Evidence for Commodity Market Risk Sharing from a Bayesian Perspective

Key points

* KF is forward algorithm so without smoother (backwards algorithm) so
* Correlation is only direction, not magnitude like beta
* Your findings confirm Gorton and Rouwenhorst (2006)! S&P and commodities negative pre-financialziation

Abstract

As financialization allowed more speculators enter the commodity market, greater risk-sharing occurs whereby commodity returns become more closely integrated with the returns of conventional assets. Previous studies have tested risk-sharing between commodity and equity markets using panel regressions or rolling correlations, which leave room for improvement on the granularity and interpretability of results. Turning to Bayesian modelling, this paper tests for co-integration between commodities and equities by the estimation of time-varying betas using Kalman filtering, which yields more comprehensive results. The findings suggest that financialization has eroded the diversification benefits of commodities for equity investors in the face of a secular convergence towards a ‘market of one’.

Structure:

1. Intro – lay down definitions, key theories and theoretical influences
2. Literature Review – say you’re ‘extending’ research that employed rolling correlation & panel regressions
3. Model – maths presentation
4. Data – say where you got it plus summary statistics/visualization
5. Empirical Implementation (results) – present and discuss, split into WTI/gas then general indices
6. Conclusion – reiterate what you’ve written so people can skip over the other sections and just read this to understand implications

**Evidence for Commodity Market Risk Sharing from a Bayesian Perspective**

1. Introduction

An investor holding a portfolio of stocks might look to diversify equity exposure by investing in the commodity market, but this line of thinking has come under empirical challenge. The original assumption that equities and commodities exhibit no positive correlation was established by studies that did not capture the period of commodity market financialization in the early 2000s, such as Gorton and Rowenhorst’s (c2006) correlation study using data from 1959 to 2004. Over the next decade, commodities such as crude oil experienced an ‘unprecedented’ boom-bust cycle along with the equity market (cFattouh, Kilian and Mahadeva, 2013), leading academics to employ rolling correlation to assess if commodity-equity correlation has changed over time (cBüyüksahin, Haigh and Robe 2010; Tang and Xiong, 2012). This paper continues in that spirit of uncovering time-trends in the commodity-equity relationship, which forms a vital part of the financialization literature.

The concept of financialization surged to prominence along with commodity prices in the run-up to 2008, igniting fears that financialization caused increased speculative fervour that was distorting markets (cBrown and Sarkozy, 2009). However, both financialization and speculation remain ambiguous terms that have been viewed differently by different papers (cFattouh, Kilian and Mahadeva, 2013). This paper will follow the definition adopted by Fattouh, Kilian and Mahadeva (ibid), such that financialization is ‘the increasing acceptance of [commodity] derivatives as a financial asset by a wide range of market participants’.

Correspondingly, the definition for speculation should capture the actions of new participants who would traditionally not have participated in the commodity market, though that is itself a challenge. Defining speculation by market actions based on expectation – as opposed to the market’s traditional role in facilitating consumption – is broad enough to include producers. For example, an oil producer may sell some stockpiles now to invest in increasing production in anticipation of rising long-run prices. Alternatively, an agent-based definition of speculation would fall unfairly strict on certain groups of participants and lax on others. All things considered, this paper will follow Working’s (c1960) definition of speculation as the volume of futures trading above that necessary to hedge spot positions. Despite the empirical critiques raised by Fattouh, Kilian and Mahadeva (2013), Working’s definition is the closest to an operational definition.

The effect of increased speculation falls roughly into three main theories in the literature. First is the idea that speculation is associated with mispricing in relation to economic fundamentals, causing asset price bubbles as the result of greed (Brown and Sarkozy 2009). Such a view of speculation is recognized by Fattouh, Kilian and Mahadeva (2013) as normative, imbuing speculation with a social dimension. Opposingly, another view is that an influx of speculators improves the market’s information discovery mechanism, allowing for new information to be better aggregated and resulting in better pricing (cCheng and Xiong, 2014). The information discovery theory thus assumes speculators are rational agents, unlike the more behavioural view of speculators by the asset price bubble theory. However, both theories remain elusive to prove empirically due to the difficulty in determining what makes prices ‘better’ or ‘worse’, especially when physical commodities are frequently affected by geopolitical factors (Masters 2008). Hence, a third theory more neutral to the pricing ability of speculators has emerged – that of risk sharing. The risk sharing theory posits that because new participants in the commodity market will have traditionally invested in established assets, their capital allocation in the commodity market will reflect their allocation in other assets, bringing a closer degree of integration between the price movement of commodities and other assets. Systematic risk is thus shared between traditionally segmented markets.

