V_{\text{D0}} + I_{\text{D0}}R + i_{\text{d}}(r_{\text{d}} + R)$$

or alternatively,

$$V_{\text{in}} = V_0 + v_{\text{in}} = V_{\text{D0}} + I_{\text{D0}}R + v_{\text{in}},$$

from which we can solve for the small-signal current,

$$i_{\text{d}} = \frac{v_{\text{in}}}{R + r_{\text{d}}}.$$

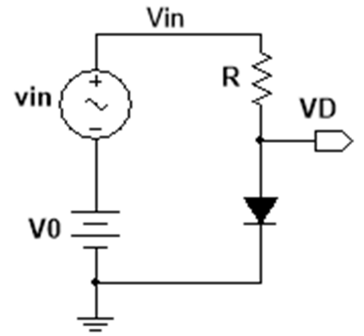


FIGURE 14. Circuit for small signal analysis.

The corresponding change in the diode voltage is the small-signal current times the small-signal diode impedance,

$$v_{\text{d}} = i_{\text{d}}r_{\text{d}} = v_{\text{in}}\frac{r_{\text{d}}}{R + r_{\text{d}}}.$$

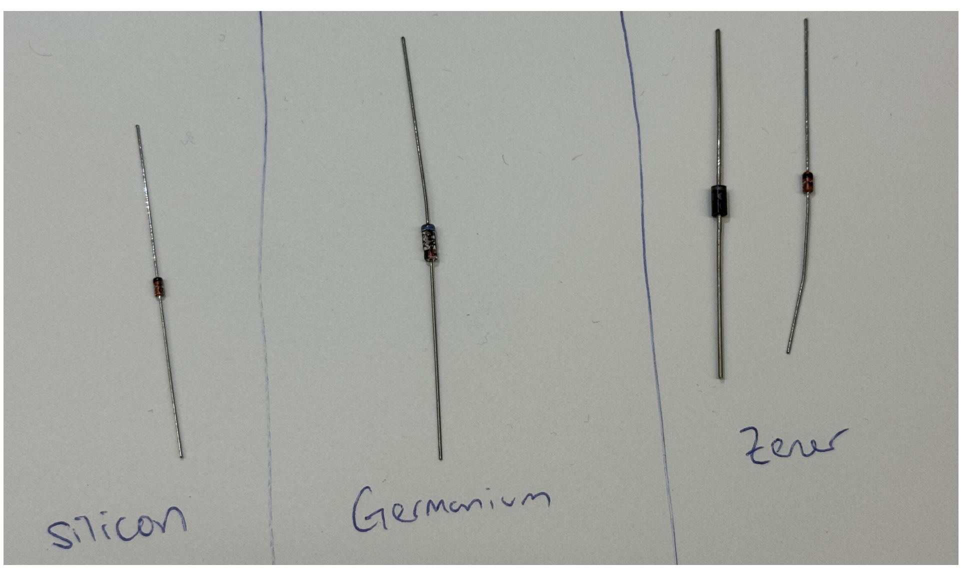
For small fluctuations in the input voltage, we can then determine the small-signal gain:

$$G = \frac{\partial V_{\text{D}}}{\partial v_{\text{in}}} = \frac{v_{\text{d}}}{v_{\text{in}}} = \frac{r_{\text{d}}}{R + r_{\text{d}}}.$$

If $R \gg r_{\text{d}}$, this circuit produces an output for which fluctuations in the input voltage are highly suppressed.

This same linear approximation can be applied to the breakdown region of Zener diodes, which we can model as $V_{\text{Z}} = V_{\text{Z0}} + ir_{\text{z}}$. Here, V_{Z0} the voltage is the operating voltage of the Zener in the breakdown region, and $r_{\text{z}} = \partial V_{\text{Z}}/\partial I_{\text{Z}}$ is the extremely low dynamic resistance at the operating point.

2.14. **Diode types.** As described above, diodes comes in many flavors based on their semiconductor material (e.g. silicon, germanium, etc) and function (regular pn diode, or Zener diode). Below is a photo of the various types of diodes we have stocked in your lab kit:



3. PRE-LAB QUESTIONS

P4.1) Show that the second term in Eq. 1 (the -1 in the brackets) may be neglected for typical operating parameters: $V_\mathrm{D} \gtrapprox 0.15\,\mathrm{V}$, $\frac{kT}{e} \approx 25.7\,\mathrm{mV}$, and $n \approx 2$.

