ALGEBRA

Laws of Exponent:

1. $a^n = a \text{ x } a \text{ x } a\ldots\text{(n factors)}$
2. $a^m a^n = a^{m+n}$
3. $\dfrac{a^m}{a^n} = a^{m-n}$
4. $(a^m)^n = a^{mn}$
5. $(ab)^n = a^n b^n$
6. $\left(\dfrac{a}{b}\right)^n = \dfrac{a^m}{a^n}$
7. $a^{m/n} = \sqrt[n]{a^m}$
8. $a^m = \dfrac{1}{a^{-m}}$ also $a^{-m} = \dfrac{1}{a^m}$
9. $a^0 = 1, \quad a \neq 0$
10. $a^m = a^n, \quad \text{then} \quad m = n, \text{provided } a \neq 0$

Laws of Radicals:

1. $\sqrt[n]{a} = a^{1/n}$
2. $\sqrt[n]{a^n} = a$
3. $\sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m$
4. $\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$
5. $\sqrt[n]{\dfrac{a}{b}} = \dfrac{\sqrt[n]{a}}{\sqrt[n]{b}}, \quad b \neq 0$

Quadratic Equation:

$$ax^2 + bx + c = 0 \text{ where a, b, c are all constants}$$

The roots are:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x_1 = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \quad \text{and} \quad x_2 = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

where: $(b^2 - 4ac)$ is called the discriminant

Nature of the roots:

1. If $b^2 - 4ac > 0$, then the roots are real and unequal
2. If $b^2 - 4ac = 0$, then the roots are real and equal (Perfect Trinomial Square)
3. If $b^2 - 4ac < 0$, then the roots are imaginary and unequal (complex conjugates)

Sum of the roots:

$$\mathbf{x_1 + x_2 = -\frac{b}{a}}$$

Product of the roots:

$$\mathbf{x_1 \bullet x_2 = \frac{c}{a}}$$

Test of Factoring:

$ax^2 + bx + c$ is factorable if,

a.) $b^2 - 4ac = 0$
b.) $b^2 - 4ac$ = perfect square

Remainder Theorem:

Remainder Theorem states that if a polynomial f(x) is divided by x – r; where r is any constant until the remainder R that is free of x is obtained, this remainder is equal to f(r).

Factor Theorem:

If x – r is a factor of f(x), then r is a root of f(x) = 0.

Binomial Theorem:

Binomial Expansion

$(a + b)^0 = 1$
$(a + b)^1 = a + b$
$(a + b)^2 = a^2 + 2ab + b^2$
$(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$
$(a + b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$

Pascal's Triangle

```
        1
      1   1
    1   2   1
  1   3   3   1
1   4   6   4   1
```

Binomial Theorem Formula:

$$(a+b)^n = a^n + n\,a^{n-1}b + \frac{n(n-1)a^{n-2}b^2}{2!} + \frac{n(n-1)(n-2)a^{n-3}b^3}{3!} + \ldots\ldots$$

$$\ldots\ldots + \frac{n(n-1)(n-2)\ldots(n-r+2)a^{n-r+1}b^{r-1}}{(r-1)!} + \ldots\ldots + n\,ab^{n-1} + b^n$$

where: $n + 1$ = total no. of terms

r = term no.

r^{th} term of $(a + b)^n$:

$$r^{th} = \frac{n(n-1)(n-2)\ldots(n-r+2)\,a^{n-r+1}\,b^{r-1}}{(r-1)!}$$

Alternate formula:

$$r^{th}\ \text{term} = {}_nC_{r-1}a^{n-r+1}b^{r-1}$$

Notes: 1. $(a + b)^n$ has middle term if n is even.

2. If r^{th} term is the middle term, $r = n/2 + 1$ or $r - 1 = n/2$

Middle term of $(a + b)^n$:

$$\text{Middle} = \text{term} = {}_nC_{n/2}\,a^{n/2}\,b^{n/2}$$

Sum of Exponents:

$$S = n(n + 1)$$

Sum of Coefficients:

Substitute 1 to every variable in each term.

Note: For n is negative or a fraction, e.g., $(a + b)^{-2}$, $(a + b)^{1/3}$,… then,

1.) Don't use alternate formula.

2.) It has infinite no. of terms.

Ratio:

The ratio a/b is usually written as a : b, a is called the antecedent and b is called the consequent.

Proportion:

Proportion is a statement of equality between two ratios.

$$a : b = c : d \text{ or } a/b = c/d$$

where: b and c are called the means.
a and d are called the extremes.
d is called the fourth proportional to a, b, and c.

Note: If the means of the proportion are equal, such as $^{a}/_{x} = {}^{x}/_{d}$, then **x** is called the mean proportional between **a** and **d**, while **d** is called the third proportional to **a** and **x**.

Variation:

Direct Variation Attention:
y is directly proportional to **x**: $y \propto x$ or $y = kx$, where k = constant of proportionality

a. Inverse Variation
y is inversely proportional to **x**: $y \propto {}^{1}/_{x}$ or $y = {}^{k}/_{x}$

b. Joint Variation
1. **y** is directly proportional to **x** and inversely proportional to **z**:

$$y \propto {}^{x}/_{z} \text{ or } y = {}^{kx}/_{z}$$

2. **y** varies jointly as **x** and **z**: $y \propto xz$ or $y = kxz$

Arithmetic Proression (A.P.)

- a sequence of numbers $a_1, a_2, a_3, \ldots, a_{n-1}, a_n$

Such that $a_2 - a_1 = a_3 - a_2 = \ldots = a_n - a_{n-1} = d$ = common difference.

For example: 1, 2, 3, 4
$d = 2 - 1 = 3 - 2 = 4 - 3 = 1$

Formulas:

1. **l or $a_n = a_1 + (n-1)d$**

2. $S_n = \frac{n}{2}(a_1 + a_n) = \frac{n}{2}[2a_1 + (n-1)d]$

3. Arithmetic Mean (A.M.) of two numbers a and b

$$A.M. = \frac{a+b}{2}$$

4. Arithmetic Mean (A.M.) of numbers a_1, a_2, a_3, ….., an

$$\mathbf{A.M. = \frac{a_1 + a_2 + a_3 + \ldots + a_n}{n}}$$

where: l or a_n = last term or nth term
a_1 = first term
S_n = sum of nth term
d = common difference
n = no. of terms

Geometric Progression (G.P.)

A sequence of numbers a_1, a_2, a_3, ….. , a_{n-1}, a_n

Such that $a_2/a_1 = a_3/a_2 = \ldots = a_n/a_{n-1} = r$ = common ratio

$$\mathbf{r = \frac{4}{2} = \frac{8}{4} = \frac{16}{8} = 2}$$

For example: 2, 4, 8, 16

Formulas:

1. l or $a_n = a_1 r^{n-1}$
2. $\mathbf{S_n = \frac{a_1\left(1 - r^n\right)}{1 - r} = \frac{\left(a_1 - a_n r\right)}{1 - r}}$
3. Geometric Mean (G.M.) of two numbers a and b

$$\mathbf{G.M. = \sqrt{ab}}$$

4. Geometric Mean of a_1, a_2, a_3, ….., a_n

$$\mathbf{G.M. = \sqrt[n]{a_1 a_2 a_3 \ldots a_n}}$$

5. Infinite Geometric Progression (I.G.P.)

$$\mathbf{S_\infty = \frac{a_1}{1 - r}}$$

Harmonic Progression (H.P.)

Sequence of numbers whose reciprocals are in A.P.

- sequence of numbers a_1, a_2, a_3, ….., a_{n-1}, a_n

Such that $\frac{1}{a_1}, \frac{1}{a_2}, \frac{1}{a_3}, \ldots, \frac{1}{a_{n-1}}, \frac{1}{a_n}$ are in A. P.

For example: 16, 8, 16/3, 4, 16/5 H.P.

$^1/_{16}$, $^1/_8$, $^3/_{16}$, $^1/_4$, $^5/_{16}$ A.P., d = 0.0625

Formulas:

$$\mathbf{H.M} = \frac{n}{\frac{1}{a_1} + \frac{1}{a_2} + \frac{1}{a_3} + \ldots + \frac{1}{a_{n-1}} + \frac{1}{a_n}}$$

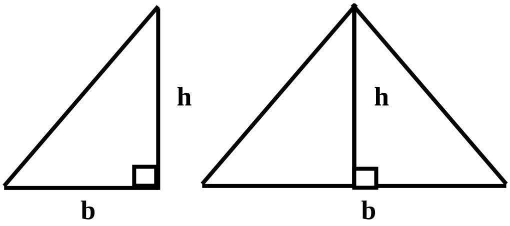
$$\mathbf{H.M} = \frac{1}{\mathbf{A.M}}$$

