Basic Mathematics and Statistics

CHAPTER 6: TRIGONOMETRIC FUNCTIONS

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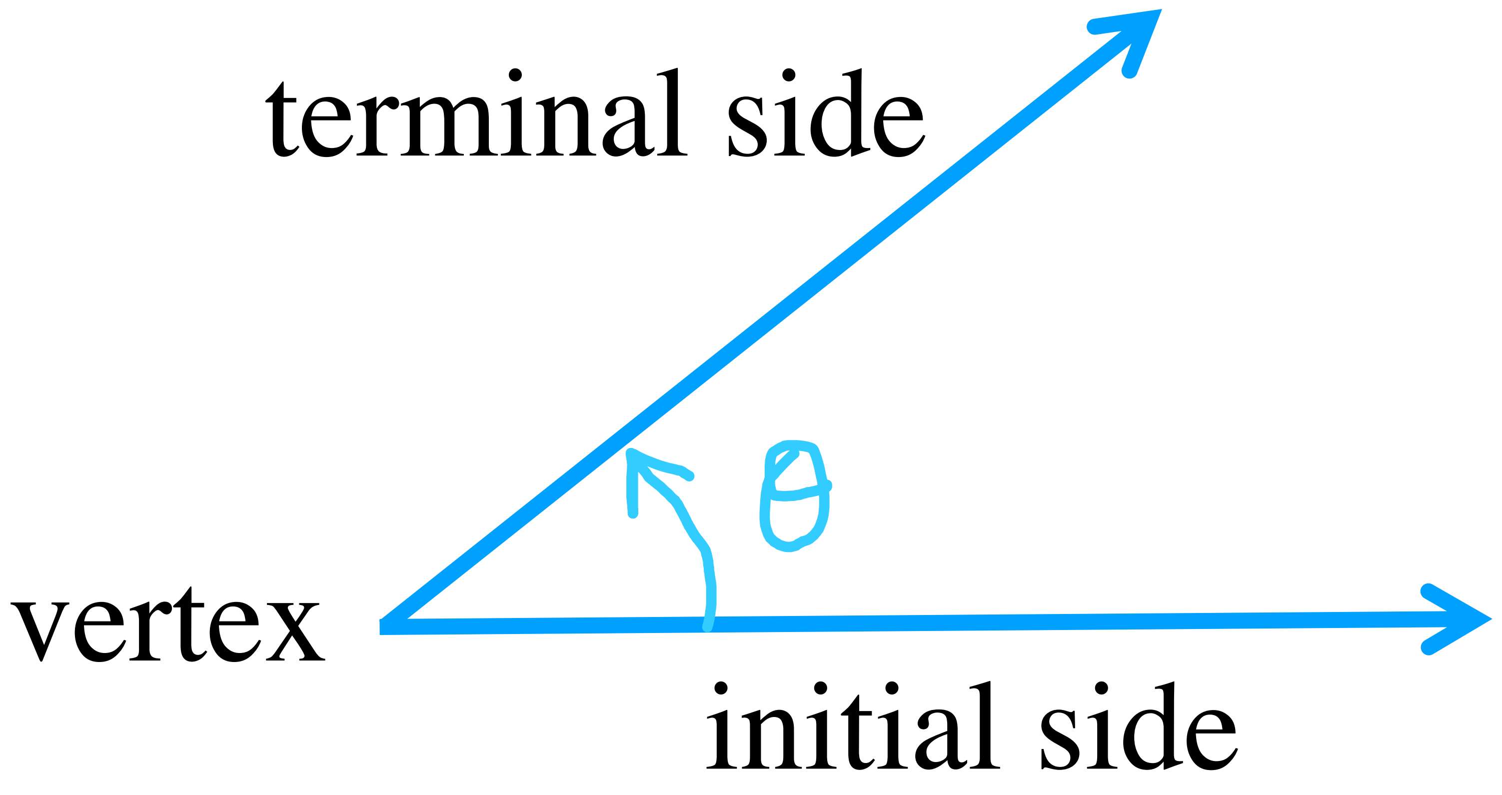
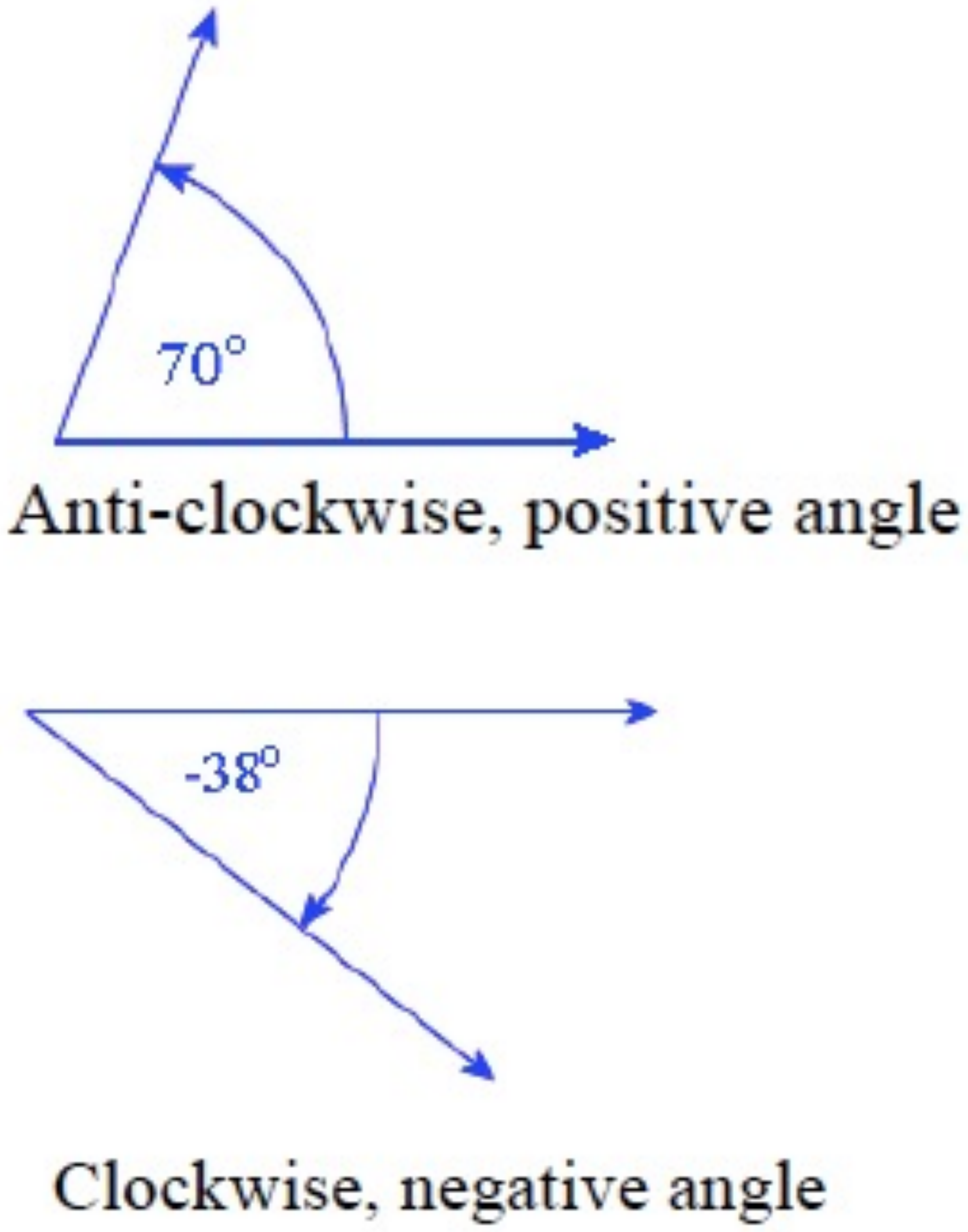
6.1 Angles and Their Measure

6.1.1 Angles

An **angle** is a measure of the amount of rotation between two-line segments.

