Forecasting Techniques

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Strategies for evaluating forecasting methods

1. Select suitable method based on forecaster's **analysis** and the **nature of data**.
2. Divide dataset into **fitting (Train) section** and **forecasting (Test) section**.
3. Develop fitted values for the initialization part of data. (Train)
4. Forecast the test part of the data and determine and evaluate the forecasting error. (Test)
5. Make decision.
   - to use the technique in its present form,
   - to modify the technique, or
   - to develop a forecast using another technique and compare the results.

Naïve Models

Naïve forecasts will assume that **recent data are the best predictors** of the future

Past	Present	Future
Past data	You are here: t	Periods to be forecast
$\ldots, Y_{t-3}, Y_{t-2}, Y_{t-1}$	Y_t	$\hat{Y}_{t+1}, \hat{Y}_{t+2}, \hat{Y}_{t+3}, \ldots$
where	Y_t is the most recent observation of a variable	$\hat{Y}_{t+1}$ is the forecast for one period in the future

Naïve Models

1. The **simplest Naïve model** $\hat{Y}_{t+1} = Y_t$ where $\hat{Y}_{t+1}$ is the forecast value at time t+1

2. **Naïve trend model** $\hat{Y}_{t+1} = Y_t + (Y_t - Y_{t-1})$

3. **Naïve rate of change model** $\hat{Y}_{t+1} = Y_t \dfrac{Y_t}{Y_{t-1}}$

4. **Naïve seasonal model for quarterly data** $\hat{Y}_{t+1} = Y_{t-3}$

5. **Naïve trend and seasonal model for quarterly data** $\hat{Y}_{t+1} = Y_{t-3} + \dfrac{Y_t - Y_{t-4}}{4}$

Naïve Models *(example)*

fitted values (training set)

forecasting (Test set)

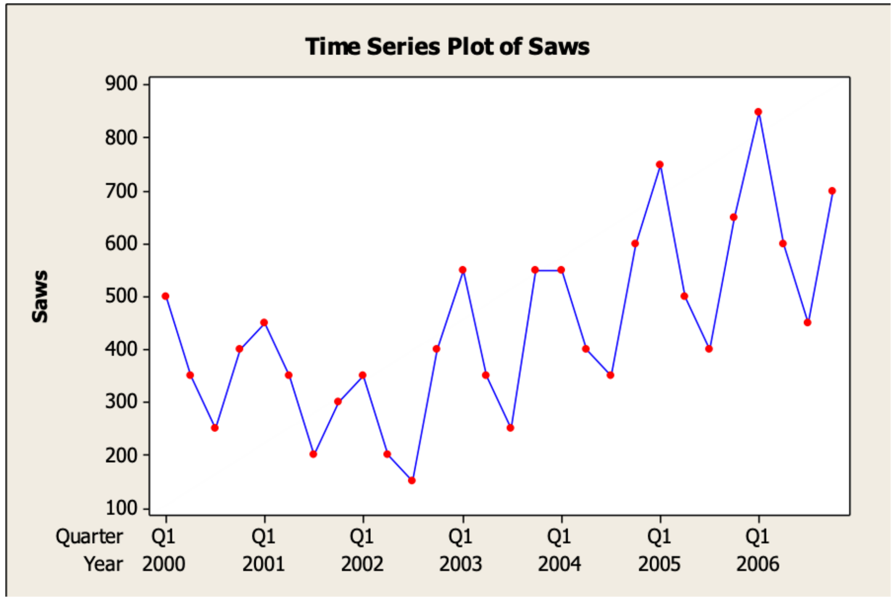
Year	Quarter	t	Saws
2000	1	1	500
	2	2	350
	3	3	250
	4	4	400
2001	1	5	450
	2	6	350
	3	7	200
	4	8	300
2002	1	9	350
	2	10	200
	3	11	150
	4	12	400
2003	1	13	550
	2	14	350
	3	15	250
	4	16	550
2004	1	17	550
	2	18	400
	3	19	350
	4	20	600
2005	1	21	750
	2	22	500
	3	23	400
	4	24	650
2006	1	25	850
	2	26	600
	3	27	450
	4	28	700

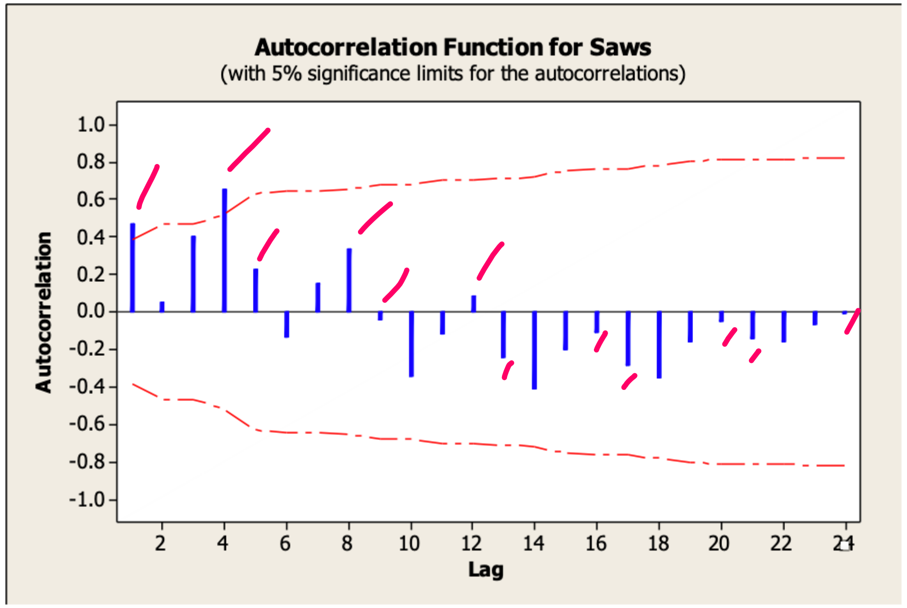
Naïve – use to forecast sales for the next quarter to be the **same as for the previous** quarter

$$\hat{Y}_{24+1} = Y_{24}$$

$$\boxed{\hat{Y}_{25} = 650}$$

$$\boxed{e_{25} = Y_{25} - \hat{Y}_{25} = 850 - 650 = 200}$$





Naïve Models *(example)*

Forecast Equation	Forecast Value	Forecast Error
1. Naïve model $\hat{Y}_{t+1} = Y_t$	$\hat{Y}_{24+1} = Y_{24} = 650$	$e_{25} = Y_{25} - \hat{Y}_{25} = 850 - 650 = 200$

Taking trend into consideration

2. Naïve trend model $\hat{Y}_{t+1} = Y_t + (Y_t - Y_{t-1})$	$\hat{Y}_{24+1} = Y_{24} + (Y_{24} - Y_{24-1}) = 650 + (650 - 400) = 900$	$e_{25} = Y_{25} - \hat{Y}_{25} = 850 - 900 = -50$

The rate of change might be more appropriate than the absolute amount of change

3. Naïve rate of change model $\hat{Y}_{t+1} = Y_t \dfrac{Y_t}{Y_{t-1}}$	$\hat{Y}_{t+1} = Y_t \dfrac{Y_t}{Y_{t-1}} = Y_{24} \dfrac{Y_{24}}{Y_{23}}$ $\hat{Y}_{25} = 650 \dfrac{650}{400} = 1{,}056$	$e_{25} = Y_{25} - \hat{Y}_{25} = 850 - 1{,}056 = -206$

Naïve Models *(example)*

Forecast Equation	Forecast Value	Forecast Error
If the seasonal pattern is strong, then an appropriate forecast for quarterly data might be		
4. Naïve seasonal model for quarterly data $\hat{Y}_{t+1} = Y_{t-3}$	$\hat{Y}_{25} = Y_{21} = 750$	$e_{25} = Y_{25} - \hat{Y}_{25} = 850 - 750 = 100$
If the trend and seasonal variation exist, the analyst can combine trend and seasonal estimates and forecast for quarterly data might be		
5. Naïve trend and seasonal model for quarterly data $\hat{Y}_{t+1} = Y_{t-3} + \frac{Y_t - Y_{t-4}}{4}$	$\hat{Y}_{24+1} = Y_{24-3} + \frac{Y_{24} - Y_{24-4}}{4}$ $\hat{Y}_{25} = Y_{21} + \frac{Y_{24} - Y_{20}}{4}$ $= 750 + \frac{650 - 600}{4} = 762.5$	$e_{25} = Y_{25} - \hat{Y}_{25}$ $= 850 - 762.5 = 87.5$

Averaging Methods

Forecasting Methods Based on Averaging

1. Simple Averages $$\hat{Y}_{t+1} = \frac{1}{t}\sum_{i=1}^{t} Y_i$$

- Appropriate to use when the series to be forecast have stabilized
- the series exists is generally unchanging

2. **Moving Averages** $$\hat{Y}_{t+1} = \frac{Y_t + Y_{t-1} + \cdots + Y_{t-k+1}}{k}$$

- equal weights are assigned to each observation
- can not handle trend or seasonality very well
- must choose the number of periods, *k*
- frequently used with quarterly or monthly data
- The larger the order of the moving average, the greater the smoothing effect.

3. Weighted Moving Averages **< Single Exponential Smoothing >**

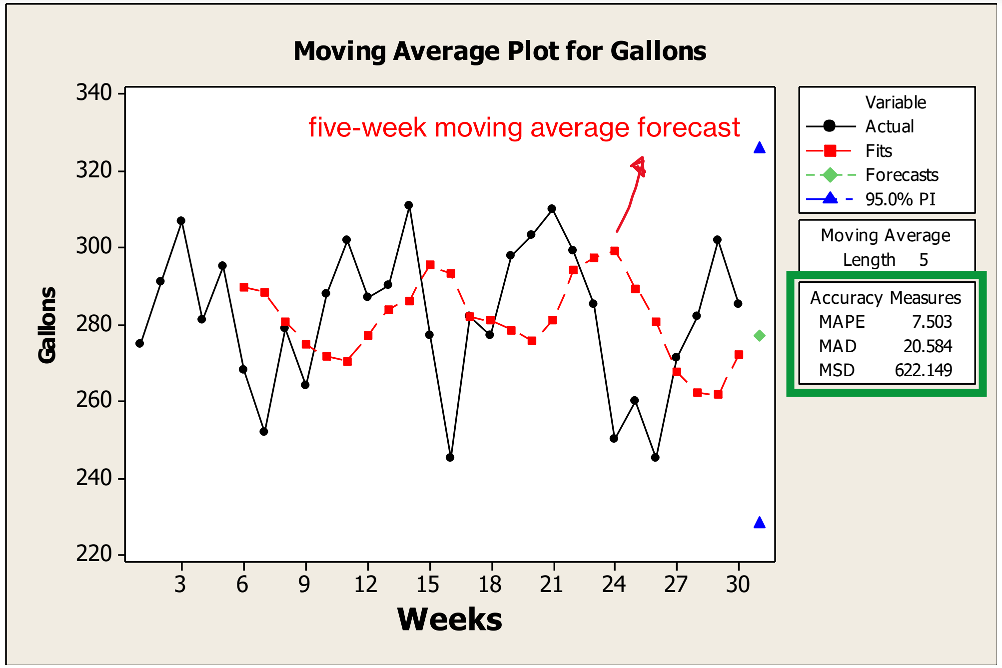
- involve selecting a different weight for each data value

Moving Averages *(example)*

Forecast

Forecast Error

Week, t	Gallons, Y_t	$\hat{Y}_t$	e_t
1	275	-	-
2	291	-	-
3	307	-	-
4	281	-	-
5	295	-	-
6	268	289.8	-21.8
7	252	288.4	-36.4
8	279	280.6	-1.6
9	264	275.0	-11.0
10	288	271.6	16.4
11	302	270.2	31.8
12	287	277.0	10.0
13	290	284.0	6.0
14	311	286.2	24.8
15	277	295.6	-18.6
16	245	293.4	-48.4
17	282	282.0	0.0
18	277	281.0	-4.0
19	298	278.4	19.6
20	303	275.8	27.2
21	310	281.0	29.0
22	299	294.0	5.0
23	285	297.4	-12.4
24	250	299.0	-49.0
25	260	289.4	-29.4
26	245	280.8	-35.8
27	271	267.8	3.2
28	282	262.2	19.8
29	302	261.6	40.4
30	285	272.0	13.0

