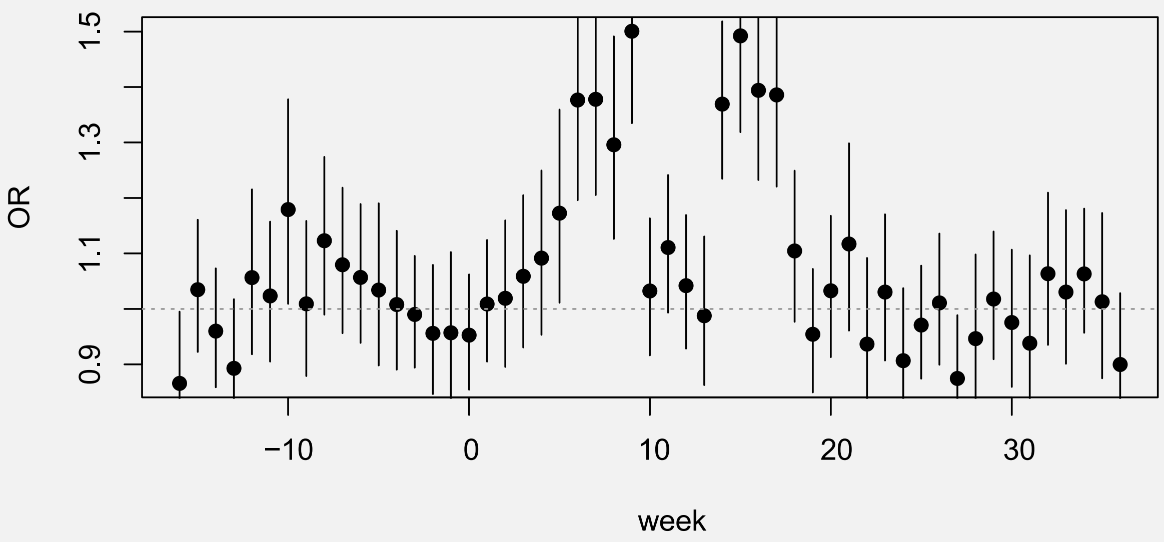
Critical Window Variable Selection (CWVS) Meta-Regression

Rogne et al. (2024), Warren et al. (2020)

Pointwise estimates of odds ratios (OR) for each week



We aim to identify which weeks constitute the critical windows

However,

- Each pointwise estimate is influenced by autocorrelation
- The issue of multiplicity arises when determining critical windows based on 95% CI

CWVS Meta-Regression is a Bayesian method designed to explicitly identify which weeks fall within critical windows

Model for observed effects

For week $t = 1, \ldots, n,$

observed effect

$$\underbrace{\hat{\alpha}_t}_{} \mid \underbrace{\alpha_t}_{} \sim \mathrm{N}(\alpha_t, \hat{\sigma}_t^2)$$

true effect (unobserved)

- $\hat{\alpha}_t \in \mathbb{R}$: Estimated effect for week t (ex. coefficient from the regression analyses)
- $\hat{\sigma}_t^2 \in \mathbb{R}$: Estimated squared standard error
- $\alpha_t \in \mathbb{R}$: True effect of interest
- $n \in \mathbb{N}$: Total number of weeks in the first stage regression analyses

Decomposition of true effects

Continuous component

$$\boxed{\theta_t = A_{11}\delta_{1t}}$$

$$\alpha_t = \underline{\theta_t}\,\underline{\gamma_t}$$

Binary component

$$\boxed{\begin{gathered}\gamma_t \mid \pi_t \sim \mathrm{Bernoulli}(\pi_t) \\ \Phi^{-1}(\pi_t) = A_{21}\delta_{1t} + A_{22}\delta_{2t}\end{gathered}}$$

- If $\gamma_t = 1$, week t constitutes a critical window.
- If $\gamma_t = 0$, week t does not.

