Basic Mathematics and Statistics

CHAPTER 3: SOLVING EQUATIONS AND INEQUALITIES

Dr. Khaing S. Htun

3.1 Linear Equations and Solving Linear Equations

Algebraic euqations - a mathematical statement that two algebraic expressions are equal

$$3x - 2 = 7, \qquad 2x - 3x + 5 = 0, \qquad \frac{1}{1+x} = \frac{x}{x-2}, \qquad \sqrt{x+4} = x - 1$$

solution set - the set of all elements in the domain of the variable that make the equation true

solution - each element of the solution set

domain - the set of numbers that are permitted to replace the variable

solve an equation - find the solution set for the equation

3.1 Linear Equations and Solving Linear Equations

3.1.1 Properties of Equality

For any real numbers a, b, and c

$$a = b$$
$$a + c = b + c$$
$$a - c = b - c$$
$$a \times c = b \times c$$
$$\frac{a}{c} = \frac{b}{c}$$

If $a = b$, then either may replace the other in any statement without changing the truth or falsity of the statement.

3.1 Linear Equations and Solving Linear Equations

3.1.2 Solving Linear Equations

Standard form of **linear equation** in one variable

$$ax + b = 0 \quad , a \neq 0$$

where *a* and *b* are real constants and *x* is a variable

General Steps of Solving Linear Equations

Step 1: Remove fractions by multiplying by *Least Common Division* (LCD).
Step 2: Remove parentheses.
Step 3: Combine like term.
Step 4: Divide by the coefficient of the variable.
Step 5: Check the solution

3.1 Linear Equations and Solving Linear Equations

3.1.2 Solving Linear Equations

Example - solve the following equations and check your answer.

$$3x - 2 = 4x + 5$$

$3x - 4x = 5 + 2$

$-x = 7$

$x = -7$

$$2(3x - 2) = 3(4x + 5)$$

$6x - 4 = 12x + 15$

$-6x = 19 \Rightarrow x = -\frac{19}{6}$

$$\frac{2}{3}(3x - 2) = \frac{3}{4}(4x + 5)$$

$6x - \frac{4}{3} = 3x + \frac{15}{4}$

$3x = \frac{45 + 16}{12} = \frac{61}{12} \Rightarrow x = \frac{61}{36}$

3.2 Linear Inequalities and Solving Linear Inequalities

Inequality - a statement that one expression's related to another expression by one of these comparisons:

< "less than"

$$x+3<8 \quad , \quad 2x-5 \geq 1 \quad , \quad x^2-4 \leq 2 \quad , \quad \frac{x+1}{x+2}>0$$

> "greater than"

≤ "less than or equal"

≥ "greater than'

≠ 'not equal"

3.2 Linear Inequalities and Solving Linear Inequalities

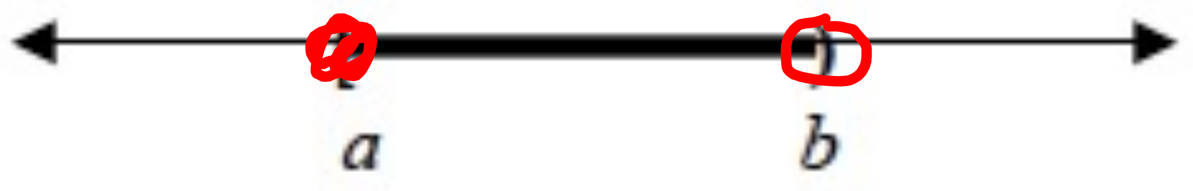
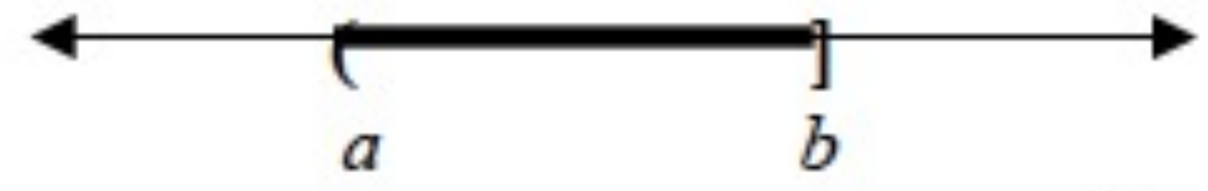
Interval - **solution set for an inequality**, the set of all real numbers of the variable that make the inequality a true statement

$(-\infty, \infty)$ $[-\infty, \infty]$ - the symbol “∞” read “infinity”.