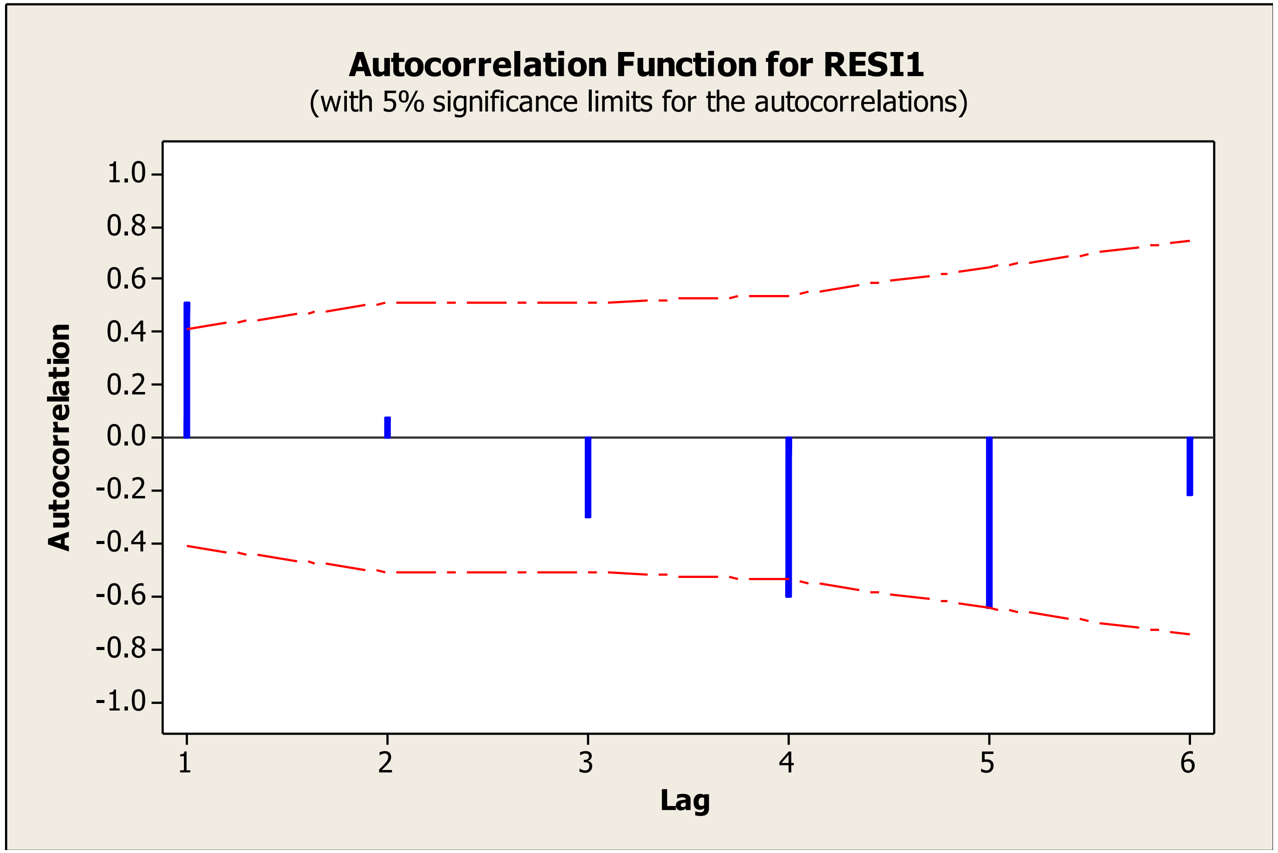


three measures of forecast accuracy

The moving average forecast for week 6 is

$$\hat{Y}_6 = \frac{Y_5 + Y_4 + Y_3 + Y_2 + Y_1}{5} = \frac{295 + 281 + 307 + 291 + 275}{5} = \frac{1449}{5} = \boxed{289.8}$$

Moving Averages *(example)*



Autocorrelation Function for the Residuals When a Five-Week Moving Average Method

Autocorrelation Function: RESI1

Lag	ACF	T	LBQ
1	0.506290	2.53	7.21
2	0.078551	0.32	7.39
3	-0.298611	-1.21	10.13
4	-0.602830	-2.31	21.81
5	-0.642632	-2.06	35.74
6	-0.219452	-0.61	37.46

Smoothing Methods

Single Exponential Smoothing

applies **unequal weights** to the time series observations.

This unequal weighting is accomplished by using a **smoothing constant** that determines how much weight is attached to each observation.

$$\boxed{\hat{y}_{t+1} = \alpha\, y_t + (1-\alpha)\, \hat{y}_t}$$

$\hat{y}_t$ = forecast of the time series for period t
y_t = actual value of the time series for period t
$\hat{y}_{t+1}$ = forecast of the time series for period t+1
α = smoothing constant ($0 \leq \alpha \leq 1$)

$$\boldsymbol{y_t = \beta_0 + e_t \;-\; no\ trend}$$

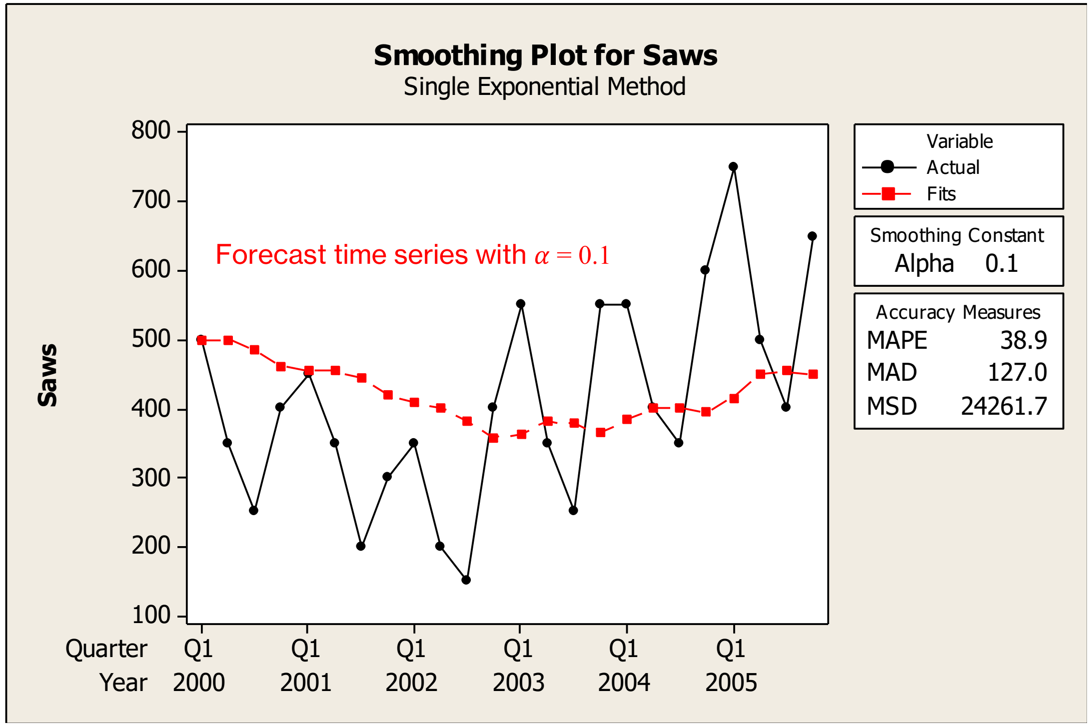
Example: Single Exponential Smoothing

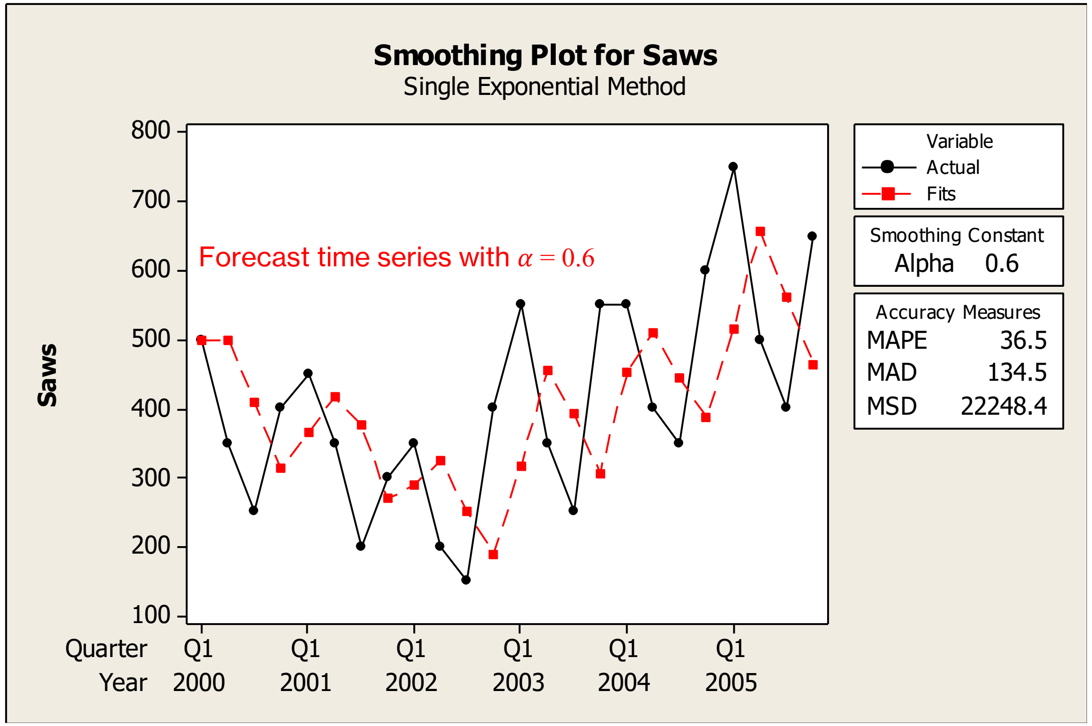
Year	Quarter	Saws, y_t	Smoothed Value, $\alpha = 0.1$ $\boxed{\hat{y}_{t+1} = \alpha\, y_t + (1-\alpha)\,\hat{y}_t}$	Forecast Error, e_t	Smoothed Value, $\alpha = 0.6$ $\boxed{\hat{y}_t = \alpha\, y_t + (1-\alpha)\,\hat{y}_{t-1}}$	Forecast Error, e_t
2000	1	500	$\hat{y}_1 = 500.0$	0.0	$\hat{y}_1 = 500.0$	0.0
	2	350	$\hat{y}_2 = 0.1\, y_1 + 0.9\,\hat{y}_1$ $= 0.1(500)+0.9(500)$ $= 500$	-150.0	$\hat{y}_2 = 0.6\, y_1 + 0.4\,\hat{y}_1$ $= 0.6(500) + 0.4(500)$ $= 500$	-150.0
	3	250	$\hat{y}_3 = 0.1\, y_2 + 0.9\,\hat{y}_2$ $= 0.1(350)+0.9(500)$ $= 485$	-235.0	$\hat{y}_3 = 0.6\, y_2 + 0.4\,\hat{y}_2$ $= 0.6(350)+0.4(500)$ $= 410$	-160.0
	4	400	461.5	-61.5	314.0	86.0

Example: Single Exponential Smoothing

Year	Quarter	Saws, y_t	Smoothed Value, $\alpha = 0.1$ $\hat{y}_{t+1} = \alpha\, y_t + (1-\alpha)\,\hat{y}_t$	Forecast Error, e_t	Smoothed Value, $\alpha = 0.6$ $\hat{y}_t = \alpha\, y_t + (1-\alpha)\,\hat{y}_{t-1}$	Forecast Error, e_t
2001	1	450	455.4	-5.4	365.6	84.4
	2	350	454.8	-104.8	416.2	-66.2
	3	200	444.3	-244.3	376.5	-176.5
	4	300	419.9	-119.9	270.6	29.4
2002	1	350	407.9	-57.9	288.2	61.8
	2	200	402.1	-202.1	325.3	-125.3
	3	150	381.9	-231.9	250.1	-100.1
	4	400	358.7	41.3	190.0	210.0
2003	1	550	362.8	187.2	316.0	234.0
	2	350	381.6	-31.6	456.4	-106.4
	3	250	378.4	-128.4	392.6	-142.6
	4	550	365.6	184.4	307.0	243.0
2004	1	550	384.0	166.0	452.8	97.2
	2	400	400.6	-0.6	511.1	-111.1
	3	350	400.5	-50.5	444.4	-94.4
	4	600	395.5	204.5	387.8	212.2
2005	1	750	415.9	334.1	515.1	234.9
	2	500	449.3	50.7	656.0	-156.0
	3	400	454.4	-54.4	562.4	-162.4
	4	650	449.0	201.0	465.0	185.0