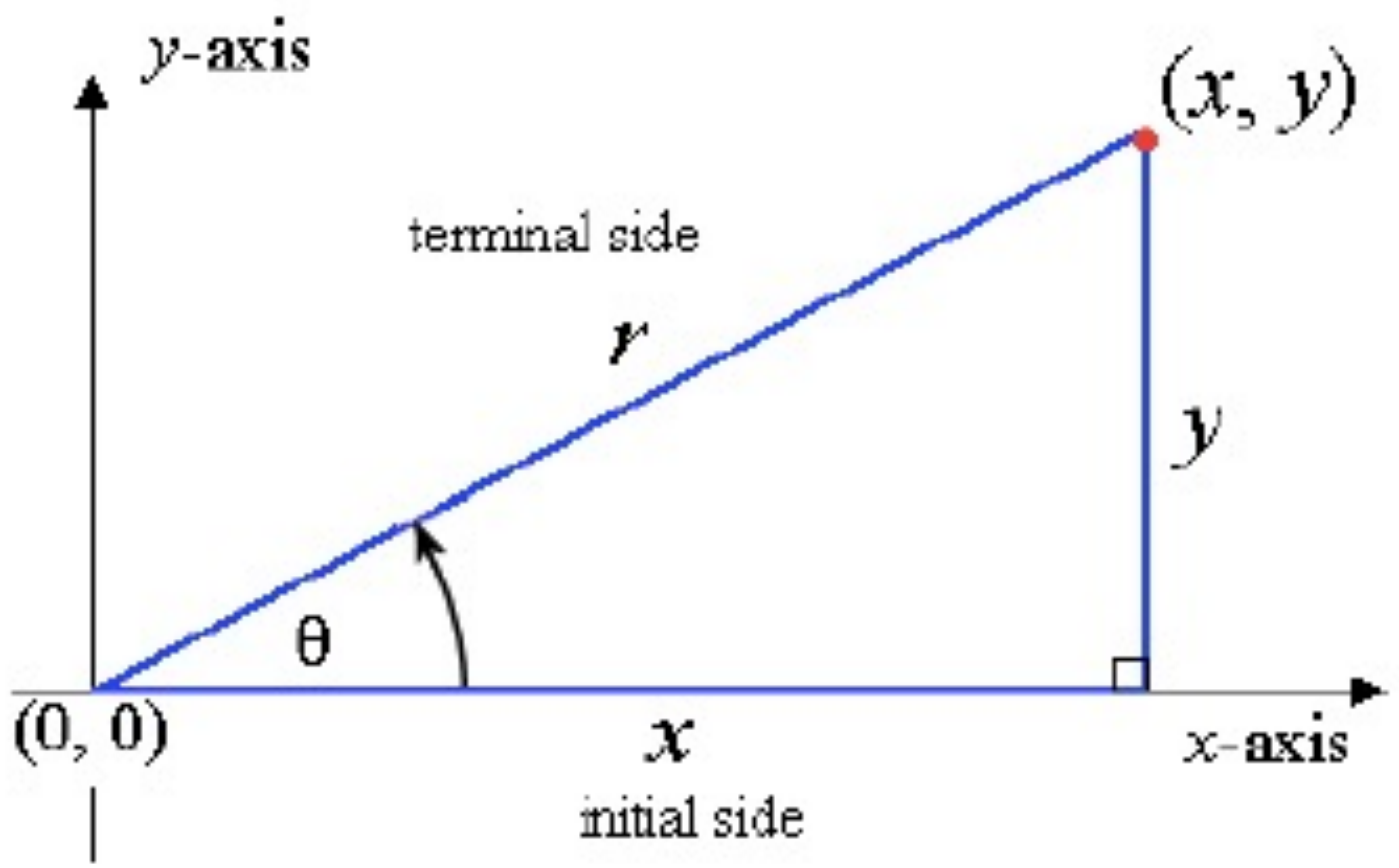


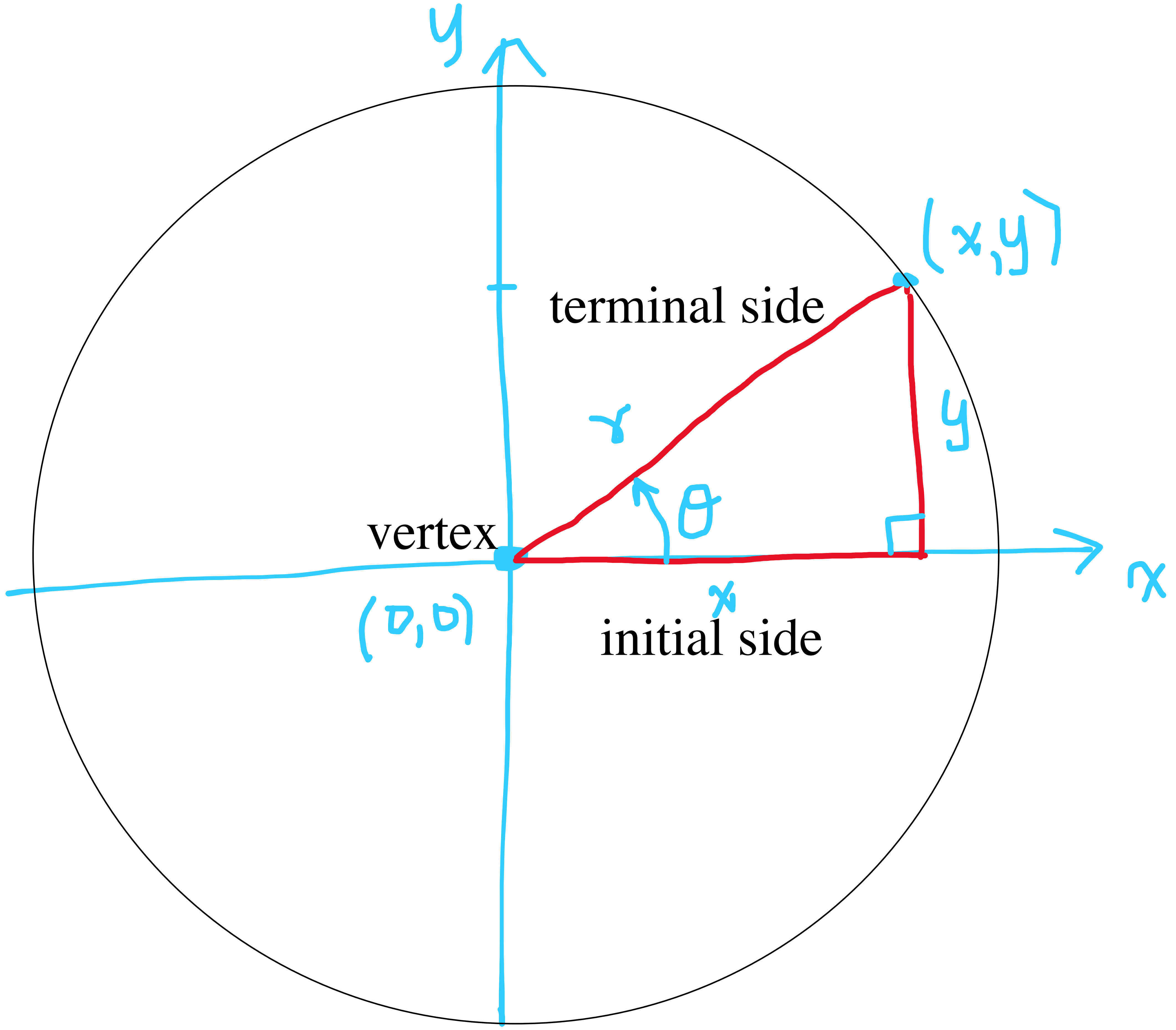


6.1 Angles and Their Measure

6.1.2 Standard Position of an Angle

An angle is in **standard position** if the initial side is the positive *x*-axis and the vertex is at the origin.





6.1 Angles and Their Measure

6.1.3 Degrees, Minutes and Seconds

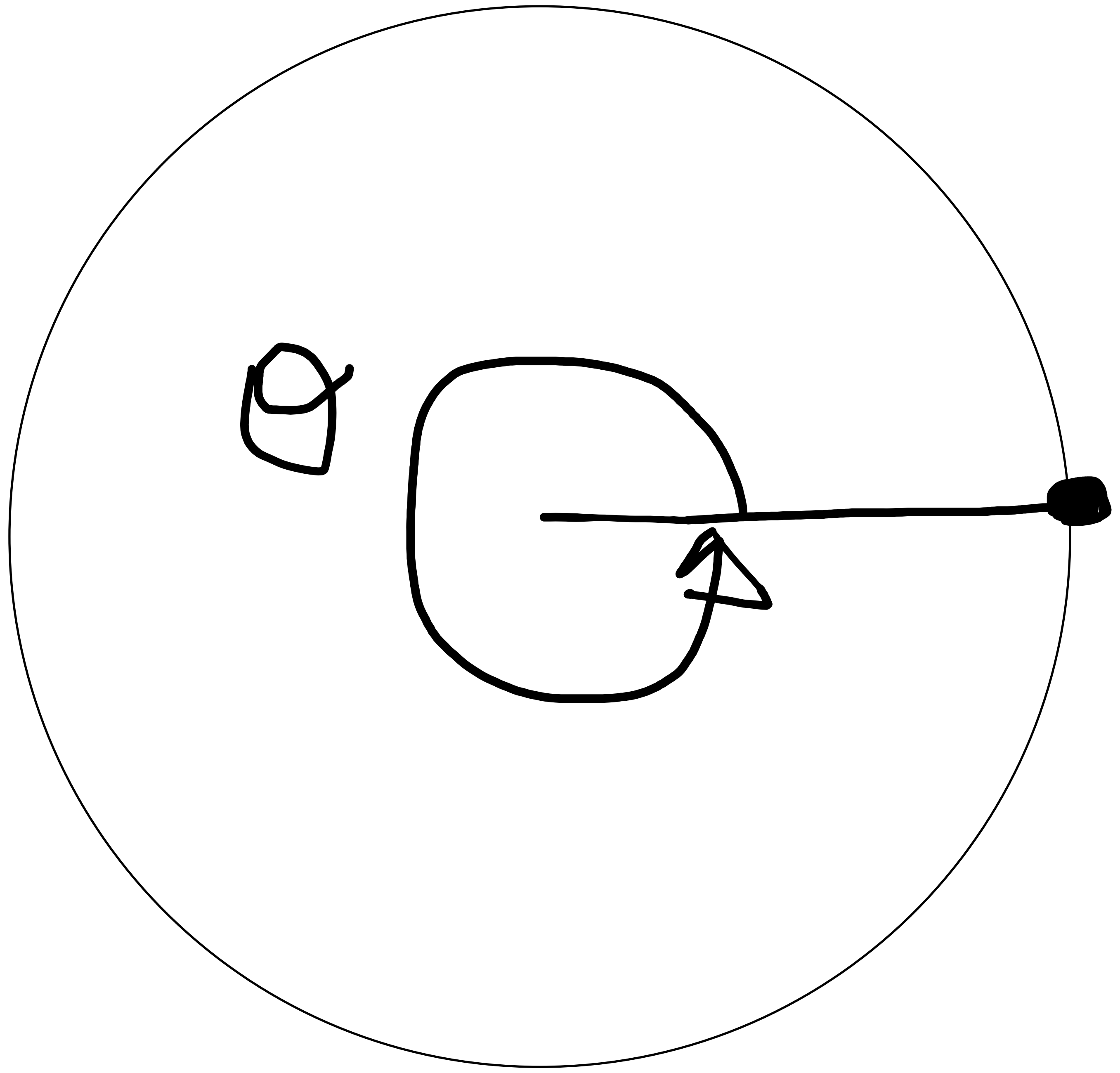
Degree (°) is divided into 60 minutes (') and a minute is divided into 60 seconds (")

Can be written as: DMS or ° ‘ “

1 counterclockwise revolution = 360°

1° = 60'

1' = 60"



Convert 50° 6' 21" to a decimal in degrees.

$$50°, 6' \to \frac{6}{60}, 21'' \to \frac{21}{60\times60} \approx 50 + 0.1 + 0.5833 = 50.105833°$$

6.1 Angles and Their Measure

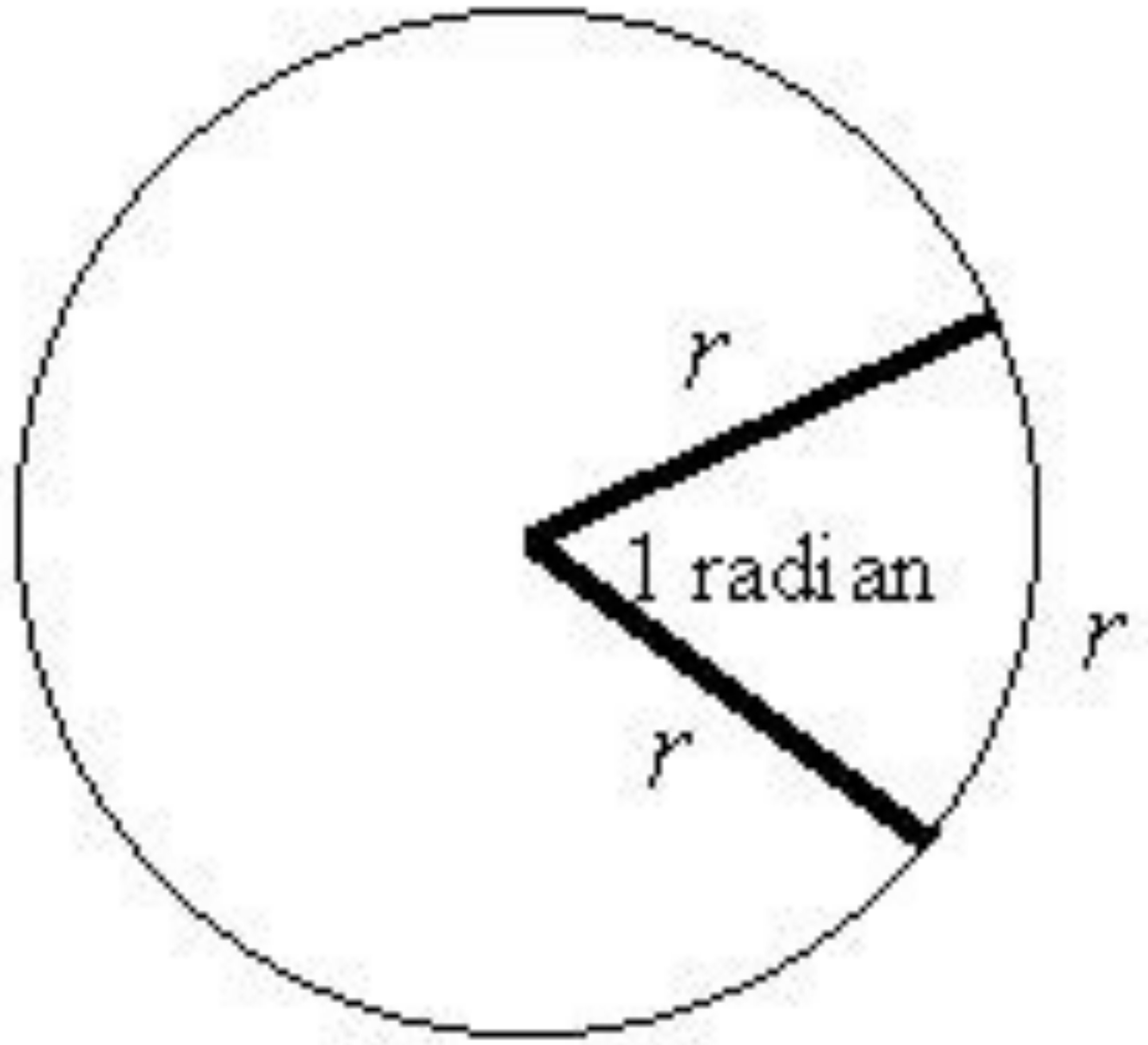
6.1.4 Radians – an Alternative Measure for Angle

In science and engineering, radians are much more convenient (and common) than degrees.