P4.2) How does a Zener diode differ from a normal diode, and what is it used for?

P4.3) Consider a silicon diode with ideality factor $n = 2$ and starting at room temperature. Show that the saturation current increases by a factor of 2 when the temperature increases by $\Delta T \approx 9\,°\mathrm{C}$.[7]

P4.4) Show that the small signal gain of the circuit shown in Fig. 15 can be expressed as

$$G = \frac{v_\mathrm{out}}{v_\mathrm{in}} = \frac{nV_T}{IR + nV_T}$$

where v_in is the amplitude of the voltage source. Remember that $V_T = kT/e$.

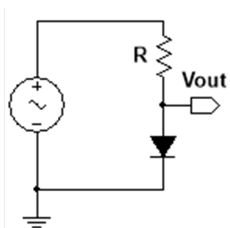


FIGURE 15. A diode divider.

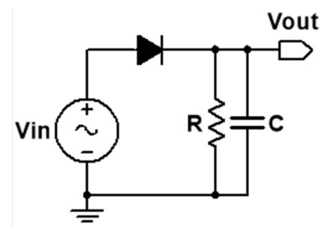


FIGURE 16. A filtered rectifier.

P4.5) Consider a diode biased with a constant current which is measured to have a voltage V_1 at temperature T_1 and voltage V_2 at T_2. What is the bandgap energy E_G?

P4.6) Consider the filtered rectifier shown in Fig. 16. Because the resistor drains the capacitor in between the peaks of the voltage drive, the output voltage is not constant. The voltage variation, which is often sawtooth-like, is called the *ripple*. Estimate the fractional peak-to-peak ripple $\Delta V/V$ of the output as a function of R, C, and the frequency f of the input sine wave, where ΔV is the voltage change over a single period. Assume that $f \gg \frac{1}{RC}$.

[7]Think about the ideal diode model for a second. When forward-biased, we want its impedance to be zero (it should just act like a short) and when reversed-biased, we want its impedance to be infinite (it shouldn't pass any current). Between Eq. 7 and what you just found here, what we've learned is that as the temperature goes up a diode is less ideal in pretty much every way!

4. LAB EXERCISES

Problem L4.1 - DMM Diode Function. Obtain a 1N4448 diode. The label 1N4448 designates the type of diode. Tens of thousands of different types of diodes are available. Many types are made by several different manufacturers; each manufacturer certifies that their diode meets the industry-wide specifications. Parts with labels that begin with 1N are always diodes, while parts that begin with 2N are transistors, but not all diodes and transistors follow this naming convention. The 1N4448 is a general-purpose switching diode. Specifications for the 1N4448 diode can be found from a quick web search for the part number, "1N4448". This lab also uses another type of diode: the 1N5234B Zener diode. They are visually very similar visually, so you will want to take care not to mix them up. If you look very closely (could use a magnifying glass), you can find the diode's part number written on it.

The DMM has a special function just for measuring the voltage drop across forward-biased diodes. Set the DMM to the diode (main dial pointing at the diode symbol). The diode function may be accessed differently depending on the DMM: *for example, the diode function could be just past the lowest scale on the resistance setting (as in Fig. 19), or from the diode setting it may be only accessible as an auxiliary function by pressing the "Select" (or yellow or "Mode") button.* For a diode measurement, the DMM puts the diode under test in series with a voltage source and resistor and reads the resulting voltage across the diode. This is functionally identical to the resistance measurements as you figured out in Lab 1. The only difference is that the value shown on the screen is the voltage across the diode rather than a calculation of its resistance from this measurement.

(a) **Draw** the electronic diagram for the measurement circuit (This is just diagramming what's in words above. As discussed above, the internal DMM circuit is identical to that of the resistance measurement from Lab 1).

(b) **Measure** the voltage **without** a diode connected to the DMM. Depending on the DMM, you will either get a voltage or an overload code (-1, OL). An overload code means that the voltage across the diode is outside of the range that the digitizer can measure (in this case the voltage is too large).

(c) **Measure** the voltage across the diode in forward bias *(anode to cathode)*.

(d) **Measure** the voltage across the diode in reverse bias *(cathode to anode)*.