Example: Single Exponential Smoothing



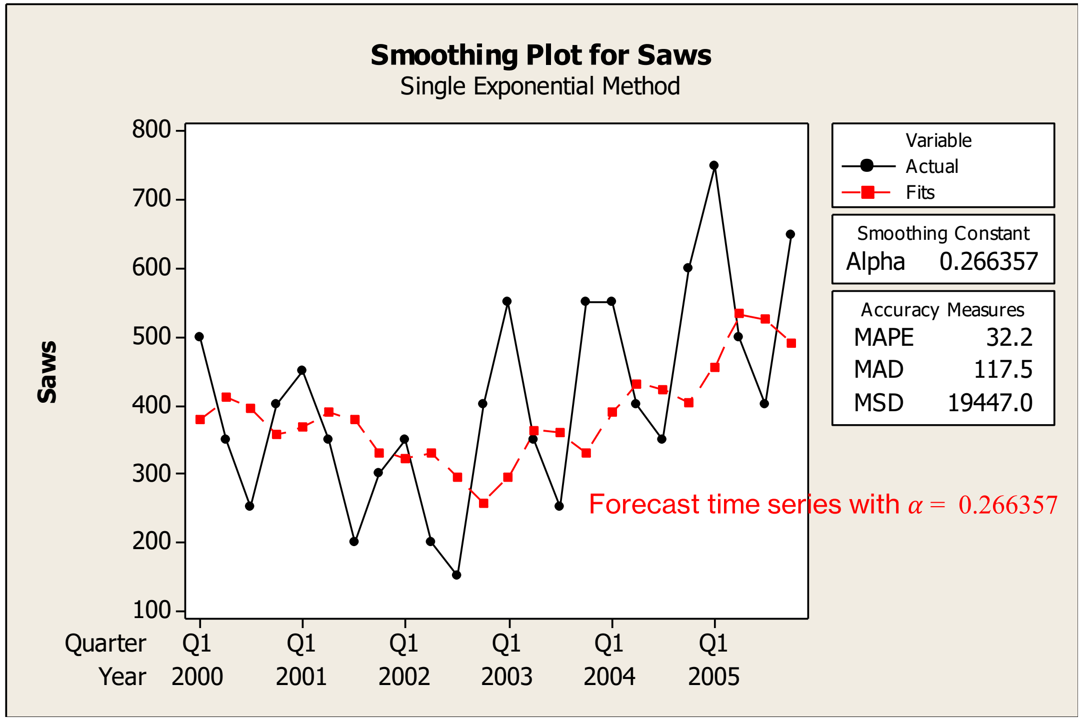


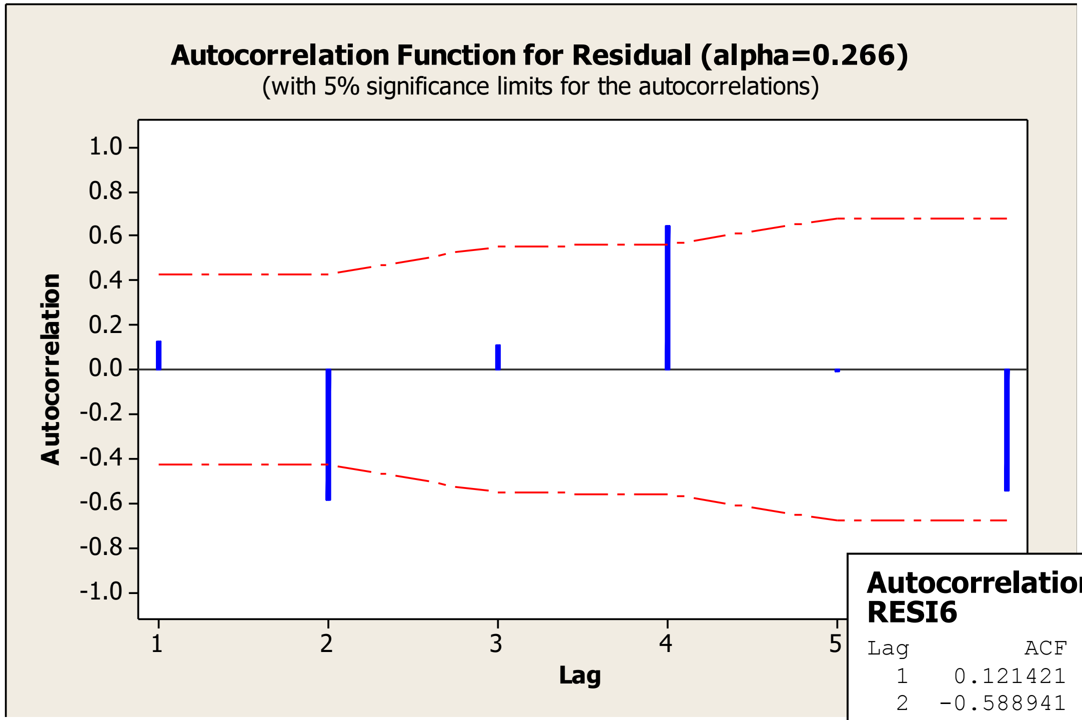
$\alpha = 0.1$ MSE = 24,261.7 MAPE = 38.9%

$\alpha = 0.6$ MSE = 22,248.4 MAPE = 36.5%

Example: Single Exponential Smoothing

$$\hat{y}_1 = \frac{1}{k}\sum_{t=1}^{k} y_t$$





Autocorrelation Function: RESI6

Lag	ACF	T	LBQ
1	0.121421	0.59	0.40
2	-0.588941	-2.84	10.24
3	0.109122	0.41	10.59
4	0.646967	2.40	23.65
5	-0.006908	-0.02	23.65
6	-0.542625	-1.65	33.86

$\alpha = 0.266357$ MSE = 19,447 MAPE = 32.2%

Double Exponential Smoothing: *Holt's Method*

- Simple exponential smoothing cannot handle trend or seasonality
- Double exponential smoothing can handle **trended data** of the form

$$\boldsymbol{y_t = \beta_0 + \beta_1 t + e_t}$$

$$\boxed{\hat{y}_{t+p} = L_t + pT_t}$$

$$L_t = \alpha\, y_t + (1-\alpha)(L_{t-1} + T_{t-1})$$

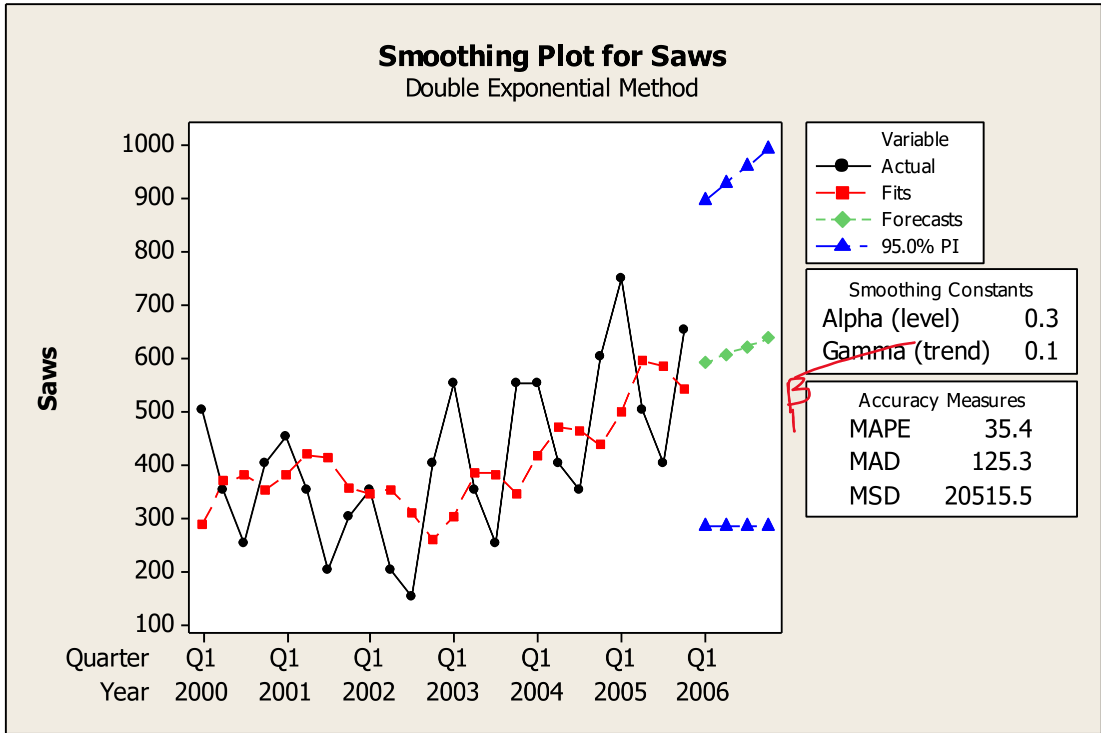
$$T_t = \beta\,(L_t - L_{t-1}) + (1-\beta)\,T_{t-1}$$

α = smoothing constant for the level $(0 < \alpha < 1)$
β = smoothing constant for trend estimate $(0 < \beta < 1)$
ρ = periods to be forecast into the future

Example: Double Exponential Smoothing

Year	Quarter	Saws, y_t	L_t	T_t	$\hat{y}_t$	e_t	**forecast for year 2006**
2000	1	500	350.8	17.0	286.8	213.2	589.5
	2	350	362.4	16.4	367.7	-17.7	605.1
	3	250	340.2	12.6	378.8	-128.8	620.7
	4	400	366.9	14.0	352.8	47.2	636.4
2001	1	450	401.6	16.1	380.9	69.1	
	2	350	397.4	14.0	417.7	-67.7	
	3	200	348.0	7.7	411.4	-211.4	
	4	300	339.0	6.0	355.7	-55.7	
2002	1	350	346.5	6.2	345.0	5.0	
	2	200	306.8	1.6	352.6	-152.6	
	3	150	260.9	-3.2	308.4	-158.4	
	4	400	300.4	1.1	257.7	142.3	
2003	1	550	376.1	8.6	301.5	248.5	
	2	350	374.2	7.5	384.6	-34.6	
	3	250	342.2	3.6	381.7	-131.7	
	4	550	407.0	9.7	345.8	204.2	
2004	1	550	456.7	13.7	416.7	133.3	
	2	400	449.3	11.6	470.4	-70.4	
	3	350	427.6	8.2	460.9	-110.9	
	4	600	485.1	13.2	435.8	164.2	
2005	1	750	573.8	20.7	498.3	251.7	
	2	500	566.2	17.9	594.5	-94.5	
	3	400	528.8	12.4	584.0	-184.0	
	4	650	573.8	15.6	541.2	108.8	

Example: Double Exponential Smoothing

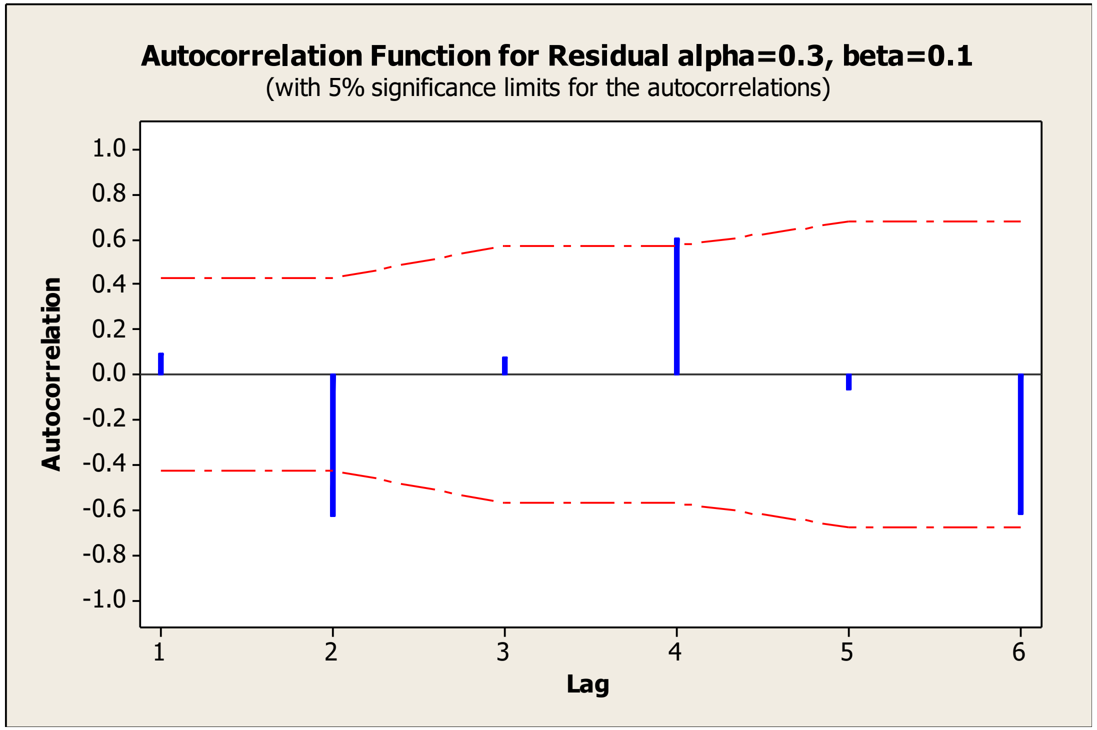


Double exponential smoothing and single exponential smoothing are comparable.