- **open interval, (a, b) -** if $a < b$, from *a* to *b* consists of all numbers between *a* and *b*, $\{x|\ a < x < b\}$
- **closed interval, [a, b] -** from *a* to *b* consists of all numbers between *a* and *b* and its endpoint(s), $\{x|\ a \leq x \leq b\}$
- **half-open interval** - intervals with one endpoints

3.2 Linear Inequalities and Solving Linear Inequalities

Table 3.1 Interval Notation on the Real Number Line

Interval Notation	Type	Inequalities Notation	Line Graph
(a,b)	Open	$a<x<b$	
$[a,b]$	Closed	$a\le x\le b$	
$[a,b)$	Half-open	$a\le x<b$	
$(a,b]$	Half-open	$a<x\le b$	
(a,∞)	Open	$a<x$ or $x>a$	
$[a,\infty)$	Closed*	$a\le x$ or $x\ge a$	
$(-\infty,b)$	Open	$x<b$	
$(-\infty,b]$	Closed*	$x\le b$	

* These intervals are closed because they contain all of their endpoints; they have only one endpoint.

3.2 Linear Inequalities and Solving Linear Inequalities

3.2.1 Properties of inequalities

For any real numbers a, b, and c:

if $a < b$ and $b < c$ then $a < c$

if $a < b$ and $a + c < b + c$

if $a < b$ and $c > 0$ then $ac < bc$ and $\frac{a}{c} < \frac{b}{c}$

if $a < b$ and $c < 0$ then $ac > bc$ and $\frac{a}{c} > \frac{b}{c}$

3.2 Linear Inequalities and Solving Linear Inequalities

3.2.2 Solving an Inequality

Finding its solution set of inequalities

$2(2x+3)-10<6(\mathrm{x}-2)$

$4x+6-10<6x-12$

$4x-4<6x-12$

$4x-6x<-12+4$

$-2x<-8$

$-x<-4$

$x>4$ (**Note:** <u>direction reverses when multiply with negative</u>)

The solution set is **(4, ∞)**

3.2 Linear Inequalities and Solving Linear Inequalities

3.2.2 Solving an Inequality

$$-5 < 3x - 2 < 1$$

$$-5 + 2 < 3x - 2 + 2 < 1 + 2$$
$$-3 < 3x < 3$$

$$-\frac{3}{3} < \frac{3x}{3} < \frac{3}{3}$$

$$-1 < x < 1$$

The solution set is **(-1, 1)**

3.2 Linear Inequalities and Solving Linear Inequalities

3.2.2 Solving an Inequality

Solve and graph

$$\frac{2x-3}{4}+6 \geq 2+\frac{4x}{3}$$

$$\frac{12(2x-3)}{4}+12(6) \geq 12(2)+\frac{12(4x)}{3} \qquad \text{multiply both sides by LCD} = 12$$

