Forcefields

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1 UFF

The theory presented here is adapted from Rappe et. al, JACS, 1992.

1.1 Bond distances

Equlibrium pairwise distances are given by

$$r_0 = r_i + r_i + r_{\text{BO}} + r_{\text{EN}} \tag{1}$$

where r_k is the covalant radius of atom k, r_{BO} a correction based on the bond order between the two atoms and r_{EN} a correction for the electronegativity.

$$r_{\rm BO} = -\lambda(r_i + r_j)\ln(n) \tag{2}$$

where $\lambda=0.1332$ and n the bond order. For example, n=1.5 for aromatic bonds.

$$r_{\rm EN} = r_i r_j \frac{(\sqrt{\chi_i} - \sqrt{\chi_j})^2}{\chi_i r_i \chi_j r_j}$$
 (3)

where χ_k is the GMP electronegativity of atom k. The energy is then a simple harmonic

$$E_b = \frac{k_{ij}}{2} (r - r_0)^2 \tag{4}$$

The derivative is

$$\frac{\partial E}{\partial X_{i,n}} = k_{ij} \left(1 - \frac{r_0}{r} \right) (X_{i,n} - X_{j,n}) \tag{5}$$

where $X_{i,n}$ is the n^{th} component of the Cartesian coordinate of atom i e.g. x, y, or z.

1.2 Bond Force Constants

Bond stretch force constants are defined as

$$k_{ij} = 664.12 \frac{Z_i^* Z_j^*}{r_{ij}^3} \tag{6}$$

where Z_k^* is the effective atomic charges in units of e and the energy in kcal mol^{-1} .

1.3 Angle Bends

In general, UFF defines the energy of an angle bend as

$$E_{\theta} = k_{ijk} \sum_{n=0}^{m} C_n \cos(n\theta) \tag{7}$$

where for linear (n = 1), trigonal-planar (n = 3), square-planar (n = 4) and octahedral (n = 4)

$$E_{\theta}^{\text{Type A}} = \frac{k_{ijk}}{n^2} (1 - \cos(n\theta)) \tag{8}$$

the derivative is evaluated using sympy¹

For other coordination environments with an equilibrum bond angle (θ_0)

$$E_{\theta}^{\text{Type B}} = k_{ijk}(C_0 + C_1 \cos(\theta) + C_2 \cos(2\theta)) \tag{9}$$

$$C_2 = \frac{1}{4\sin^2(\theta_0)}$$
 ; $C_1 = -4C_2\cos(\theta_0)$; $C_0 = C_2(2\cos^2(\theta_0) + 1)$ (10)

1.4 Angle Force Constants

Angle force constants are defined as

$$k_{ijk} = \beta \frac{Z_i^* Z_k^*}{r_{ik}^5} r_{ij} r_{jk} \left[r_{ij} r_{jk} (1 - \cos^2(\theta_0)) - r_{ik}^2 \cos(\theta_0) \right]$$
 (11)

$$\beta = \frac{664.12}{r_{ij}r_{jk}} \tag{12}$$

¹https://docs.sympy.org/

1.5 Torisional Dihedrals

For a sequence of bonded atoms i-j-k-l the torsional energy is given by

$$E_{\phi} = \frac{V_{\phi}}{2} [1 - \cos(n_{\phi}\phi_0)\cos(n_{\phi}\phi)] \tag{13}$$

where ϕ is the torsinal angle, V_{ϕ} a force constant and n_{ϕ} the multiplicity. The general cases are

- 1. sp³-sp³: $n_{\phi}=3; \phi_0=180^{\circ}$ unless j,k are group 16 atoms where $n_{\phi}=2; \phi_0=90^{\circ}$.
- 2. $\mathrm{sp}^2 \mathrm{sp}^3$: $n_{\phi} = 6$; $\phi_0 = 0^{\circ}$
- 3. sp^2 - sp^2 : $n_{\phi} = 2$; $\phi_0 = 180^{\circ}$
- 4. $\mathrm{sp^2-sp^2-sp^3-X}$: $V_0 = 2.0$; $n_{\phi} = 3$; $\phi_0 = 180^{\circ}$

and

$$V_{\rm sp^3} = \sqrt{V_j V_k} \tag{14}$$

where V_m are tabulated values, while bonds containing for ${\rm sp}^2$ centres

$$V_{\rm sp^2} = 5\sqrt{U_j U_k} (1 + 4.18 \ln(n_{BO,jk})) \tag{15}$$

where U_m are values based on the period of the atom (indexed by m) and $n_{BO,jk}$ is the value of the bond order between atoms j and k.

Torsional potentials are only considered where the central bonds are main group and the atoms non sp hybridised.

If either of the bond angles approaches 180° then the potential is set to zero.