PLANE AND SOLID GEOMETRY

Areas of Triangles

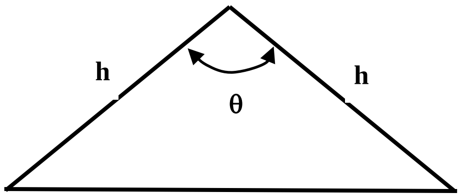
Given base and altitude:

$$\mathbf{A = \frac{1}{2} bh}$$



Given two sides and included angle:

$$\mathbf{A = \frac{1}{2} ab \sin\theta}$$

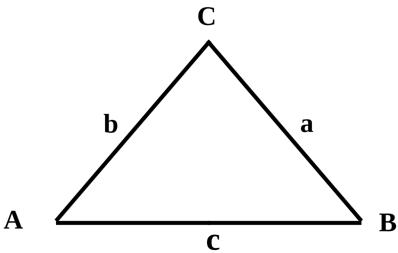


Given two angles and included side:

$$\mathbf{A = \frac{1}{2} \frac{(b^2 \sin A \sin C)}{\sin B}}$$

$$\mathbf{A = \frac{1}{2} \frac{(a^2 \sin C \sin B)}{\sin A}}$$

$$\mathbf{A = \frac{1}{2} \frac{(c^2 \sin A \sin B)}{\sin C}}$$

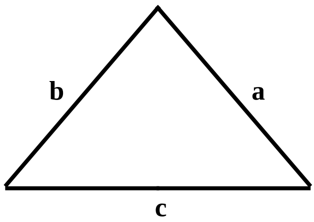


Given the three sides:
Hero's Formula:

$$\mathbf{A = \sqrt{s(s-a)(s-b)(s-c)}}$$

where :

$$\mathbf{s = \frac{a+b+c}{2}}$$

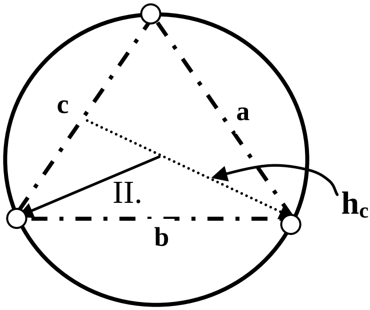


Circle Circumscribing aTriangle (Circumcircle)

$$\mathbf{A_T = \frac{abc}{4R}}$$

$$\mathbf{d = \frac{ab}{h_c}}$$

where: R = radius of circle
A_T = area of triangle
d = diameter of circle
h_c = altitude of the third side
R = radius of the circle

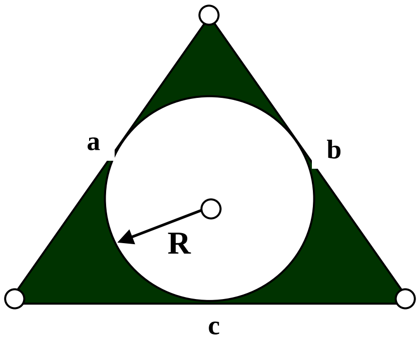


Circle Inscribed in a Triangle (Incircle)

$$\mathbf{A_t = Rs}$$

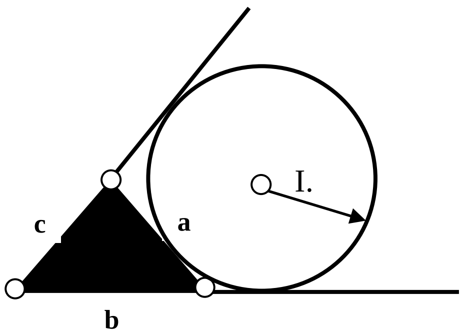
where:

$$\mathbf{s = \frac{a+b+c}{2}}$$



Circle Escribed in a Triangle (Excircle)

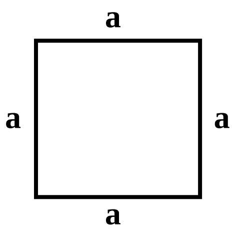
$$\mathbf{A = R(s - a)}$$



Quadrilaterals

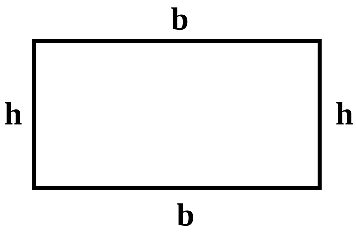
Square:

$A = a^2$
$P = 4a$



Rectangle:

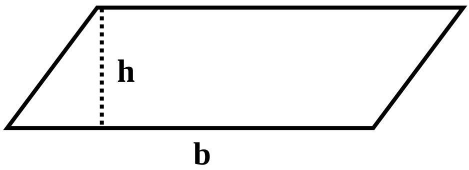
$A = bh$
$P = 2(b + h)$



Paralleleogram:

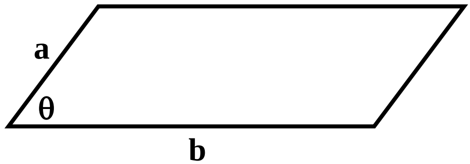
Given base and altitude:

$A = bh$



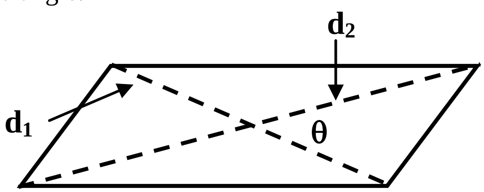
Given two sides and included angle:

$A = ab \sin\theta$
$P = 2a + 2b$



Given diagonals and their included angle:

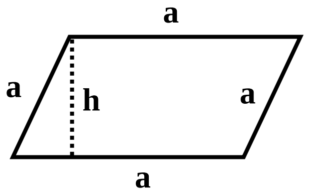
$$A = \frac{1}{2} d_1 d_2 \sin\theta$$



Rhombus:

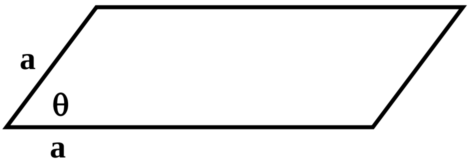
Given base and altitude

A = ah



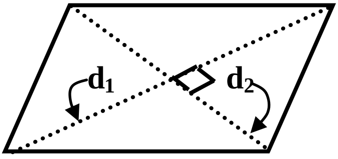
Given side and included angle

$\mathbf{A = a^2 \sin\theta}$
P = 4a

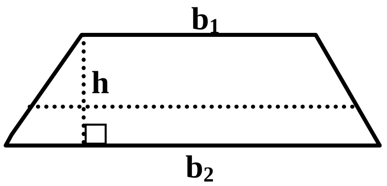


Given diagonals

$$\mathbf{A} = \frac{1}{2}\mathbf{d}_1\mathbf{d}_2$$



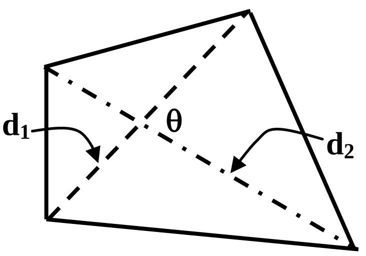
$$\mathbf{A} = \frac{1}{2}(\mathbf{b}_1 + \mathbf{b}_2)$$



General Quadrilateral: (Unequal sides)

Given diagonals

$$\mathbf{A} = \frac{1}{2}\mathbf{d}_1\mathbf{d}_2 \sin\theta$$



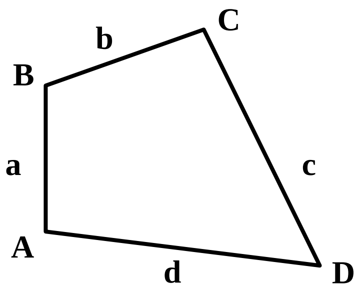
Given four sides and sum of opposite angles

$$A_a = \sqrt{(s-a)(s-b)(s-c)(s-d) - abcd\cos^2\theta}$$

where:

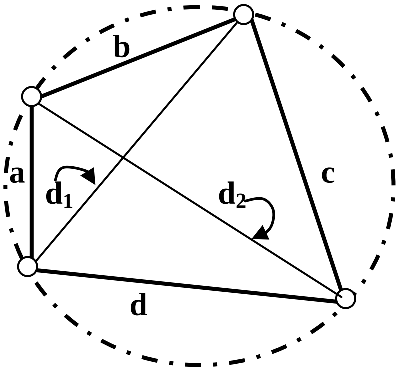
$$\theta = \frac{A+C}{2} \quad \text{or} \quad \theta = \frac{B+D}{2}$$

$$s = \frac{a+b+c+d}{2}$$



Ptolemy's Theorem

$$d_1 d_2 = ac + bd$$



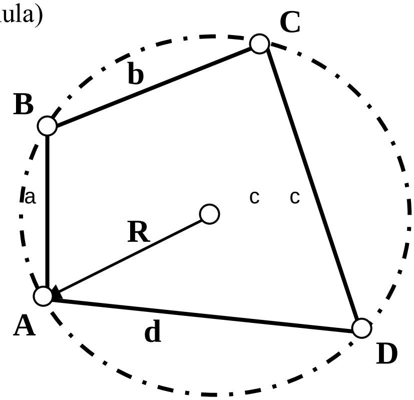
Cyclic Quadrilateral: (Bramaguptha's Formula)

$$A_q = \sqrt{(s-a)(s-b)(s-c)(s-d)}$$

$$R = \frac{\sqrt{(ab+cd)(ac+bd)(ad+bc)}}{4A}$$

where :

