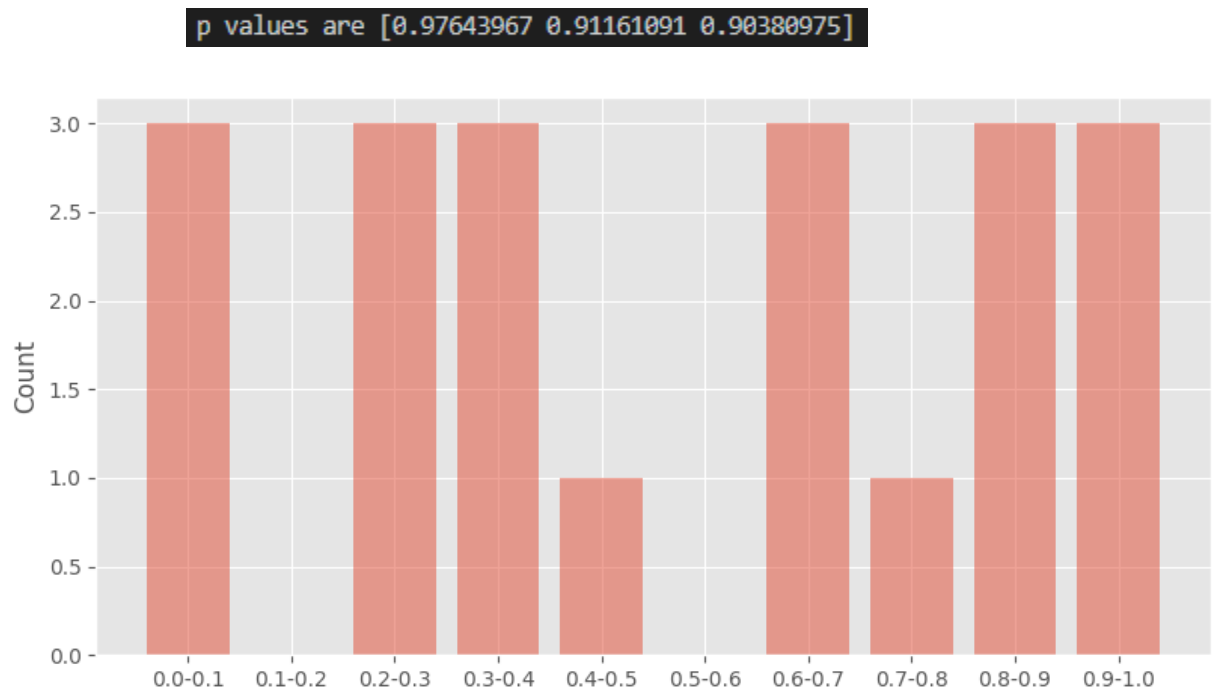


Homework 3

1. Chi-Squared Test

- a. For this problem, I created a program with the hard-coded numbers given and first used the matplotlib package to display the histogram. Next, I used sklearn package's chi-squared calculator to calculate the p values and compared them to the alpha. Since the p values are greater than the alpha, the null hypothesis holds true.



- b. Using the same program from part a, I calculated the p values of the grade distribution and found that, while it was close to being statistically different, the set was also independent since the p values were above the alpha.

p values are [0.06788915 0.63941185 0.05157586 0.41421618 0.56370286]

2. For this section, I made a simple program in python that took the list of customers and serviced them based on their time. This first screenshot was done with only one server.

```
Done. Time elapsed: 549
Customer 1 left at 51
Customer 2 left at 83
Customer 3 left at 138
Customer 4 left at 186
Customer 5 left at 204
Customer 6 left at 254
Customer 7 left at 301
Customer 8 left at 319
Customer 9 left at 347
Customer 10 left at 401
Customer 11 left at 450
Customer 12 left at 526
Customer 13 left at 548
Server usage: 92.896%
```

This screenshot was done with two servers.

```
Done. Time elapsed: 549
Customer 1 left at 51
Customer 2 left at 62
Customer 3 left at 117
Customer 5 left at 135
Customer 4 left at 142
Customer 6 left at 203
Customer 8 left at 238
Customer 7 left at 244
Customer 9 left at 331
Customer 10 left at 399
Customer 11 left at 450
Customer 12 left at 526
Customer 13 left at 548
Server 1 usage: 68.852%
Server 2 usage: 22.587%
```

For these two servers, server 2 acted as a back up for server 1. If server 1 had a client, they would go to server 2. Otherwise, they go to server 1.

