Lecture: Data Analysis and Machine Learning Theory

KTH AI Student

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About me

- Jag heter Martín! I am from Chile, did my Bachelor's Degree at Universidad de Chile, doing a year-long exchange at KTH.
- I am currently interning at Hopsworks, an AI Lakehouse in Stockholm. Working on ML pipelines and LLMs.
- I enjoy running, hiking, and trying to improve my awful Swedish.

In this lecture

- We will review key concepts in data analysis and machine learning theory.
  - Descriptive Statistics and Data Visualization.
  - Probability Theory and Simulation.
  - Correlation and Regression Analysis.
  - A/B Testing and Hypothesis Testing.

Question 1

Imagine we have a big dataset, and we want to summarize it.
What are some ways we can do this?

Example: Student Test Scores

- **Dataset:** Contains scores of students.
- **Goals:**
  - Compute key descriptive statistics to summarize performance.
  - Visualize score distributions to identify trends or outliers.
  - Provide actionable insights to improve teaching methods.

Key Concepts

Descriptive Statistics: Summarize and describe the main features of a dataset.

- **Mean**: The average value of a dataset.

$$\bar{x} = \frac{1}{n} \sum_{i=1}^{n} x_i$$

- **Median**: The middle value when data is sorted.

$$x_{\text{median}} = \begin{cases} x_{(n+1)/2} & \text{if } n \text{ is odd} \\ \frac{1}{2}\left(x_{n/2} + x_{n/2+1}\right) & \text{if } n \text{ is even} \end{cases}$$

- **Mode**: The most frequently occurring value.

$$x_{\text{mode}} = \text{value with highest frequency}$$

Key Concepts

- **Variance**: Measures the spread of data points from the mean.

$$\sigma^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2$$

- **Standard Deviation**: Square root of variance, represents data dispersion.

$$\text{SD} = \sqrt{\sigma^2}$$

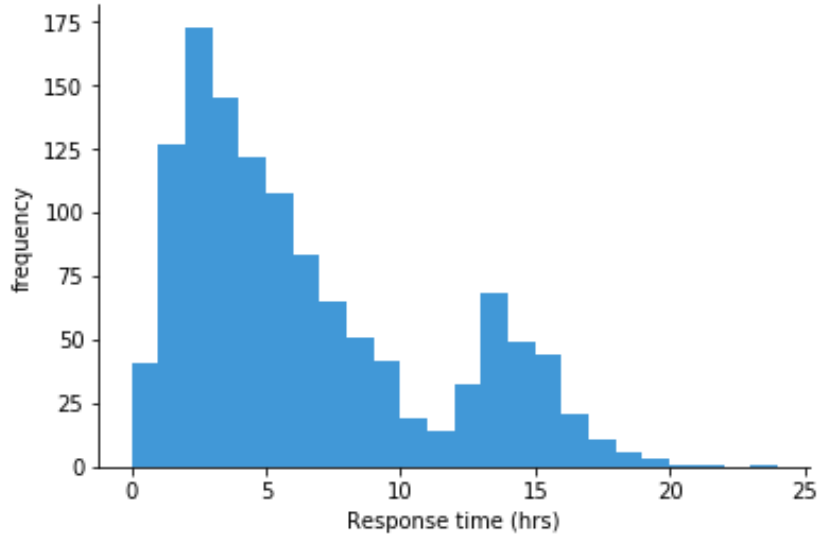
- **Range**: Difference between the maximum and minimum values.