(e) Are your results roughly consistent with the voltage offset diode model?

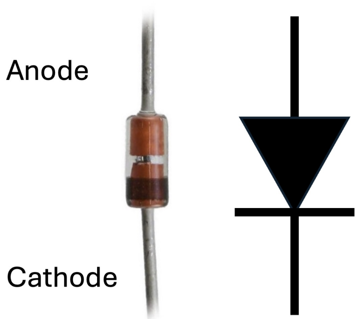


FIGURE 17. Typical diode with black line indicating the cathode.

FIGURE 18. DMM diode function.

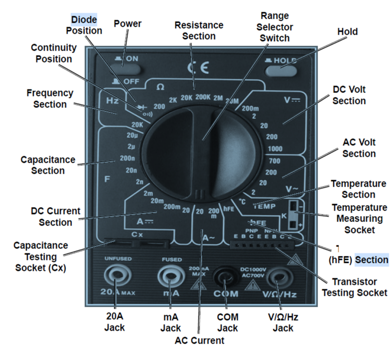
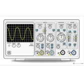


FIGURE 19. DMM diode function.


Problem L4.2 - Diode characteristic curve and parameters. In the local 111A lab, there is a dedicated piece of equipment used to measure the characteristic I-V curves of semiconductors called a *curve tracer*. Here we will use the ADS to duplicate the function of the curve tracer. To characterize a diode, we want to measure the relationship between current through the diode and voltage across the diode over a wide range of currents. We put the diode in series with a resistor and measure the voltage drop across the resistor to determine the current. We ramp the applied voltage to explore a range of currents and use the resulting data to solve for the diode ideality factor n and the saturation current I_S.

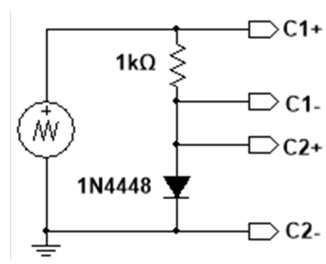


FIGURE 20. Circuit for diode characterization.

(a) Look at the circuit in Fig. 20. Channel 1 measures the voltage across the resistor and Channel 2 measures the voltage across the diode. With these measurements, how can you estimate the current through the diode?

(b) **Build** the circuit shown in Fig. 20. **Measure** the voltage across the resistor with Channel 1 and the voltage across the diode with Channel 2. **Use** the following settings:

- Wavegen
  - i) Type = Ramp Up
  - ii) Frequency = 9.8 Hz
  - iii) Amplitude = 1.75 V
  - iv) Offset = 1.25 V
  - v) **Select** Synchronized from the Menu at the top
  - vi) **Check** Auto in the Trigger Menu.
  - vii) Finally, click Run All in the upper left.
- Scope
  - i) C1 Range = 500 mV/div. C1 Offset = 0
  - ii) C2 Range = 200 mV/div. C2 Offset = 0
  - iii) Time Base = 10 ms/div
  - iv) Position = 51 ms (so that the transition before the beginning of the ramp is off the screen).
  - v) Trigger Source = Wavegen C1

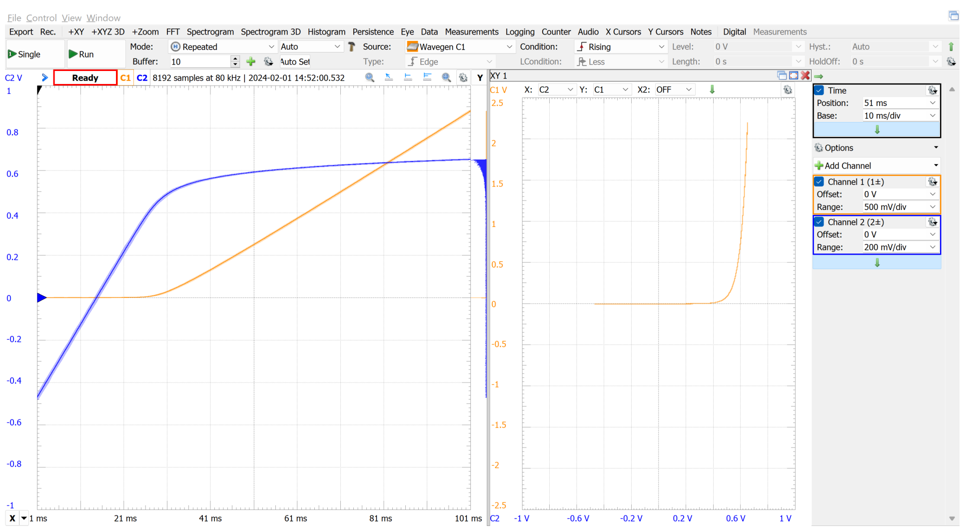


FIGURE 21. Scope trace for diode characterization.