To summarize:

$\alpha = 0.266$	MSE = 19,447	MAPE = 32.2%
$\alpha = 0.30$ and $\beta = 0.1$	MSE = 19,447	MAPE = 32.2%

Example: Double Exponential Smoothing



Autocorrelation Function: RESI4

Lag	ACF	T	LBQ
1	0.092547	0.45	0.23
2	-0.629408	-3.06	11.47
3	0.074355	0.27	11.63
4	0.601371	2.18	22.92
5	-0.070132	-0.22	23.08
6	-0.618267	-1.90	36.33

Multiplicative Winters' Method: *Holt's-Winter*

- Double exponential smoothing cannot handle seasonality
- Multiplicative Winters' method can handle trend and **seasonal data** of the form

$$\boldsymbol{y_t = (\beta_0 + \beta_1 t) \cdot SN_t + e_t}$$

$$\boxed{\hat{y}_{t+p} = (L_t + pT_t)S_{t-s+p}}$$

$$L_t = \alpha \frac{y_t}{S_{t-s}} + (1-\alpha)(L_{t-1} + T_{t-1})$$

$$T_t = \beta(L_t - L_{t-1}) + (1-\beta)T_{t-1}$$

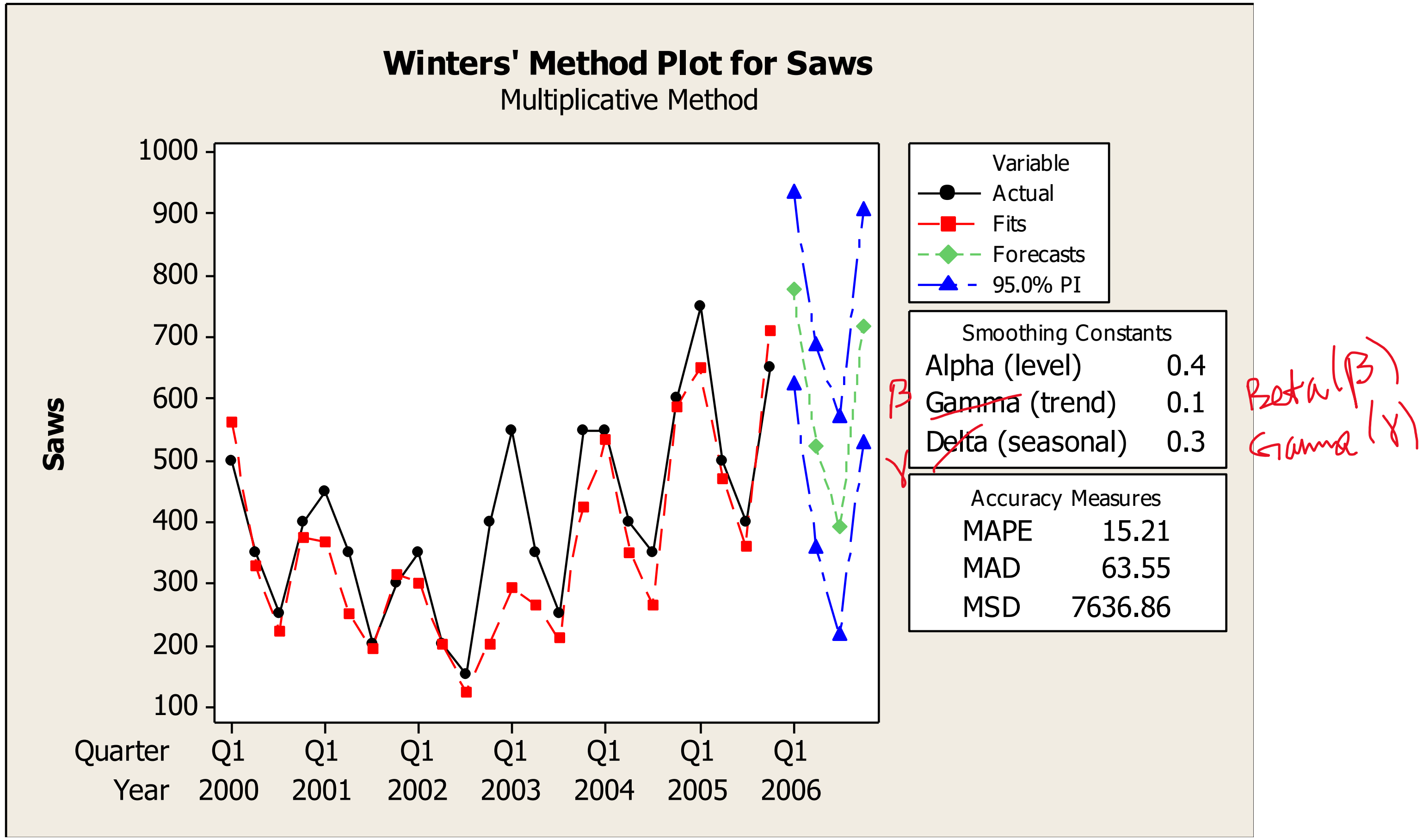
$$S_t = \gamma \frac{y_t}{L_t} + (1-\gamma)S_{t-s}$$

α = smoothing constant for the level ($0 < \alpha < 1$)
β = smoothing constant for trend estimate (0 < β < 1)
γ = smoothing constant for seasonality estimate ($0 < \gamma < 1$)
s = length of the seasonality
ρ = periods to be forecast into the future

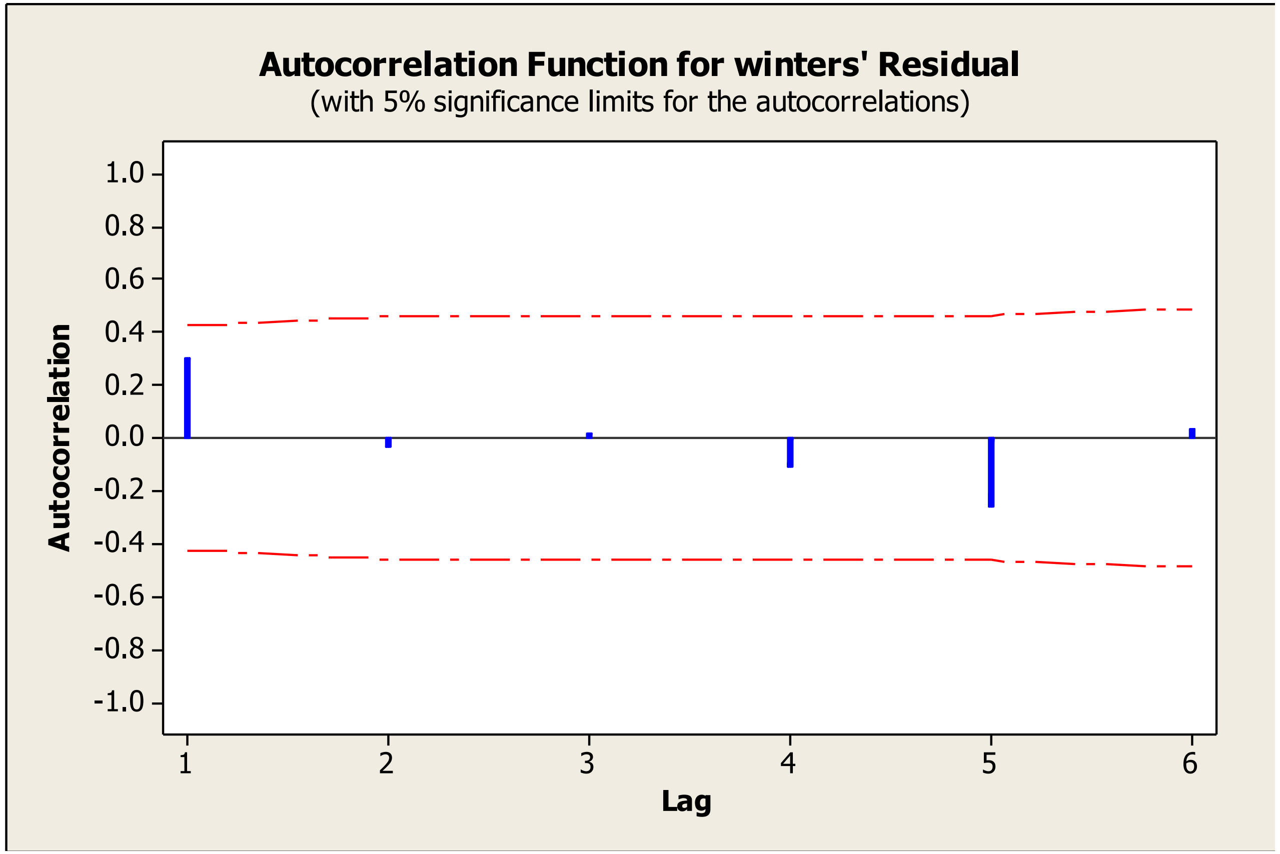
Example: Multiplicative Winters' Method

Year	Quarter	Saws, y_t	L_t	T_t	S_t	$\hat{y}_t$	e_t	**forecast for year 2006**
2000	1	500	415.5	-42.0	1.3	563.3	-63.3	778.2
	2	350	383.1	-41.0	0.9	328.9	21.1	521.9
	3	250	359.0	-39.3	0.7	222.6	27.4	393.4
	4	400	328.1	-38.5	1.2	375.3	24.7	716.7
2001	1	450	315.8	-35.8	1.3	367.1	82.9	
	2	350	325.2	-31.3	0.9	249.3	100.7	
	3	200	296.7	-31.0	0.7	195.2	4.8	
	4	300	260.5	-31.6	1.2	315.6	-15.6	
2002	1	350	243.8	-30.1	1.4	300.9	49.1	
	2	200	212.8	-30.2	0.9	202.3	-2.3	
	3	150	199.5	-28.5	0.7	121.9	28.1	
	4	400	238.6	-21.7	1.3	201.3	198.7	
2003	1	550	293.0	-14.1	1.5	292.9	257.1	
	2	350	315.6	-10.4	1.0	263.3	86.7	
	3	250	327.5	-8.2	0.7	211.3	38.7	
	4	550	357.4	-4.4	1.4	423.6	126.4	
2004	1	550	357.6	-3.9	1.5	532.6	17.4	
	2	400	373.2	-2.0	1.0	351.4	48.6	
	3	350	418.9	2.8	0.8	265.0	85.0	
	4	600	425.6	3.2	1.4	586.3	13.7	
2005	1	750	454.9	5.8	1.6	650.7	99.3	
	2	500	473.1	7.0	1.0	468.6	31.4	
	3	400	501.3	9.1	0.8	360.3	39.7	
	4	650	492.5	7.4	1.4	712.7	-62.7	

Example: Multiplicative Winters' Method



Example: Multiplicative Winters' Method



Autocorrelation Function: RESI5

Lag	ACF	T	LBQ
1	0.297749	1.46	2.41
2	-0.031543	-0.14	2.43
3	0.017610	0.08	2.44
4	-0.107084	-0.48	2.80
5	-0.257066	-1.15	4.97
6	0.033280	0.14	5.01

Class Assignment

Advantages and limitations of

- Naïve Methods
- Averaging Methods
- Smoothing Methods

Present it to class

Time Series Regression

Time series Regression:
Modeling trend components

Trend Projection

Linear

- **Linear trend regression**
- With time as independent variable

Non-linear

- **Quadratic trend**
- **Exponential trend**

Linear trend regression

$$T_t = b_0 + b_1 t$$

where

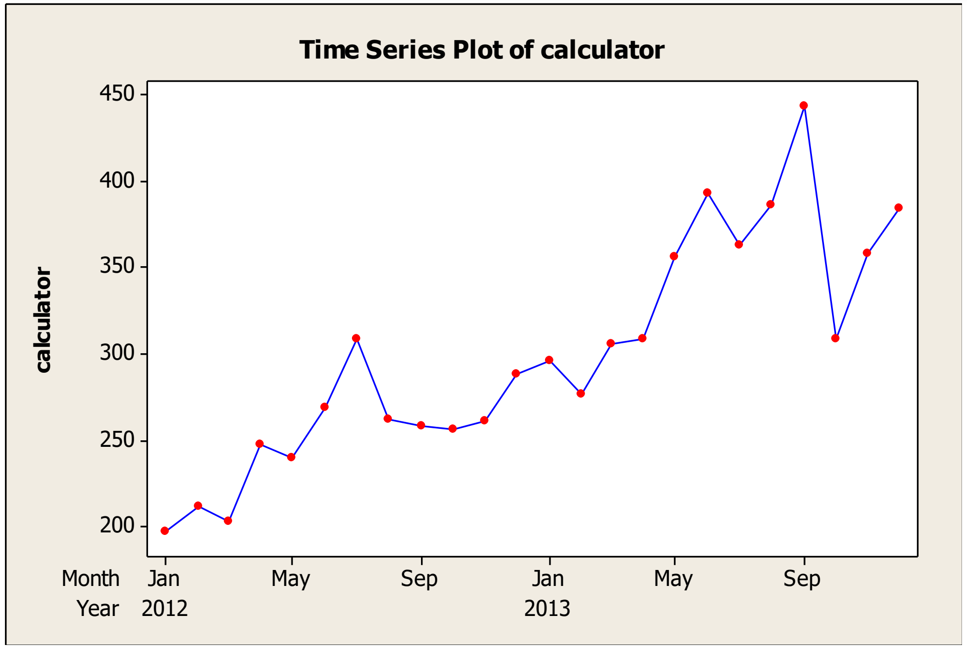
- T_t = linear trend forecast in period t
- b_0 = intercept of the linear trend line
- b_1 = slope of the linear trend line
- t = time period
- n = number of periods in time series