$$6x-9+72 \geq 24+16x$$

$$6x+63 \geq 24+16x$$

$$6x-16x \geq 24-63$$

$$-10x \geq -39$$

$$10x \leq 39$$

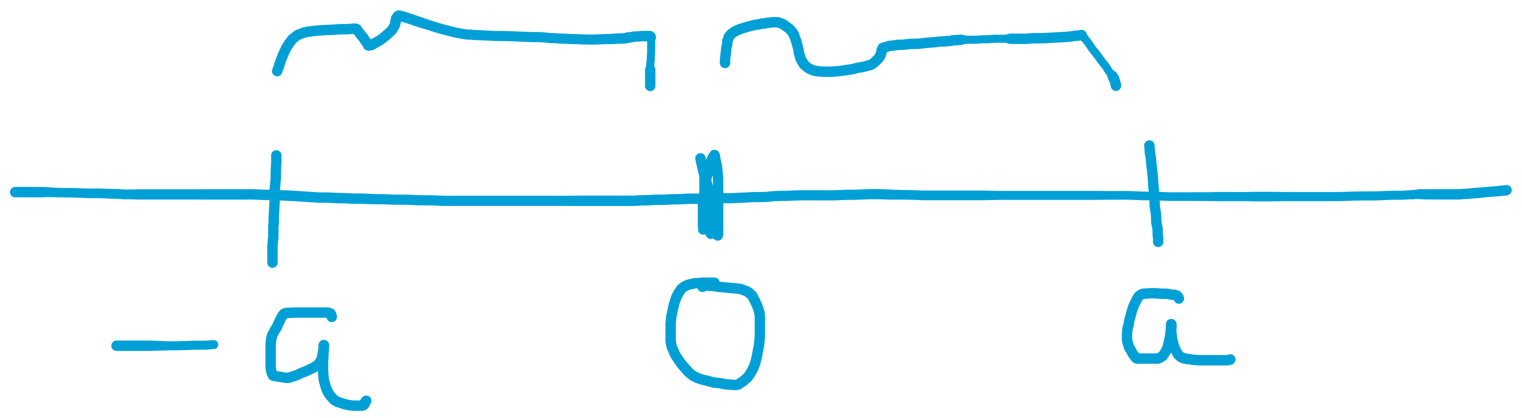
$$x \leq 3.9$$

The solution set is (-∞, **3.9]**

3.3 Absolute Value in Equations and Inequalities

absolute value | a | - the distance between zero and the number (a) on the number line

$$|a| = \begin{cases} -a & ; \quad a < 0 \\ a & ; \quad a \geq 0 \end{cases}$$



(a) $|5| =$ **(b)** $|-6| =$ **(c)** $|3-7| =$

(d) $-|-2| =$ **(e)** $-|6-9|-|15-17| =$

3.3 Absolute Value in Equations and Inequalities

3.3.1 Properties of Equations and Inequalities Involving $|ax + b|$

For c > 0:

1.	$\lvert ax+b \rvert = c$ is equivalent to $ax+b=c$ or $ax+b=-c$.
2.	$\lvert ax+b \rvert < c$ is equivalent to $-c < ax+b < c$.
3.	$\lvert ax+b \rvert > c$ is equivalent to $ax+b < -c$ or $ax+b > c$.

3.3 Absolute Value in Equations and Inequalities

3.3.2 Solving absolute value equations

$$2|4x - 5| + 3 = 15$$

$$2|4x - 5| = 15 - 3$$

$$2|4x - 5| = 12$$

$$|4x - 5| = \frac{12}{2}$$

$$|4x - 5| = 6$$

$$4x - 5 = 6 \qquad\qquad 4x - 5 = -6$$

$$4x = 11 \qquad\qquad 4x = -1$$

$$x = \frac{11}{4} \qquad\qquad x = -\frac{1}{4}$$

Check:

$$2|4(11/4) - 5| + 3 = 15$$
$$2|6| + 3 = 15$$
$$2(6) + 3 = 15$$
$$15 = 15 \rightarrow \textbf{True}$$

$$2|4(-1/4) - 5| + 3 = 15$$
$$2|-6| + 3 = 15$$
$$2(6) + 3 = 15$$
$$15 = 15 \rightarrow \textbf{True}$$

The solution set are $\boldsymbol{x = \frac{11}{4}}$ and $\boldsymbol{x = -\frac{1}{4}}$