3. A.

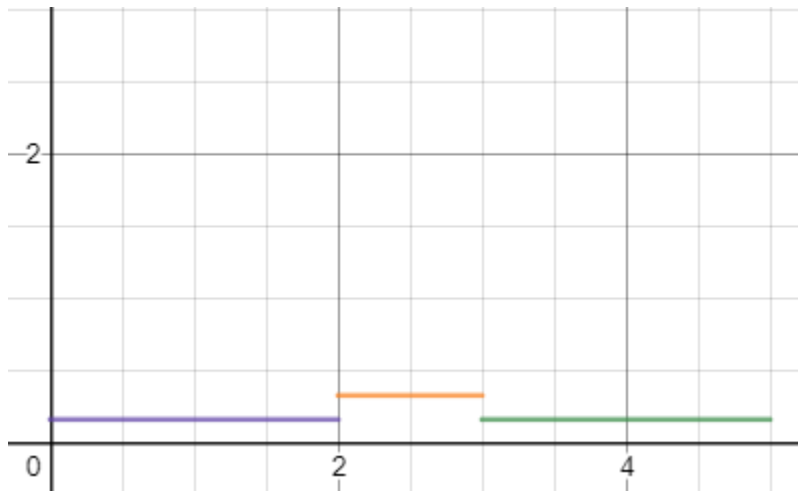
$$\int_0^2 f(x)dx = \int_0^2 c_1 dx = c_1 x_0^2 = 2c_1$$

Since c_1 has to be equal to $1/3$, **$c_1 = 1/6$** .

$$\int_2^3 c_2 dx = c_2 x_2^3 = c_1$$

Since c_2 has to be equal to $1/3$, **$c_2 = 1/3$** .

$$f(x) = \begin{cases} \frac{1}{6}, & 0 \leq x < 2 \\ \frac{1}{3}, & 2 \leq x < 3 \\ \frac{1}{6}, & 3 \leq x < 5 \end{cases}$$



$$\begin{aligned} \text{b. } E(X) &= \int_0^2 \frac{1}{6} x dx + \int_2^3 \frac{1}{3} x dx + \int_3^5 \frac{1}{6} x dx = \frac{1}{12} x^2 \Big|_0^2 + \frac{1}{6} x^2 \Big|_2^3 + \frac{1}{12} x^2 \Big|_3^5 = \\ &= \left[\frac{4}{12} - 0 \right] + \left[\frac{9}{6} - \frac{4}{6} \right] + \left[\frac{25}{12} - \frac{9}{12} \right] = \frac{2}{6} + \frac{5}{6} + \frac{8}{6} = \frac{5}{2} \end{aligned}$$

$$\begin{aligned} E(X^2) &= \int_0^2 \frac{1}{6} x^2 dx + \int_2^3 \frac{1}{3} x^2 dx + \int_3^5 \frac{1}{6} x^2 dx = \frac{1}{18} x^3 \Big|_0^2 + \frac{1}{9} x^3 \Big|_2^3 + \frac{1}{18} x^3 \Big|_3^5 = \\ &= \left[\frac{8}{18} - 0 \right] + \left[\frac{27}{9} - \frac{8}{9} \right] + \left[\frac{125}{18} - \frac{27}{18} \right] = \frac{4}{9} + \frac{19}{9} + \frac{49}{9} = \frac{72}{9} = 8 \end{aligned}$$

$$V(X) = E(X^2) - E(X)^2$$

$$V(X) = 8 - (5/2)^2 = 8 - (25/4) = 32 - 25/4 = \mathbf{7/4}$$

$$\text{c. } P\{1 < X < 3\}$$

$$\int_1^2 \frac{1}{6} dx + \int_2^3 \frac{1}{3} dx = \frac{x}{6} \Big|_1^2 + \frac{x}{3} \Big|_2^3 = \frac{1}{6}(2 - 1) + \frac{1}{3}(3 - 2) = \frac{1}{6} + \frac{1}{3} = \mathbf{\frac{1}{2}}$$

$$\text{d. } P\{X > 3 | X > 1\}$$

$$P(X > 3) = \int_3^5 \frac{1}{6} dx = \frac{x}{6} \Big|_3^5 = \frac{5 - 3}{6} = \frac{1}{3}$$

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$$\begin{aligned} P(x > 1) &= \int_1^2 \frac{1}{6} dx \\ &+ \int_2^3 \frac{1}{3} dx + \int_3^5 \frac{1}{6} dx = \frac{x}{6} \Big|_1^2 + \frac{x}{3} \Big|_2^3 + \frac{x}{6} \Big|_3^5 = \frac{2-1}{6} + \frac{3-2}{3} + \frac{5-3}{6} \\ &= \frac{1}{6} + \frac{1}{3} + \frac{1}{3} = \frac{5}{6} \end{aligned}$$

Since $P\{X>3 | X>1\} = P(x>3)/P(x>1) = (1/3)/(5/6) = \mathbf{2/5}$