Computing the slope, b_0 and intercept, b_1 for a linear trend

$$b_0 = \bar{y} - b_1 \bar{t}$$

$$b_1 = \frac{\sum t y_t - \frac{\left(\sum t \sum y_t\right)}{n}}{\sum t^2 - \frac{\left(\sum t\right)^2}{n}}$$

Example: Linear Trend Regression



$$T_t = b_0 + b_1 t$$

$$T_t = 198.0296 + 8.0743t$$

$$b_0 = \bar{y} - b_1 \bar{t}$$

$$b_1 = \frac{\sum t y_t - \frac{\left(\sum t \sum y_t\right)}{n}}{\sum t^2 - \frac{\left(\sum t\right)^2}{n}}$$

Trend calculations of calculator sales data

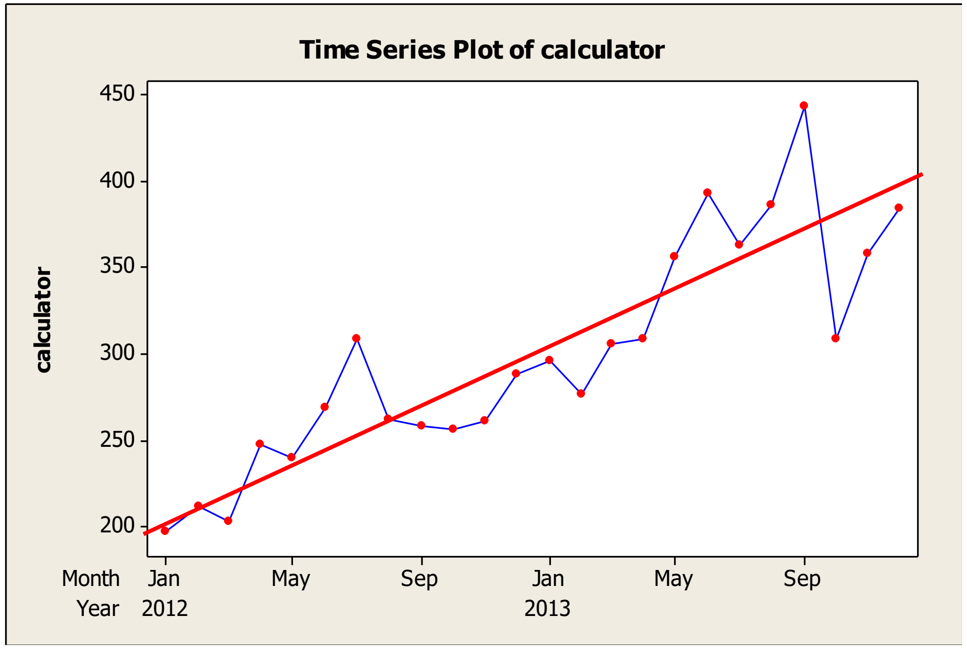
Year	Month	Calculator Sales, y_t	t	t^2	ty_t
2012	Jan.	197	1	1	197
	Feb.	211	2	4	422
	Mar.	203	3	9	609
	Apr.	247	4	16	988
	May	239	5	25	1195
	Jun.	269	6	36	1614
	Jul.	308	7	49	2156
	Aug.	262	8	64	2096
	Sep.	258	9	81	2322
	Oct.	256	10	100	2560
	Nov.	261	11	121	2871
	Dec.	288	12	144	3456
2013	Jan.	296	13	169	3848
	Feb.	276	14	196	3864
	Mar.	305	15	225	4575
	Apr.	308	16	256	4928
	May	356	17	289	6052
	Jun.	393	18	324	7074
	Jul.	363	19	361	6897
	Aug.	386	20	400	7720
	Sep.	443	21	441	9303
	Oct.	308	22	484	6776
	Nov.	358	23	529	8234
	Dec.	384	24	576	9216
	Total	7175	300	4900	98973
	Mean	298.9583	12.50		

$$b_1 = \frac{98{,}973 - \frac{(300)(7{,}175)}{24}}{4{,}900 - \frac{(300)^2}{24}} = \frac{98{,}973 - 89{,}687.5}{4{,}900 - 3{,}750} = \frac{9{,}285.5}{1{,}150} = \boxed{8.0743}$$

$$b_0 = 298.9583 - 8.0743(12.5) = 298.9583 - 100.92875 = \boxed{198.0296}$$

Example: Linear Trend Regression

$T_t = 198.0296 + 8.0743t$



T_{25}
$= 198.0296 + 8.0743(25)$
$= 399.8871$

T_{26}
$= 198.0296 + 8.0743(26)$
$= 407.9614$

trend pattern seem to be curvilinear

Year	Month	Calculator Sales, y_t	t	t^2	ty_t
2012	Jan.	197	1	1	197
	Feb.	211	2	4	422
	Mar.	203	3	9	609
	Apr.	247	4	16	988
	May	239	5	25	1195
	Jun.	269	6	36	1614
	Jul.	308	7	49	2156
	Aug.	262	8	64	2096
	Sep.	258	9	81	2322
	Oct.	256	10	100	2560
	Nov.	261	11	121	2871
	Dec.	288	12	144	3456
2013	Jan.	296	13	169	3848
	Feb.	276	14	196	3864
	Mar.	305	15	225	4575
	Apr.	308	16	256	4928
	May	356	17	289	6052
	Jun.	393	18	324	7074
	Jul.	363	19	361	6897
	Aug.	386	20	400	7720
	Sep.	443	21	441	9303
	Oct.	308	22	484	6776
	Nov.	358	23	529	8234
	Dec.	384	24	576	9216
2014	**Jan**	399.8871	25		
	Feb	407.9614	26		

Example: Linear Trend Regression

Model 1: **Linear trend model** $y_t = \beta_0 + \beta_1 t + e_t$

Regression Analysis: calculator sales versus t

The regression equation is calculator sales = 198 + 8.07 t

Predictor	Coef	SE Coef	T	P
Constant	198.03	13.34	14.84	**0.000**
t	8.0743	0.9339	8.65	**0.000**

Has the significance

S = 31.67 **R-Sq = 77.3%** R-Sq(adj) = 76.2%

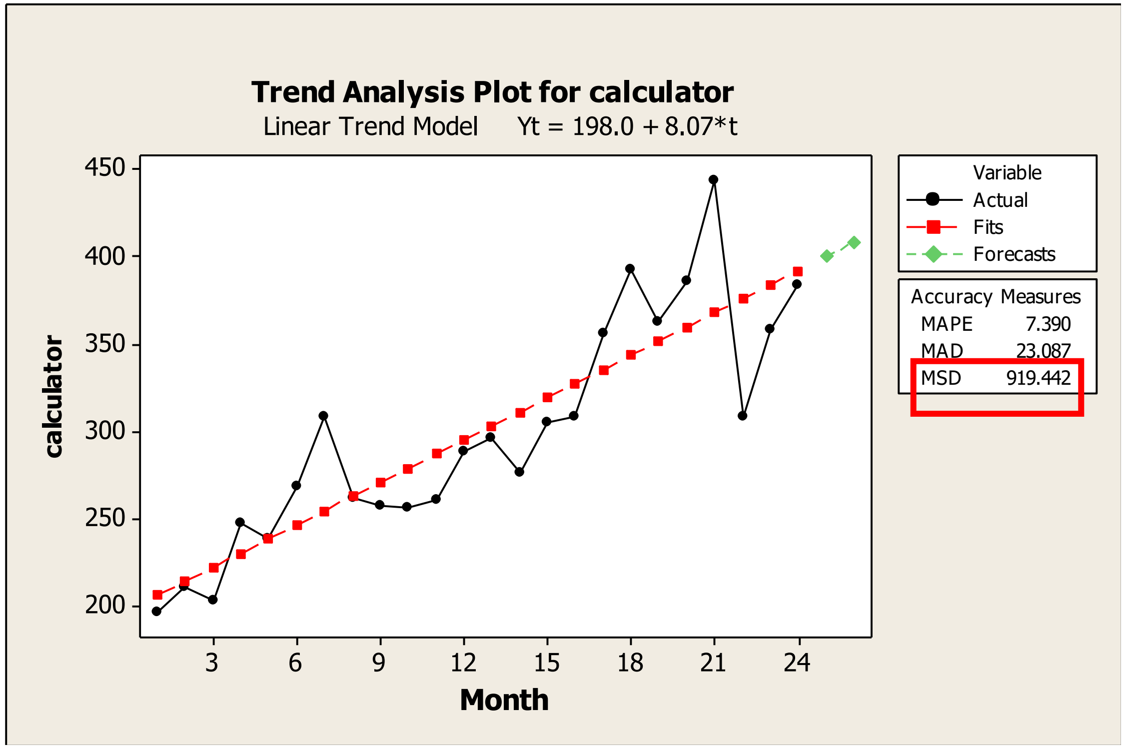
Analysis of Variance

Source	DF	SS	MS	F	P
Regression	1	74974	74974	74.75	0.000
Residual Error	22	22067	1003		
Total	23	97041			

Unusual Observations

Obs	t	calculator	Fit	SE Fit	Residual	St Resid
21	21.0	443.00	367.59	10.24	75.41	2.52R
22	22.0	308.00	375.66	10.98	-67.66	-2.28R

R denotes an observation with a large standardized residual



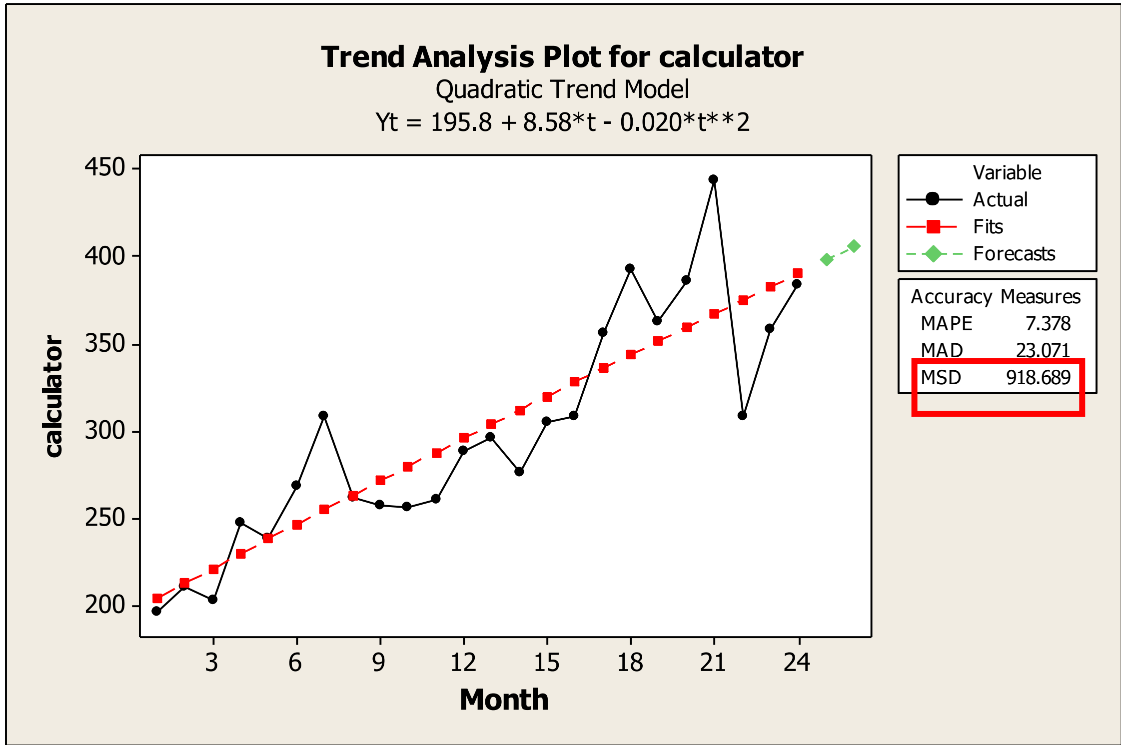
Example: Linear Trend Regression

$T_t = 198.0296 + 8.0743t$

Model 2: **Quadratic trend model** $y_t = \beta_0 + \beta_1 t + \beta_2 t^2 + e_t$

```
Regression Analysis: calculator sales versus t, t_square

The regression equation is calculator sales = 196 + 8.58 t - 0.020 t_square

Predictor        Coef    SE Coef       T       P
Constant       195.83      21.62    9.06   0.000
t               8.582      3.985    2.15   0.043
t_square      -0.0203     0.1547   -0.13   0.897

S = 32.40       R-Sq = 77.3%     R-Sq(adj) = 75.1%

Analysis of Variance
Source            DF        SS       MS       F       P
Regression         2     74992    37496   35.71   0.000
Residual Error    21     22049     1050
Total             23     97041

Source       DF     Seq SS
t             1      74974
t_square      1         18

Unusual Observations
Obs        t    calculat       Fit    SE Fit    Residual    St Resid
 21     21.0      443.00    367.10     11.13       75.90       2.49R
 22     22.0      308.00    374.81     13.00      -66.81      -2.25R

R denotes an observation with a large standardized residual
```