3.3 Absolute Value in Equations and Inequalities

3.3.3 Solving Absolute Value Inequalities

Solve inequality, write solutions in both inequality and interval notation

$$|2x+4| \leq 3$$

$$-3 \leq 2x+4 \leq 3$$

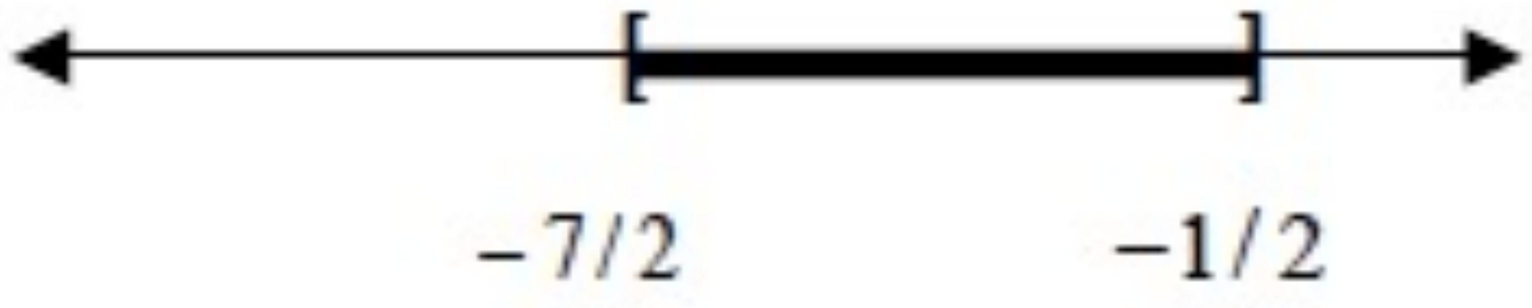
$$-3-4 \leq 2x+4-4 \leq 3-4$$

$$-7 \leq 2x \leq -1$$

$$-\frac{7}{2} \leq \frac{2x}{2} \leq -\frac{1}{2}$$

$$-\frac{7}{2} \leq x \leq -\frac{1}{2}$$

The solution set is $\left[-\frac{7}{2}, -\frac{1}{2}\right]$



3.3 Absolute Value in Equations and Inequalities

3.3.3 Solving Absolute Value Inequalities

Solve inequality, write solutions in both inequality and interval notation

$2|2x-5|-3>3$

.

.

$x<1 \quad or \quad x>4$

The solution set are (–∞,1) ∪ (4, ∞)

3.4 Using Absolute Value to Solve Radical Inequalities

$\sqrt{x^2} = |x|$ - for any real number x

$$\sqrt{(x-2)^2} \leq 5$$

$$|x-2| \leq 5$$

$$-5 \leq x - 2 \leq 5$$

$$-3 \leq x \leq 7$$

The solution set is $[-3, 7]$

Exercise

Exercise 3.1

Exercise 3.2

3.5 Quadratic Equations and Solving Quadratic Equations

3.5.1 Using Factoring to Solve Quadratic Equations

Standard form of **quadratic equation** in one variable

$$ax^2 + bx + c = 0 \quad , \mathrm{a} \neq 0$$

where *a, b* and *c* are constants

3.5 Quadratic Equations and Solving Quadratic Equations

3.5.1 Using Factoring to Solve Quadratic Equations

$3x^2 + 4x + 1 = 0$

$(3\mathrm{x} + 1)(x + 1) = 0$ **The Zero Factor Property**

$(3\mathrm{x} + 1) = 0;\ x = -\dfrac{1}{3}$

$(x + 1) = 0;\ x = -1$

Three methods to find solution of quadratic equation.