The problem of risk sharing has already been noted above for investors who want to diversify their portfolio by investing in the commodity market, but ironically the act of diversifying into commodities itself exacerbates the phenomenon. Recall that our definition of speculation covers any type of activity in the futures market that is not hedging, including portfolio diversification using index-tracking futures products as commodities become more established as a portfolio asset. Masters (c2008) points to the rise of commodity index speculation, such as buying products tracking the Goldman Sachs Commodity Index (GSCI), as the cause of commodity price surges, but captures only the partial picture in his assessment. Cheng and Xiong (2014) note that subsequent papers fail to establish a linkage between changes in index speculation volume and commodity futures returns, finding instead more evidence that index speculation increases the level of integration of returns with previously segmented markets, including between different commodity sectors. It has since been generally accepted that institutional index speculation is the main cause of risk sharing (cBasak and Pavolova, 2016).

This paper will seek to extend the conventional analysis of risk sharing, comparing the degree of equity market integration for individual commodities and commodity indices to determine whether the theory of index speculation holds. However, instead of testing how commodity-equity integration has changed over time using rolling correlation, a Bayesian approach will be used where time-varying systematic risk coefficients are estimated using Kalman filters. This construction has already been applied to analyse systematic risk in the equity market (cFaff, Hillier and Hillier, 2003; Renzi-Ricci, 2016), and offers several advantages over rolling correlation – in particular improving the granularity and interpretability of results. With the wealth of now-accumulated data and a new statistical technique, this paper hopes to give a more comprehensive picture of the time-varying level of commodity-equity integration, which should be of interest for researchers, investors and policy makers.

The structure of this paper shall be as follows: [write something about structure and findings – consistent but new evidence or smth] [briefly explain models used]

1. Literature Review

Previous literature has focused on the correlation of commodity and equity markets as a test for the degree of integration. [SIlvennoinen and that]

[Further, financial shocks appear to be important predictors of correlation dynamics. For 11 of the 24 commodities, high expected stock market volatility raises correlations with equities. Somewhat surprisingly, the correlation regime switch for oil futures and stocks occurred during the GFC, rather than earlier in the decade, and preferred models do not include the VIX transition variable.] – Silvennoinen include?

[also find some other frequentist methods like Isleimeyyeh’s regression, Kilian’s paper on EM stocks]

[highlight the data period used by critics – paper Arie sent you and the ‘market of one’ one]

1. Model

In this section we shall first develop the intuition behind the linear model we apply to test for the level of commodity-equity integration. Then, we shall transform the linear model into state-space representation using the Kalman filter, so that the linear coefficient can be modelled as a time-dependent state.

1. Linear Model

Our linear model is simple and can be derived from first principles. To begin, returns may be used to build a general picture of price movements, expressed as $y$ and $x$. Integration can then be assumed to manifest through the proportionality $y \prop x$, such that $y = \beta x$ where $\beta$ is non-zero. A perfect degree of integration will therefore result in a $\beta$ of one such that returns are expected to co-move in the same direction and magnitude, while a $\beta$ of negative one expects returns co-move in opposite directions but with the same magnitude. A near-zero $\beta$ suggests that we expect little co-movement. If two assets are integrated under the risk sharing hypothesis, a positive $\beta$ should exist, and the level of integration will be observable from the closeness of $\beta$ to one. Note that $\beta$ beyond -1 and 1 is possible, but it is unlikely that commodity and equity returns will be so sensitive to each other.

The benefit of our $\beta$ measure for integration over standard correlation testing is that correlation only tests for the strength of co-movement through direction, whereas $\beta$ adjusts correlation to incorporate magnitude. From our intuitive derivation from proportionality above, the $\beta$ measure is also much easier to interpret because it essentially the value that maps the expected values of y onto x. Formally, a time-invariant $\beta$ can be written in terms of correlation $\rho$ and standard deviation $sigma$ such that:

[\beta$ = \rho\_{xy} \frac{\sigma\_y}{\sigma\_x}

However, although the benefits of $\beta$ over correlation remain the same, the above equation is solely for a constant $\beta$. To observe whether the level of equity-commodity integration changes over time, we need $\beta$ to be expressed as a time-varying coefficient for returns at different time points. To express our final model, we introduce time indexing and the market noise variable $\epsilon$:

[y\_t = \beta\_t x\_t + \epsilon\_t]

For equation [CITE], let $y$ denote commodity asset return and $x$ denote equity asset return. Note that unless specified, commodity assets include commodity indices, and equity assets include equity indices, given the availability of index-tracking products as discussed in the introduction.