$$\text{Range} = \max(x) - \min(x)$$

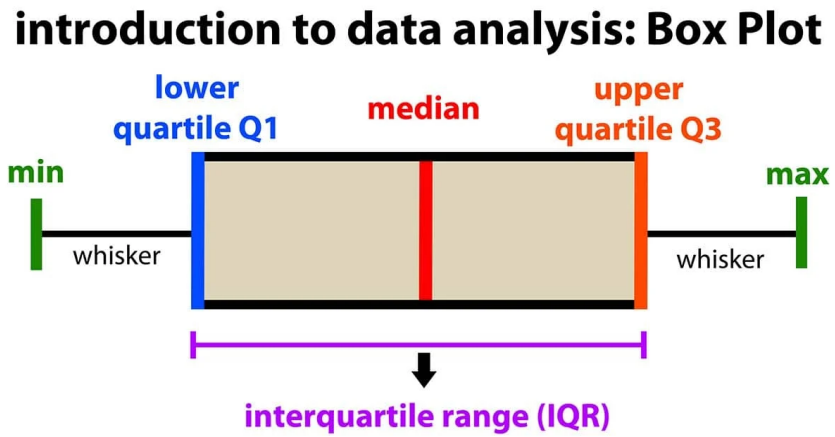
Key Concepts

Data Visualization: Graphical representation of data.

- **Histograms**: Show frequency distribution of data.

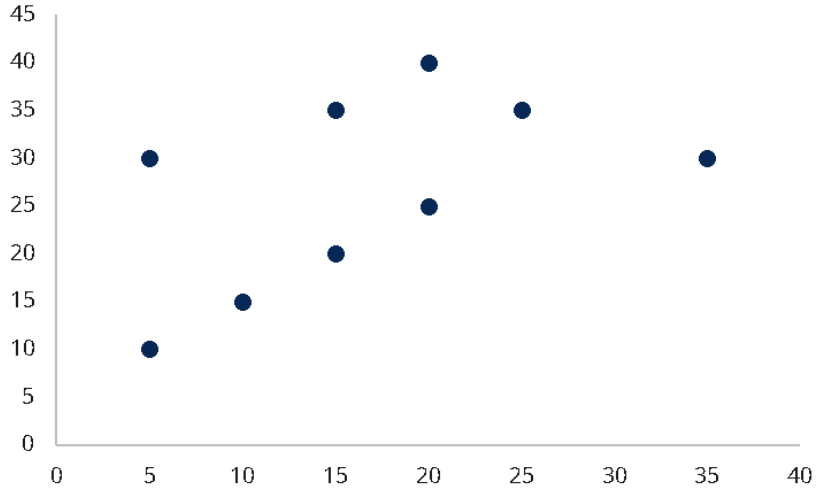


- **Box Plots**: Visualize data spread and identify outliers.

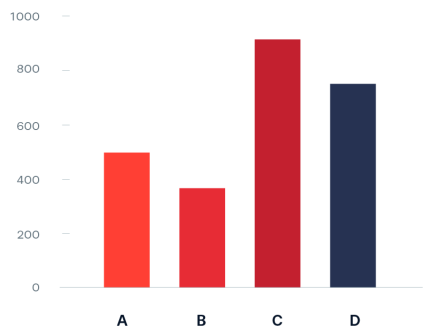


Key Concepts

- **Scatter Plots**: Display relationships between two variables.



- **Bar Charts**: Compare categorical data.



Question 2

We have a dataset, but does this dataset represent the real world?
How can we estimate the probability of events?

Example: Simulation Tasks

- Simulate 1000 coin tosses to calculate the probability of heads and compare with theoretical value.
- Simulate 1000 dice rolls to calculate:
  - Probability of rolling a prime number.
  - Conditional probability of a prime given the number is odd.
- Use Monte Carlo simulation to estimate π.

Key Concepts: Probability

- **Probability:** Study of the likelihood of events.
  - **Theoretical Probability:** Based on known outcomes (e.g., coin toss).

$$P(A) = \frac{\text{Number of favorable outcomes}}{\text{Total number of outcomes}}$$

  - **Simulated Probability:** Estimated by running experiments or simulations.
  - **Bayes' Theorem:** Describes conditional probability, updates beliefs based on evidence.

$$P(A|B) = \frac{P(B|A) \cdot P(A)}{P(B)}$$

Key Concepts: Probability Distributions

- **Probability Distributions:** Represent how probabilities are distributed over values.
  - **Uniform Distribution:** All outcomes are equally likely.

$$P(x) = \frac{1}{n} \quad \text{for } x \in \{1, 2, \ldots, n\}$$

  - **Binomial Distribution:** Number of successes in fixed trials.

$$P(X = k) = \binom{n}{k} p^k (1-p)^{n-k}$$

  - **Normal Distribution:** Bell-shaped curve, common in natural data.