A **radian** is defined as the angle between 2 radii (radiuses) of a circle where the arc between them has length of one radius.

Another way of putting it is: "a radian is the angle subtended by an arc of length *r* (the radius)".

$$\mathbf{1\ \mathit{radian} = 57.3^\circ}$$



Radians are especially useful in **calculus**

where we want to interchange angles and other quantities (e.g. length).

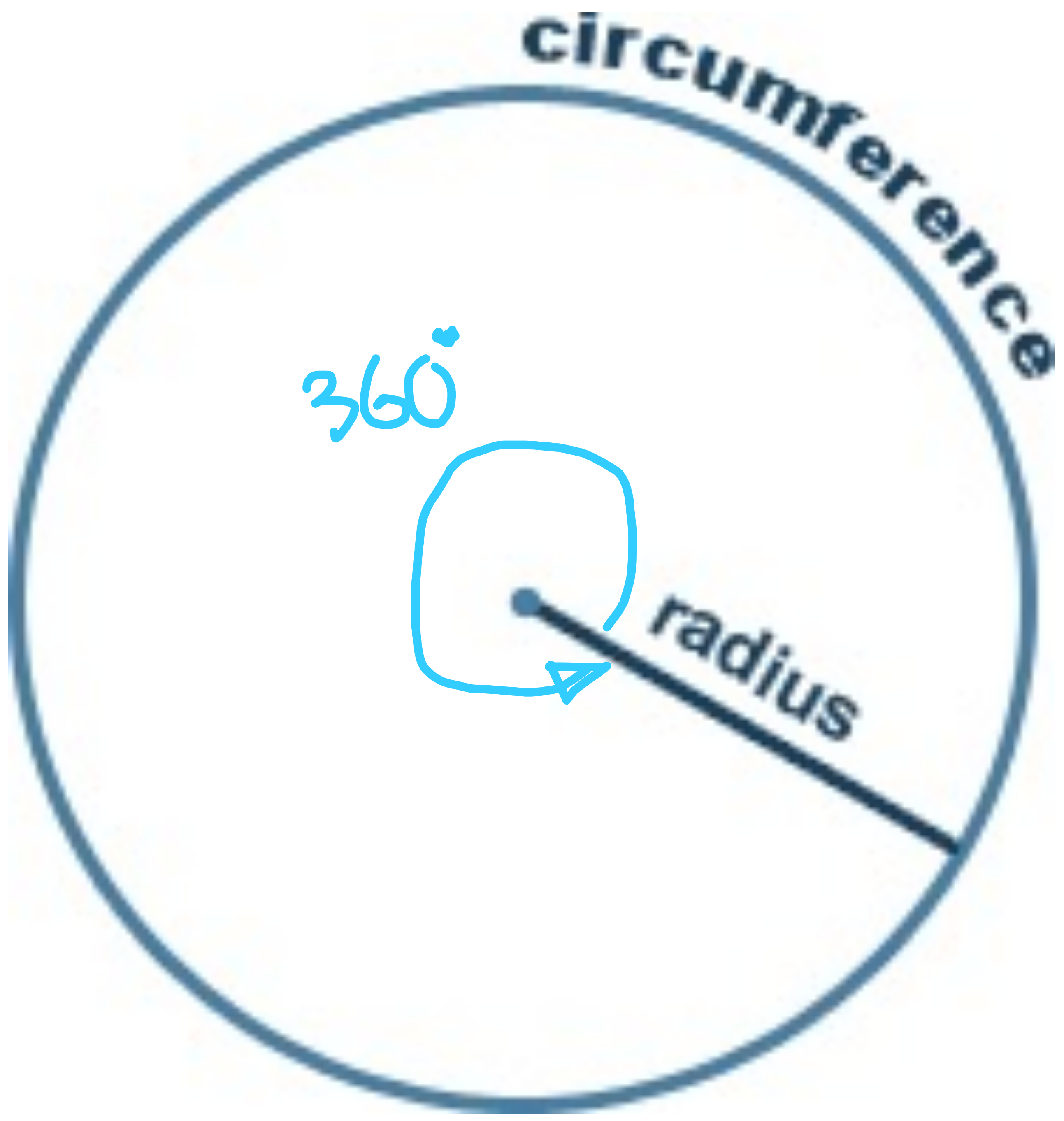
6.1 Angles and Their Measure

6.1.4 Radians – an Alternative Measure for Angle

Converting Degrees to Radians

circumference of a circle $= 2\pi r$

one revolution of a circle is 360°



2π radians = 360°

$$\boldsymbol{\pi = 180°}$$

Converting to degrees:

$$1 \text{ radian} = \frac{180°}{\pi} = 57.29578°$$

$$2 \text{ radians} = \frac{2 \times 180°}{\pi} = 114.59156°$$

Converting to radians:

$$50° = 50 \times \frac{\pi}{180°} = 0.8727\, radian$$

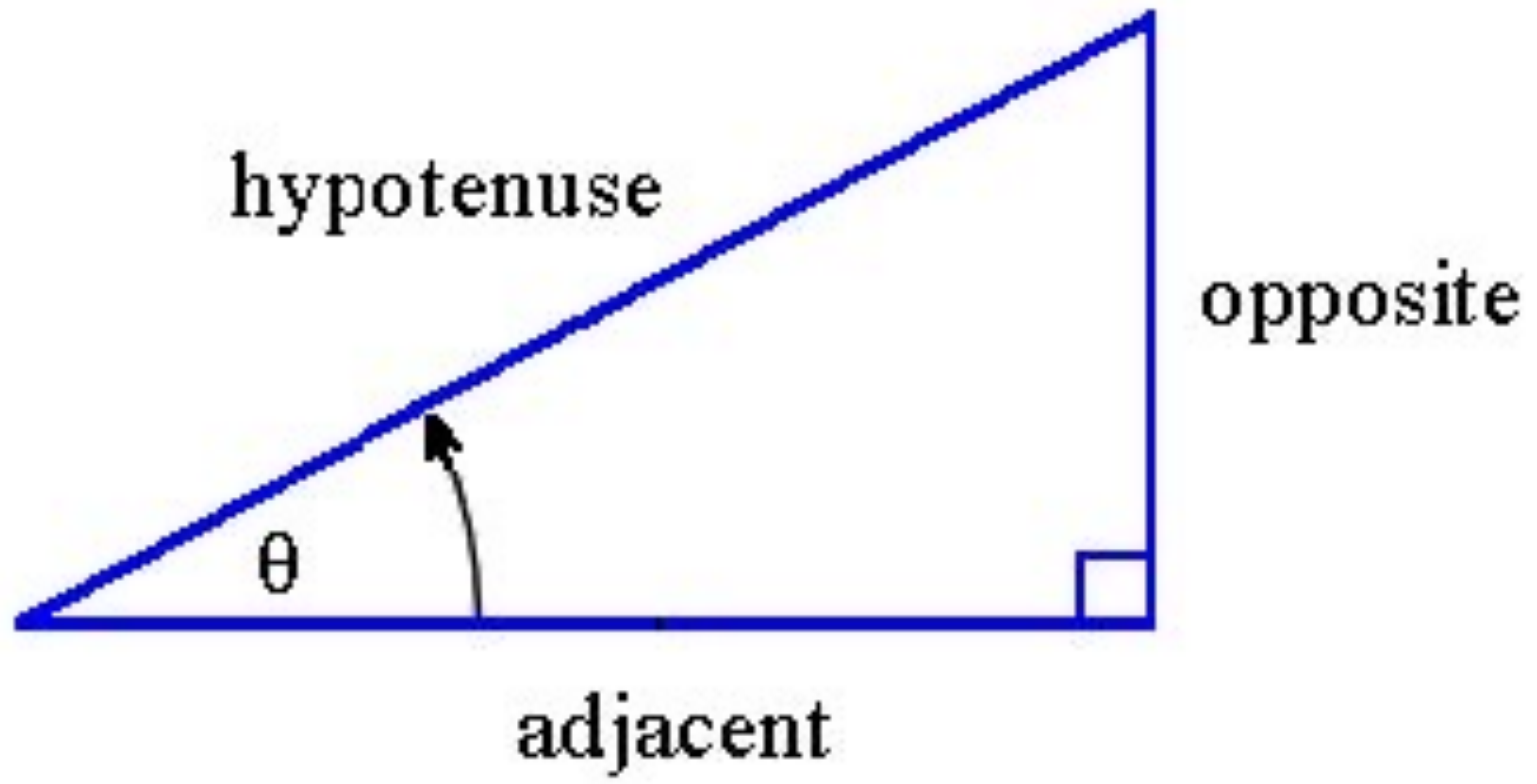
$$357° = 357 \times \frac{\pi}{180°} = 6.2308\, radians$$

6.2 Basic Trigonometric Functions

6.2.1 Sine, Cosine, Tangent and the Reciprocal Ratios

For the **angle θ** in a **right-angled triangle** as shown, the sides are named as below:

- Hypotenuse - the longest side of a right-angled triangle, opposite the right angle
- Opposite – side opposite the angle **θ**
- Adjacent – side next to angle **θ**



6.2 Basic Trigonometric Functions

6.2.1 Sine, Cosine, Tangent and the Reciprocal Ratios

There are **6 trigonometric ratios**:

The Reciprocal Trigonometric Ratios

- sine θ

  $\sin\theta = \dfrac{opposite}{hypotenuse}$

- cosine θ

  $\cos\theta = \dfrac{adjacent}{hypotenuse}$

- tangent θ

  $\tan\theta = \dfrac{opposite}{adjacent} = \dfrac{\sin\theta}{\cos\theta}$

- cosecant θ

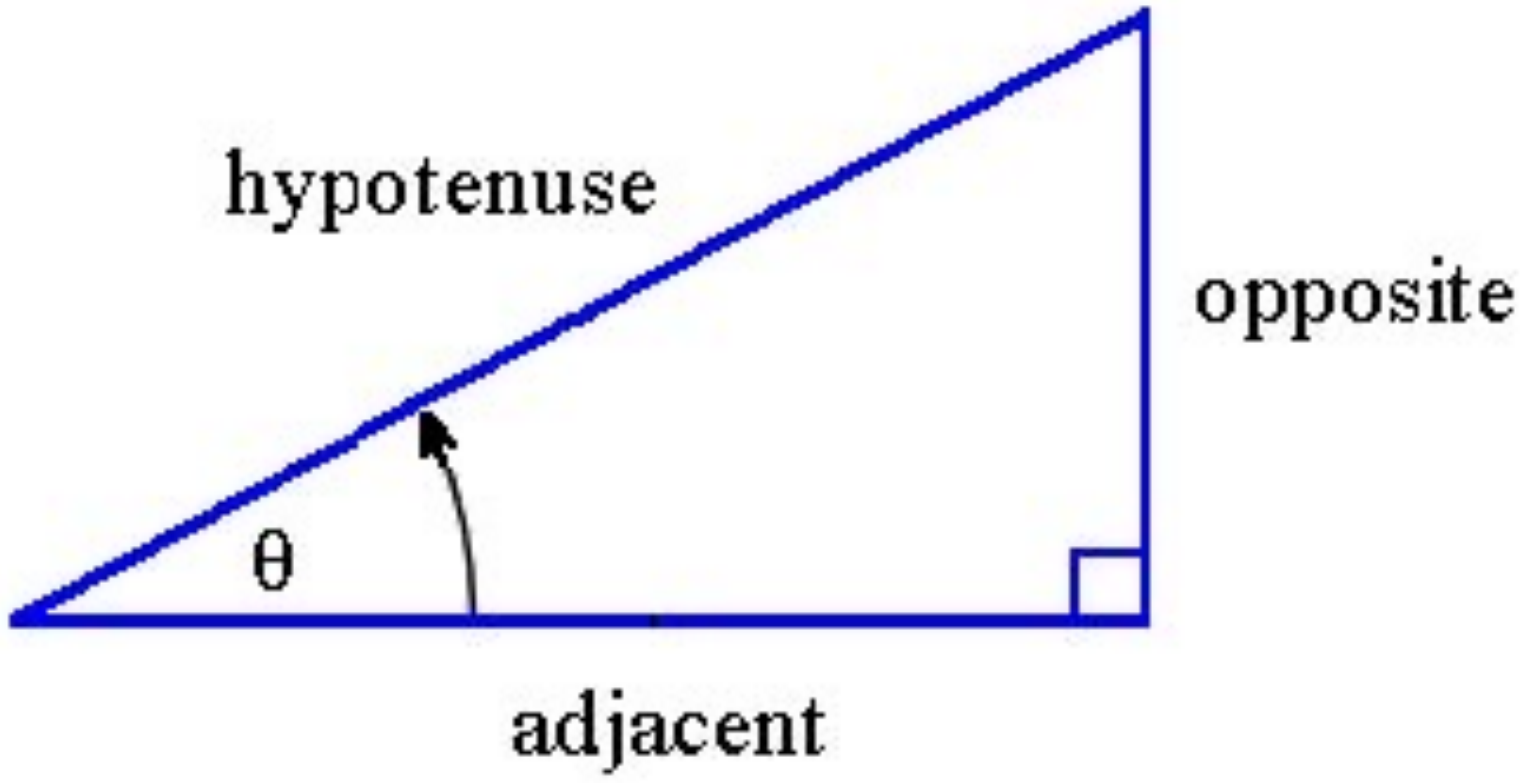
  $\csc\theta = \dfrac{1}{\sin\theta}$

- secant θ

  $\sec\theta = \dfrac{1}{\cos\theta}$

- cotangent θ

  $\cot\theta = \dfrac{1}{\tan\theta} = \dfrac{\cos\theta}{\sin\theta}$



6.2 Basic Trigonometric Functions

6.2.1 Sine, Cosine, Tangent and the Reciprocal Ratios

Important note:

There is a **big difference** between csc θ and $\sin^{-1}\theta$.

1. The first one is a reciprocal: csc $\theta = \frac{1}{\sin\theta}$
2. The second one involves finding an **angle** whose sine is θ. ($\sin^{-1}\theta$ – ***arcsin*** *or* sine inverse)

6.2 Basic Trigonometric Functions

6.2.1 Sine, Cosine, Tangent and the Reciprocal Ratios

For an angle in **standard position**, the trigonometric ratios are defined in terms of x, y and r as follow:

(1) $\sin\theta = \frac{y}{r}$

$= \frac{opposite}{hypotenuse}$

(2) $\cos\theta = \frac{x}{r}$

$= \frac{adjacent}{hypotenuse}$

(3) $\tan\theta = \frac{y}{x}$

$= \frac{opposite}{adjacent} = \frac{\sin\theta}{\cos\theta}$

(4) $\csc\theta = \frac{r}{y}$

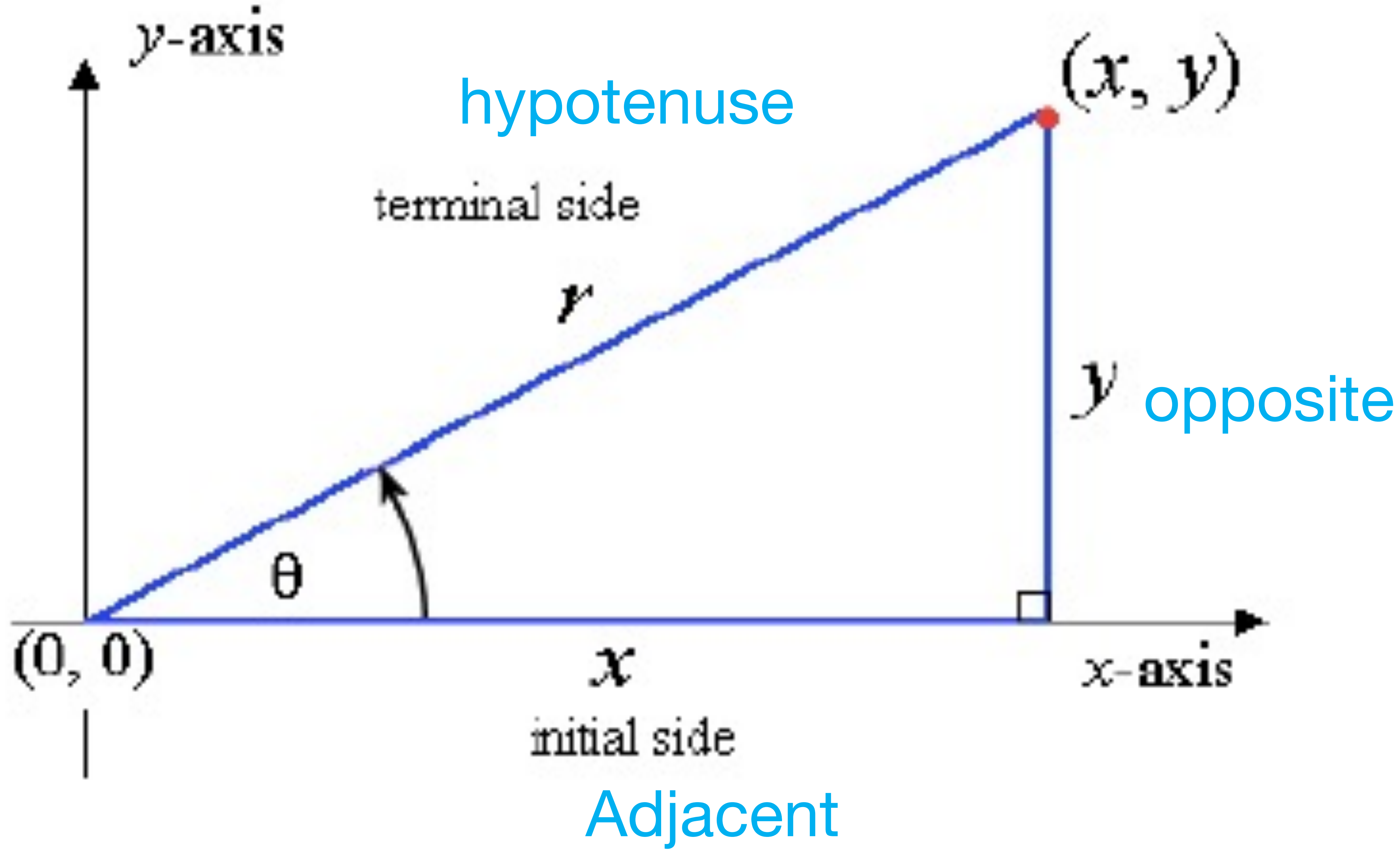
$= \frac{1}{\sin\theta}$

(5) $\sec\theta = \frac{r}{x}$

$= \frac{1}{\cos\theta}$

(6) $\cot\theta = \frac{x}{y}$

$= \frac{1}{\tan\theta} = \frac{\cos\theta}{\sin\theta}$

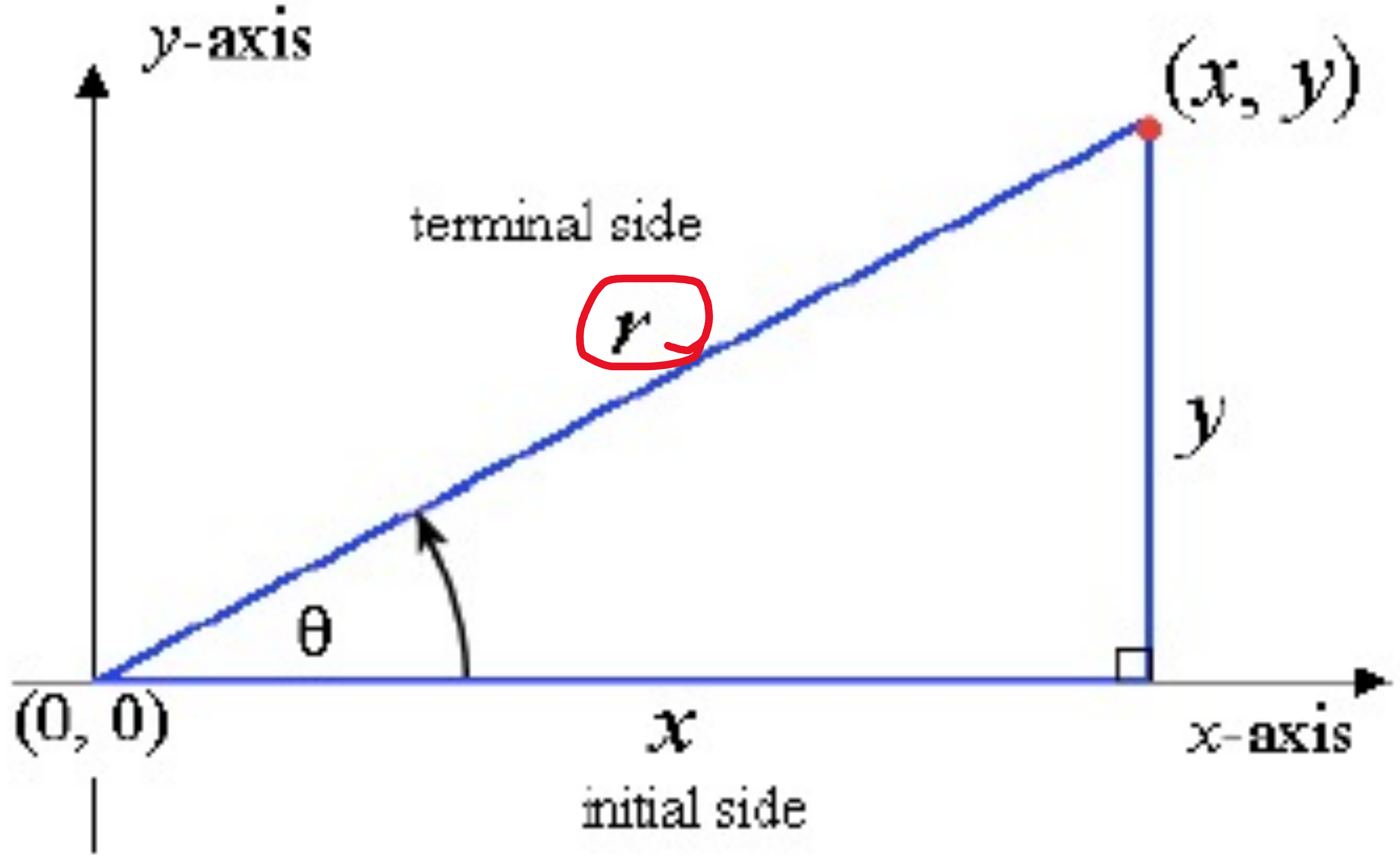


6.2 Basic Trigonometric Functions

6.2.1 Sine, Cosine, Tangent and the Reciprocal Ratios

using ***Pythagoras' Theorem***

$$r^2 = x^2 + y^2$$
$$r = \sqrt{x^2 + y^2}$$



Note: $\sin^2 = \frac{y^2}{r^2}$ $\cos^2 = \frac{x^2}{r^2}$

$$(1)\ \sin^2\theta + \cos^2\theta = \frac{y^2 + x^2}{r^2} = 1$$

$$\frac{y^2}{r^2} + \frac{x^2}{r^2} = \frac{y^2 + x^2}{r^2} = \frac{\cancel{y^2 + x^2}}{\cancel{x^2 + y^2}} = 1$$

$$(2)\ \cot\theta = \frac{1}{\tan\theta};\ \tan\theta \neq 0$$

6.2 Basic Trigonometric Functions

6.2.1 Sine, Cosine, Tangent and the Reciprocal Ratios

Example 6.4 Find the values of the six trigonometric functions of an angle θ if θ is in standard position with the endpoint $P(-3,4)$.

