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CSC137
HW3

Problem 2.14

a) $f(w,x,y,z) = \Sigma(0,2,8,10) + \Sigma_d(12,14)$

K-map:

wx \ yz	00	01	11	10
00	1			1
01				
11	d			d
10	1			1

Minimal SOP: $\boxed{f(w,x,y,z) = \bar{w}\bar{x}\bar{z} + w\bar{z}}$

b $g(a,b,c,d) = \Sigma(5,7,13,15) + \Sigma_d(6,14)$

ab \ cd	00	01	11	10
00				
01		1	1	d
11		1	1	d
10				

Minimal SOP: $\boxed{g(a,b,c,d) = bd + b.c}$

$\boxed{c)}$ $h(w,x,y,z) = \Pi(0,2,8,10) + \Pi_d(12,14)$

wx \ yz	00	01	11	10
00	0			0
01				
11	d			d
10	0			0

$$\text{Pos of } h = (w+x+z)(\bar{w}+z)$$

$$\cancel{\Rightarrow \text{dual of } \bar{h} = (\bar{w}+\bar{x}+\bar{z})(w+\bar{z})}$$

$$\cancel{\Rightarrow \text{SoP of } \bar{h} = (\bar{w}.\bar{x}.\bar{z}) + (w.\bar{z})}$$

$$\rightarrow \text{SOP of } h = w.\bar{w} + w.z + x.\bar{w} + xz + z\bar{w} + z.z$$

$$= \boxed{w.z + x.\bar{w} + xz + z\bar{w} + z}$$

$\boxed{d)}$ $t(a,b,c,d) = \Pi(5,7,13,15) + \Pi_d(6,14)$

ab \ cd	00	01	11	10
00				
01		0	0	d
11		0	0	d
10				

$$\text{Pos of } t = (\bar{b}+\bar{d})(\bar{b}+\bar{c})$$

$$\rightarrow \text{SOP of } t = \underbrace{\bar{b}.\bar{b}}_{0} + \bar{b}.\bar{c} + \bar{d}.\bar{b} + \bar{d}.\bar{c}$$

$$= \boxed{\bar{b}.\bar{c} + \bar{d}.\bar{b} + \bar{d}.\bar{c}}$$

Problem 2.24

Design POS expression for the 2-to-1 MUX in Fig 2.31

$r(s, x, y) = \Sigma(1, 3, 6, 7)$

Kmap:

s \ xy	00	01	11	10
0		1	1	
1			1	1

SOP expression: $r = \bar{s}y + sx$

$\bar{r} = \overline{\bar{s}y + sx}$

$= (\overline{\bar{s}y}) \cdot (\overline{sx})$

$= (s + \bar{y}) \cdot (\bar{s} + \bar{x})$

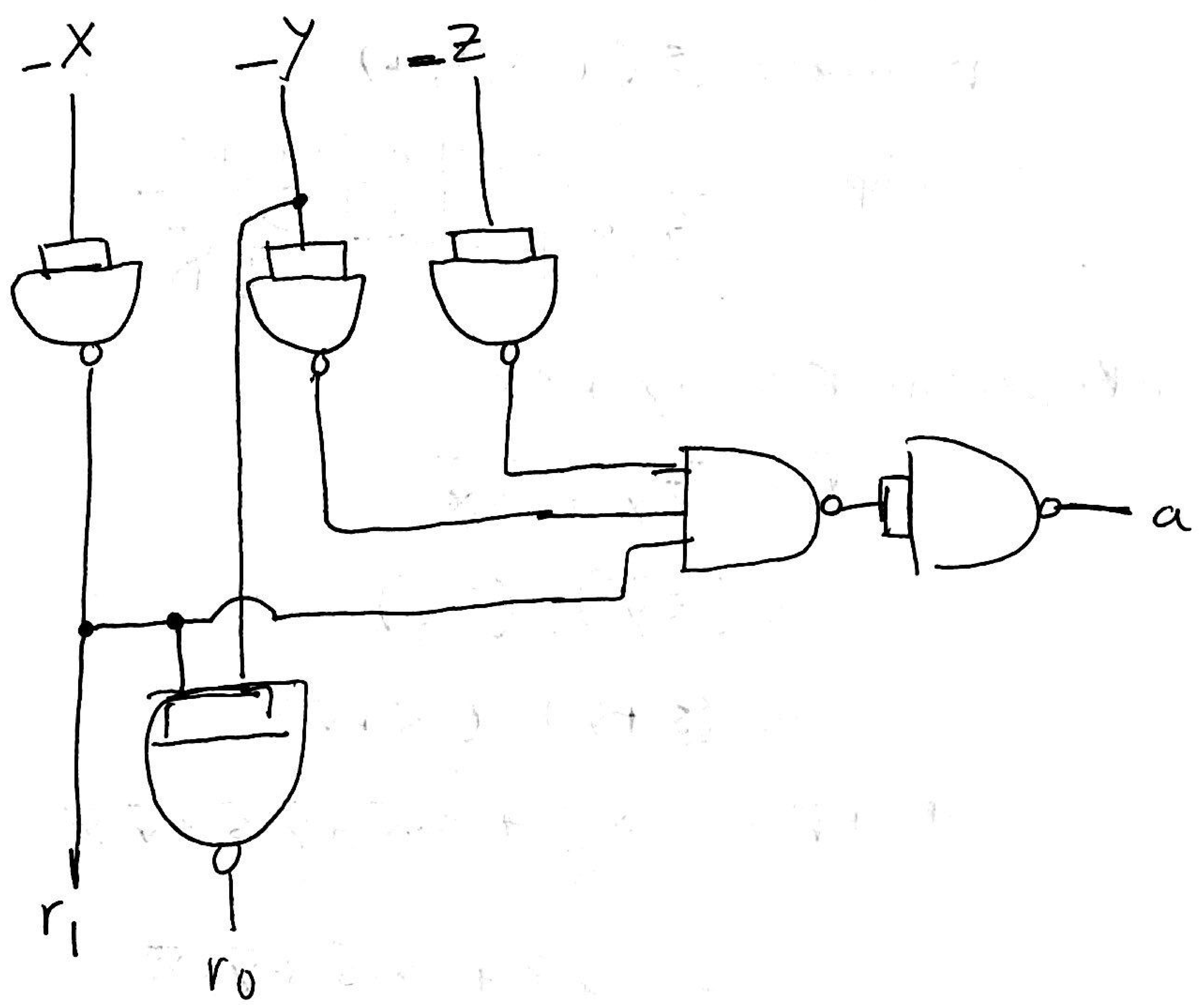
SOP of $\bar{r}$ = $\underbrace{s \cdot \bar{s}}_{0} + s \cdot \bar{x} + \bar{y} \cdot \bar{s} + \bar{y} \cdot \bar{x}$

$= s \cdot \bar{x} + \bar{y} \cdot \bar{s} + \bar{y} \cdot \bar{x}$

Dual of $\bar{r}$ = $(s + \bar{x})(\bar{y} + \bar{s})(\bar{y} + \bar{x})$

$\boxed{\text{POS expression of } r = (\bar{s} + x)(y + s)(y + x)}$

Problem 2.27



Problem 3.2

a)

$$
\begin{array}{lr}
A: & 11001100 \\
+\ B: & 00110100 \\
\hline
 & \boxed{1\,00000000}
\end{array}
$$

carryout (not overflow bit)

d)

$$
\begin{array}{lr}
A: & 11111000 \\
-\ B: & 00001000 \\
\hline
 & ?
\end{array}
$$

$$
\begin{array}{lr}
(B)_{1s} = & 11110111 \\
+ & 1 \\
\hline
-(B)_{2s} = & 11111000
\end{array}
$$

$\Rightarrow$

$$
\begin{array}{lr}
A: & 11111000 \\
+\ B: & 11111000 \\
\hline
 & \boxed{1\,11110000}
\end{array}
$$

carryout = 1 (not overflow bit)

e)

$$
\begin{array}{lr}
A: & 10000010 \\
-\ B: & 00000011 \\
\hline
 & ?
\end{array}
$$

$$
\begin{array}{lr}
(B)_{1s} = & 11111100 \\
+ & 1 \\
\hline
-(B)_{2s} = & 11111101
\end{array}
$$

$\Rightarrow$

$$
\begin{array}{lr}
A: & 10000010 \\
+\ B: & 11111101 \\
\hline
 & \boxed{1\,01111111}
\end{array}
$$

carryout = 1 (not overflow bit)

f)

$$
\begin{array}{lr}
A: & 01111101 \\
-\ B: & 11111010 \\
\hline
 & ?
\end{array}
$$

$$
\begin{array}{lr}
(B)_{1s} = & 00000101 \\
+ & 1 \\
\hline
-(B)_{2s} = & 00000110
\end{array}
$$