These settings will create a voltage ramp from $-0.5\,\mathrm{V}$ to $+3.0\,\mathrm{V}$. The Frequency of 9.8 Hz is chosen so that the return to the minimum voltage after the ramp is also off the screen. The maximum voltage of the ramp is 3 V, and the voltage across the diode at the maximum current $V_D \approx 0.65\,\mathrm{V}$. So, the maximum current will be $I = (3\,\mathrm{V} - V_D)/(1\,\mathrm{k\Omega}) \approx 2.35\,\mathrm{mA}$.[8]

[8] We could make the ramp amplitude larger, but then we would also need to increase the scale for Channel 1. If you configure the Scope with $5 \times (\text{Volts/div}) + |\text{offset}| > 2.6\,\mathrm{V}$, then the scope will default to the low sensitivity setting, which will compromise the signal-to-noise ratio of the measurement. This is one place that you will want to follow the instructions for configuration carefully.

Your scope trace should look like Fig. 21. Just for fun, I added an X-Y[9] plot (V_R vs. V_D) of the data. **Export an image of your Scope trace, and save the data to a file.**

(c) We can analyze the I-V data to determine the best fit parameters for the diode I-V characteristic (Eq. 1). In the limit where $V_D \gg nV_T = nkT/e$, we have $I_D \cong I_S\, e^{V_D/nV_T}$. Taking the logarithm of this expression, we have,

$$V_D = nV_T \ln\left(\frac{I_D}{I_S}\right) = nV_T \ln(I_D) - nV_T \ln(I_S)$$

We can rearrange this to write,

$$\ln(I_D) = \frac{V_D}{nV_T} + \ln(I_S)$$

In a plot of $\ln(I_D)$ vs. V_D, the **slope is** $1/nV_T$ **and the intercept is** $\ln(I_S)$. Use your exported data and the fact that $I_D = V_R/R$ to create a plot of $\ln(I_D)$ vs. V_D. Your data should closely follow a straight line. Restricting your data to $V_D > 0.3\,\text{V}$, fit a line to your data (using Python, Excel, or other software of your choice) and **determine the values of** n **and** I_S. Remember to use a temperature corresponding to room temperature.[10]

(d) **Write** the diode equation (Eq. 1) with the parameters you have found in (c). Use this equation to **calculate** the current for diode voltages $V_D = 0.4\,\text{V}$, $0.5\,\text{V}$ and $0.6\,\text{V}$. **Compare** these predictions with the closest measurements in your data file and comment on the accuracy. Now that you have characterized this diode, **mark** it with a piece of tape; we will use it in later exercises.

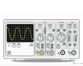
FIGURE 22. Diode (left end) on a stick. Note the difference between a regular diode and an LED. The red wire is connected to the diode anode, while the black wire is connected to the cathode.

(e) Obtain a diode-on-a-stick (see Fig. 22) from the GSIs. **Repeat** the measurements and analysis from (b) and (c) for this diode-on-a-stick. The constants you determine should be close to, but not necessarily identical to, those with your original diode.

Then **obtain** some liquid nitrogen (LN2) in a Styrofoam cup from the GSIs. **Please be careful with the liquid nitrogen.** *You must wear goggles while interacting with LN2, and do not let it come into contact with your eyes, skin, or clothing.*

Immerse the diode in the LN2 and let the boiling subside. **Repeat** the measurements in part (a). **Plot** the warm diode and the cold diode current vs. voltage curves on the same graph. How does the data **shift**? Does the forward voltage required to get an appreciable current **change in a predictable manner**?

Then **redo** the analysis in part (b) for this now cold diode, but this time using a $V_T = kT/e$ appropriate for $T = 77\,\text{K}$. You will likely have to fit only to data for which the forward voltage is greater than about 1 V. Does n change significantly? The saturation current may not make sense.