$$s = \frac{a+b+c+d}{2}$$

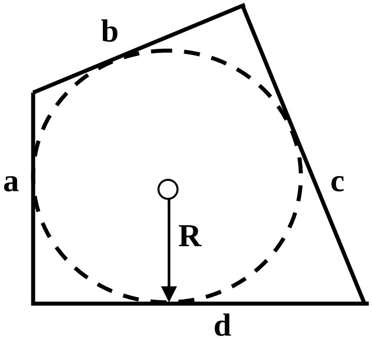


Circle inscribed in a quadrilateral

$$\mathbf{A_q = Rs = \sqrt{abcd}}$$

where :

$$\mathbf{s = \frac{a+b+c+d}{2}}$$

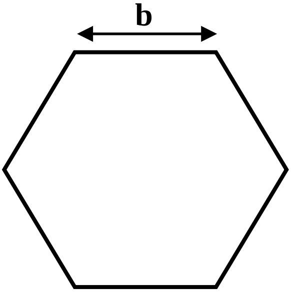


Area of Regular Polygon

$$\mathbf{A = \frac{1}{4} nb^2 \cot\left(\frac{180°}{n}\right)}$$

$$\mathbf{P = nb}$$

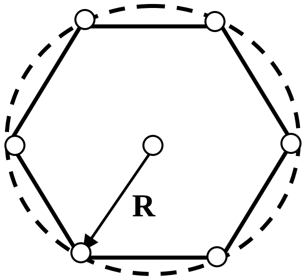
where: n = number of sides



Regular Polygon Inscribed in a Circle

$$\mathbf{A = \frac{1}{2} nR^2 \sin\left(\frac{360°}{n}\right)}$$

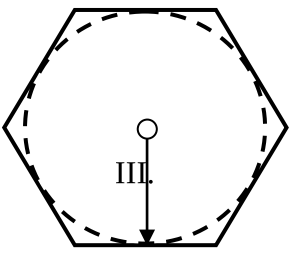
$$\mathbf{P = 2nR \sin\left(\frac{180°}{n}\right)}$$



Regular Polygon Circumscribing a Circle

$$\mathbf{A = nR^2 \tan\left(\frac{180°}{n}\right)}$$

$$\mathbf{P = 2nR \tan\left(\frac{180°}{n}\right)}$$



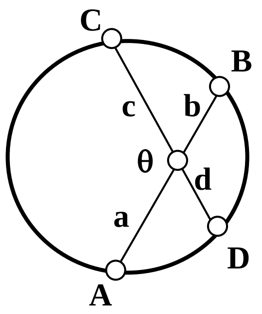
Central Angle and Inscribed Angle: (θ)

$$\theta = 2\beta$$

Intersecting Chords

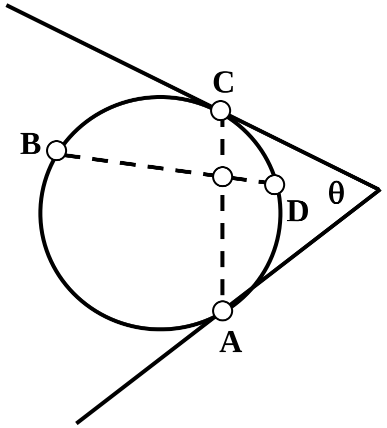
$$\theta = \frac{1}{2}(\text{arc AC} + \text{arc BD})$$

$$ab = cd$$



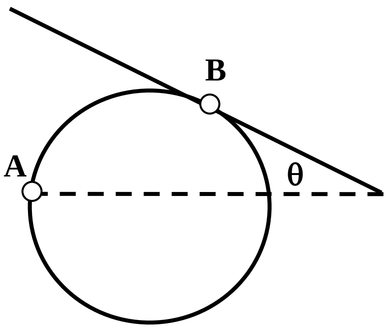
Intersecting Tangents

$$\theta = \frac{1}{2}(\text{arc ABC} - \text{arc ADC})$$



Intersecting Tangent and Chord

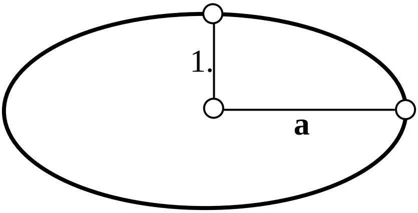
$$\theta = \frac{1}{2}\text{arc AB}$$



Ellipse

$$A = \pi ab$$

$$P = 2\pi\sqrt{\frac{(a^2 + b^2)}{2}}$$



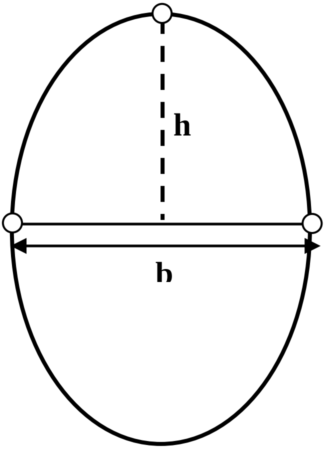
Parabolic Segment

$$A = \frac{2}{3}ab$$

$$P = \frac{c}{2} + \frac{b^2}{8h}\ln\left(\frac{4h + c}{b}\right) + b$$

where :

$$c = \sqrt{b^2 + 16h^2}$$



ANALYTIC GEOMETRY

Distance Between Points $P_1(x_1,y_1)$ and $P_2(x_2,y_2)$:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Distance from a Point to a Line

LINE: $Ax + By + C = 0$

POINT: $P_1(x_1,y_1)$

$$d = \frac{Ax_1 + By_1 + C}{\pm\sqrt{A^2 + B^2}}$$

Where the ambiguous sign $\pm$ follows the sign of B (or A when B=0).

Notes: 1. d is (+), if the point is above the line.
2. d is (–), if the point is below the line.

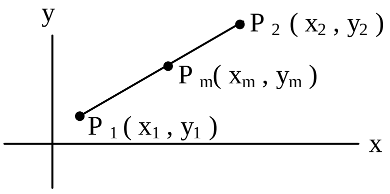
Distance Between Two Parallel Lines

L_1: $Ax + By + C_1 = 0$

L_2: $Ax + By + C_2 = 0$

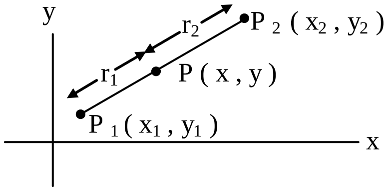
$$d = \left| \frac{C_2 - C_1}{\sqrt{A^2 + B^2}} \right|$$

Midpoint of Line Segment P_1P_2:



$$x_m = \frac{x_1 + x_2}{2} \qquad y_m = \frac{y_1 + y_2}{2}$$

Division of Line Segments $P_1 P_2$:



$$x = \frac{r_1 x_2 + r_2 x_1}{r_1 + r_2} \qquad y = \frac{r_1 y_2 + r_2 y_1}{r_1 + r_2}$$

where: $\dfrac{r_1}{r_2} = \dfrac{\overline{P_1 P}}{\overline{PP_2}}$

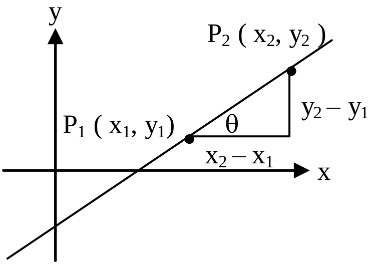
Alternate Formula:

$$x = x_1 + k\,(x_2 - x_1)$$
$$y = y_1 + k\,(y_2 - y_1)$$

where: $k = \dfrac{\overline{P_1 P}}{\overline{PP_2}}$

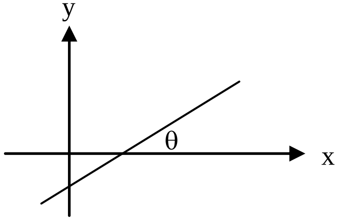
Inclination and Slope of a Line:

In General:



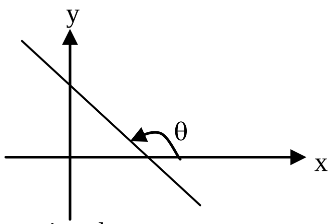
$$m = \tan\theta = \frac{y_2 - y_1}{x_2 - x_1}$$

1. A line sloping upward to the right



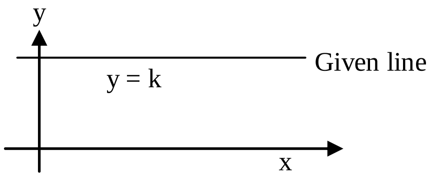
positive slope

2. A line sloping downward to the right

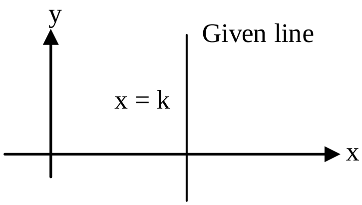


negative slope

3. If the given line is parallel to the x- axis so that $y_2 = y_1$; m = 0



4. If the given line is parallel to the y- axis so that $x_2 = x_1$: m is meaningless



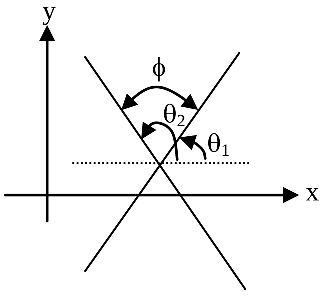
Angle Between Two Lines

$$\mathbf{m_1 = \tan\theta_1; \quad m_2 = \tan\theta_2}$$

$$\boldsymbol{\phi = \theta_2 - \theta_1}$$

$$\boldsymbol{\tan\phi = \tan(\theta_2 - \theta_1) = \frac{\tan\theta_2 - \tan\theta_1}{1 + \tan\theta_2 \tan\theta_1}}$$

$$\boldsymbol{\tan\phi = \frac{m_2 - m_1}{1 + m_1 m_2}}$$



Notes: 1. If L_1 is parallel to L_2 ; then $m_1 = m_2$
2. If L_1 is perpendicular to L_2 ; then $m_1\ m_2 = -1$