$\Rightarrow$

$$
\begin{array}{lr}
A: & 0111101 \\
+\ B: & 00000110 \\
\hline
 & \boxed{10000011}
\end{array}
$$

Problem 3.4

$$\Delta CPA(n) = (n-1) * \Delta FA_C + \Delta FA_S$$

With $\Delta FA_C = 0.5ns$, $\Delta FA_S = 0.3ns$

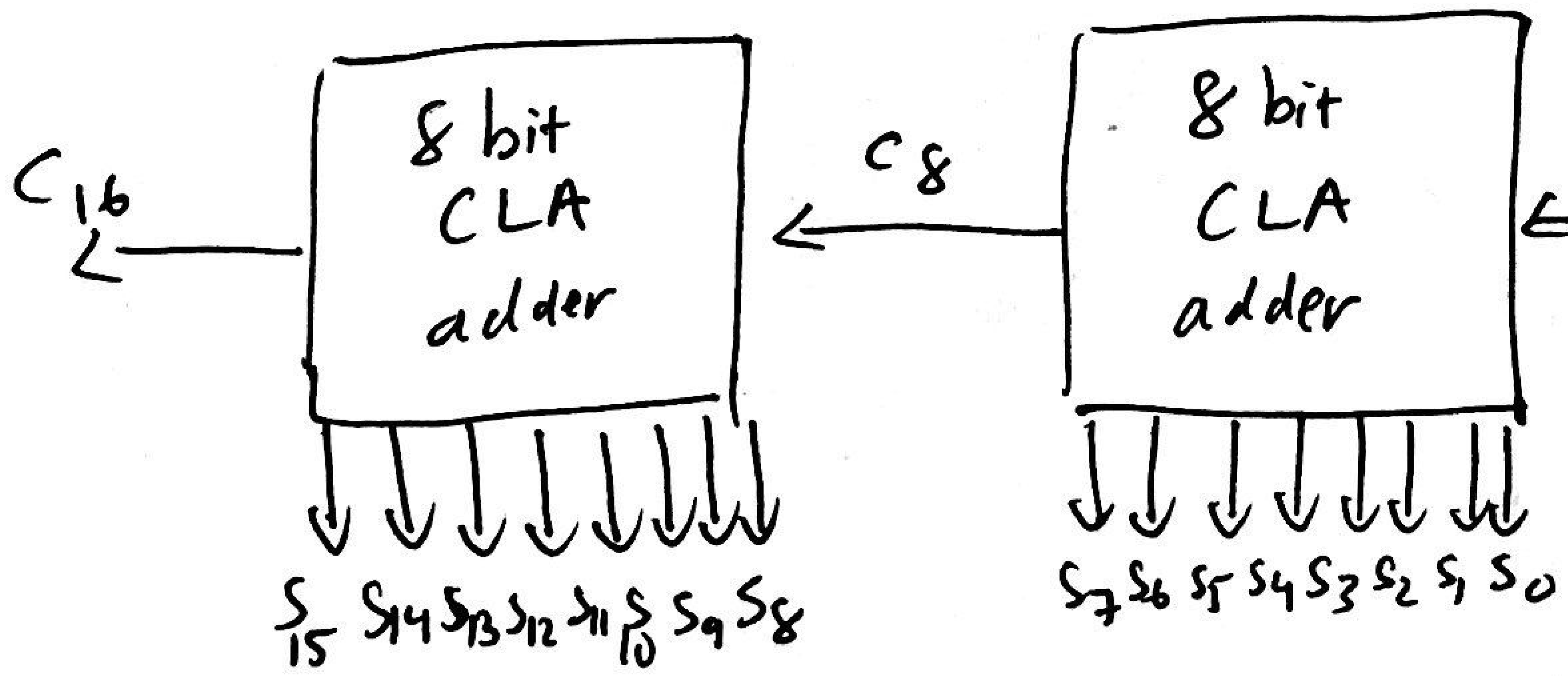
we have $\Delta CPA(8) = \Delta CPA(4) + \Delta CPA(4)$