(1) Solution by Factoring

(2) Solution by Completing the Square

(3) Solution by Quadratic Formula

3.5 Quadratic Equations and Solving Quadratic Equations

3.5.2 Steps of Solving Quadratic Equations by Factoring

Step 1: Areange the quadratic equation in standard form $ax^2 + bx + c$

Step 2: Factor $ax^2 + bx + c$

Step 3: Apply the Zero Factor property and solve for the variable

Step 4: Check the answer

3.5 Quadratic Equations and Solving Quadratic Equations

3.5.2 Steps of Solving Quadratic Equations by Factoring

$$2m^2 = 23m - 63$$

$$2m^2 - 23m + 63 = 0$$

$$(2m-9)(m-7) = 0$$

$$2m - 9 = 0 \Rightarrow m = \frac{9}{2}$$

$$m - 7 = 0 \Rightarrow m = 7$$

3.5 Quadratic Equations and Solving Quadratic Equations

3.5.3 Fractional Equations

$$\frac{x+2}{x} = \frac{6x}{x+1}$$

$$(x+2)(x+1) = 6x \cdot x$$

$$x^2 + x + 2x + 2 = 6x^2$$

$$x^2 + 3x + 2 - 6x^2 = 0$$

$$-5x^2 + 3x + 2 = 0$$

$$5x^2 - 3x - 2 = 0 \Rightarrow (5x+2)(x-1) = 0$$

$$5x+2=0, \quad x = -\frac{2}{5}$$

$$x-1=0, \quad x=1$$

3.5 Quadratic Equations and Solving Quadratic Equations

3.5.4 Equation with Radical

$$a = \sqrt{\frac{6-13a}{5}}$$

$$a^2 = \frac{6-13a}{5}$$

$$5a^2 = 6-13a$$

$$5a^2 + 13a - 6 = 0$$

$$(5a-2)(a+3) = 0$$

$$5a - 2 = 0 \qquad a + 3 = 0$$

$$a = \frac{2}{5} \qquad \cancel{a = -3}$$

Caution: Squaring both sides of an equation to remove the radical may lead to extraneous roots. Always check your answer.

Since – 3 does not exist,

the solution is *a* = 2/5

3.5 Quadratic Equations and Solving Quadratic Equations

3.5.5 Solution by Completing the Square

Step 1: Arrange the equation in $ax^2 + bx = c$.

Step 2: If $a \neq 1$, divide each term of the equation by a, giving $x^2 + \frac{b}{a}x = \frac{c}{a}$

Step 3: Complete the square by *adding* the magnitude (omit the sign) of $\left(\frac{b}{2a}\right)^2$ to both sides of the equation.

Step 4: Write the left-hand side of the equation as a perfect square:

$$\left(\sqrt{\textit{first term}} \pm \sqrt{\textit{third term}}\right)^2$$

Step 5: Apply the Square Root Property.

Step 6: Solve the resulting linear equation for the variable.

3.5 Quadratic Equations and Solving Quadratic Equations

3.5.5 Solution by Completing the Square

$2x^2 + 12x - 54 = 0$

$2x^2 + 12x = 54$

$x^2 + 6x = 27 \Rightarrow a = 1,\ b = 6$

$\Rightarrow \left(\frac{b}{2a}\right)^2 = \left(\frac{6}{2(1)}\right)^2 = 3^2 = 9$

$x^2 + 6x + 9 = 27 + 9$

$(x+3)^2 = 36$

$\sqrt{(x+3)^2} = \pm\sqrt{36}$

$x + 3 = \pm 6$

$x = 6 - 3 = 3$

$x = -6 - 3 = -9$

3.5 Quadratic Equations and Solving Quadratic Equations

3.5.6 Solution by the Quadratic Formula

$$\boldsymbol{ax^2 + bx + c = 0} \quad , \mathrm{a} \neq 0$$

$$\downarrow$$

$$\boldsymbol{x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$$

$$x = \frac{-b+\sqrt{b^2-4ac}}{2a} \qquad x = \frac{-b-\sqrt{b^2-4ac}}{2a}$$

3.5 Quadratic Equations and Solving Quadratic Equations

3.5.6 Solution by the Quadratic Formula

Steps of solving a quadratic equation using the formula:

Step 1: Arrange the equation in standard form, $ax^2 + bx + c = 0$

Step 2: Identify a, b, and c. The coefficient of the squared term is a; the coefficient of the linear term is b; and the constant term is c.

Step 3: Substitute the values of a, b, and c into the formula.

Step 4: Simplify the expression.

Note : The coefficient a, b, and c *include their signs.*

3.5 Quadratic Equations and Solving Quadratic Equations

3.5.6 Solution by the Quadratic Formula

$5x^2 - 6x = 3$

$$ax^2 + bx + c = 0 \rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$5x^2 - 6x - 3 = 0$

a b c

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-6) \pm \sqrt{(-6)^2 - 4(5)(-3)}}{2(5)}$$

$$= \frac{6 \pm \sqrt{36 + 60}}{10} = \frac{6 \pm \sqrt{96}}{10} = \frac{6 \pm 4\sqrt{6}}{10} = \frac{3 \pm 2\sqrt{6}}{5}$$