Straight Line Equation Form:

1. General Form : **Ax + By + C = 0**
2. Standard Form:

   a. Point slope form: $\mathbf{y - y_1 = m(x - x_1)}$

   b. Slope intercept form: $\mathbf{y = mx + b}$

   c. Intercept form:

   $$\mathbf{\frac{x}{a} + \frac{y}{b} = 1}$$

   d. Two point form:

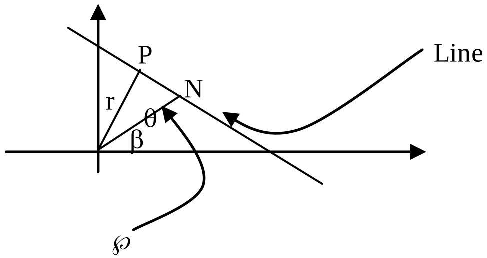
   $$\mathbf{\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1}}$$

   e. Area Form:

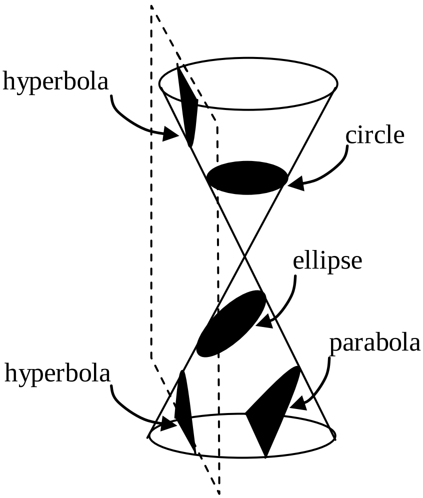
   $$\begin{vmatrix} \mathbf{x} & \mathbf{y} & \mathbf{1} \\ \mathbf{x_1} & \mathbf{y_1} & \mathbf{1} \\ \mathbf{x_2} & \mathbf{y_2} & \mathbf{1} \end{vmatrix}$$

   f. Normal Form:
   $x \cos\beta + y \sin\beta = \wp$

   g. Polar Form:
   $r \cos(\theta - \beta) = \wp$



Conics



General Equation of a Conic Section :

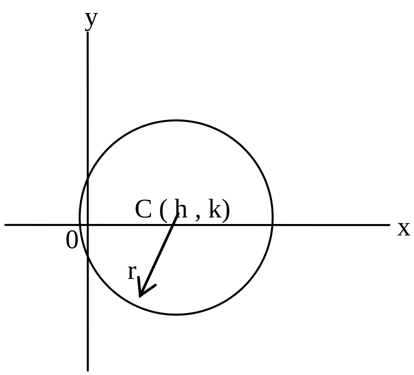
$$\mathbf{Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0}$$

where: A, B, and C are not all zero.

Note: If $B \neq 0$, this will represent a conic section whose axis or axes are not parallel to the x and y axes.

Circle (eccentricity $e \Rightarrow 0$)

A **circle** is the locus of a point moving in a plane in such a way that its distance from a fixed point remains constant



I. General Equation:

$$\mathbf{Ax^2 + Cy^2 + Dx + Ey + F = 0}$$

where: A = C, A and C are with same sign

II. Standard Equations:

1. Center at (h , k)

$$\mathbf{(x - h)^2 + (y - k)^2 = r^2}$$

2. Center at (0,0)

$$\mathbf{x^2 + y^2 = r^2}$$

III. Radius of the circle:

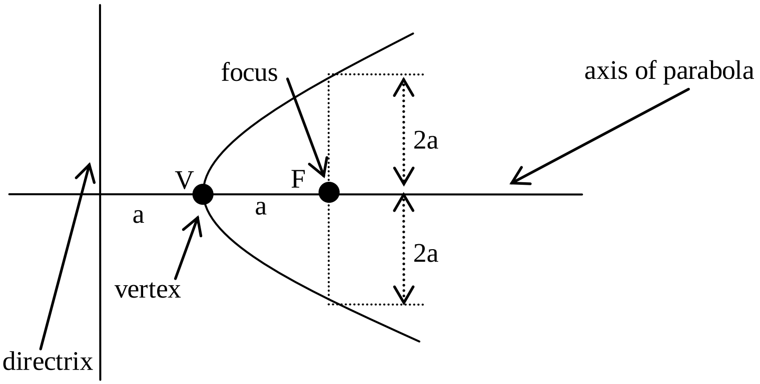
$$\mathbf{r = \sqrt{\frac{D^2 + E^2 - 4FA}{4A^2}}}$$

IV. Abscissa and Ordinate of the center : (h , k)

$$\mathbf{h = -D / 2A \quad , \quad k = -E / 2A}$$

PARABOLA (e = 1)

Parabola is the locus of a point which moves so that the distance from a fixed point called focus and a fixed line called directrix are equal.



where: Focal distance = FV = a
Length of latus rectum (LR) = 4a

I. General Equations:

a. Axis Vertical

$$\mathbf{A\,x^2 + Dx + Ey + F = 0}$$

b. Axis Horizontal

$$\mathbf{Cy^2 + Dx + Ey + F = 0}$$

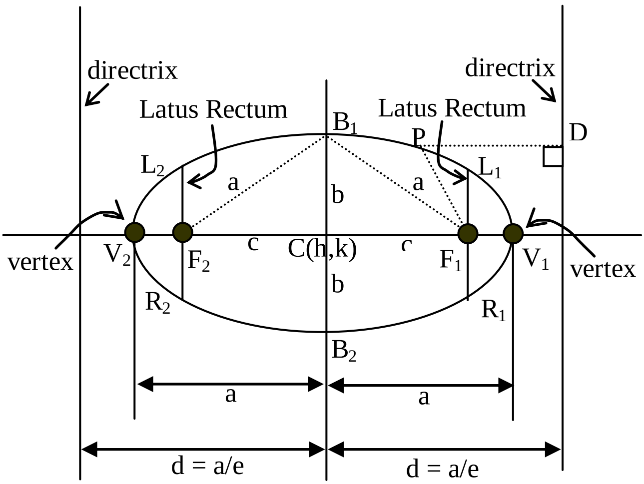
II. Standard Equations:

a. vertex at V (h, k)
1. $(x - h)^2 = 4a\,(y - k)$, opens upward
2. $(x - h)^2 = -4a(y - k)$, opens downward
3. $(y - k)^2 = 4a\,(x - h)$, opens to the right
4. $(y - k)^2 = -4a\,(x - h)$, opens to the left

b. vertex at origin : V (0, 0)
1. $x^2 = 4ay$, opens upward
2. $x^2 = -4ay$, opens downward
3. $y^2 = 4ax$, opens to the right
4. $y^2 = -4ax$, opens to the left

ELLIPSE (e<1)

Ellipse is the locus of a pt. which moves so that the sum of the distances from two fixed points called foci is constant.



$$F_1V_1 + F_2V_1 = F_1V_1 + F_1V_2$$

But: $F_1V_1 = F_2V_2$

Therefore: $\mathbf{F_1V_1 + F_2V_1 = F_1V_1 + F_1V_2 = V_1V_2 = 2a}$

Note : the constant sum = 2a

$$e = \frac{\overline{F_1P}}{\overline{\overline{PD}}} < 1$$

I. General equation:

$$Ax^2 + Cy^2 + Dx + Ey + F = 0$$

where: $A \neq C$ and (A and C have the same sign)

II. Standard Equation:

A. Center at (h, k)

1. $$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1 \quad , \quad \text{MA : Horizontal}$$

2. $$\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1 \quad , \quad \text{MA : Vertical}$$

B. Center at (0, 0)

1. $$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad , \quad \text{MA : Horizontal}$$

2. $$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1 \quad , \quad \text{MA : Vertical}$$

Relations of constants: $\mathbf{a^2 = b^2 + c^2}$

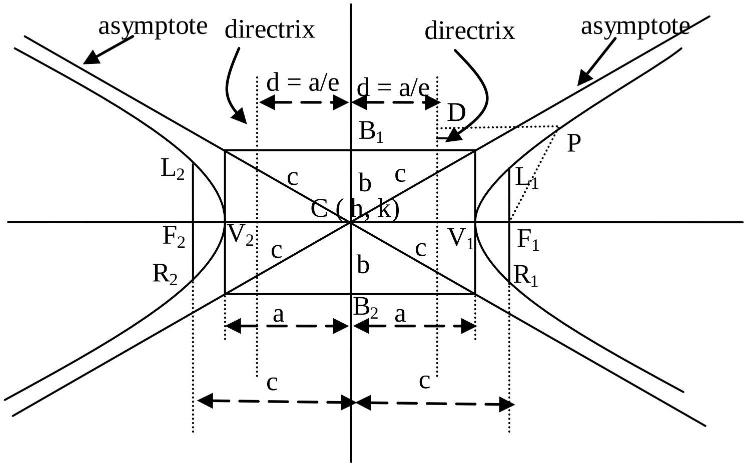
1. Semi - Major Axis = $CV_1 = CV_2 = a$
2. Semi- Minor Axis = $CB_1 = CB_2 = b$
3. $C = CF_1 = CF_2$
4. MA = Major Axis = $V_1V_2 = 2a$
5. ma = minor axis = $B_1B_2 = 2b$
6. $LR = 2b^2 / a$