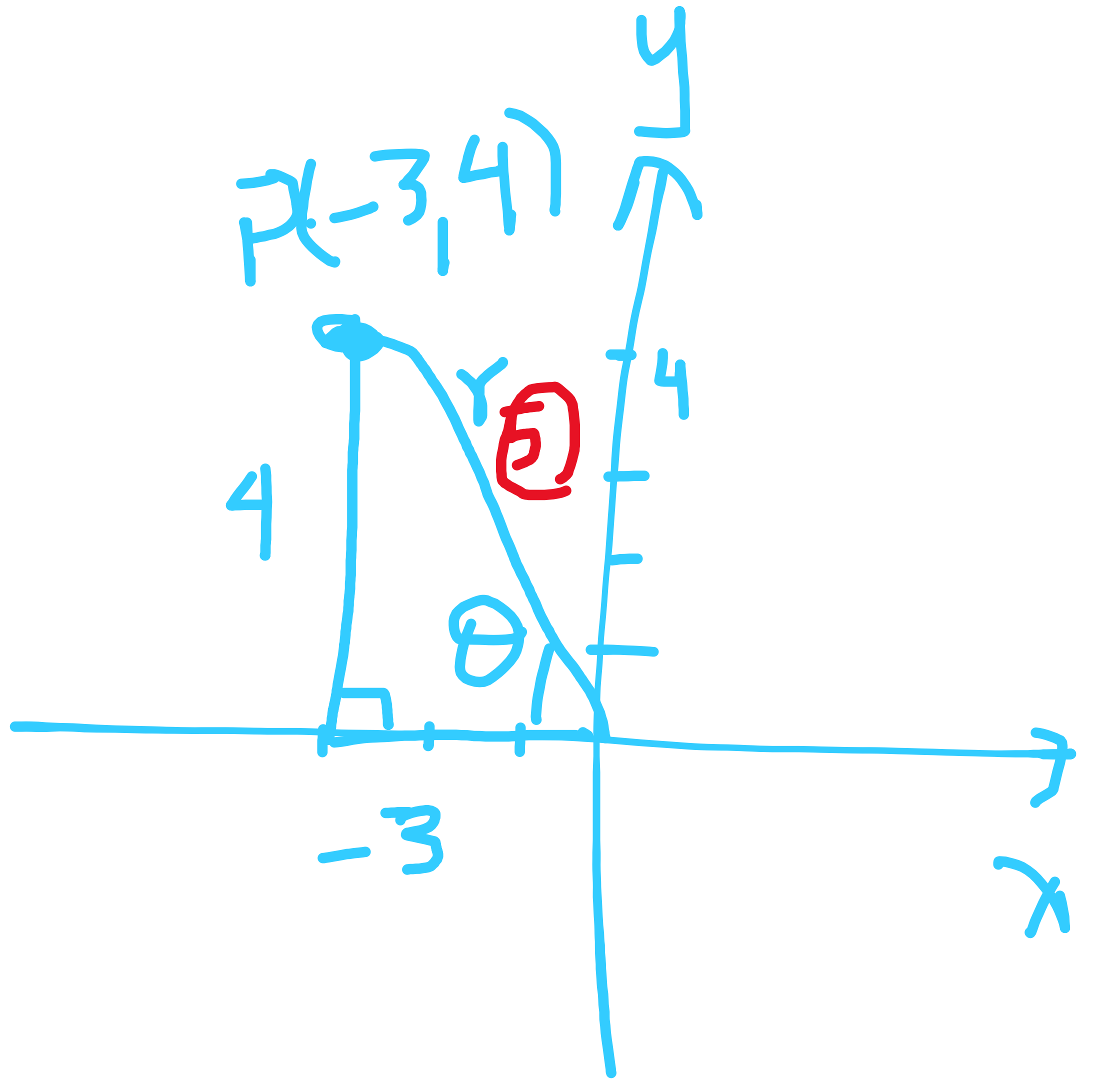
Solution From the point $P(-3,4)$ we have $x = -3$, $y = 4$ with $r^2 = x^2 + y^2$

$$r = \sqrt{x^2 + y^2} = \sqrt{9+16} = 5 \text{ , then}$$

$\frac{o}{h} = \frac{4}{5}$ $\quad$ $= \frac{-3}{5}$

$$\sin\theta = \frac{y}{r} = \boxed{\frac{4}{5}} \quad , \quad \cos\theta = \frac{x}{r} = -\frac{3}{5} \quad , \quad \tan\theta = \frac{\sin\theta}{\cos\theta} = -\frac{4}{3}$$

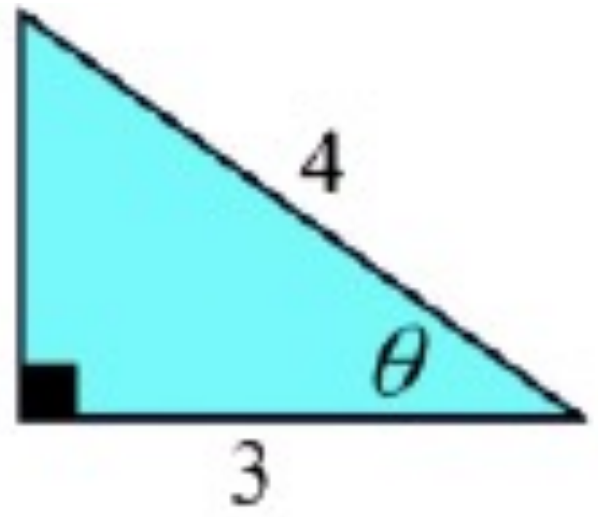
$$\cot\theta = \frac{1}{\tan\theta} = -\frac{3}{4} \quad , \quad \sec\theta = \frac{1}{\cos\theta} = -\frac{5}{3} \quad , \quad \csc\theta = \frac{1}{\sin\theta} = \frac{5}{4}$$



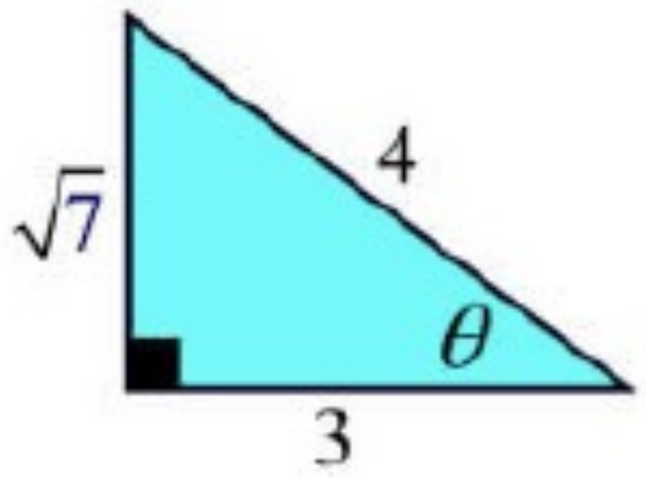
6.2 Basic Trigonometric Functions

6.2.1 Sine, Cosine, Tangent and the Reciprocal Ratios

Examples 6.5 Given the triangle at the right, express the exact value of the six trig functions in relation to theta.



Solution Let $h = 4$, $a = 3$, o is unknown. Finding the missing side o of the right triangle will use the Pythagorean Theorem. Then, using the diagram, express each function as a ratio of the lengths of the sides. *Do not "estimate" the answers.*



Pythagorean Theorem

$$a^2 + o^2 = h^2$$
$$3^2 + o^2 = 4^2$$
$$o^2 = 16 - 9 = 7$$
$$o = \sqrt{7}$$

Then, $\sin\theta = \frac{o}{h} = \frac{\sqrt{7}}{4}$ $\qquad \csc\theta = \frac{h}{o} = \frac{4}{\sqrt{7}} = \frac{4\sqrt{7}}{7}$

$$\cos\theta = \frac{a}{h} = \frac{3}{4} \qquad \sec\theta = \frac{h}{a} = \frac{4}{3}$$

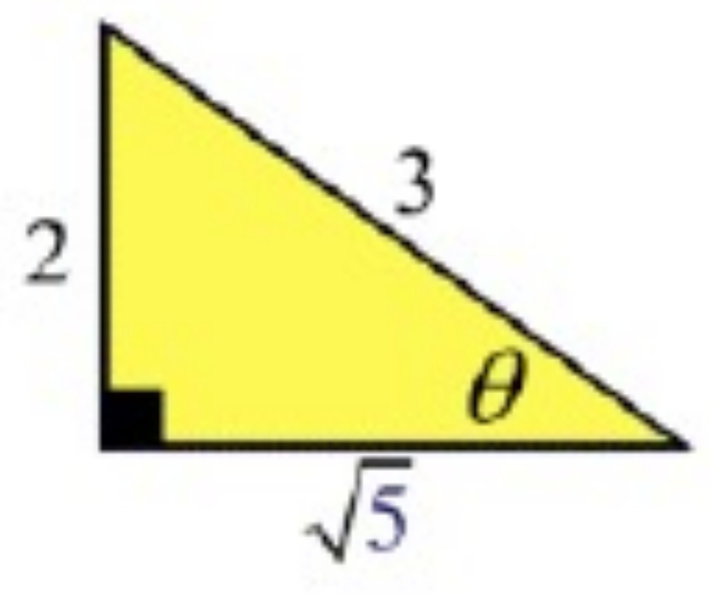
$$\tan\theta = \frac{o}{a} = \frac{\sqrt{7}}{3} \qquad \cot\theta = \frac{a}{o} = \frac{3}{\sqrt{7}} = \frac{3\sqrt{7}}{7}$$

6.2 Basic Trigonometric Functions

6.2.1 Sine, Cosine, Tangent and the Reciprocal Ratios

Example 6.6 Find $\sec\theta$ and $\cot\theta$, given $\sin\theta = \frac{2}{3}$ and $\cos\theta = \frac{\sqrt{5}}{3}$

Solution Draw a diagram to get a better understanding of the given information.



Since sine is opposite over hypotenuse, position the 2 and the 3 accordingly in relation to the angle theta. Now, since cosine is adjacent over hypotenuse, position these values (the 3 should already be properly placed). Be sure that the largest value is on the hypotenuse and that the Pythagorean Theorem is true for these values. (If you are not given the third side, use the Pythagorean Theorem to find it.)