For traditionalists, our linear model can also be derived using the Capital Asset Pricing Model (CAPM). From Fama and French (c2004), the original Sharpe-Lintner CAPM is expressed as:

[E[R\_i] = R\_f + (E[R\_m] – R\_f)\beta]

Where $R\_i$ is the return of a single stock, $R\_f$ is the risk free rate, and $R\_m$ is the equity market return. The CAPM model thus tests for systematic risk within the equity market such that $\beta$ represents the undiversifiable equity market risk inherent in a given stock. By altering the assumption for the risk sharing theory such that systematic risk may exist between different asset classes, we can use our notation of $y$ and $x$ such that $E[y] = R\_f + (E[x] – R\_f)\beta$, which results in the time-varying version of $y\_t = R\_{f, t} + (x\_t – R\_{f, t})\beta\_t + \epsilon\_t$. If using time-series data of sufficiently high frequency, the inter-period risk-free rate should also be negligible, rendering the final relationship the same as equation 1.

1. State-Space Transform

Finding $\beta\_t$ is not as simple as dividing $y$ by $x$ for each $t$ due to our recognition of market noise $\epsilon$, which necessitates some consideration for how $\beta\_t$ evolves given $\beta\_{t-1}$ to separate trend from noise. Although techniques like rolling regression can be used to estimate $\beta\_t$ given a span of past values, it can be slow to respond when $\beta$ changes (Renzi-Ricci 2016) and results in data loss at the start of the overall sample period. Such qualities motivate our departure from frequentist statistics, using Bayesian inference to estimate the most likely value of $\beta\_t$ for each point in time instead. This paper will use Kalman filtering to estimate $\beta\_t$.

The Kalman filter is a recursive model that estimates the most likely value of hidden states given 1) prior predictions at $t-1$, and 2) new observations at $t$. The general algorithm is as follows:

\begin{verbatim}

1. Initiate model inputs, which represent posteriors at time t = 0
2. For t = 1 to t = T:
3. Predict the prior for t using the posterior estimate at t-1
4. Compute the posterior estimate using the prior and the observation at t

\end{verbatim}

$\beta\_t$ is thus estimated by posterior values for each timestep. To formalize subsequent notation, let $\hat\beta\_t$ denote estimated $\beta\_t$ such that $\hat\beta\_{t|t-1}$ represents the predicted priors, and $\hat\beta{t|t}$ represents the posterior estimates. We can let $y\_t$ and $x\_t$ remain as per prior notation, since we are only constructing a one-dimensional Kalman filter and so can forego matrix representation. Kalman filtering also assumes that $\hat\beta\_t$ and $y\_t$ can be modelled using Gaussian distributions, where the distribution for the state variable $\hat\beta\_t$ is time-dependent and the distribution for observed variable $y\_t$ time-invariant. As such:

\hat\beta\_t \sim N(\hat\beta\_t, P\_t)

y\_t \sim N(y, R)

To predict $\hat\beta\_{t|t-1}$, we will follow Faff, Hillier and Hillier (2003) in assuming that $\beta$ evolves through a random walk process, which was found to yield more accurate $\beta$ estimates for the CAPM. Hence for the predictive step we use the following set of equations:

\hat\beta\_{t|t-1} = \hat\beta\_{t-1|t-1}

P\_{t|t-1} = P\_{t-1|t-1} + Q

Where $Q$ is a time-invariant assumption of process noise. Essentially, a lower value of $Q$ restricts the variance around the prior, which necessitates the posterior to give greater consideration to the prior than the observation. Hence a lower $Q$ implies a higher $\epsilon\_t$ in our linear model, and $\epsilon$ is not incorporated directly within the Kalman filter. This follows since if we assume the predicted $\hat\beta\_t$ is without noise (error), then $\beta\_t$ is deterministic and the fluctuations of $y\_t$ and $x\_t$ can be explained by market noise $\epsilon\_t$ alone.

To compute our posterior estimates we use the following:

K = \frac{P\_{t|t-1}}{P\_{t|t-1} + R}

\hat\beta\_{t|t} = \hat\beta\_{t|t-1} + K(y\_t - \hat\beta\_{t|t-1}{x\_t})

P\_{t|t} = (1 – K) P\_{t|t-1}

$K$ above is commonly known as the ‘Kalman gain’ between each time-step, which decides how much weight should be given to the new residual computed using the prior prediction. Intuitively, it controls how the error of the prediction is used to update the posterior.

1. Implementation

This section will focus on developing the practical methodology used to obtain our results. First will be a discussion of general strategy around building the models using empirical data, before an explanation of how data was obtained.