7. Eccentricity $e = c / a$

8. d = distance of the directrix from the center = a / e

Hyperbola ($e > 1$)

Hyperbola is the locus, of a point which moves so that the difference of its distances from two fixed points is constant.



$$F_2V_2 = F_1V_1$$
$$F_2V_1 - F_1V_1 = V_1V_2 = 2a$$

Where constant length = 2a

$$e = \frac{\overline{F_1P}}{PD} > 1$$

I General Equation:

$$Ax^2 + Cy^2 + Dx + Ey + F = 0$$

where: A and C have opposite signs

II. Standard Form :

A. Center at (h, k)

1. $$\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}=1 \quad , \quad \text{TA : Horizontal}$$

2. $$\frac{(x-h)^2}{b^2}-\frac{(y-k)^2}{a^2}=1 \quad , \quad \text{TA : Vertical}$$

B. Center at (0,0)

1. $$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1 \quad , \quad \text{TA : Horizontal}$$

2. $$\frac{x^2}{b^2}-\frac{y^2}{a^2}=1 \quad , \quad \text{TA : Vertical}$$

Notes:

1. Relations of constants : $a^2 + b^2 = c^2$, $a > b$ or $a = b$ or $a < b$
2. SEMI – TA = $CV_1 = CV_2 = a$
3. SEMI – CA = $CB_1 = CB_2 = b$
4. TA = $V_1V_2 = 2a$
5. CA = $B_1B_2 = 2b$
6. $F_1F_2 = 2c$
7. L. R. = $2b^2 / a$
8. $e = c/a > 1$
9. $d = a / e$
10. Equations of asymptote:

a. $$y-k=\pm\frac{b}{a}(x-h), \text{ for horizontal TA}$$

b. $$y-k=\pm\frac{a}{b}(x-h), \text{ for vertical TA}$$

Legend:

TA --- Transverse axis

CA --- Conjugate axis

DIFFERENTIAL CALCULUS

Differentiation Formulas

1. $\frac{d}{dx}(c) = 0, \quad \text{where c is any constant}$

2. $\frac{d}{dx}(x) = 1$

3. $\frac{d}{dx}\left(u^n\right) = n \bullet u^{n-1}\frac{du}{dx}$

4. $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$

5. $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

6. $\frac{d}{dx}\left(\sqrt{u}\right) = \frac{1}{2\sqrt{u}}\frac{du}{dx}$

7. $\frac{d}{dx}\left(\frac{a}{u^n}\right) = \frac{a(-n)}{u^{n+1}}\frac{du}{dx}$

8. $\frac{d}{dx}\left(e^u\right) = e^u\frac{du}{dx}$

9. $\frac{d}{dx}\left(a^u\right) = a^u(\ln a)\frac{du}{dx}$

10. $\frac{d}{dx}\left(u^v\right) = u^v\left(\frac{v}{u}\frac{du}{dx} + \ln u\frac{dv}{dx}\right)$

11. $\frac{d}{dx}(\ln u) = \frac{1}{u}\frac{du}{dx}$

12. $\frac{d}{dx}(\log_b u) = \frac{1}{u}(\log_b e)\frac{du}{dx}$

13. $\frac{d}{dx}(\sin u) = \cos u\frac{du}{dx}$

14. $\frac{d}{dx}(\tan u) = -\sin u\frac{du}{dx}$

15. $\frac{d}{dx}(\tan u) = \sec^2 u\frac{du}{dx}$

16. $\frac{d}{dx}(\cot u) = \csc^2 u\frac{du}{dx}$

17. $\dfrac{d}{dx}(\sec u) = \sec u \tan u \dfrac{du}{dx}$

18. $\dfrac{d}{dx}(\csc u) = -\csc u \cot u \dfrac{du}{dx}$

19. $\dfrac{d}{dx}(\arcsin u) = \dfrac{1}{\sqrt{1-u^2}} \dfrac{du}{dx}$

20. $\dfrac{d}{dx}(\arccos u) = \dfrac{-1}{\sqrt{1-u^2}} \dfrac{du}{dx}$

21. $\dfrac{d}{dx}(\arctan u) = \dfrac{1}{1+u^2} \dfrac{du}{dx}$

22. $\dfrac{d}{dx}(\operatorname{arc\,cot} u) = \dfrac{-1}{1+u^2} \dfrac{du}{dx}$

23. $\dfrac{d}{dx}(\operatorname{arc\,sec} u) = \dfrac{1}{u\sqrt{u^2-1}} \dfrac{du}{dx}$

24. $\dfrac{d}{dx}(\operatorname{arc\,csc} u) = \dfrac{-1}{u\sqrt{u^2-1}} \dfrac{du}{dx}$

25. $\dfrac{d}{dx}(\sinh u) = \cosh u \dfrac{du}{dx}$

26. $\dfrac{d}{dx}(\cosh u) = \sinh u \dfrac{du}{dx}$

27. $\dfrac{d}{dx}(\tanh u) = \operatorname{sech}^2 u \dfrac{du}{dx}$

28. $\dfrac{d}{dx}(\coth u) = -\operatorname{csch}^2 u \dfrac{du}{dx}$

29. $\dfrac{d}{dx}(\operatorname{sech} u) = -\operatorname{sech} u \tanh u \dfrac{du}{dx}$

30. $\dfrac{d}{dx}(\operatorname{csch} u) = -\operatorname{csch} u \coth u \dfrac{du}{dx}$

31. $\dfrac{d}{dx}(\operatorname{arc\,sinh} u) = \dfrac{1}{\sqrt{1+u^2}} \dfrac{du}{dx}$

32. $\dfrac{d}{dx}(\operatorname{arc\,cosh} u) = \dfrac{1}{\sqrt{u^2-1}} \dfrac{du}{dx}; \quad (u>1)$

33. $\dfrac{d}{dx}(\operatorname{arc\,tanh} u) = \dfrac{1}{1-u^2} \dfrac{du}{dx}; \quad (u^2<1)$

34. $$\frac{d}{dx}\left(\operatorname{arc\,coth} u\right)=\frac{1}{1-u^2}\,\frac{du}{dx}; \quad \left(u^2>1\right)$$

35. $$\frac{d}{dx}\left(\operatorname{arc\,sech} u\right)=\frac{-1}{u\sqrt{1-u^2}}\,\frac{du}{dx}; \quad \left(0<u^2<1\right)$$

36. $$\frac{d}{dx}\left(\operatorname{arc\,csch} u\right)=\frac{-1}{u\sqrt{1+u^2}}\,\frac{du}{dx}; \quad \left(u\neq 0\right)$$

INTEGRAL CALCULUS

Integration Formulas

1. $\int a\,du = a\int du = au + C$, where a = constant
2. $\int u^n\,du = \dfrac{u^{n+1}}{n+1} + C$, where $n \neq -1$
3. $\int \dfrac{du}{u} = \ln u\,du + C$
4. $\int \ln u\,du = u\ln u - u + C$
5. $\int e^u\,du = e^u + C$
6. $\int a^u\,du = \dfrac{a^u}{\ln a} + C$
7. $\int u\,dv = uv - \int v\,du$ (Integration by Parts)
8. $\int \sin u\,du = -\cos u + C$
9. $\int \cos u\,du = \sin u + C$
10. $\int \tan u\,du = \ln \sec u + C = -\ln \cos u + C$
11. $\int \cot u\,du = \ln \sin u + C = -\ln \csc u + C$
12. $\int \sec u\,du = \ln(\sec u + \tan u) + C$
13. $\int \csc u\,du = \ln(\csc u - \cot u) + C = -\ln(\csc u + \cot u) + C$
14. $\int \sec^2 u\,du = \tan u + C$
15. $\int \csc^2 u\,du = -\cot u + C$
16. $\int \sec u \tan u\,du = \sec u + C$
17. $\int \csc u \cot u\,du = -\csc u + C$
18. $\int \dfrac{du}{\sqrt{a^2 - u^2}} = \arcsin \dfrac{u}{a} + C$
19. $\int \dfrac{du}{u^2 + a^2} = \dfrac{1}{a} \arctan \dfrac{u}{a} + C$
20. $\int \dfrac{du}{u\sqrt{u^2 - a^2}} = \dfrac{1}{a}\,\mathrm{arc\,sec}\dfrac{u}{a} + C$
21. $\int \sinh u\,du = \cosh u + C$

22. $\int \cosh u \, du = \sinh u + C$

23. $\int \operatorname{sech}^2 u \, du = \tanh u + C$

24. $\int \operatorname{csch}^2 u \, du = -\coth u + C$

25. $\int \operatorname{sech} u \tanh u \, du = -\operatorname{sech} u + C$

26. $\int \operatorname{csch} u \coth u \, du = -\operatorname{csch} u + C$

27. $\int \frac{du}{u^2 - a^2} = \frac{1}{2a} \ln\left(\frac{u-a}{u+a}\right) + C$, if $u^2 > a^2$