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

Key Concepts: Monte Carlo Simulation

- **Monte Carlo Simulation:** Uses random sampling to estimate mathematical results.
  - Example: Estimate π by generating random points in a square and calculating the ratio inside a quarter circle.

$$\pi \approx 4 \times \frac{\text{Number of points inside circle}}{\text{Total number of points}}$$

Question 3

We have two variables, how can we determine if they are related?
How can we predict one variable based on the other?

Example: Car Prices and Mileage

- **Dataset:** Contains car prices and mileage.
- **Tasks:**
  - Compute the correlation coefficient to assess the strength and direction of the relationship.
  - Build a simple linear regression model to predict prices based on mileage.
  - Visualize the data and regression line to interpret the results.

Key Concepts: Correlation

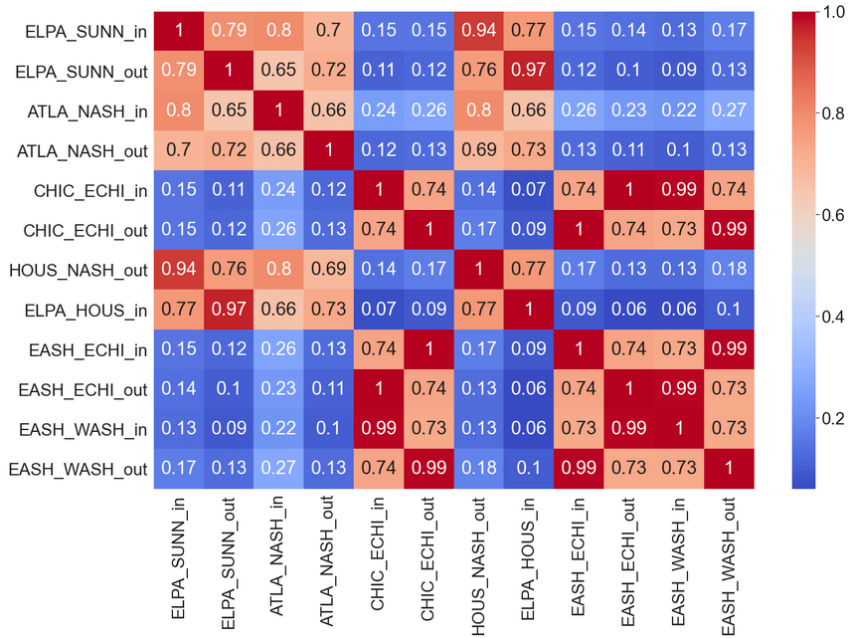
- **Correlation:** Measures the strength and direction of the linear relationship between two variables.
  - **Range:** Values range from -1 to 1.
  - **Interpretation:**
    - 1: Perfect positive correlation.
    - -1: Perfect negative correlation.
    - 0: No linear correlation.

Key Concepts: Correlation

- **Correlation Coefficient:** Denoted by r.