Exercise

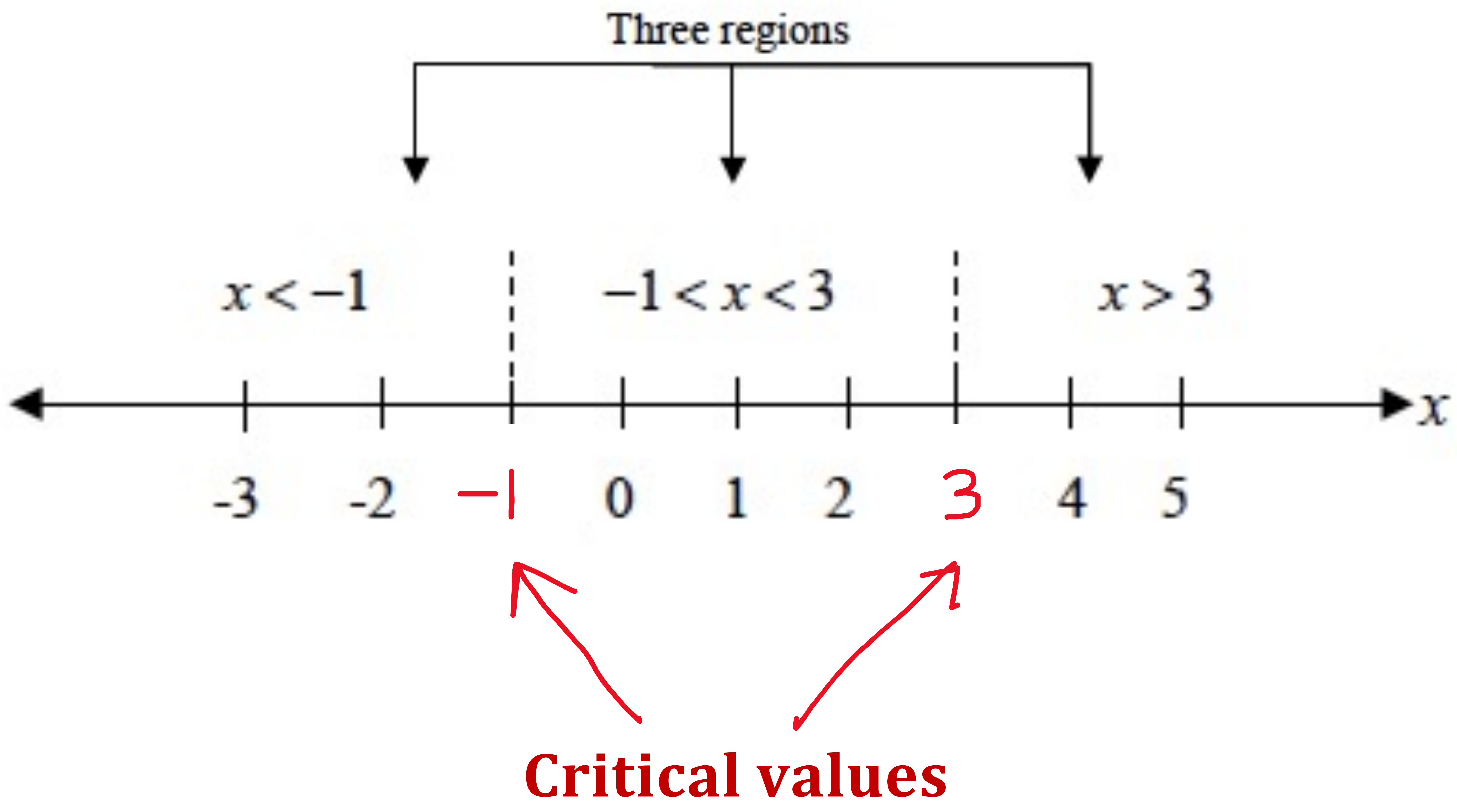
Exercise 3.3

3.6 Nonlinear Inequalities

$x^2 - 2x - 3 = 0$

$(x - 3)(x + 1) = 0$

$x = 3$ and $x = -1$



3.6 Nonlinear Inequalities

3.6.1 Solving Quadratic Inequalities

$2x^2 \leq 15 - x$

$2x^2 + x - 15 \leq 0$ change to equation $2x^2 + x - 15 = 0$

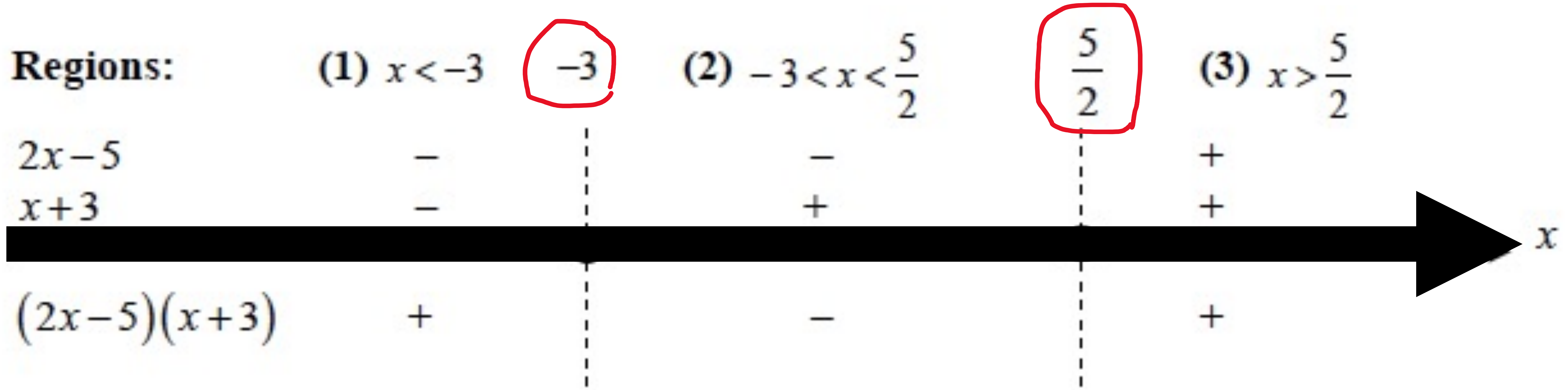
$(2x - 5)(x + 3) = 0$

$2x - 5 = 0 \quad or \quad x + 3 = 0$

$\boldsymbol{x = \frac{5}{2}} \qquad \boldsymbol{x = -3}$

3.6 Nonlinear Inequalities

3.6.1 Solving Quadratic Inequalities



Since the original inequality is less than or equal to zero and region (2) gives the solutions set which corresponds with inequality $2x^2 \leq 15 - x$.

So, the solution of this inequality is $-3 \leq x \leq \frac{\mathbf{5}}{\mathbf{2}}$.

The critical values are included in the solution because the original inequality contains the “equal to” symbol.

3.6 Nonlinear Inequalities

3.6.2 Rational Inequalities