Now, using the diagram, read off the values for the $\sec\theta = \frac{h}{a} = \frac{3}{\sqrt{5}}$ and $\cot\theta = \frac{a}{o} = \frac{\sqrt{5}}{2}$ #

6.2 Basic Trigonometric Functions

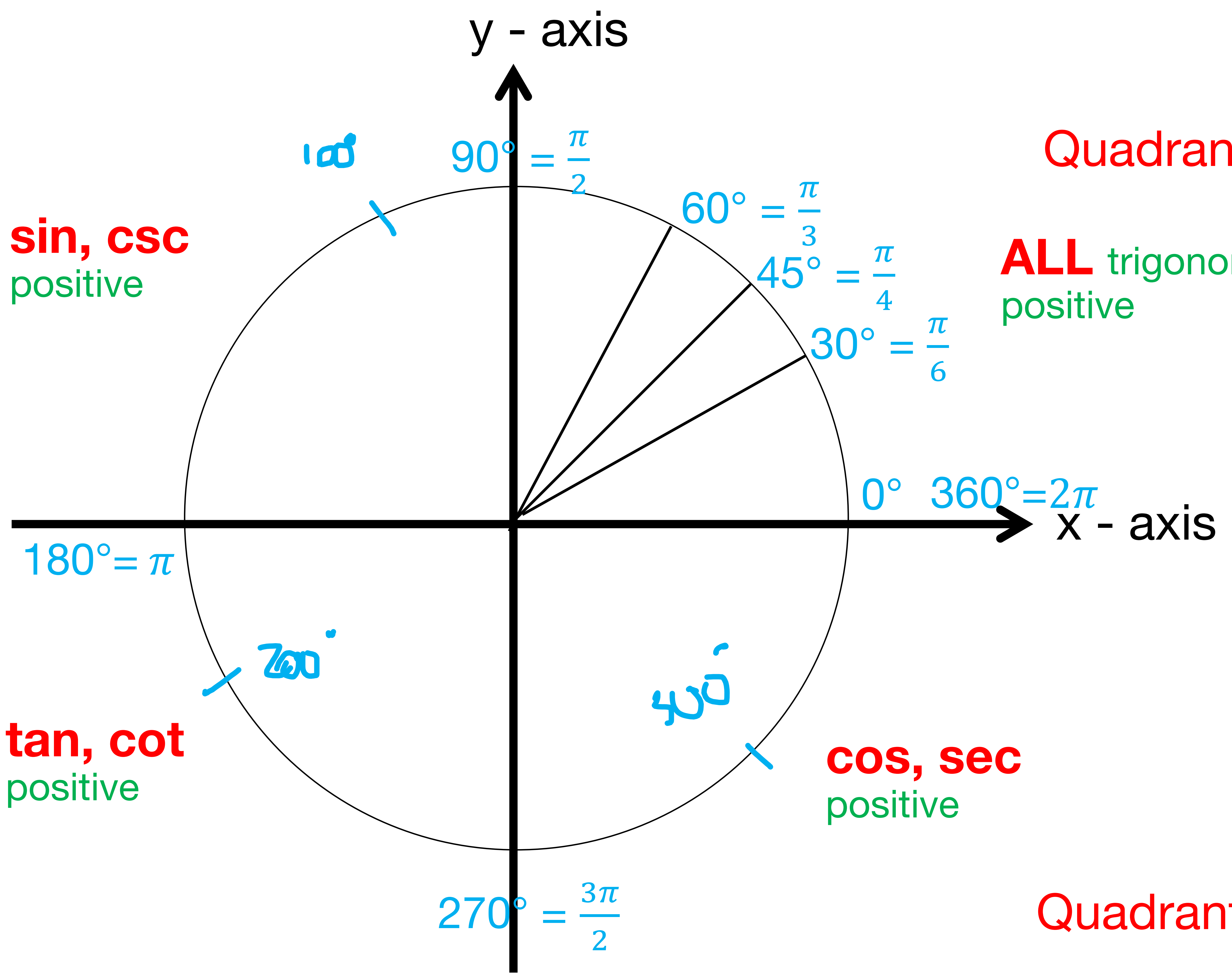
6.2.2 Triginometric Functions of Any Angle

Quadrant II

$\sin 100^\circ = +$
$\cos 100^\circ = -$
$\tan 100^\circ = -$

Quadrant I

ALL trigonometric functions positive



$\sin 200^\circ = -$
$\cos 200^\circ = -$
$\tan 200^\circ = +$

$\sin 300^\circ = -$
$\cos 300^\circ = +$
$\tan 300^\circ = -$

Quadrant III

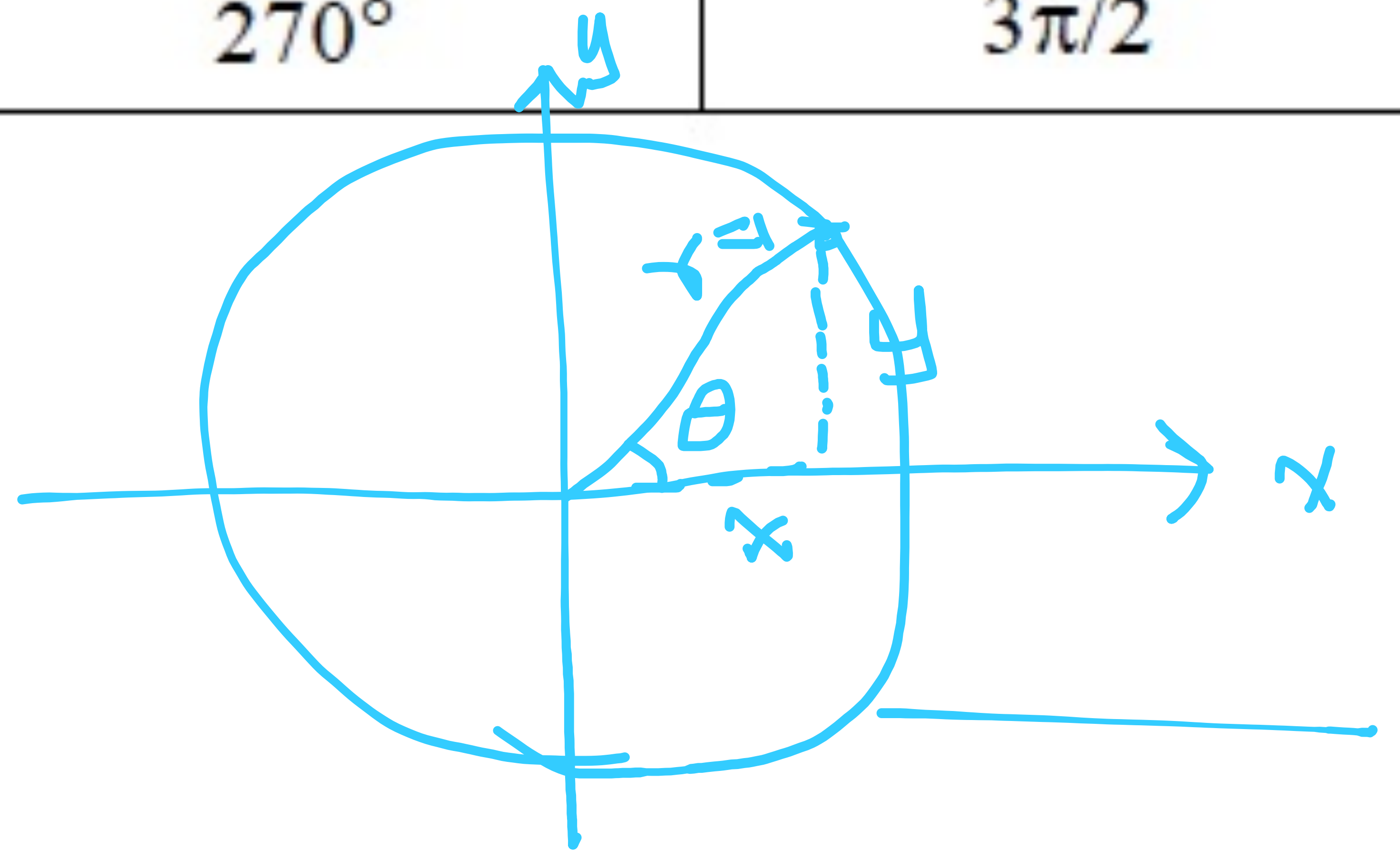
Quadrant IV

6.2 Basic Trigonometric Functions

6.2.2 Triginometric Functions of Any Angle

Table 6.1 summarizes the trigonometric functional values of some special angles.

θ (degrees)	θ (radians)	$\sin\theta$	$\cos\theta$	$\tan\theta$
0° 360	0	0	1	0
30°	$\pi/6$	$1/2$	$\sqrt{3}/2$	$\sqrt{3}/3$
45°	$\pi/4$	$1/\sqrt{2}$	$1/\sqrt{2}$	1
60°	$\pi/3$	$\sqrt{3}/2$	$1/2$	$\sqrt{3}$
90° 450	$\pi/2$	1	0	*undefined*
180° 540	π	0	-1	0
270°	$3\pi/2$	-1	0	*undefined*



unit circle $(r=1)$

$\sin 30^\circ = \frac{1}{2}$

$\cos 60^\circ = \frac{1}{2}$

$\tan 45^\circ = 1$

6.2 Basic Trigonometric Functions

6.2.2 Triginometric Functions of Any Angle

Some Identities

1. $\sin(-x) = -\sin x$, $\cos(-x) = \cos x$, $\tan(-x) = -\tan x$,
$\csc(-x) = -\csc x$, $\sec(-x) = \sec x$, $\cot(-x) = -\cot x$

90°

2. $\sin\left(\frac{\pi}{2} - x\right) = \cos x$, $\cos\left(\frac{\pi}{2} - x\right) = \sin x$, $\tan\left(\frac{\pi}{2} - x\right) = \cot x$
$\csc\left(\frac{\pi}{2} - x\right) = \sec x$, $\sec\left(\frac{\pi}{2} - x\right) = \csc x$, $\cot\left(\frac{\pi}{2} - x\right) = \tan x$

180°

3. $\sin(\pi - x) = \sin x$, $\cos(\pi - x) = -\cos x$, $\tan(\pi - x) = -\tan x$
$\csc(\pi - x) = \csc x$, $\sec(\pi - x) = -\sec x$, $\cot(\pi - x) = -\cot x$

90°

4. $\sin\left(\frac{\pi}{2} + x\right) = \cos x$, $\cos\left(\frac{\pi}{2} + x\right) = -\sin x$, $\tan\left(\frac{\pi}{2} + x\right) = -\cot x$
$\csc\left(\frac{\pi}{2} + x\right) = \sec x$, $\sec\left(\frac{\pi}{2} + x\right) = -\csc x$, $\cot\left(\frac{\pi}{2} + x\right) = -\tan x$