Problem L4.3 - Diode Equilibrium. In this problem, we will compare our calculated equilibrium values for diode current and voltage with measured values when biased with constant voltage though a series resistor. In this experiment we will use the same circuit, diode, and oscilloscope connections as in L4.2.

[9] Click +XY on the top menu bar in the Scope window to add an XY plot.
[10] If $V_D \lesssim 0.3\,\text{V}$, the voltage across the resistor becomes getting too small to measure accurately. We could extend the range to lower current by taking another set of data with a larger series resistor. The curve tracer in the lab automatically switches between several widely spaced resistors to get results over a very wide range of currents. You can try other series resistors if you want, but it is not necessary to get good diode parameters.

(a) **Build** the circuit shown in Fig. 23. Open the Wavegen and **set** the Type to DC. Then enter the desired DC voltage as an Offset. **Start** with Offset = 1 V. Open the Voltmeter instrument and click on the gear to set Update = 1 s. **Configure** the two scope channels to measure the voltage across the diode V_{D} and resistor V_R. **Sum** V_{D} and V_R to determine the total applied voltage rather than assuming it is the value requested from the Wavegen. The output impedance of the Wavegen makes the applied voltage slightly less than the requested value. **Use** the voltage across the resistor to compute the current through the diode $I_{\mathrm{D}} = V_R/R$. **Determine** V_{D}, I_{D}, and the total applied voltage for programmed voltages 1 V, 3 V, and 5 V.

(b) **Replace** the 1 kΩ resistor with a 10 kΩ resistor. Again **measure** V_{D}, I_{D} and the total applied voltage for programmed voltages 1 V, 3 V, and 5 V.

(c) Using the diode equation (Eq. 1) with the parameters measured in L4.2c, calculate the equilibrium values of V_{D} and I_{D} when the total voltage is what was measured with the 1 kΩ series resistor and Wavegen output programmed to 3 V. **Make** a load-line plot and visually determine the equilibrium values. **Verify** your plot by calculating the root numerically using software of your choice.

Compare your measurements from (a) with these calculations. **Note: don't put your circuit away— you'll use the same set up for the next problem!**

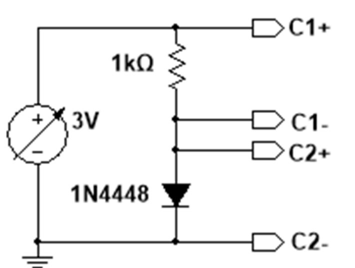


FIGURE 23. Circuit for diode equilibrium.

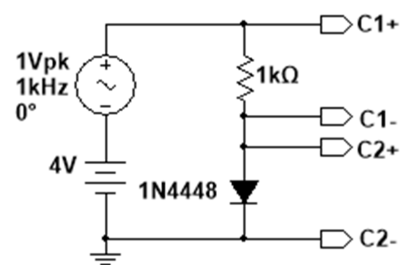
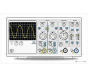


FIGURE 24. Small-signal circuit.



Problem L4.4 - Small-Signal Behavior. In this problem we investigate the response of the diode to small perturbations from equilibrium. We will be working in the small signal limit where we can approximate the response of the diode to small changes in voltage or current as linear. We will use a circuit similar to that from L4.3 with the series resistor equal to 1 kΩ as shown in Fig. 24. Open the Wavegen and **set** the Function = Sine, Frequency = 1 kHz, Amplitude = 1 V, and Offset = 4 V.

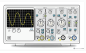
(a) Open the Scope and **set** the Time Base to 100 µs/div. **Use** the Measurements function to determine the average DC voltage V_{D0} and amplitude of the AC voltage v_{d} on the diode. From the voltage across the resistor **compute** the average DC current I_{D0} and AC current i_{d}. **Determine** the small signal impedance $r_{\mathrm{d}} = v_{\mathrm{d}}/i_{\mathrm{d}}$ of the diode at the operating point. [11]

(b) **Change** the series resistor to 10 kΩ. **Repeat** your measurement and analysis from (a) to determine the small signal impedance of the diode, r_{d}.

(c) **Find the ratio** of the AC diode voltage measured in (a) and (b), to the input AC voltage. **Compare** these results with the expectations for a voltage divider with the measured diode dynamic impedance r_{d}. Are these **consistent**?