has no the significance



Time series Regression: Modeling Seasonal Components

Treat the season as categorical variable

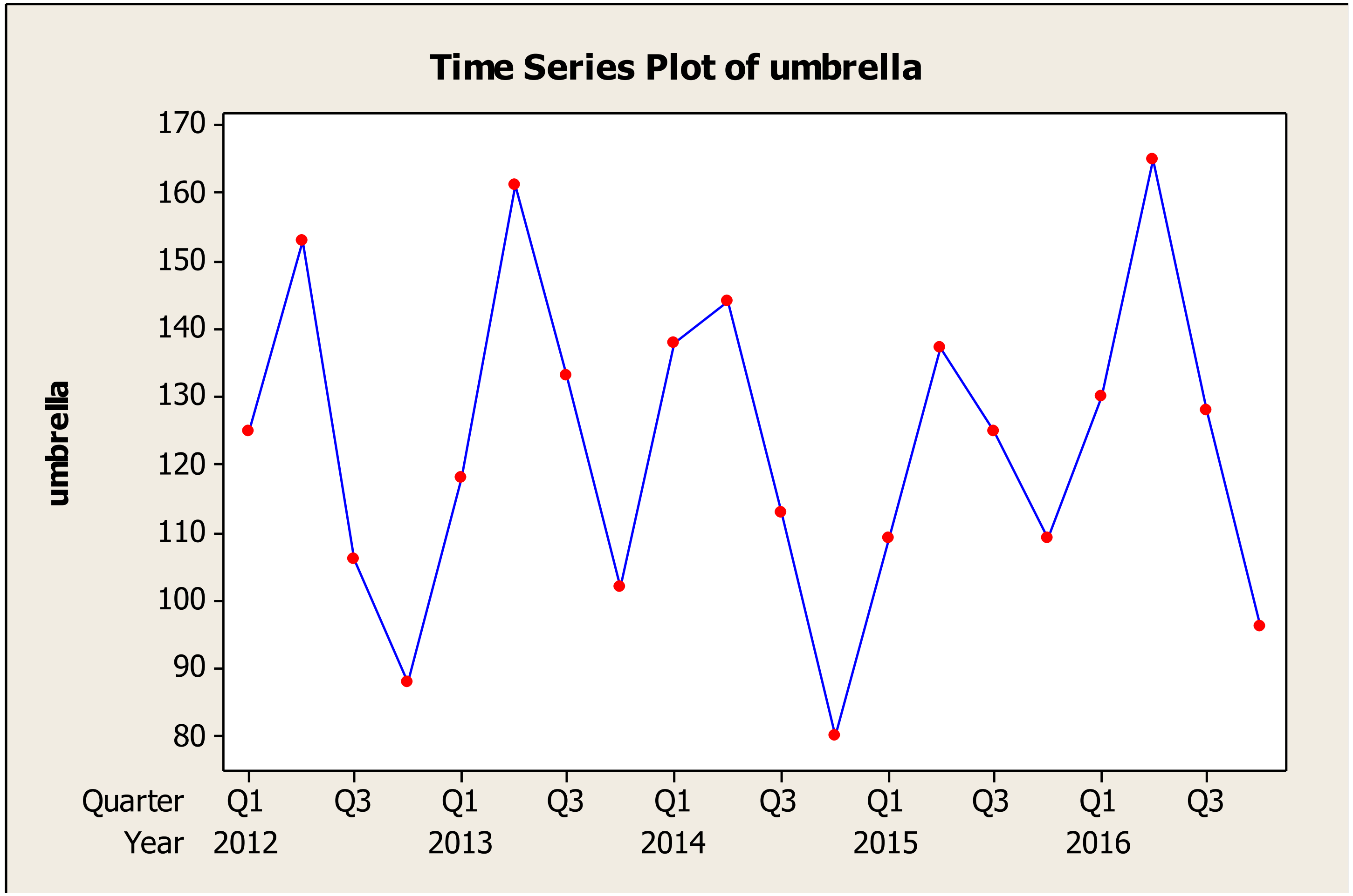
- dummy variables for categorical independent variables (multiple regression model)
- **k – 1**

1. **Seasonality Without Trend**
2. **Seasonality and Trend**

Seasonality Without Trend

Umbrella Sales

Year	Quarter	Sales,
2012	1	**125**
	2	**153**
	3	**106**
	4	**88**
2013	1	**118**
	2	**161**
	3	**133**
	4	**102**
2014	1	**138**
	2	**144**
	3	**113**
	4	**80**
2015	1	**109**
	2	**137**
	3	**125**
	4	**109**
2016	1	**130**
	2	**165**
	3	**128**
	4	**96**



Example: Seasonality Without Trend

k – 1 = 4 – 1 = 3 dummy variables

$$Q1 = \begin{cases} 1 & \text{if quarter1} \\ 0 & \text{otherwise} \end{cases}$$

$$Q2 = \begin{cases} 1 & \text{if quarter2} \\ 0 & \text{otherwise} \end{cases}$$

$$Q3 = \begin{cases} 1 & \text{if quarter3} \\ 0 & \text{otherwise} \end{cases}$$

$$\boxed{\hat{Y} = b_0 + b_1 Q1 + b_2 Q2 + b_3 Q3}$$

Year	Quarter	Sales,	Q1	Q2	Q3
2012	1	125	1	0	0
	2	153	0	1	0
	3	106	0	0	1
	4	88	0	0	0
2013	1	118	1	0	0
	2	161	0	1	0
	3	133	0	0	1
	4	102	0	0	0
2014	1	138	1	0	0
	2	144	0	1	0
	3	113	0	0	1
	4	80	0	0	0
2015	1	109	1	0	0
	2	137	0	1	0
	3	125	0	0	1
	4	109	0	0	0
2016	1	130	1	0	0
	2	165	0	1	0
	3	128	0	0	1
	4	96	0	0	0

Example: Seasonality Without Trend

```
The regression equation is umbrella = 95.0 + 29.0 Q1 + 57.0 Q2 + 26.0 Q3

Predictor      Coef   SE Coef      T      P
Constant     95.000     5.065  18.76  0.000
Q1           29.000     7.162   4.05  0.001
Q2           57.000     7.162   7.96  0.000
Q3           26.000     7.162   3.63  0.002

S = 11.3248   R-Sq = 79.9%   R-Sq(adj) = 76.1%
```

$$\hat{Y} = b_0 + b_1 Q1 + b_2 Q2 + b_3 Q3$$

Sales = 95 + 29Q1 + 57Q2 + 26Q3

Quarter 1: Sales = 95 + 29(1) + 57(0) + 26(0) = 124

Quarter 2: Sales = 95 + 29(0) + 57(1) + 26(0) = 152

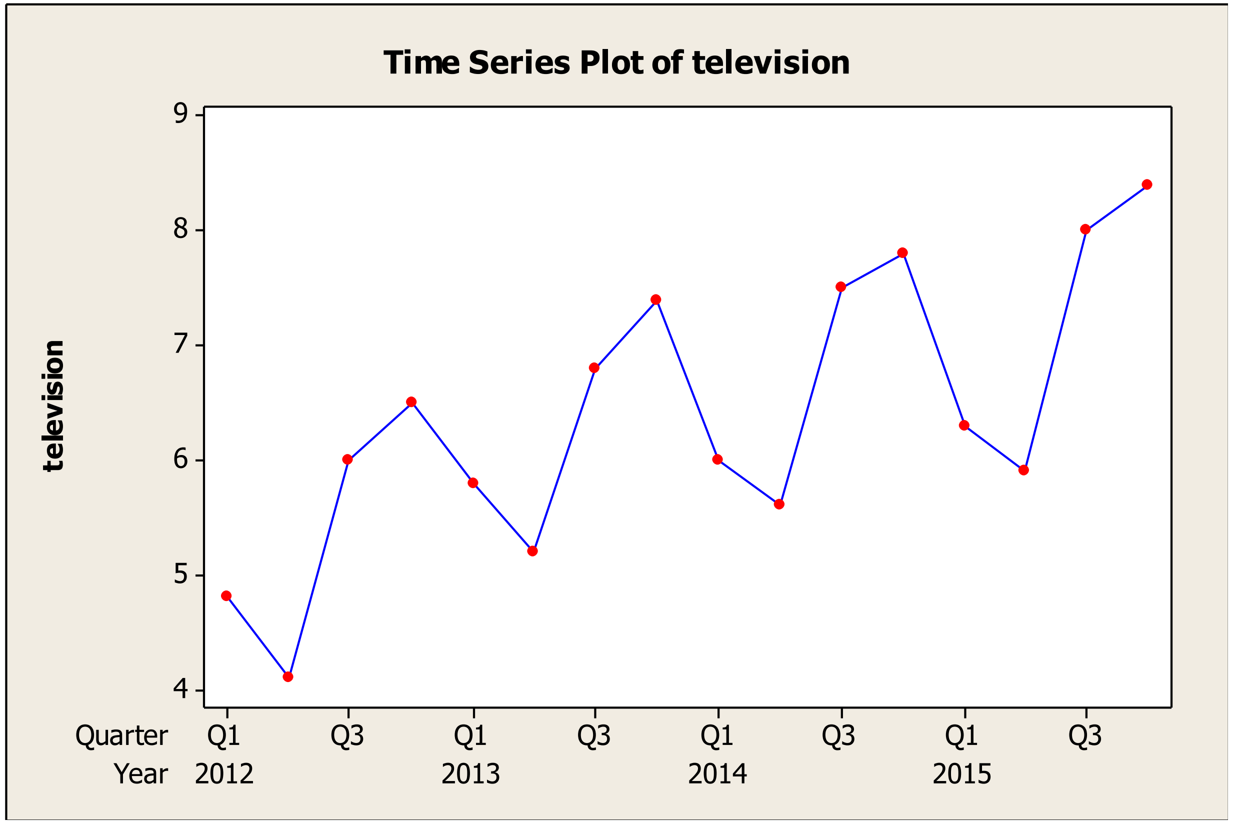
Quarter 3: Sales = 95 + 29(0) + 57(0) + 26(1) = 121

Quarter 4: Sales = 95 + 29(0) + 57(0) + 26(0) = 95

Seasonality and Trend

$$\hat{Y} = b_0 + b_1 Q1 + b_2 Q2 + b_3 Q3 + b_4 t$$

Year	Quarter	Television Set,
2012	1	4.8
	2	4.1
	3	6.0
	4	6.5
2013	1	5.8
	2	5.2
	3	6.8
	4	7.4
2014	1	6.0
	2	5.6
	3	7.5
	4	7.8
2015	1	6.3
	2	5.9
	3	8.0
	4	8.4



Example: Seasonality and Trend

Year	Quarter	Television Set Sales, y_t	Q1	Q2	Q3	t
2012	1	4.8	1	0	0	1
	2	4.1	0	1	0	2
	3	6.0	0	0	1	3
	4	6.5	0	0	0	4
2013	1	5.8	1	0	0	5
	2	5.2	0	1	0	6
	3	6.8	0	0	1	7
	4	7.4	0	0	0	8
2014	1	6.0	1	0	0	9
	2	5.6	0	1	0	10
	3	7.5	0	0	1	11
	4	7.8	0	0	0	12
2015	1	6.3	1	0	0	13
	2	5.9	0	1	0	14
	3	8.0	0	0	1	15
	4	8.4	0	0	0	16

Regression Analysis: television versus Q1_1, Q2_1, Q3_1, t_1

```
The regression equation is
television = 6.07 - 1.36 Q1 - 2.03 Q2 - 0.304 Q3 + 0.146 t

Predictor      Coef   SE Coef       T      P
Constant     6.0688    0.1625   37.35  0.000
Q1_1        -1.3631    0.1575   -8.66  0.000
Q2_1        -2.0337    0.1551  -13.11  0.000
Q3_1        -0.3044    0.1537   -1.98  0.073
t_1         0.14562   0.01211   12.02  0.000

S = 0.216664   R-Sq = 97.6%   R-Sq(adj) = 96.8%
```

Sales = 6.0688 – 1.3631Q1 – 2.0337Q2 – 0.3044Q3 + 0.14562*t*

Example: Seasonality and Trend

Sales = 6.0688 – 1.3631Q1 – 2.0337Q2 – 0.3044Q3 + 0.14562*t*

Forecast for Time Period 17 (Quarter 1 in Year 2016)

Sales = 6.0688 – 1.3631(1) – 2.0337(0) – 0.3044(0) + 0.14562(17) = 7.18124

Forecast for Time Period 18 (Quarter 2 in Year 2016)

Sales = 6.0688 – 1.3631(0) – 2.0337(1) – 0.3044(0) + 0.14562(18) = 6.65626

Forecast for Time Period 19 (Quarter 3 in Year 2016)

Sales = 6.0688 – 1.3631(0) – 2.0337(0) – 0.3044(1) + 0.14562(19) = 8.53118

Forecast for Time Period 20 (Quarter 4 in Year 2016)

Sales = 6.0688 – 1.3631(0) – 2.0337(0) – 0.3044(0) + 0.14562(20) = 8.9812

Example: Seasonality and Trend

The dummy variables in the estimated multiple regression equation provide four estimated multiple regression equations, one for each quarter.