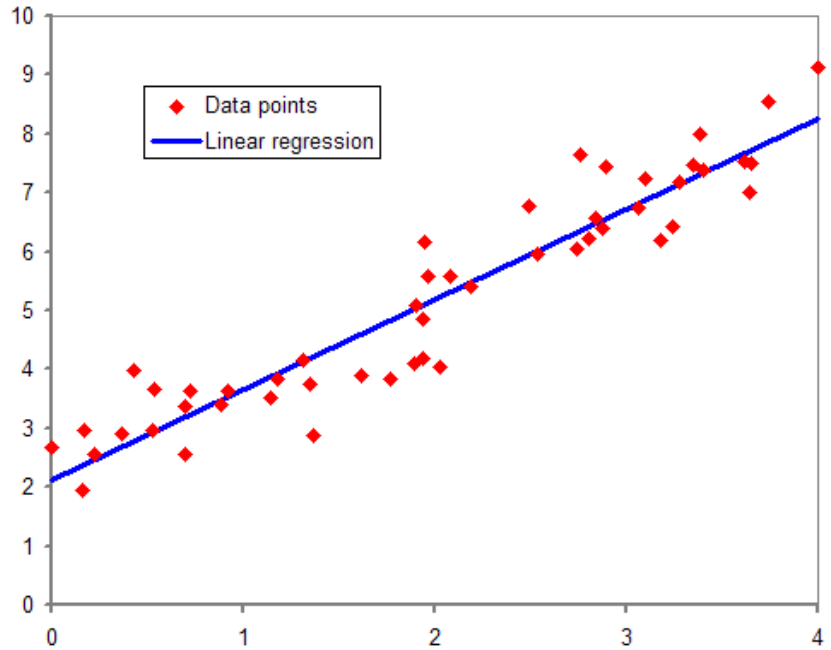
$$r = \frac{\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^{n}(x_i - \bar{x})^2}\sqrt{\sum_{i=1}^{n}(y_i - \bar{y})^2}}$$

- **Correlation Matrix:** Displays pairwise correlations between variables.



Key Concepts: Regression Analysis

- **Regression Analysis:** Models the relationship between a dependent variable and one or more independent variables.
  - **Simple Linear Regression:** $y = \beta_0 + \beta_1 x + \epsilon$
  - **Goals:**
    - Estimate the coefficients (β_0, β_1).
    - Minimize prediction error (ϵ).
  - **Evaluation Metrics:** Assess model fit using metrics such as Mean Squared Error (MSE).



Question 4

We have two groups, how can we determine if they are significantly different? How can we validate our assumptions?

Example: Website Redesign A/B Test

- **Dataset:** User engagement metrics for old and new designs.
- **Tasks:**
  - Perform a t-test to compare engagement levels.
  - Calculate and interpret the p-value.
  - Determine whether the new design significantly improves engagement.

Key Concepts: Hypothesis Testing

- **Hypothesis Testing:** Framework to evaluate whether observed data provides sufficient evidence to reject a null hypothesis (H_0).
  - **Null Hypothesis (H_0):** Assumes no effect or difference.
  - **Alternative Hypothesis (H_a):** Suggests a significant effect or difference.
- **t-Test:** Compares means of two groups.
  - **t-statistic:** Quantifies the difference relative to variability.
  - **p-value:** Probability of observing results as extreme as the data, assuming H_0 is true.
- **Significance Level:** Common threshold $\alpha = 0.05$.

Key Concepts: Hypothesis Testing

- **Interpretation:**
  - **p-value** $< \alpha$**:** Reject H_0, evidence supports H_a.
  - **p-value** $\geq \alpha$**:** Fail to reject H_0, insufficient evidence.
- **Type I Error:** Incorrectly reject H_0 (false positive).
- **Type II Error:** Incorrectly fail to reject H_0 (false negative).
- **Power:** Probability of correctly rejecting H_0.

Key Concepts: Hypothesis Testing

How to perform a t-test:

1. Define null and alternative hypotheses.
2. Choose a significance level α.
3. Calculate the t-statistic.

$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

4. Calculate the degrees of freedom.

$$\text{df} = \frac{(s_1^2/n_1 + s_2^2/n_2)^2}{\frac{(s_1^2/n_1)^2}{n_1 - 1} + \frac{(s_2^2/n_2)^2}{n_2 - 1}}$$

5. Calculate the p-value.
6. Make a decision based on the p-value.

Summary

- Reviewed essential concepts in data analysis and machine learning:
  - Descriptive statistics and visualization to summarize and understand data.
  - Probability and simulation to estimate theoretical and practical outcomes.
  - Regression analysis to model relationships and make predictions.
  - Hypothesis testing to assess differences and validate assumptions.
- Emphasized critical thinking and interpretation of results for data-driven decisions.

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