$$= \left[(4-1)(0.5ns) + 0.3ns\right] + \left[(4-1)(0.5ns) + 0.3ns\right]$$

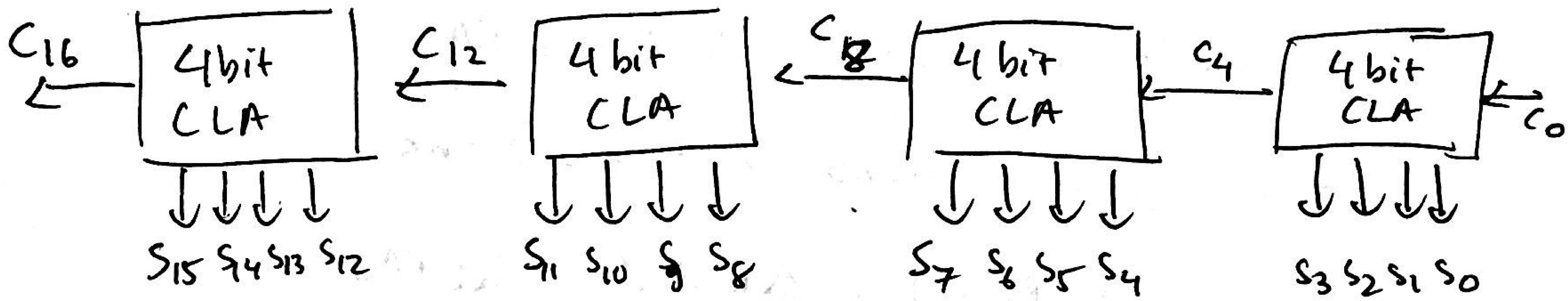
$$= 3.6ns$$

Problem 3.5

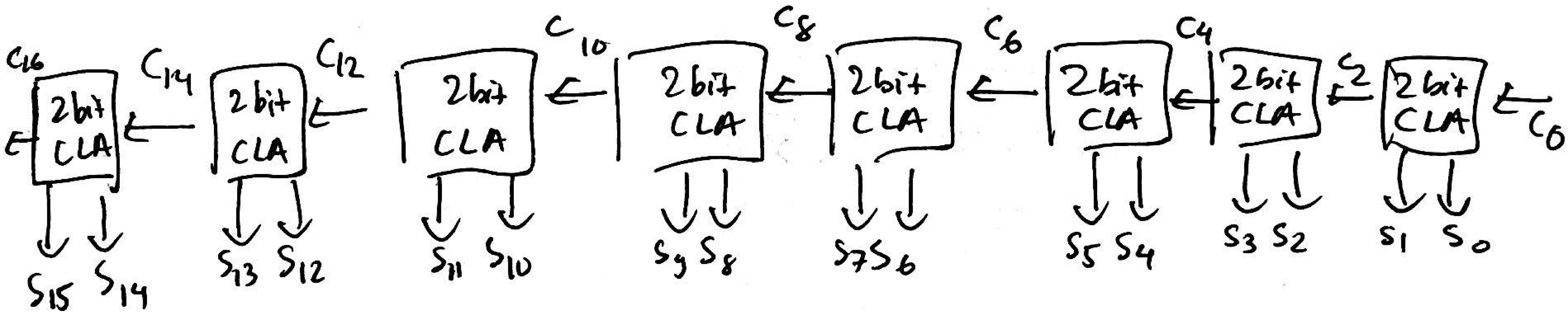
a)



b)


C_{16}
4bit CLA
C_{12}
4 bit CLA
C_8
4 bit CLA
C_4
4bit CLA
C_0
S_{15} S_{14} S_{13} S_{12}
S_{11} S_{10} S_9 S_8
S_7 S_6 S_5 S_4
S_3 S_2 S_1 S_0

c)


C_{16}
2bit CLA
C_{14}
2 bit CLA
C_{12}
2bit CLA
C_{10}
2bit CLA
C_8
2bit CLA
C_6
2bit CLA
C_4
2bit CLA
C_2
2bit CLA
C_0
S_{15} S_{14}
S_{13} S_{12}
S_{11} S_{10}
S_9 S_8
S_7 S_6
S_5 S_4
S_3 S_2
S_1 S_0