28. $\int \frac{du}{a^2 - u^2} = \frac{1}{2a} \ln\left(\frac{u+a}{u-a}\right) + C$, if $u^2 < a^2$

29. $\int \frac{du}{\sqrt{u^2 - a^2}} = \ln\left(u + \sqrt{u^2 + a^2}\right) + C$

30. $\int \frac{du}{\sqrt{a^2 - u^2}} = \ln\left(u + \sqrt{u^2 + a^2}\right) + C$

31. $\int \sqrt{a^2 - u^2} = \frac{u}{2}\sqrt{a^2 - u^2} + \frac{a^2}{2} \operatorname{arc\,sin} \frac{u}{a} + C$

32. $\int \sqrt{u^2 \pm a^2} = \frac{u}{2}\sqrt{u^2 \pm a^2} \pm \frac{a^2}{2} \ln\left(u + \sqrt{u^2 \pm a^2}\right) + C$

Walli's Formula

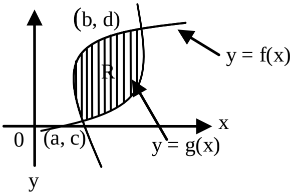
$$\int_0^{\pi/2} \sin^2\theta \cos^n\theta \, d\theta = \left(\frac{[(m-1)(m-3)\ldots(2 \text{ or } 1)][(n-1)(n-3)\ldots(2 \text{ or } 1)]}{(m+n)(m+n-2)(m+n-4)\ldots(2 \text{ or } 1)}\right)\alpha$$

where: m and m are non-negative integers (0, 1, 2, 3, 4, ……)
$\alpha = \pi/2$, if both m and n are even, or one is zero and the other is even
$\alpha = 1$, if otherwise

Note: If m and n equals 1 or 0, apply the following:

Rule: If the factor $(m - 1)$ or $(n - 1)$ in the numerator is 0 or –1, replace the product in which this occurs by unity.

Plane Areas in Rectangular Coordinates System

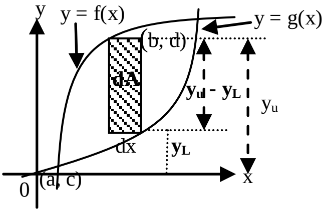


I. Using a vertical rectangular element (vertical stripping)

$$\mathbf{dA = \left(y_u - y_L\right)dx}$$

$$\mathbf{A = \int_a^b \left(y_u - y_L\right)dx}$$

$$\mathbf{A = \int_a^b \left[f(x) - g(x)\right]dx}$$



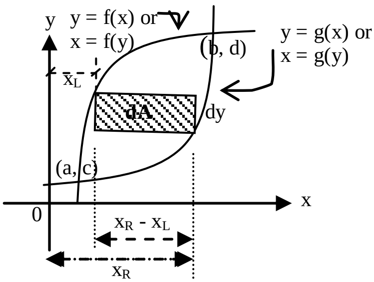
where: U – upper
L – lower

II. Using a horizontal rectangular element (horizontal stripping)

$$\mathbf{dA = \left(x_R - x_L\right)dy}$$

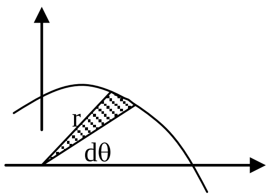
$$\mathbf{A = \int_c^d \left(x_R - x_L\right)dy}$$

$$\mathbf{A = \int_c^d \left[g(y) - f(y)\right]dy}$$

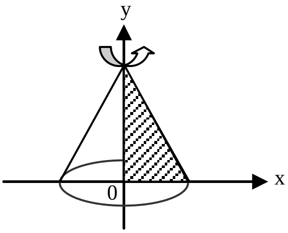


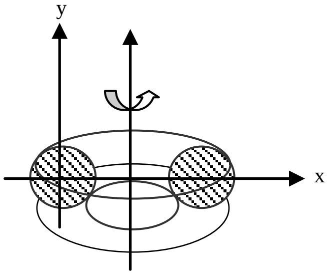
Using Polar Coordinates

$$\mathbf{A = \frac{1}{2}\int_{\theta_1}^{\theta_2} r^2\, d\theta}$$



Volume of Solid of Revolution



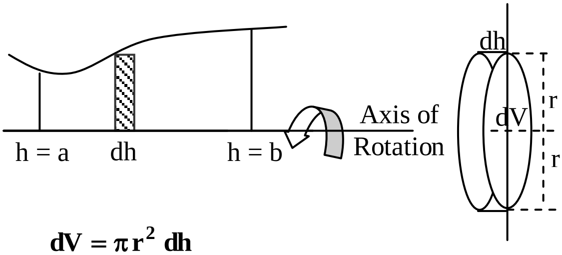


Torus

Methods of Finding the Volumes of Solid of Revolution

I. Disk Method

Rules: 1. The axis of rotation is a part of the boundary of the plane area.
2. The element chosen must be parallel to the axis of rotation.

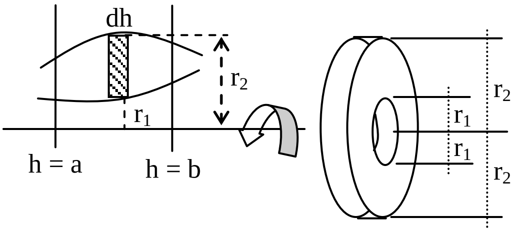


$$\mathbf{dV = \pi r^2\, dh}$$

$$\mathbf{V = \pi \int_a^b r^2\, dh}$$

II. Ring or Washer Method

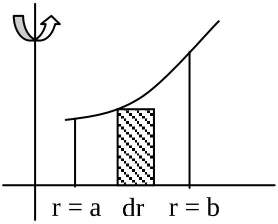
Rules: 1. The axis of rotation is not a part of the boundary of the plane area.
2. The element chosen must be parallel to the axis of rotation.

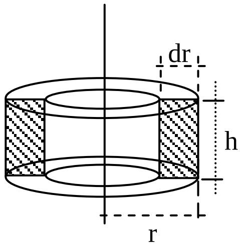


$$dV = \left(\pi r_2^{\,2} - \pi r_1^{\,2}\right)dh$$
$$dV = \pi\left(r_2^{\,2} - r_1^{\,2}\right)dh$$
$$V = \pi\int_a^b \left(r_2^{\,2} - r_1^{\,2}\right)dh$$

III. Cylindrical Shell Method

Rules:
1. The axis of rotation may or may not be a part of the boundary of the rotated area.
2. The element chosen must be parallel to the axis of rotation.



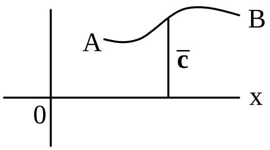


$$dV = 2\pi rh\,dr$$
$$V = 2\pi\int_a^b rh\,dr$$

Pappus Theorem

First Proposition
The surface area of revolution is equal to the length of the generating arc times the circumference of the circle described by the centroid of the arc, provided the axis of revolution does not cross the generating arc.

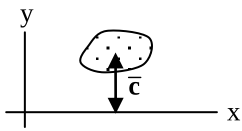
$$A_s = CL$$
$$A_s = 2\pi\bar{c}L$$



where: L – length of the arc

Second Proposition
The volume of the solid revolution is equal to the generating area times the circumference of the circle described by the centroid of the area, provided the axis of revolution does not cross the generating arc.

$$V = AC = A(2\pi\bar{c})$$
$$V = 2\pi\bar{c}A$$

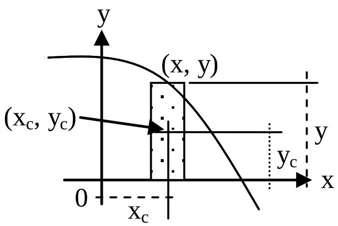


Centroid of a Plane Area

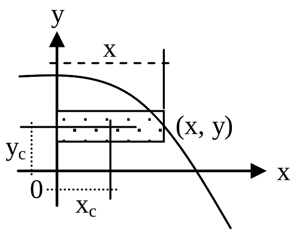
$$A\bar{x} = \int x_c \, dA$$
$$A\bar{y} = \int y_c \, dA$$

where: x_c and y_c are coordinates of the centroid of rectangular element
$\bar{x}$ and $\bar{y}$ are coordinates of the centroid of area A