(d) **Compare** your measurements of the diode dynamic impedance from (a) and (b) with the expectation from the diode I-V model with your measured DC parameters.

[11] Note that if the Scope Range is improperly set, measurements may saturate and be erroneous. An erroneous measurement will be shown with a red background like this. **Fix** this issue by adjusting the Range of the offending channel to be greater than 500 mV/div. Even with this fix, the signal trace itself may not necessarily be visible. It is always best to confirm that your signal is correct by viewing its trace. You should be able to bring the trace into view by adjusting the Range and Offset.

Problem L4.5 - Zener Diodes. Circuits frequently require precise DC voltages less than the power supply voltage. Such voltages can be produced with voltage dividers, but dividers do not have a low output impedance and their output voltage will decrease when loaded. Furthermore, the divider voltage will follow any power supply voltage fluctuations. Better schemes use a device called a Zener diode. Zener diodes are deliberately optimized for use in the reverse breakdown region where they produce a voltage that depends very weakly on the current through the diode. Using a Zener diode in series with a resistor as in Fig. 25, it is possible to make voltage source that maintains a nearly constant voltage despite a changing load.

(a) When the Zener is in breakdown, qualitatively what is the zener's small signal dynamic impedance, $r_z = \partial V_Z / \partial I_Z$: (zero, ∞)?

(b) When the Zener is reversed biased but doesn't have a large enough reverse voltage to be in breakdown, qualitatively, what is the Zener's small signal dynamic impedance, r_z: (zero, ∞)?

(c) **Configure** the ADS to measure the current through the Zener, and the voltage across the Zener. **Measure** circuit quantities with no load resistor, R_L. Is the Zener in breakdown?

(d) What is the minimum load resistor R_L that can be connected across the Zener, and have the Zener remain in breakdown?

(e) Add a $R_L = 10\,\text{k}\Omega$ resistor in parallel, and measure the change in the output voltage. Calculate the change in current, and voltage across the Zener when R_L is added. From this, **compute the Zener diode's dynamic impedance**, $r_z = \partial V_Z / \partial I_Z$.

(f) If you were to make a resistor voltage divider to produce the same output voltage and dynamic impedance as you found in (e), how much current would flow though it? Why wouldn't it work? **(Do not try to construct this circuit!)**

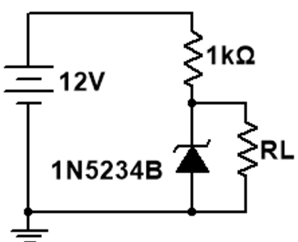


FIGURE 25. Zener test circuit.

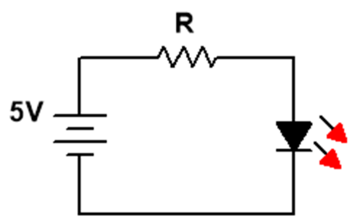


FIGURE 26. LED equilibrium circuit.

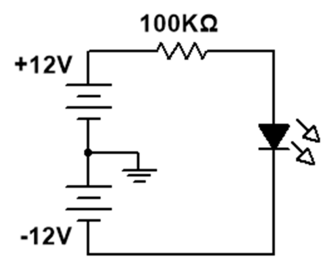


FIGURE 27. Current bias for temperature dependence.

Problem L4.6 - Light-Emitting Diodes (LEDs). The light emitted by an LED is proportional to the current passing through the device. The forward voltage at a fixed current is related to details of construction and the bandgap energy of the semiconductor. The forward voltage at a given current is much larger than in silicon diodes like the 1N4448 and generally increases with the frequency of the light emitted by the LED. For LEDs that are forward-biased with a few mA, the voltage drop is typically $\sim 1.5\,\text{V}$ for red and $\sim 2.5\,\text{V}$ for blue LEDs. (Note that the diode function on the DMM is not able to read the large forward voltage of LEDs and will read "1".)

In operation, LEDs are typically biased through a series resistor that limits the current. **Never connect a LED directly to a voltage source!** The equilibrium voltage and current for an LED can be solved for exactly given the parameters of the LED. However, often we just want the LED to operate at a fixed current, where the typical voltage drop at that current can be read from a data sheet for the device.