In a rational expression, the critical values occur where the numerator or denominator equals zero. A rational expression changes signs only at its critical values.

In solving a rational inequality, remember domain restrictions necessary to **avoid division by zero.**

3.6 Nonlinear Inequalities

3.6.2 Rational Inequalities

$\dfrac{(x-5)^2(x+3)}{(x-1)} \geq 0$ → the critical values are x = 5, 1, and -3

Regions:	(1) $x<-3$	-3	(2) $-3<x<1$	1	(3) $1<x<5$	5	(4) $x>5$
$(x-5)^2$	$+$		$+$		$+$		$+$
$(x+3)$	$-$		$+$		$+$		$+$
$(x-1)$	$-$		$-$		$+$		$+$
$\dfrac{(x-5)^2(x+3)}{(x-1)}$	$+$		$-$		$+$		$+$

x

Since the original inequality is greater than or equal to zero, regions (1), (3) and (4) give the solutions set which corresponds with inequality. So, the solutions set are is $x \leq -3$ or $x > 1$

Exercise

Exercise 3.4

Assignment

Deadline for submission: next week Monday

Exercise 3.1- 9, 27, 29

Exercise 3.2 – 5, 14, 26

Deadline for submission: next week Monday

Exercise 3.3 –15, 22, 36, 44

Exercise 3.4 – 3, 12