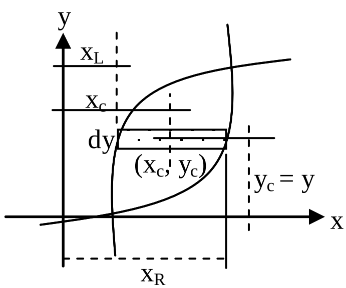
$$x_c = x$$
$$y_c = \frac{1}{2}y$$



$$x_c = \frac{1}{2}x$$
$$y_c = y$$



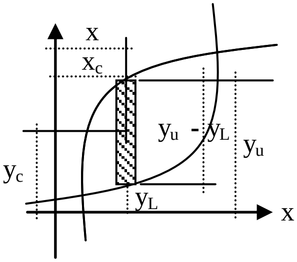
$$y_c = y$$
$$x_c = \frac{x_R - x_L}{2} + x_L$$
$$x_c = \frac{x_R + x_L}{2}$$



$$\mathbf{x_c = x}$$

$$\mathbf{y_c = \frac{y_u - y_L}{2} + y_L}$$

$$\mathbf{y_c = \frac{y_u + y_L}{2}}$$



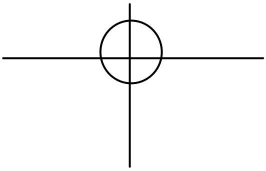
Centroid of a Solid of Revolution

$$\mathbf{V\bar{x} = \int x_c \, dV}$$

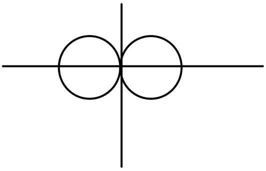
$$\mathbf{V\bar{y} = \int y_c \, dV}$$

dV → SHELL
→ DISK
→ WASHER

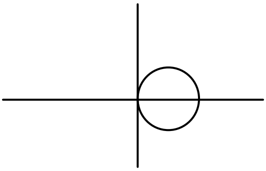
Area of Common Polar Curves



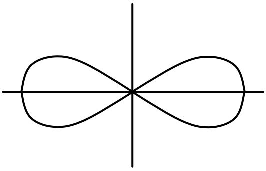
$\mathbf{r = 2a\sin\theta, \quad A = \pi a^2}$



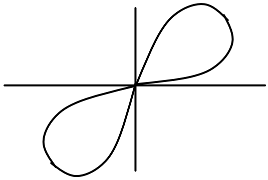
$\mathbf{r^2 = a^2\cos\theta, \quad A = 2\pi a^2}$



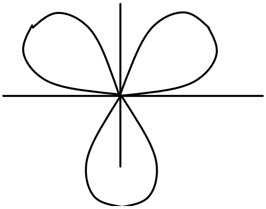
$\mathbf{r = 2a\cos\theta, \quad A = \pi a^2}$



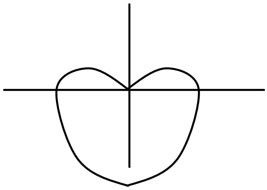
$r^2 = a^2 \cos 2\theta, \quad A = a^2$



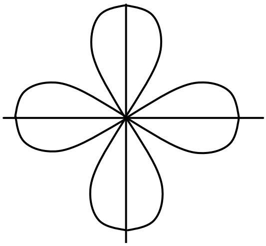
$r^2 = a^2 \sin 2\theta, \quad A = a^2$



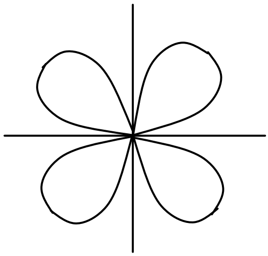
$r = a\sin 3\theta, \quad A = \dfrac{\pi a^2}{12}$



$r = a\left(1 - \sin\theta\right), \quad A = \dfrac{3\pi a^2}{2}$



$r = a\cos 2\theta, \quad A = \dfrac{1}{2}\pi a^2$



$r = a\sin 2\theta, \quad A = \dfrac{1}{2}\pi a^2$

ADVANCE ENGINEERING MATHEMATICS

Complex Numbers:

$$z = x + jy = r(\cos\theta + j\sin\theta) = r\ \text{cjs}\ \theta = r\ \angle\theta = r\ e^{j\theta}$$

where: $j^2 = -1$ or $j = \sqrt{-1}$

Power of Complex Numbers:

$$z^n = (x + jy)^n = r^n(\cos n\theta + j\sin n\theta) = r^n\ \text{cjs}\ n\theta = r^n\ \angle n\theta = r^n\ e^{jn\theta}$$

Roots of Complex Numbers:

Since for any whole number k,

$$\sin(\theta + 360^o k) = \sin\theta$$
$$\cos(\theta + 360^o k) = \cos\theta$$

Therefore in general form,

$$\sqrt[n]{z} = z^{\left(\frac{1}{n}\right)} = r^{\left(\frac{1}{n}\right)}\text{cis}\left(\frac{\theta + 360\,°k}{n}\right) = r^{\left(\frac{1}{n}\right)}\angle\left(\frac{\theta + 360\,°k}{n}\right) = r^{\left(\frac{1}{n}\right)}e^{j\left(\frac{\theta + 2\pi k}{n}\right)}$$

where: k = 0 , when taking the first root
k = 1, when taking the second root
k = 2, when taking the third root
k = n - 1, when taking the n^{th} root

Trigonometric and Hyperbolic Functions of Complex Numbers:

1. $\cosh(j\theta) = \cos\theta$
2. $\sinh(j\theta) = j\sin\theta$
3. $\cos ju = \cosh u$
4. $\sin ju = j\sinh u$
5. $\sin(x \pm jy) = \sin x \cos jy \pm \cos x \sin jy$
   $= \sin x \cosh y \pm j\cos x \sinh y$
6. $\cos(x \pm jy) = \cos x \cos jy \mp \sin x \sin jy$
   $= \cos x \cosh y \mp j\sin x \sinh y$

Laplace Transform:

The Laplace transform of a function f(t), denoted by £ [f(t)], is defined as a function of a variable s by the integral.

$$F(s)=\pounds[f(t)]=\int_0^{\infty} f(t)e^{-st}dt$$

where $t > 0$ and s is any number, real or complex.

Formulas:

1. $\pounds[a]=\frac{a}{s}$
2. $\pounds[t^n]=\frac{n!}{s^{n+1}}, n=1,2,\ldots$
3. $\pounds[e^{\pm at}]=\frac{1}{s\mp a}$
4. $\pounds[\sin at]=\frac{a}{s^2+a^2}$
5. $\pounds[\cos at]=\frac{s}{s^2+a^2}$
6. $\pounds[\sinh at]=\frac{a}{s^2-a^2}$
7. $\pounds[\cosh at]=\frac{s}{s^2-a^2}$
8. $\pounds[t^p]=\frac{\Gamma(p+1)}{s^{p+1}}$

where $p > -1$ and non – integrals

Important Theorems on Laplace Transforms.

Theorem 1. Linearity Theorem

$$\pounds\,[a\, f(t) + b\, g(t)] = a\,\pounds[f(t)] + b\,\pounds[g(t)]$$

Theorem 2. First ShiftingTheorem

$$\pounds\,[e^{at} f(t)] = \pounds[f(t)]_{s\to s-a}$$

Theorem 3. Second Shifting Theorem

$$\pounds\,[f(t-a)\,u(t-a)] = e^{-as}\,\pounds[f(t)]$$

Theorem 4. Transforms of Derivative

$$\pounds[f'(t)] = s\,\pounds[f(t)] - f(0)$$

$$\pounds[f''(t)] = s^2\,\pounds[f(t)] - [s\,f(0) + f'(0)]$$

$$\pounds[f'''(t)] = s^3\,\pounds[f(t)] - [s^2 f(0) + sf'(0) + f''(0)]$$

$$\pounds[f^{(n)}(t)] = s^n\,\pounds\,[f(t)] - [s^{n-1} f(0) + s^{n-2} f'(0) + \ldots + s\, f^{n-2}(0) + f^{n-1}(0)]$$

Theorem 5. Transform of Integral

If f(t) is of exponential order and at least piecewise continuous, then

$$\pounds\left[\int_c^t f(t)dt\right]=\frac{1}{s}\pounds[f(t)]+\frac{1}{s}\int_c^0 f(t)dt$$

Theorem 6.

$$\text{If } \pounds[f(t)] = \phi(s), \text{ then } \pounds[t\, f(t)] = -\phi'(s)$$

Theorem 7.

$$\text{If } \lim_{t \to 0} \frac{f(t)}{t} \text{ exists, and if } \pounds[f(t)] = \phi(s), \text{ then } \pounds\left[\frac{f(t)}{t}\right] = \int_0^\infty \phi(s)\,ds$$

Inverse Laplace Transform:

$$\text{If } \pounds[f(t)] = F(s), \text{ then } f(t) = \pounds^{-1}[F(s)] \text{ provided that } \pounds[f(t)] \text{ exists}$$

Gamma Function:

The gamma function denoted by $\Gamma(n)$ is defined by $\int_0^\infty t^{n-1} e^{-t}\,dt$ which is convergent for $n > 0$. A recursion or recurrence formula for the gamma function is $\boldsymbol{\Gamma(n+1) = n\Gamma(n) = n!}$

Notes:

1. If $n < 0$ but $n \neq -1, -2, -3, \ldots$..
   Use $\Gamma(n) = \dfrac{\Gamma(n+1)}{n}$
2. $\Gamma(1/2) = \sqrt{\pi}$
3. For $n = 0, -1, -2, -3, \ldots\ldots\ldots$
   $\Gamma(n) = \infty$

Sequence and Series:

Sequence of Numbers – defined as a succession of numbers formed according to some fixed rule.

For example:

n^{th} term

1. 2, 4, 6, 8, 10, 12,………., $2n$,
2. 1, 4 , 9, 16, 25,…………, n^2,….

Series – defined as the indicated sum of a sequence of numbers.