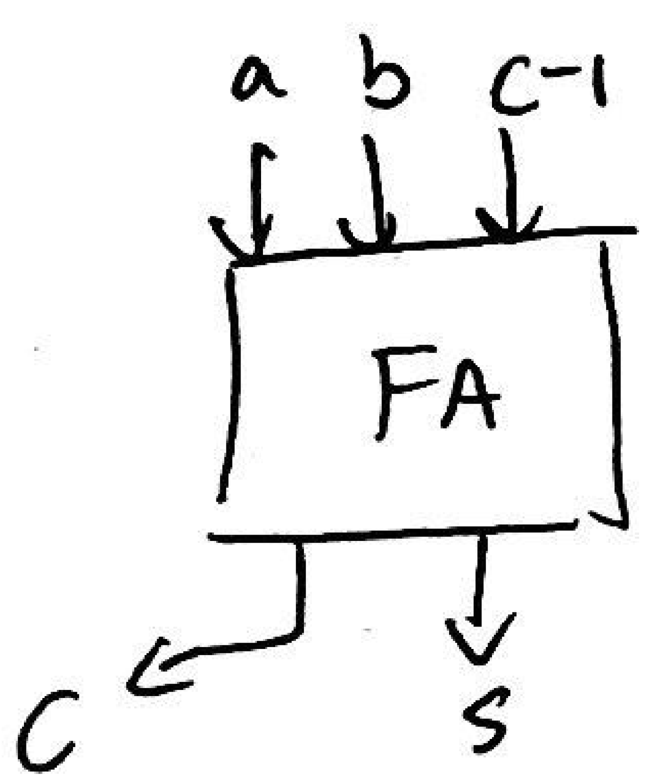
Problem 3.6

ratio of $\dfrac{\Delta CPA(8)}{\Delta CLA(8)} = \dfrac{1.7ns}{0.8ns} = 2.125$

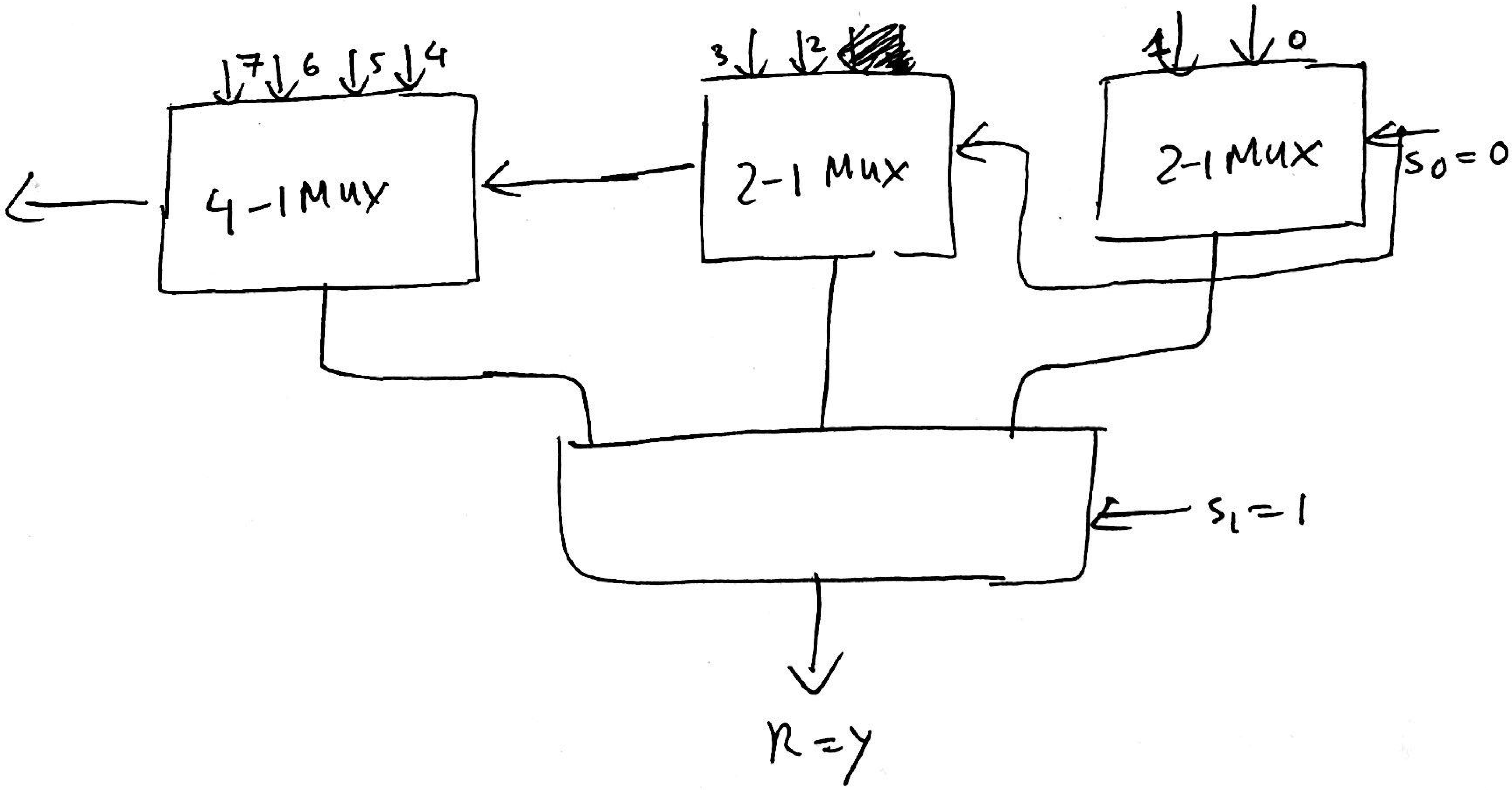
=> The CLA(8) is more than 2 times faster than the CPA(8)