180°

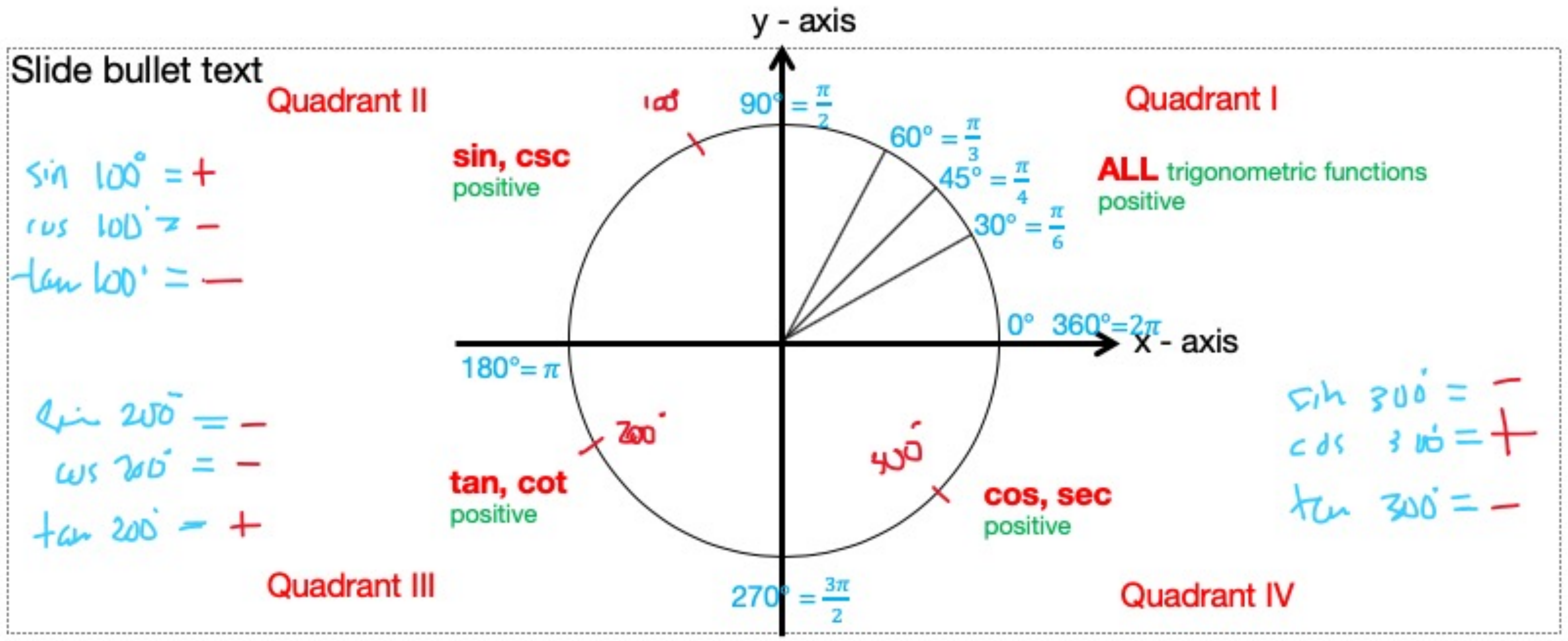
5. $\sin(\pi + x) = -\sin x$, $\cos(\pi + x) = -\cos x$, $\tan(\pi + x) = \tan x$
$\csc(\pi + x) = -\csc x$, $\sec(\pi + x) = -\sec x$, $\cot(\pi + x) = \cot x$

360°

6. $\sin(2\pi + x) = \sin x$, $\cos(2\pi + x) = \cos x$, $\tan(2\pi + x) = \tan x$
$\sec(2\pi + x) = \sec x$, $\csc(2\pi + x) = \csc x$, $\cot(2\pi + x) = \cot x$

6.2 Basic Trigonometric Functions

6.2.2 Triginometric Functions of Any Angle



Examples: Evaluate

(a) cos 225°

$$= \cos(180° + 45°) \quad = -\cos 45° \quad = -\frac{1}{\sqrt{2}}$$

(b) sin (– 480°)

$$= -\sin 480° \quad = -\sin(540° - 60°) \quad = -\sin 60° \quad = -\frac{\sqrt{3}}{2}$$

(c) $\cos\frac{15\pi}{4}$

$$= \cos\left(4\pi - \frac{\pi}{4}\right) \quad = \cos\frac{\pi}{4} \quad = \frac{1}{\sqrt{2}}$$

Table 6.1 summarizes the trigonometric functional values of some special angles.

θ (degrees)	θ (radians)	$\sin\theta$	$\cos\theta$	$\tan\theta$
0°	0	0	1	0
30°	$\pi/6$	$1/2$	$\sqrt{3}/2$	$\sqrt{3}/3$
45°	$\pi/4$	$1/\sqrt{2}$	$1/\sqrt{2}$	1
60°	$\pi/3$	$\sqrt{3}/2$	$1/2$	$\sqrt{3}$
90°	$\pi/2$	1	0	*undefined*
180°	π	0	-1	0
270°	$3\pi/2$	-1	0	*undefined*

Exercise

- Exercises 6.1

6.3 Inverse Trigonometric Functions

Definition: The inverse of trigonometric functions are defined by

1. $\boldsymbol{y = \arcsin x}$ if and only if $x = \sin y$, where $-1 \leq x \leq 1$ and $-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$

$$y = \sin^{-1} x \quad \rightarrow \quad x = \sin y$$

2. $\boldsymbol{y = \arccos x}$ if and only if $x = \cos y$, where $-1 \leq x \leq 1$ and $0 \leq y \leq \pi$

$$y = \cos^{-1} x \quad \rightarrow \quad x = \cos y$$

3. $\boldsymbol{y = \arctan x}$ if and only if $x = \tan y$, where $x \in R$ and $-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$

$$y = tan^{-1} x \quad \rightarrow \quad x = \tan y$$

6.3 Inverse Trigonometric Functions

Table 6.1 summarizes the trigonometric functional values of some special angles.

θ (degrees)	θ (radians)	$\sin\theta$	$\cos\theta$	$\tan\theta$
0°	0	0	1	0
30°	$\pi/6$	$1/2$	$\sqrt{3}/2$	$\sqrt{3}/3$
45°	$\pi/4$	$1/\sqrt{2}$	$1/\sqrt{2}$	1
60°	$\pi/3$	$\sqrt{3}/2$	$1/2$	$\sqrt{3}$
90°	$\pi/2$	1	0	*undefined*
180°	π	0	-1	0
270°	$3\pi/2$	-1	0	*undefined*

Examples: Evaluate

(a) $\arccos\left(-\dfrac{\sqrt{3}}{2}\right)$

Let $\theta = \arccos\left(-\dfrac{\sqrt{3}}{2}\right)$

$$\cos\theta = -\frac{\sqrt{3}}{2}$$

$$\theta = \arccos\left(-\frac{\sqrt{3}}{2}\right)$$

$$= \frac{5\pi}{6} \text{ or } 150°$$

(b) $\sin\left(\arccos\dfrac{1}{2}\right)$

Let $\theta = \arccos\dfrac{1}{2}$

$$\cos\theta = \frac{1}{2}$$

$$\theta = \arccos\frac{1}{2}$$

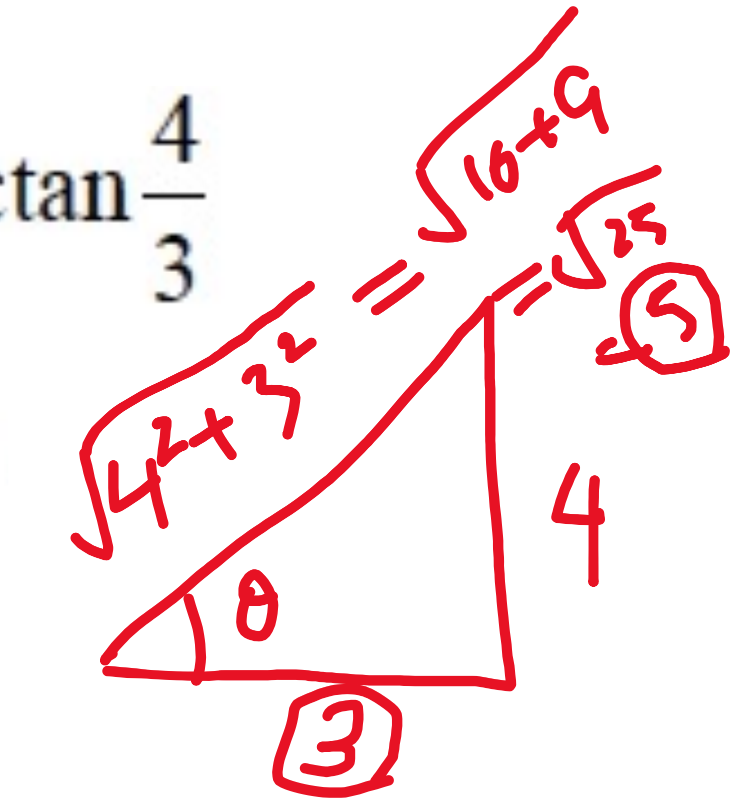
$$= \frac{\pi}{3} \text{ or } 60°$$

Then $\sin\left(\arccos\dfrac{1}{2}\right) = \sin\dfrac{\pi}{3} = \dfrac{\sqrt{3}}{2}$

(c) $\cos\left(\arctan\dfrac{4}{3}\right)$

Let $\theta = \arctan\dfrac{4}{3}$

$$\tan\theta = \frac{4}{3}$$



then $\cos\left(\arctan\dfrac{4}{3}\right) = \cos\theta = \dfrac{3}{5}$

$$\cos\theta = \frac{A}{H} = \frac{3}{5}$$

Exercise

- Exercises 6.2

6.4 Trigonometric Identities

Some Important Identities

(1) $\sin^2\theta + \cos^2\theta = 1$

(2) $1 + \tan^2\theta = \sec^2\theta$

(3) $1 + \cot^2\theta = \csc^2\theta$

6.4 Trigonometric Identities

Sum and Difference Formula

(4) $\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta$

(5) $\cos(\alpha-\beta)=\cos\alpha\cos\beta+\sin\alpha\sin\beta$

(6) $\sin(\alpha+\beta)=\sin\alpha\cos\beta+\cos\alpha\sin\beta$

(7) $\sin(\alpha-\beta)=\sin\alpha\cos\beta-\cos\alpha\sin\beta$

(8) $\tan(\alpha+\beta)=\dfrac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$

(9) $\tan(\alpha-\beta)=\dfrac{\tan\alpha-\tan\beta}{1+\tan\alpha\tan\beta}$

6.4 Trigonometric Identities

Double Angle Formula

$$(10)\quad \sin 2\theta = 2\sin\theta\cos\theta$$

$$(11)\quad \cos 2\theta = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$$

$$(12)\quad \tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$$

6.4 Trigonometric Identities

Examples: Verify the identity

(a) $\sec\theta\sin\theta = \tan\theta$

LS.

$$= \frac{1}{\cos\theta}\times\sin\theta$$

$$= \frac{\sin\theta}{\cos\theta}$$

$$= \tan\theta = \text{RS}$$

$(\sin\theta)\times$ $\quad\times(1+\cos\theta)$

(b) $\dfrac{\sin\theta}{1+\cos\theta} + \dfrac{1+\cos\theta}{\sin\theta} = 2\csc\theta$

LS.

$$= \frac{\sin^2\theta + (1+\cos\theta)^2}{\sin\theta(1+\cos\theta)}$$

$=1$

$$= \frac{\sin^2\theta + 1 + 2\cos\theta + \cos^2\theta}{\sin\theta(1+\cos\theta)}$$

$$= \frac{2+2\cos\theta}{\sin\theta(1+\cos\theta)}$$

$$= \frac{2\cancel{(1+\cos\theta)}}{\sin\theta\cancel{(1+\cos\theta)}} = 2\csc\theta = \text{RS}.$$

(c) $\dfrac{\sin(\alpha+\beta)}{\cos\alpha\cos\beta} = \tan\alpha + \tan\beta$

LS.

$$= \frac{\sin\alpha\cos\beta + \cos\alpha\sin\beta}{\cos\alpha\cos\beta}$$

$$= \frac{\sin\alpha}{\cos\alpha} + \frac{\sin\beta}{\cos\beta}$$

$$= \tan\alpha + \tan\beta = \text{RS}.$$

6.4 Trigonometric Identities

Examples: Evaluate

(a) $\cos 75°$

$= \cos(45° + 30°)$

$= \cos 45° \cos 30° - \sin 45° \sin 30°$

$$= \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right)$$

$$= \frac{\sqrt{6}-\sqrt{2}}{4}$$

(b) $\sin\left(\arccos\frac{1}{2} + \arcsin\frac{3}{5}\right)$

Let $\alpha = \arccos\frac{1}{2}$ $\qquad \beta = \arcsin\frac{3}{5}$

$\cos\alpha = \frac{1}{2}$ $\qquad \sin\beta = \frac{3}{5}$

Then $\sin\left(\arccos\frac{1}{2} + \arcsin\frac{3}{5}\right)$

$$= \sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$

$$= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{4}{5}\right) + \left(\frac{1}{2}\right)\left(\frac{3}{5}\right) = \frac{4\sqrt{3}+3}{10}$$