- $\theta_t \in \mathbb{R}$: Continuous component – "magnitude of effect"
- $\gamma_t \in \{0,1\}$: Binary component – "whether week t constitute a critical window "
- $\delta_{jt} \in \mathbb{R}$: Parameters that account for correlation structures
- $A_{11},\ A_{21},\ A_{22} \in \mathbb{R}_{\geq 0}$: Scaling parameters
- $\Phi: \mathbb{R} \to [0,1]$: CDF of standard normal distribution (probit link function)

Consideration of temporal correlation structures

For weeks $t = 2, \dots, n$ and continuous/binary processes $j = 1, 2$,

$$\delta_{j1} | \rho_j \sim \mathrm{N}\left(0, 1 - \rho_j^2\right)$$

$$\underline{\delta_{jt}} | \rho_j, \delta_{j,t-1} \sim \mathrm{N}\left(\underline{\rho_j \delta_{j,t-1}}, 1\right)$$

Assuming that each process has AR(1) temporal correlation structure

- $\rho_j \in [0,1]$: Temporal correlation parameters

Hyper-priors

For continuous/binary processes $j = 1, 2,$

$$\rho_j \sim \mathrm{Uniform}(0,1)$$
$$A_{jj} \sim \mathrm{Uniform}(0,100)$$
$$A_{21} \sim \mathrm{N}(0, 100^2)$$

Setting weakly informative hyper-priors

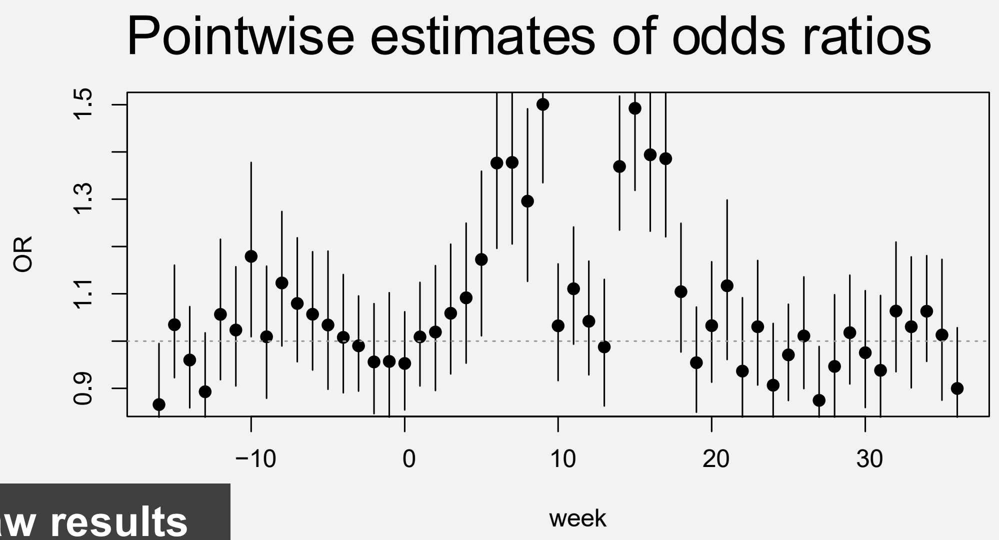
Sampling algorithm

- We implemented Metropolis-Hastings algorithm (one of Markov Chain Monte Carlo methods) to sample from posterior distribution

- Code is available from https://github.com/t-yui/CWVS-MetaRegression

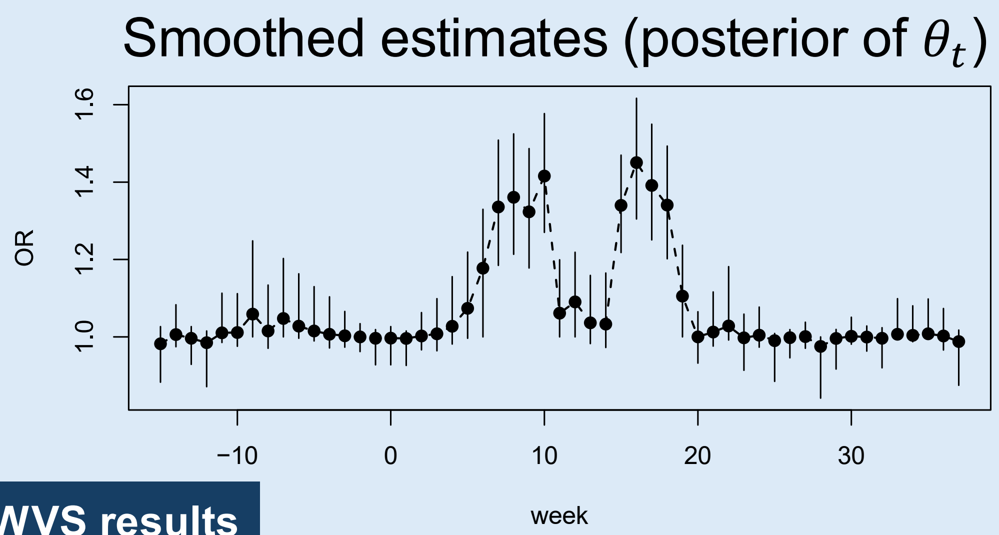
- Detailed sampling settigs for example data:
  - Iteration : 100,000
  - Burn-in : 4,000
  - Thinning : 1 out of 20