For example, for the sequence $a_1, a_2, a_3, \ldots\ldots, a_n$

The corresponding series is $a_1 + a_2 + a_3 + \ldots\ldots + a_n$

Type of Series:

1. **Finite Series** – the number of terms is limited.
2. **Infinite Series** – the number of terms is unlimited

Power Series:

A power series in x – a has the form

$$\sum_{n=0}^{+\infty} c_n (x-a)^n = c_0 + c_1(x-a) + c_2(x-a)^2 + \dots + c_n(x-a)^n + \dots$$

When a = 0, and the series become a power in x, which is

$$\sum_{n=0}^{+\infty} C_n \mathbf{x}^n = C_0 + C_1\mathbf{x} + C_2\mathbf{x}^2 + \dots + C_n\mathbf{x}^n + \dots$$

For example:

$$\frac{\mathbf{1}}{\mathbf{1-x}} = \mathbf{1 + x + x^2 + x^3} + \dots$$

General Method: for expanding a function in a power series in x and in (x – a) is given below. Note the requirement that the function and its derivatives of all orders must exist at x = 0 or at x = a.

This $\frac{\mathbf{1}}{\mathbf{x}}$, ln x, and cot x cannot be expanded in power of x.

Maclaurin's Series – power series expansion of f(x) about x = 0.
– power series in x.

$$\mathbf{f(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 + \dots + \frac{f^n(0)}{n!}x^n}$$

OR

$$\mathbf{f(x) = \sum_{n=0}^{\infty} \frac{f^n(0)}{n!}x^n}$$

Taylor's Series – power series in x – a.

- power series expansion of f(x) about x = a. This series, which includes Maclaurin's series as a special case (a = 0).

$$f(x)=f(a)+f'(a)(x-a)+\frac{f''(a)}{2!}(x-a)^2+\frac{f'''(a)}{3!}(x-a)^3+\ldots+\frac{f^n(a)}{n!}(x-a)^n+\ldots$$

OR

$$f(x)=\sum_{n=0}^{\infty}\frac{f^n(a)}{n!}(x-a)^n$$

Fourier Series – is a series used to represent a periodic wave in either exponential of trigonometric form. The trigonometric form is in terms in sine and cosine functions. The series has the form,

$$F(t) = \frac{A_0}{2} + A_1 \cos t + A_2 \cos 2t + A_3 \cos 3t + \ldots\ldots + A_n \cos nt + B_1 \sin t$$
$$+ B_2 \sin 2t + B_3 \sin 3t + \ldots + B_n \sin nt$$

For a particular period wave, the coefficient of the series are determined by means of the following equations.

$$A_o = \frac{1}{\pi}\int_0^{2\pi} F(t)\,dt$$

$$A_n = \frac{1}{\pi}\int_0^{2\pi} F(t)\cos nt\,dt$$

$$B_n = \frac{1}{\pi}\int_0^{2\pi} F(t)\sin nt\,dt$$

PROBABILITY AND STATISTICS

Arithmetic Mean (A. M.)

$$\mathbf{A.M.} = \frac{\mathbf{a_1 + a_2 + a_3 \ldots\ldots + a_n}}{\mathbf{n}}$$

Median is the middle value when all data are arranged in increasing or decreasing order.

Mode is the value that occurs most frequently.

Range

$$\textbf{Range} = \textbf{Maximum Value} - \textbf{Minimum Value}$$

Variance

The variance of a set of numbers is defined by

$$\sigma^2 = \frac{\sum (x_i - \bar{x})^2}{n}$$

where: $\bar{x}$ – arithmetic mean

Standard Deviation

$$\textbf{Standard Deviation} = \sqrt{\frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n}}$$

OR

$$\textbf{Standard Deviation} = \sqrt{\frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n-1}}$$

Note: When $n < 30$, $(n - 1)$ is more exact.

Fundamental Principle

If an event can happen in any one of n_1 ways, and if this has occurred, another event can happen in one of n_2 ways, then the number of ways in which both events can happen in the specified order is $n_1 n_2$.

In general, for k events,

$$\mathbf{n_T = n_1 \bullet n_2 \bullet n_3 \ldots\ldots n_k}$$

Permutation (P)
The grouping of things in a definite order. To permute a set of things means to arrange them in a definite order.

1. Permutation of n different elements taken r at a time

$$\mathbf{{}_nP_r = \frac{n!}{(n-r)!}}$$

Note: ${}_nP_r = P(n, r) = P_r^{\,n}$

Illustration:
For letters a, b, c, the number of permutation taken two (2) at a time is

$$\mathbf{{}_3P_2 = \frac{3!}{(3-2)!} = 6}$$

Another way,

ab, ba, ac, ca, bc, cb = 6

OR by Fundamental Principle,

$$\mathbf{3 \bullet 2 = 6}$$

2. Permutation of n different elements taken all (r = n) at a time

$$\mathbf{{}_nP_n = n!}$$

Illustration:
For a, b, c, the number of permutation taken all at a time is

$$\mathbf{{}_3P_3 = 3! = 6}$$

Another way,

abc, acb, bac, bca, cba, cab = 6

OR by Fundamental Principle

$$\mathbf{3 \bullet 2 \bullet 1 = 6}$$

3. Permutation of n elements some of which are alike

$$_{n}P_{n-s} = \frac{n!}{[n-(n-s)]!} = \frac{n!}{s!}$$

where: s – the number of times the element is repeated in the set

Illustration:
For letters a, a, c, the number of permutation taken all at a time is

$$P = \frac{n!}{s!} = \frac{3!}{2!} = 3$$

Another way,

aac, aca, caa = 3

4. Permutation of n elements not all different taken all at a time

$$P = \frac{n!}{n_1!n_2!n_3!\ldots\ldots n_k!}$$

where: n_1, n_2, n_3, n_k – number of elements which are alike
n – total number of elements in a given set

5. Theorem on Partitioning. The number of ways of partitioning a set of n objects into r cells with n_1 elements in the first cell, n_2 elements in the second cell and so forth is,

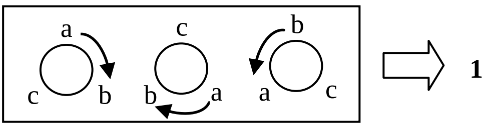
$$\begin{pmatrix} n \\ n_1, n_2, \ldots\ldots, n_r \end{pmatrix} = \frac{n!}{n_1!\,n_2!\ldots\ldots n_r!}$$

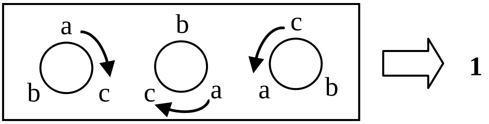
where: $n = n_1 + n_2 + \ldots\ldots\ldots\ldots + n_r$

6. Cyclic Permutations. The number of permutations of n different objects arranged in a circle is,

$$P_c = (n-1)!$$

Illustration: For the letters a, b, c arranged in a circle.





$$\mathbf{P_c = 1 + 1 = 2}$$

By Formula,

$$\mathbf{P_c = (3-1)! = 2! = 2 \bullet 1 = 2}$$

Combination (C). A selection of things considered without regard to order or the grouping of things where arrangement is immaterial.

1. Combination of n objects taken r at a time

$$\mathbf{{}_nC_r = \frac{{}_nP_r}{r!} = \frac{n!}{r!(n-r)!}}$$

Note: ${}_nC_r = C\ (n, r) = C_r^{\ n}$

2. Combination of n objects taken all (n = r) at a time

$$\mathbf{{}_nC_n = 1}$$

3. Combinations that can be made taking successively 1 at a time, 2 at a time, 3 at a time and so on up to n at a time

$$\mathbf{C = {}_nC_1 + {}_nC_2 + {}_nC_3 + \ldots\ldots + {}_nC_n = 2^n - 1}$$

Probability (p)

$$\mathbf{Pr\ obability = \frac{number\ of\ favorable\ outcomes}{number\ of\ possible\ outcomes}}$$

Probability of Success + Probability of Failure = 1

1. **Probability in Single Event.** If an event can happen in h ways and can fail in f ways are equally likely, then in a single tr5ial, the probability will happen that it will happen is given by,

$$\mathbf{p = \frac{h}{h+f}}$$

and the probability that it will fail is given by,

$$\mathbf{p = \frac{f}{h + f}}$$

2. **Mutually Exclusive Events.** Two or more events are mutually exclusive if not more than one of them can happen in a given trial. The probability that some one or other of a set of mutually exclusive events will happen in a single trial is the sum of their separate probabilities of happening.

$$\mathbf{p = p_1 + p_2 + \ldots\ldots + p_n}$$

3. **Independent Events.** Two or more events are said to be independent if the happening of one does not affect the happening of the others. The probability that two or more independent events will happen is the product of their separate probabilities.

$$\mathbf{p = p_1 \bullet p_2 \bullet \ldots\ldots \bullet p_n}$$

4. **Dependent Events.** Two or more events are said to be dependent if the happening of one affects the probability that the other will happen. If p_1 is the probability that an event will happen, and after it has happened the second will occur with probability p_2, then the probability that the first event and then the second event will happen is the product of p_1 and p_2.

5. **Probability for Repeated Trials (Binomial Density Distribution)**

$$\mathbf{p = {}_nC_r P_1^r \left(1 - p_1\right)^{n-r}}$$

where: n – number of trials
r – number of desired successful outcome
p_1 – probability of a successful outcome in a trial