1. Quarter 1: Sales = 6.0688 – 1.3631(1) + 0.14562t

   Sales = 4.7057 + 0.14562t

2. Quarter 2: Sales = 6.0688 – 2.0337(1) + 0.14562t

   Sales = 4.0351 + 0.14562t

3. Quarter 3: Sales = 6.0688 – 0.3044(1) + 0.14562t = 5.7644 + 0.14562t

   Sales = 4.0351 + 0.14562t

4. Quarter 4: **Sales = 6.0688 + 0.14562t**

a growth in sales of about 146 sets per quarter

sales in quarter 1 are **4.7057** – **6.0688** = –1.3631 or 1363 sets less than in quarter 4.

Models Based on Monthly Data

For monthly **(k =12)- k – 1 = 12 – 1 = 11 dummy variables**

$$\text{month1} = \begin{cases} 1 & \text{if January} \\ 0 & \text{otherwise} \end{cases}$$

$$\text{month2} = \begin{cases} 1 & \text{if February} \\ 0 & \text{otherwise} \end{cases}$$

$$\vdots$$

$$\text{month11} = \begin{cases} 1 & \text{if November} \\ 0 & \text{otherwise} \end{cases}$$

Time Series Decomposition

1. Additive Decomposition Model

$$Y_t = \text{Trend}_t + \text{Seasonal}_t + \text{Irregular}_t$$

2. Multiplicatibe Decomposition Model

$$Y_t = \text{Trend}_t \times \text{Seasonal}_t \times \text{Irregular}_t$$

Multiplicative Decomposition: Separating of time series component

Multiplicative Decomposition Method -

- Trend (T)
- ~~Seasonal (S)~~
- Cyclical (C)
- Irregular (I)

Deseasonalizing - remove the seasonal effects from a time series

Calculating the Seasonal Indexes

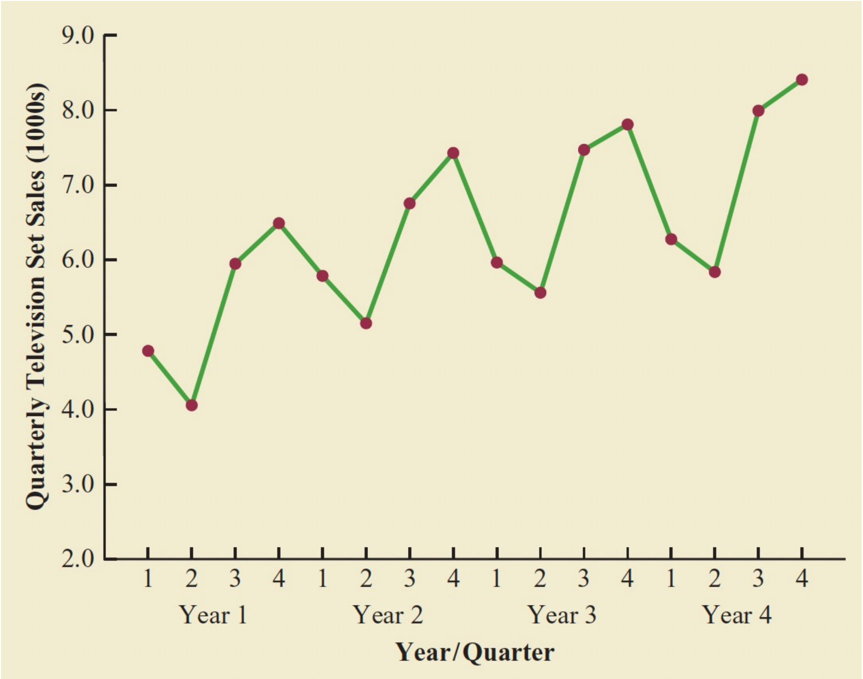
$$Y_t = \text{Trend}_t \times \text{Seasonal}_t \times \text{Irregular}_t$$

$$\frac{Y_t}{\text{Trend}_t} = \frac{\text{Trend}_t \times \text{Seasonal}_t \times \text{Irregular}_t}{\text{Trend}_t} = \text{Seasonal}_t \times \text{Irregular}_t$$

Compute Seasonal Indexes for each season

For example, in this case there are 4 quarters, therefore

- Seasonal indexes for Q1
- Seasonal indexes for Q2
- Seasonal indexes for Q3
- Seasonal indexes for Q4



Seasonal Indexes = average of each season $\text{Seasonal}_t \times \text{Irregular}_t$

Multiplicative Decomposition: Separating of time series component

Steps to Multiplicative Decomposition

1. Compute a moving average of y_t by using a centering moving average
   - This eliminates the seasonality and irregularity, so the component of time series which remains in the time series data is trend and cyclical components or $T \times C$.
   - Taking a period moving averaging, the average needs to be matched up with a specific period as following steps:
     1. Consider a 4-period moving average (4MA), that is the average of quarter 1, 2, 3 and 4 of year 2012 is 5.35, this does not match any period.
     2. Consider a 2-period moving average of 4MA, that is the average of 5.35 and 5.60 is 5.475, this matches up with period 3.

2. For the same period compute $\frac{y_t}{T \times C}$, which the results are the values of $S \times I$.
   - The first and last few periods do not have a value from Step 1, these periods are skipped.

Multiplicative Decomposition: Separating of time series component

Steps to Multiplicative Decomposition

3. All of the values from Step 2 for season 1 are averaged together to form seasonal factor for season 1
   - This is repeated for every season.
   - If there are four seasons, there will be four factors or four values of seasonal indexes. This means that **we can obtain the seasonal component.**

4. Deseasonalizing the time series to obtain $y_t = T \times C \times I$ and develop a linear trend equation $T = b_0 + b_1 t$. This means that **we can obtain the trend component.**

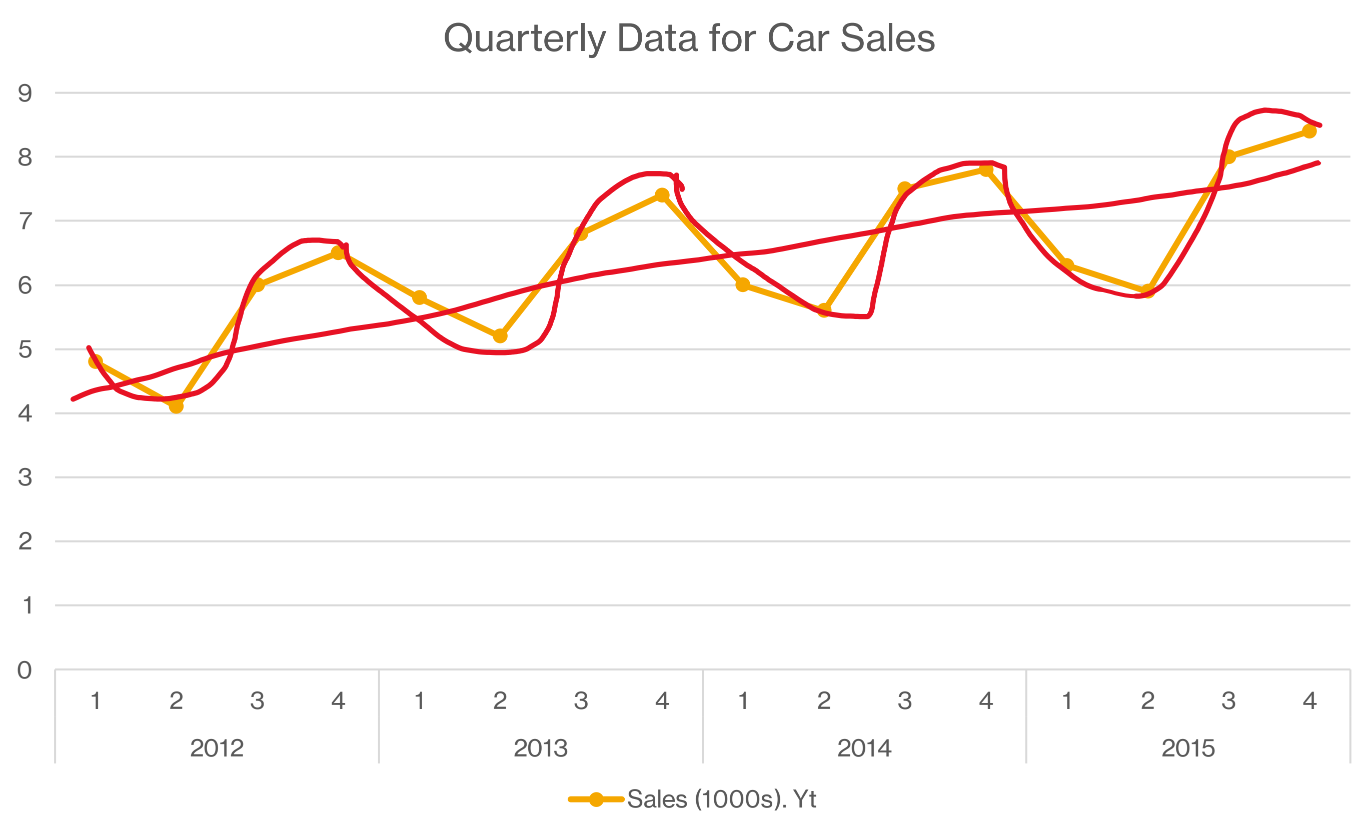
5. Compute $C \times I = \dfrac{T \times C \times I}{T}$.