SOP expression for FA: $S_i = P_i \oplus C_{i-1}$

$C_i = g_i + P_i C_{i-1}$



Problem 3.9



Problem 3.10

$$\Delta ALU(8) = \Delta NAND + \Delta OR + \Delta MUX + \Delta AM + \Delta MASK + \Delta MAP$$

$$= (0.1ns) + (0.8ns)$$

$$= 0.9ns$$

$\boxed{\text{Problem 3.22}}$

Represent 6.725 to IEEE Floating Point

$\boxed{\text{a)}}$ Single Precision

$$6.725 = (110.1011100)_2$$

$$= (110.1011100)_2 \times 2^0$$

$$= (1.101011100)_2 \times 2^2$$

$$= (1.101011100)_2 \times 2^{2+127}$$

$$= (1.101011100)_2 \times 2^{129}$$

⇒ Sign 1 bit = 0

⇒ 8 bit biased exponent $E = 129 = (1000\ 0001)_2$

⇒ 23-bit fraction F = 1010 11100 1100 1100 110011

⇒ $\boxed{\text{IEEE Floating Point:}}$

0	1000 0001	10101110011001100110011

or in hex: $\boxed{0\text{x}40D73333}$

$\boxed{b)}$ Double precision

$$6.725 = (110.1011100)_2$$
$$= (110.1011100)_2 \times 2^0$$
$$= (1.101011100)_2 \times 2^2$$

In double precision: bias = 1023

$$\Rightarrow (1.101011100)_2 \times 2^2$$
$$= (1.101011100)_2 \times 2^{2+1023}$$
$$= (1.101011100)_2 \times 2^{1025}$$

+> Signbit = 0

+> 11 bit biased exponent $E = 1025 = (100\ 0000\ 0001)_2$

+> 52 bit fraction F= 101011100 1100 1100 1100 1100 1100 1100 1100 1100 1100...

=> $\boxed{\text{IEEE Floating Point:}}$

0	100 0000 0001	101011100 1100 1100 1100 1100 1100 1100 1100 1100 1100 110

Problem 3.24

Represent IEEE single precision FP $0x\,41DD\,0000$ in decimal

$0x\,41DD\,0000$

0100 0001 1101 1101 0000 0000 0000 000

$\Rightarrow (0, 10000011, 1011101\,00000000\,0000\,000)$

With single precision, we have bias = 127

The biased 8 bit exponent $= (10000011)_2 = 131$

$$\text{Exponent} = (\text{biased exponent} - \text{bias})$$
$$= 131 - 127$$
$$= 4$$

So, $(1.1011101)_2 * 2^{131} = (1.1011101)_2 \times 2^{4+127}$

$= (1.1011101)_2 \times 2^4 = (11011.101)_2 \times 2^0$

$$11011.101 = 27 + (0.1)_2 + (0.001)_2$$
$$= 27 + \frac{1}{2} + \frac{1}{8}$$
$$= 27 + 0.5 + 0.125$$

$\boxed{\text{Decimal}} = \boxed{27.625}$