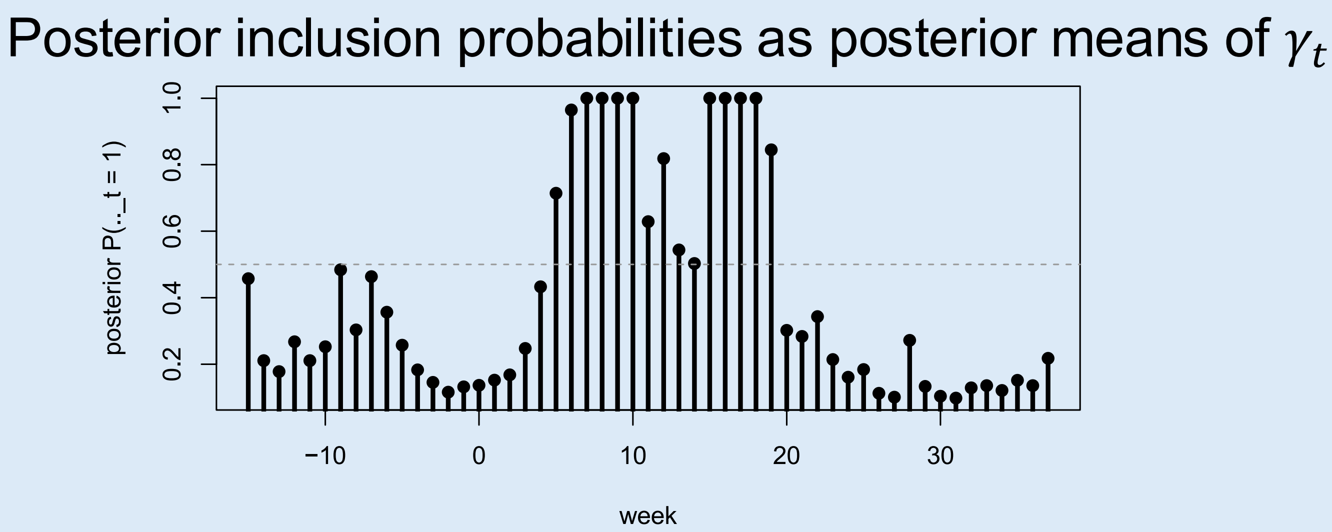
Results obtained from CWVS Meta-Regression



Raw results

We can obtain
- smoothed estimates
- inclusion probability to critical windows for each week





CWVS results

How is different from DLM / DLNM?

	Distributed Lag (Non-linear) Model (DLM / DLNM)	Critical Window Variable Selection (CWVS)
Components of the model	**Smooth curve only** : a single lag-effect curve (often splines). ➢ No latent "selection" layer	**Two-layer design**: separates a smooth effect curve from a Bernoulli selection process and lets them share correlation.
Definition of critical windows	**Post-hoc rule** : after fitting, mark lags whose 95 % CI excludes zero as "critical."	**Inside the model** : the selection variable is estimated directly.
<u>Empirical</u>* performance	Tends to over-smooth, giving wider CIs and missing sharp risk peaks. ➢ **Lower sensitivity, more false negatives**	Recovers true windows more often, with **lower error ratio** and **shorter intervals**; remains stable even when exposures are highly autocorrelated.

* Based on simulation study in Warren et al. (2020)

References

Rogne, Tormod, et al. "High ambient temperature in pregnancy and risk of childhood acute lymphoblastic leukaemia: an observational study." *The Lancet Planetary Health* 8.7 (2024): e506-e514.

Warren, Joshua L., et al. "Critical window variable selection: estimating the impact of air pollution on very preterm birth." *Biostatistics* 21.4 (2020): 790-806.