1. Strategy

To implement our model, we shall use series of monthly returns for $x$ and $y$ to make our linear model compatible with the aforementioned assumption of negligible inter-period risk-free rate. This is a realistic assumption since the US Federal Funds rate has remained below 0.5% per month since the 1980s (see Appendix CITE). Returns shall also be standardized at an annual basis to remove any leptokurtosis that undermines our normality assumption, as well as de-seasonalized to remove the influence of seasonal demand/supply patterns. De-seasonalization will involve calculating the difference between actual returns and in-sample monthly averages. It matters little whether we use simple or exponential returns due to the two-step transformation performed. For this paper simple returns will be used.

Recall also that we need to specify the initial parameters $\hat\beta\_0$, $P\_0$, $R$ and $Q$ for the Kalman filter. Since we are testing for integration, we shall first assume no integration with $\hat\beta\_0 = 0$, and let the model convince us otherwise. $R$ is easy to specify as the in-sample variance of $y$, which should remain consistent throughout the period given our data cleaning. $P\_0$ will be assumed to be the same as $R$, though the initial specification matters little given that it is a dynamic variable, which will be altered with each iteration. This paper shall use 0.01 for $Q$, which reflects the assumption that the level of integration between markets is not volatile given that financialization is a gradual process.

For computation this paper uses the Python package FilterPy, version 1.4.5, by Labbe (c2015) to implement the Kalman filter. Recall that the Kalman filter is a forward (in time) algorithm that calculates the most likely position of $\beta\_t$ given data at $t-1$, making it useful for real-time updating. However, for inference it is sometimes desirable to compute the most likely evolution of $\beta\_t$ given observations at $t+1$ as well, which requires adjusting the Kalman filter estimates using a backward algorithm known as a ‘smoother’. For the analysis below smoothing will refer to the Rauch-Tung-Striebal smoother implemented by FilterPy, which we shall not detail here given its standard nature and implementation.

1. Data

The data used for this paper falls into two categories: single securities and indices. Data is collected as time-series at monthly intervals as detailed above, utilizing first-of-month values. We collected the maximum historical sample available for all data, with cut-off points being January 1986 and December 2021. Prices were all collected in US dollar denomination, and deflated to December 2021 terms using the monthly US Consumer Price Index available on the St. Louis Federal Reserve Economic Data website.

Single securities data are used for comparative analysis against commodity indices. The historical prices of front-month futures for oil (WTI) and natural gas (Henry Hub) were collected from the US Energy Information Administration. Front-month future prices for generic contracts in gold, hog, wheat, lumber, oats, palladium and rubber were collected from a Bloomberg Terminal.

Indices data include both commodity and equity indices, all collected from Bloomberg. Commodity indices collected are the GSCI (including energy and non-energy components), Dow Jones Commodity Index (DJCI), Bloomberg Commodity Index (BCI), and Commodity Research Bureau Index (CRBI). Equity indices used are the S&P 500, MSCI World and MSCI Emerging Markets. The methodology and constituents of these indices and their significance will be detailed in our discussion of results below.

1. Results

In this section we detail our findings in two parts. First, we present the level of integration found between commodity and equity indices. Then, we analyse individual commodities, comparing the effect of financialization on popular (by index inclusion) commodities and non-popular commodities.

1. Indices
   1. Exploratory Analysis

[graph of indices]

[maybe table of return distribtions?]

Figure [CITE] visualizes the real price and return distributions of the different indices used in this paper, which are due to index construction. GSCI is a production-weighted index, with weights determined by five-year average world production data (cS&P Global 2022). As such, it is a momentum-driven construction that has seen significant historical exposure to energy commodities such as crude oil, which leads to critique of over-exposure to idiosyncratic event risks, evident from the high standard deviation of returns (cAnson 2008). By contrast, DJCI is a liquidity-weighted index based on five-year average trading volumes, and is designed to maximise diversification with a weight cap of 32% (cS&P Global 2021). Similarly, BCI

[impact on individual commodities in indices aren’t that great because weights adjust each year, so level of integration differs. This is shown by evidence of sporadic integration for individual commodities but consistent level of integration between GSCI and DJCI despite different methodologies – which implies investors buy the indexed basket of assets not the asset itself. Hence fears of index driving decoupling from fundamentals is overblown, because ultimately index weights decide how much each commodity is impacted, which is decided by fundamental factors like production or liquidity. So index more of an amplification effect than anything

However, for non-indexed commodities the level of integration is more homogenous because investors are more homogenous?]

[maybe present from indices first, then segue into individual assets? helpful to explain weights and index construction first] – actually perhaps even index first then individual then NECI vs ECI