Exercise

- Exercises 6.3

6.4 Trigonometric Identities

6.4.1 Trigonometric Equations

Examples- Let $0 \le \theta < 2\pi$; solve the equation

(a) $2\sin^2\theta - 3\sin\theta + 1 = 0$ $\quad x = \sin\theta$

$2x^2 - 3x + 1 = 0$ $\quad (2x-1)(x-1)$

$(2\sin\theta - 1)(\sin\theta - 1) = 0$ $\quad (2\sin\theta - 1)(\sin\theta - 1)$

$\therefore \quad 2\sin\theta - 1 = 0 \quad$ **or** $\quad \sin\theta - 1 = 0$

$\sin\theta = \dfrac{1}{2} \qquad\qquad \sin\theta = 1$

$\theta = \dfrac{\pi}{6}, \dfrac{5\pi}{6} \qquad\qquad \theta = \dfrac{\pi}{2}$

(b) $\cos 2\theta + 3 = 5\cos\theta$

$(2\cos^2\theta - 1) + 3 = 5\cos\theta$ $\quad x = \cos\theta$

$2\cos^2\theta - 5\cos\theta + 2 = 0$ $\quad 2x^2 - 5x + 2 = 0$

$(\cos\theta - 2)(2\cos\theta - 1) = 0$ $\quad (2x-1)(x-2) = 0$

$\therefore \quad \cos\theta - 2 = 0 \quad$ or $\quad 2\cos\theta - 1 = 0$

$\cos\theta = 2 \qquad\qquad \cos\theta = \dfrac{1}{2}$

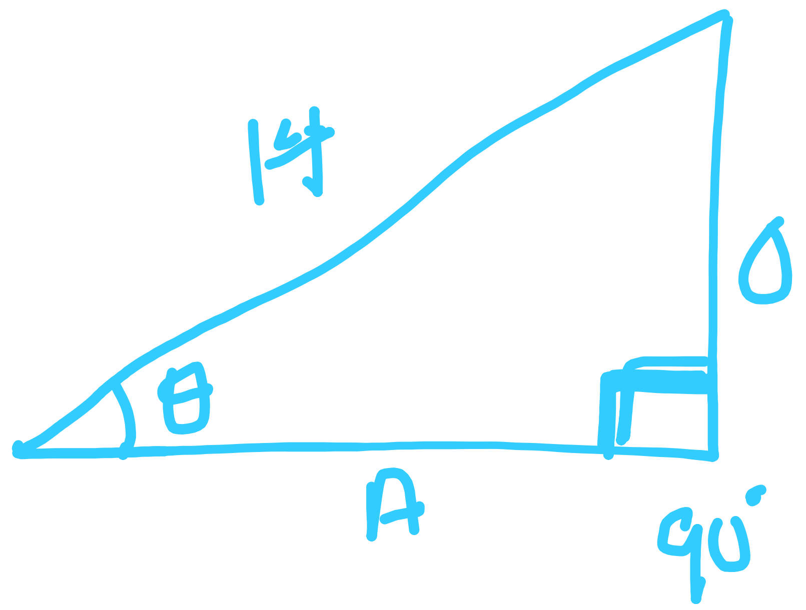
Impossible because $-1 \le \cos\theta < 1$ $\qquad \theta = \dfrac{\pi}{3}, \dfrac{5\pi}{3}$

Exercise

- Exercises 6.4

6.5 Solving Oblique Triangles

Laws of Cosines and Sines



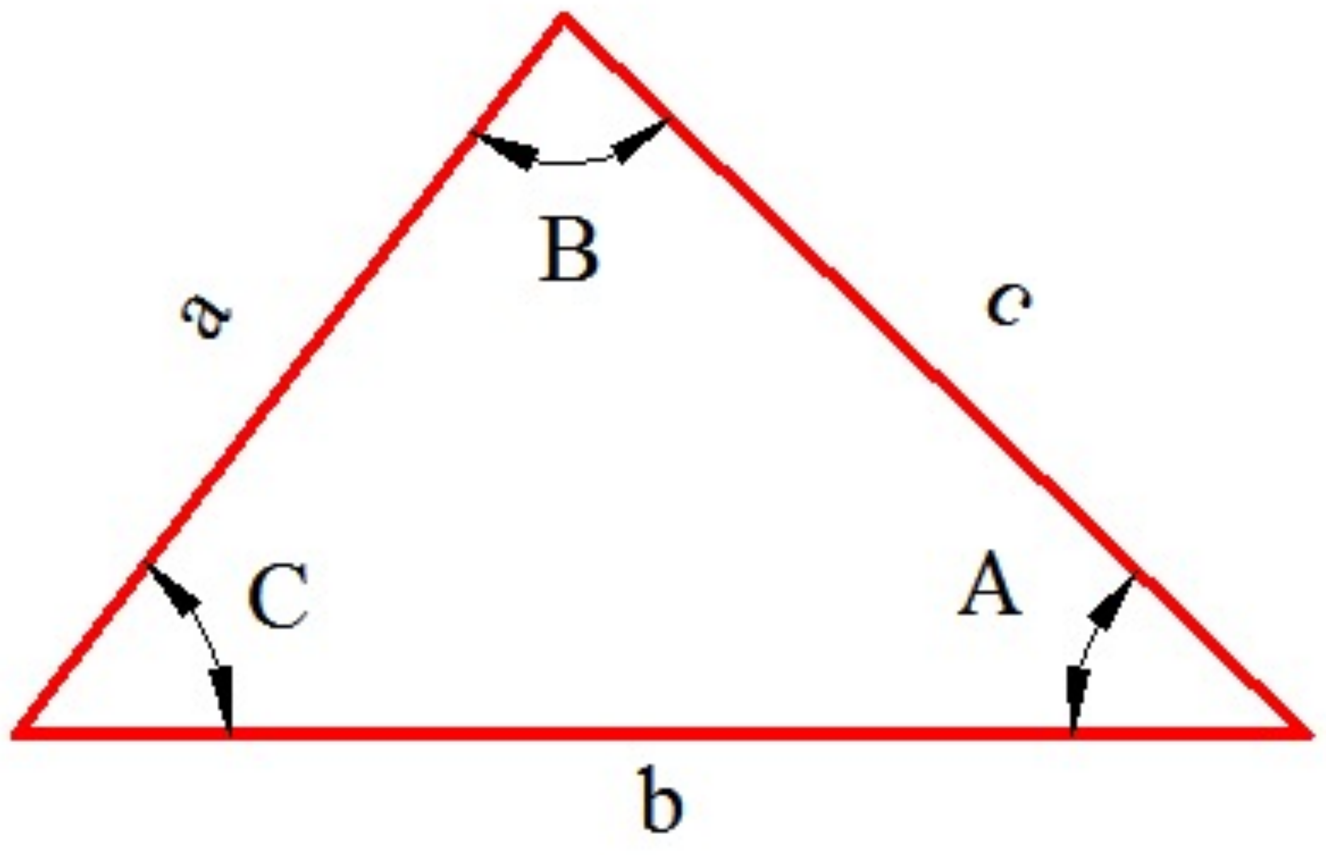
In any triangle *ABC* that has of length *a* , *b* and *c* , the following relationships are true

Laws of Cosines

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$b^2 = a^2 + c^2 - 2ac\cos B$$

$$c^2 = a^2 + b^2 - 2ab\cos C$$

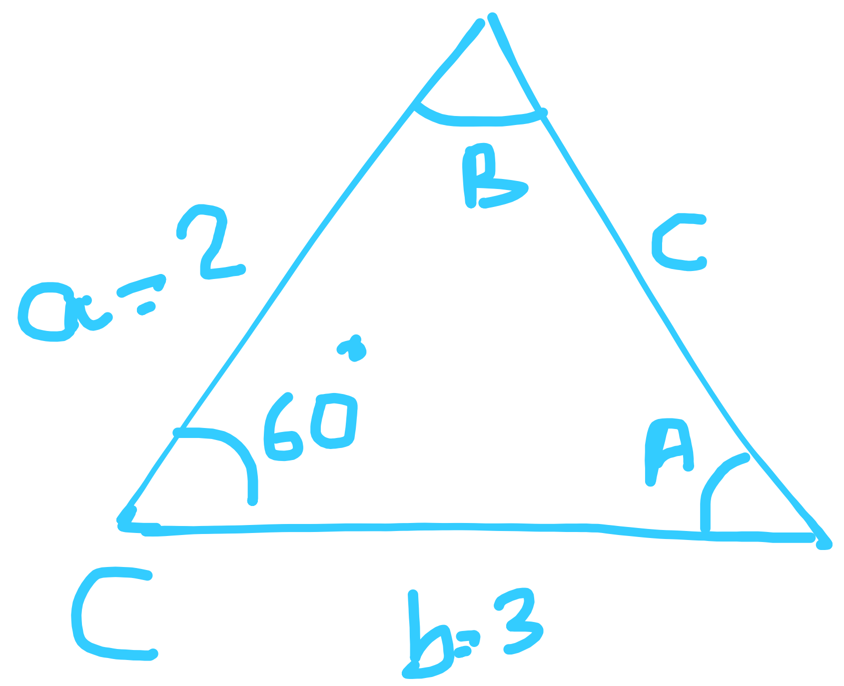


Laws of Sines

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

6.5 Solving Oblique Triangles

Laws of Cosines and Sines



Example 6.12 Solve the triangle $a = 2$, $b = 3$ and $C = 60°$.

Solution

From Laws of Cosines $$c^2 = a^2 + b^2 - 2ab\cos C$$

$$c^2 = 4 + 9 - 2(2)(3)\cos 60°$$

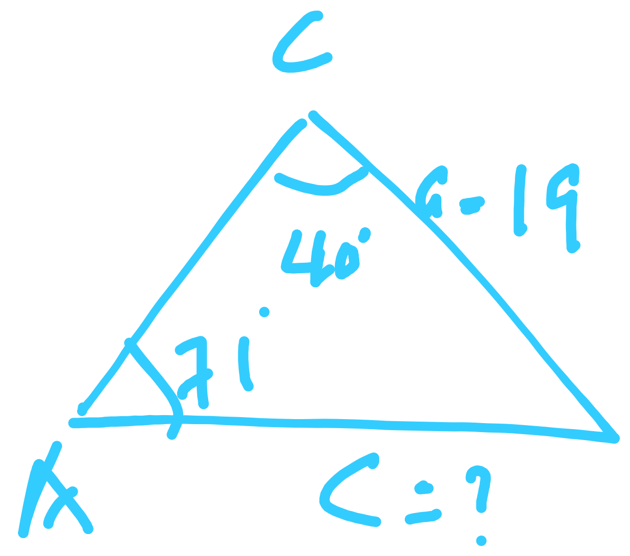
$$c^2 = 13 - 12\left(\frac{1}{2}\right) = 7$$

$$c = \sqrt{7}$$ #

$$\boxed{\frac{c}{\sin C} = \frac{a}{\sin A} = \frac{b}{\sin B}}$$

6.5 Solving Oblique Triangles

Laws of Cosines and Sines



Example 6.13 Suppose that in triangle ABC , A = 71° , C = 40° and $a = 19$ cms.

Find c to the nearest tenth of a centimeter .

Solution From Laws of Sines $\boxed{\dfrac{a}{\sin A} = \dfrac{c}{\sin C}}$

$$\frac{19}{\sin 71^\circ} = \frac{c}{\sin 40^\circ}$$

$$\therefore \qquad c = \frac{19 \sin 40^\circ}{\sin 71^\circ} \approx 12.9 \text{ cms.}$$

Assignment

Deadline for submission: Wednesday August, 2020

- Exercises 6.1
- Exercises 6.2
- Exercises 6.3
- Exercises 6.4
- Exercises 6.5