(a) **Choose the values of the resistor** R such that the circuit in Fig. 26 sources 10 mA across one of the red LEDs you were provided in your parts kit. **Start** with a guess of a constant 1.5 V forward drop. **Select** the closest value resistor value to give the desired 10 mA. **Measure** the voltage across the diode and **refine** your value for the resistor if necessary. Notice how quickly this converges to an accuracy better than the spacing between resistor values.

(b) The bandgap energy E_{G} in an LED is exactly equal to the energy of the emitted photons. In P4.5, you explored the temperature dependence of the diode voltage at fixed current, and its relation to the bandgap energy E_{G}. Thus, by measuring this temperature dependence in an LED emitting light at wavelength λ while it is driven at fixed current, then using the relation $h = E_{\mathrm{G}}\lambda/c$, you can measure Planck's constant h, one of the fundamental constants of physics.

The circuit for this measurement is shown above in Fig. 27. We create a nearly fixed current by using the largest potential difference available and a relatively large resistor. Approximately **what** will be the current through the red ($\lambda = 650\,\mathrm{nm}$) LED in this circuit?

Use your DMM to **measure** the voltage across the LED and **watch** that voltage change when you hold the LED between your thumb and finger to heat it up. If we measure the LED voltage with fixed current at two known temperatures, we can use that to determine the bandgap energy of the LED (Sec. 2.8).

To make a more accurate measurement, use the largest temperature difference available to you. **If you are in lab, ask one of the GSIs for a LED-on-a-stick and cool the LED in liquid nitrogen, as in L4.2e. Calculate** the bandgap energy E_{G} and Planck's constant h.

Report your measurement of h, including your measurement uncertainty. Compare experiment to theory: what value (noting units) of Planck's constant h did you expect to measure?

Problem L4.7 - Half-wave Rectification. Since diodes carry significant current only in one direction, they can be used to *rectify* AC signals; rectify means to convert an AC signal (flowing in both directions) into DC (flowing in one direction).

Build the half-wave rectifier circuit in Fig. 28. Use the Wavegen to **generate** a 2.5 V amplitude, 100 Hz sine wave. **Display** the output of the Wavegen on Channel 1 of the Scope, and the voltage across the resistor on Channel 2. **Trigger** the Scope to trigger Synchronously from the Wavegen.

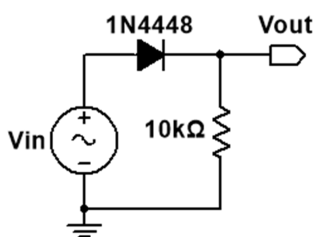


FIGURE 28. Rectifier circuit.

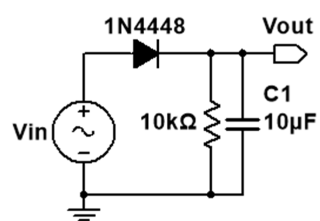


FIGURE 29. Filtered rectifier.

(a) **What** is the voltage drop between the input and output at the peak of the waveform? **Compare** this with your rough expectation from the diode I-V model.

(b) Keeping the Wavegen settings the same as in (a), **measure** the voltage drop[12] across the diode when the output voltage across the resistor (V_{out}) is 100 mV. How does the diode voltage differ from that in part (a)?

Rectification, as provided by the previous circuit, is only the first step in converting AC power into DC power. The large fluctuations in the signal produced by the above circuit needs to be smoothed out, typically by a high-capacitance filter capacitor.

(c) **Add** a 10 µF ceramic capacitor to your circuit as shown in Fig. 29 to create a filtered rectifier.[13] **Save** images of the output waveform. **Note** the peak-to-peak voltage of the ripple (ΔV) divided by the DC offset of the waveform (V). **Compare** it with the fractional peak-to-peak ripple $\Delta V/V$ that you calculated in P4.6. **How** does the ripple change when you: i) **double** the input frequency? ii) **double** the filter capacitance?

[12] For the purpose of DC power generation, the exact value of voltage drop across the diode is not critical and the simple "constant voltage drop" model is adequate. However, for many signal processing applications, we want a rectifier that outputs the exact rectified waveform. Later in the course, we will develop active rectifiers that approach ideal performance.

[13] If instead you use an electrolytic capacitor, make sure you obey the polarity markings on the capacitor body.