PHYS20161 Final assignment: Z^0 boson

November 23, 2021

The Z^0 boson is an elementary particle that mediates the weak nuclear interaction. It is studied at particle physics facilities such as Fermilab and CERN where beams of particles collide and their products are analysed.

In this assignment you are tasked with deducing the mass and lifetime of the Z^0 boson by studying data from electron-positron collisions resulting in electron-positron products; i.e. $e^-e^+ \to Z^0 \to e^-e^+$ events. If possible, you should also compute the uncertainty on these values. Your work should produce some graphics to support your analysis.

1 Theory

In a general scattering experiment, a beam is incident onto a target and the production rates of the final states are counted. Often the final states differ in particle type to the beam and the target. It is clear the rate of a specific reaction, W_r, will be proportional to the flux of the beam, J, (the number of particles passing through a surface at the target perpendicular to the beam direction) and the number of target particles illuminated by the beam, N;

$$W_r \propto JN. \tag{1}$$

In practice, there will be a plethora of reactions that can take place after each collision. Events with particular products and energies can be selected to form a data set to study the reaction of interest. We define the constant of proportionality between the reaction rate, flux and number of particles to be the cross-section, σ_r, such that

$$\sigma_r \equiv \frac{W_r}{JN}. \tag{2}$$

σ_r has dimensions of area and the typical unit is the *barn* which corresponds to 10^{-28} m^2 (100 fm^2). Studying the cross-section eliminates the dependency on the number of particles in the beam and the target. However, a large number of events are required to transform these individual events into a continuous distribution.

Often in particle physics experiments, the target is not stationary and is another beam. This makes larger centre-of-mass energies accessible to study. Examples of such facilities include the Large Electron Positron collider (LEP) which was operational at CERN between 1989 and 2000 with a maximum collision energy of 209 GeV. This was replaced by the Large Hadron Collider (LHC) which went live in 2008 and collides two beams of protons; its current maximum collision energy is 13 TeV.

The reaction we are interested in is $e^-e^+ \to Z^0 \to e^-e^+$, a Feynman diagram for this process is given in figure 1. This diagram represents a quantum mechanical amplitude which in turn is related to the observable cross-section, details on how to perform this calculation will be taught in Quantum Field Theory and Gauge Theories in your fourth year. In short, the key components are the *vertices* and the *propagator*. A vertex is where different particle lines (or *legs)* meet. Here, we have a $e^+e^- \to Z^0$ vertex which by symmetry is identical to the $Z^0 \to e^+e^-$ vertex. There is a likelihood for each vertex interaction occurring, this is associated to the *partial width*

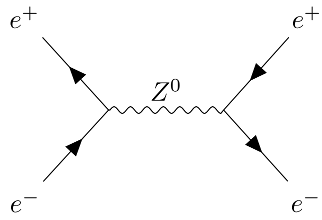

Figure 1: Lowest order Feynman diagram depicting Z^0 boson resonance in e^+e^- annihilation. The boson then decays to e^+e^-. Time increases on the horizontal axis, anti-fermion lines point backwards.

a measurable quantity that gives the decay rate into different products. For instance the partial width for $Z^0 \to e^+e^-$, which we will denote Γ_{ee}, is 83.91 MeV. Whilst the partial width for Z^0 decaying into hadrons is 1744 MeV; a reflection that a $Z^0 \to$ hadrons event is significantly more likely than $Z^0 \to e^+e^-$.

A propagator is a particle line that begins an ends at vertices (an *internal leg*). This contributes the kinematics to the expression, such as the centre-of-mass energy which we will denote as E. For the case of a massive and unstable propagator, such as the Z^0, it will introduce its mass, m_Z, and its width, Γ_Z. The width is related to the lifetime, τ, by

$$\Gamma = \frac{\hbar}{\tau} \tag{3}$$

and has units of energy.

The cross-section for $e^-e^+ \to Z^0 \to e^-e^+$ is given by

$$\sigma = \frac{12\pi}{m_Z^2} \frac{E^2}{\left(E^2 - m_Z^2\right)^2 + m_Z^2 \Gamma_Z^2} \Gamma_{ee}^2, \tag{4}$$

in natural units[1], GeV^{-2}. Here we have assumed the incoming and outgoing particle masses are negligible. The form of this cross-section is often called a *Breit-Wigner* expression as it describes a resonant state. This could be approximated to a gaussian distribution centred over m_Z with full-width at half-maximum given by Γ_Z.

2 Project description

An experiment has taken place to determine the mass and width of the Z^0 boson by measuring the cross-section at different energies. Two detectors were used intermittently over different energy ranges. The data can be found in `z_boson_data_1.csv` and `z_boson_data_2.csv`. Unfortunately, there were some faults that led to invalid data points which you must filter.

You are tasked with obtaining m_Z and Γ_Z from this data by fitting to equation 4.

Do not attempt to turn this into a linear problem and no not try and fit the parameters independently. This will significantly over-complicate the problem and will almost certainly return the wrong result.

Previous studies suggest that $m_Z = 90$ GeV/c^2 and $\Gamma_Z = 3$ GeV. To convert equation 4 from natural units, you should use

$$\frac{\hbar^2 c^2}{\mathrm{GeV}^2} = \frac{197.33^2\,\mathrm{MeV}^2\,\mathrm{fm}^2}{\mathrm{GeV}^2} = \frac{197.33^2\,\mathrm{fm}^2}{10^6} = 0.03894\,\mathrm{fm}^2 = 0.3894\,\mathrm{mb}, \tag{5}$$

where mb are mili-barns.

Your programme should:

[1] $\hbar = c = 1$.

- Read in, validate and combine both data files.
- Perform a minimised χ^2 fit by simultaneously varying m_Z and Γ_Z.
- Calculate both m_Z and Γ_Z to four significant figures in GeV/c^2 and GeV respectively.
- Calculate $\chi^2_{\text{red.}}$ to 3 decimal places.
- Calculate τ_Z to 3 significant figures in seconds.
- Produce a useful plot of your result.
- Ideally, you should also find the uncertainties on m_Z, Γ_Z and τ_Z to the appropriate precision.

With regards to style, in addition to what was asked for in the previous assignment, we expect your code:

- To have a useful file check that halts the code neatly if there is an issue.
- Read in data using inbuilt functions; do not ask the user to input the file names.
- Use inbuilt functions to perform the minimisation.
- Be versatile and applicable to data with similar validation issues.
- To make any plots by attaching axes attributes to figure objects.
- Save any plots as a `.png` file.
- To achieve a linter score of at least 9.80/10.00 (maximum penalty of 10 marks[2].)

Additional marks are available for extra features. You do not need to incude them all to get full marks for this aspect. Can you display extra information in these plots? Can you format these plots nicely? Can it be applied to systems with different decay products? Could it be applied to different files with *different validation issues*? Can you make the initial guess on m_Z and Γ_Z general? Could it work without knowing the initial value for Γ_{ee}?

More detail on how the mark is split can be found in the illustrative rubric on BlackBoard.

[2]PyLint 2.5.3 using the configuration file provided. A score of 8.33/10.00 corresponds to a deduction of 1.57 marks and -2.40 corresponds to a deduction of 10.00 marks.