6. Taking of 3MA of $C \times I$, this means that we can obtain the **cyclical component**.

7. Compute $I = \dfrac{C \times I}{C}$, this means that we can obtain the **irregular component**.

Multiplicative Decomposition: Separating of time series component *(example)*

Year	Quarter	Sales (1000s). Yt
2012	1	4.8
	2	4.1
	3	6
	4	6.5
2013	1	5.8
	2	5.2
	3	6.8
	4	7.4
2014	1	6
	2	5.6
	3	7.5
	4	7.8
2015	1	6.3
	2	5.9
	3	8
	4	8.4



Multiplicative Decomposition: Separating of time series component (*example*)

Smooth CMA - **Centered** Moving Average

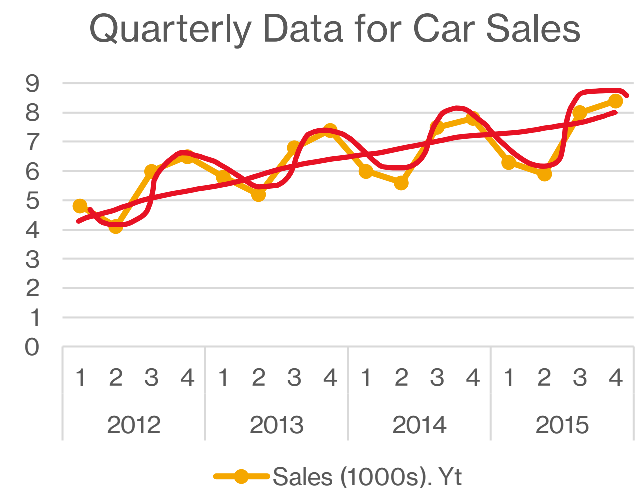
S_t - Seasonal index

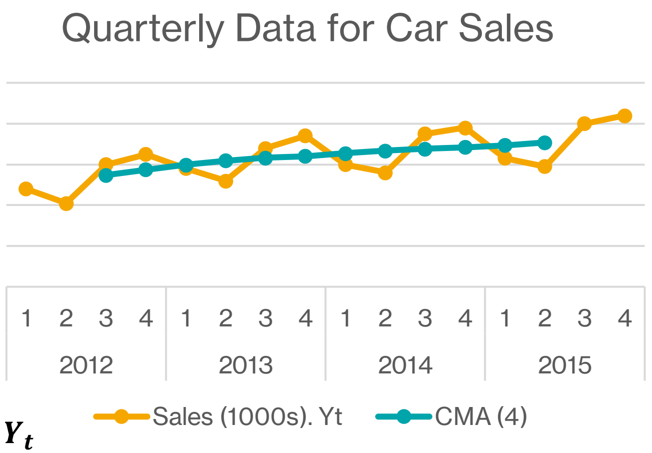
t	Year	Quarter	Sales (1000s). Y_t	MA (4)	CMA (4)	S_t x I_t	S_t	Deseasonalized
1	2012	1	4.8				0.93	5.15
2		2	4.1				0.84	4.89
3		3	6	5.4	5.5	1.10	1.09	5.49
4		4	6.5	5.6	5.7	1.13	1.15	5.66
5	2013	1	5.8	5.9	6.0	0.97	0.93	6.22
6		2	5.2	6.1	6.2	0.84	0.84	6.21
7		3	6.8	6.3	6.3	1.08	1.09	6.23
8		4	7.4	6.4	6.4	1.16	1.15	6.44
9	2014	1	6	6.5	6.5	0.92	0.93	6.44
10		2	5.6	6.6	6.7	0.84	0.84	6.68
11		3	7.5	6.7	6.8	1.11	1.09	6.87
12		4	7.8	6.8	6.8	1.14	1.15	6.79
13	2015	1	6.3	6.9	6.9	0.91	0.93	6.76
14		2	5.9	7.0	7.1	0.83	0.84	7.04
15		3	8	7.2			1.09	7.33
16		4	8.4				1.15	7.31

Y_t $\quad T\times C \quad S\times I = \frac{Y_t}{T\times C}$

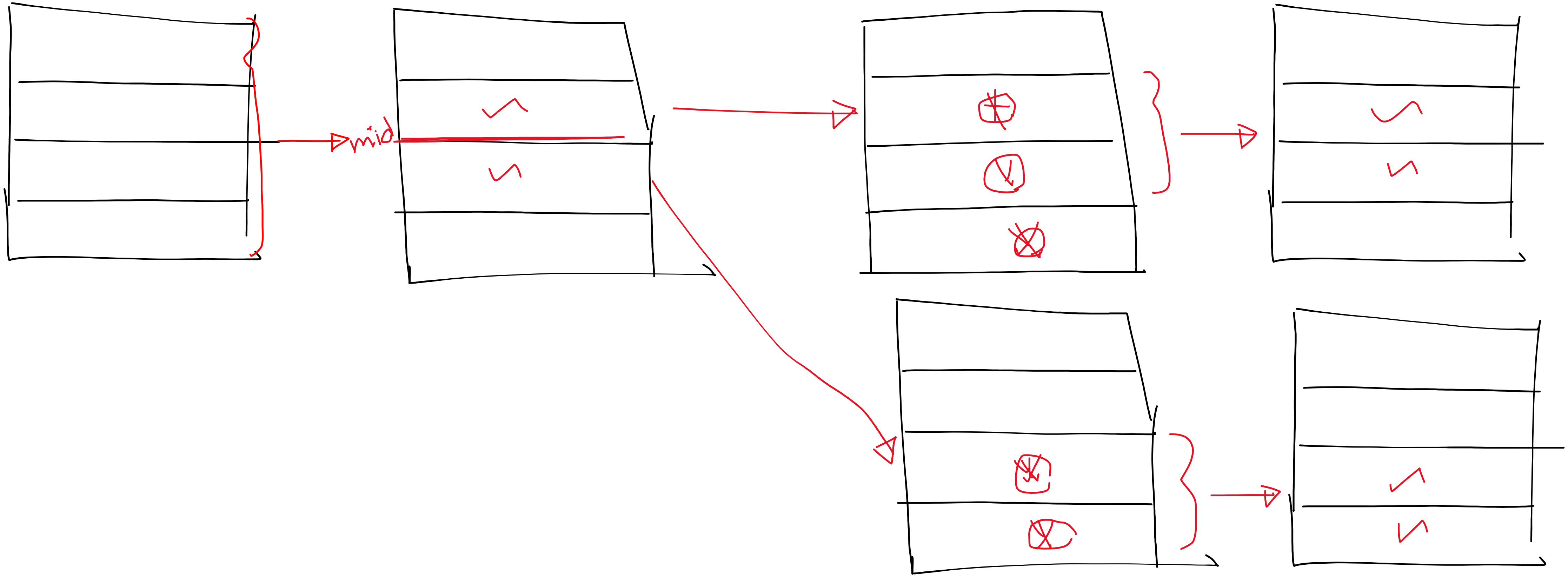
$Deseasonalized = \frac{Y_t}{S_t}$

$Y_t = T\times C\times I$ (no seasonal)

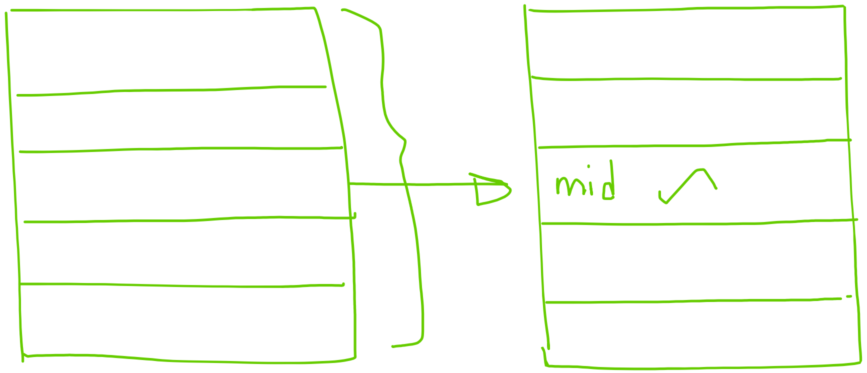




4MA



5MA



Multiplicative Decomposition: Separating of time series component *(example)*

Calculating the Seasonal Indexes

Quarter	S_t
1	0.93
2	0.84
3	1.09
4	1.15

$$\text{Average} = \frac{2013\,Q1 + 2014\,Q1 + 2015\,Q1}{3}$$

$$\text{Average} = \frac{2013\,Q2 + 2014\,Q2 + 2015\,Q2}{3}$$

$$\text{Average} = \frac{2012\,Q3 + 2013\,Q3 + 2014\,Q3}{3}$$

$$\text{Average} = \frac{2012\,Q4 + 2013\,Q4 + 2014\,Q4}{3}$$

Year	Quarter	S_t, I_t	S_t
2012	1		0.93
	2		0.84
	3	1.10	1.09
	4	1.13	1.15
2013	1	0.97	0.93
	2	0.84	0.84
	3	1.08	1.09
	4	1.16	1.15
2014	1	0.92	0.93
	2	0.84	0.84
	3	1.11	1.09
	4	1.14	1.15
2015	1	0.91	0.93
	2	0.83	0.84
	3		1.09
	4		1.15

Multiplicative Decomposition: Separating of time series component *(example)*

$Deseasonalized = \frac{Y_t}{S_t}$

$Y_t = T \times C \times I \times S$ | $T \times C$ | $S \times I = \frac{Y_t}{T \times C}$ | $Y_t = T \times C \times I$

t	Year	Quarter	Sales (1000s). Y_t	MA (4)	CMA (4)	S_t x I_t	S_t	Deseasonalize	T_t
1	2012	1	4.8				0.93	5.15	5.25
2		2	4.1				0.84	4.89	5.39
3		3	6	5.4	5.5	1.10	1.09	5.49	5.54
4		4	6.5	5.6	5.7	1.13	1.15	5.66	5.69
5	2013	1	5.8	5.9	6.0	0.97	0.93	6.22	5.83
6		2	5.2	6.1	6.2	0.84	0.84	6.21	5.98
7		3	6.8	6.3	6.3	1.08	1.09	6.23	6.12
8		4	7.4	6.4	6.4	1.16	1.15	6.44	6.27
9	2014	1	6	6.5	6.5	0.92	0.93	6.44	6.42
10		2	5.6	6.6	6.7	0.84	0.84	6.68	6.56
11		3	7.5	6.7	6.8	1.11	1.09	6.87	6.71
12		4	7.8	6.8	6.8	1.14	1.15	6.79	6.86
13	2015	1	6.3	6.9	6.9	0.91	0.93	6.76	7.00
14		2	5.9	7.0	7.1	0.83	0.84	7.04	7.15
15		3	8	7.2			1.09	7.33	7.30
16		4	8.4				1.15	7.31	7.44

Simple Linear Regression

$$T_t = b_0 + b_1 t$$

$$T_t = \underline{5.10} + \underline{0.15}t$$

Multiplicative Decomposition: Separating of time series component *(example)*

Simple Linear Regression

SUMMARY OUTPUT

Regression Statistics	
Multiple R	0.95899309
R Square	0.91966774
Adjusted R S	0.91392972
Standard Err	0.21345682
Observations	16

$$T_t = 5.10 + 0.15t$$

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	7.302793978	7.30279398	160.276184	4.6903E-09
Residual	14	0.637893375	0.04556381		
Total	15	7.940687353			

		Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*
b_0	Intercept	5.09898609	0.111937699	45.5520003	1.2757E-16	4.85890361	5.33906858	4.858903608	5.33906858
b_1	t	0.14655649	0.011576321	12.660023	4.6903E-09	0.12172775	0.17138523	0.121727754	0.17138523

Multiplicative Decomposition: Separating of time series component *(example)*

$$b_1 = \frac{\sum t y_t - \frac{\left(\sum t \sum y_t\right)}{n}}{\sum t^2 - \frac{\left(\sum t\right)^2}{n}}$$

$$= \frac{915.1748 - \frac{(136)(101.7727)}{16}}{1496 - \frac{(136)^2}{16}} = \frac{50.10685}{340} = 0.147$$

$$b_0 = \bar{y} - b_1 \bar{t}$$

$$= 6.3608 - 0.147(8.5) = 5.111$$

Year	Quarter	Deseasonalized Sales, $TCI = \frac{y_t}{S}$	t	t^2	ty_t	$T = 5.111 + 0.147t$	$CI = \frac{TCI}{T}$	C	$I = \frac{CI}{C}$
2012	1	5.1580	1	1	5.1580	5.258	0.9810	-	-
	2	4.9020	2	4	9.8040	5.405	0.9069	0.9593	0.9454
	3	5.4965	3	9	16.4895	5.552	0.9900	0.9654	1.0255
	4	5.6948	4	16	22.7792	5.699	0.9993	1.0185	0.9811
2013	1	6.2325	5	25	31.1625	5.846	1.0661	1.0343	1.0307
	2	6.2171	6	36	37.3026	5.993	1.0374	1.0394	0.9981
	3	6.2294	7	49	43.6058	6.140	1.0146	1.0277	0.9873
	4	6.4833	8	64	51.8664	6.287	1.0312	1.016	1.015
2014	1	6.4475	9	81	58.0275	6.434	1.0021	1.0169	0.9854
	2	6.6954	10	100	66.9540	6.581	1.0174	1.0136	1.0037
	3	6.8706	11	121	75.5766	6.728	1.0212	1.0109	1.0102
	4	6.8337	12	144	82.0044	6.875	0.9940	0.9931	1.0009
2015	1	6.7698	13	169	88.0074	7.022	0.9641	0.9807	0.9831
	2	7.0540	14	196	98.7560	7.169	0.9840	0.9833	1.0007
	3	7.3287	15	225	109.9305	7.316	1.0017	0.9906	1.0112
	4	7.3594	16	256	117.7504	7.463	0.9861		
Total		101.7727	136	1496					

$$T = 5.111 + 0.147t$$

Multiplicative Decomposition: Separating of time series component *(example)*

$Deseasonalized = \frac{Y_t}{S_t}$ $\quad C\times I = \frac{T\times C\times I}{T}$ $\quad I = \frac{C\times I}{C}$

$Y_t = T\times C\times S\times I$ $\quad T\times C$ $\quad S\times I = \frac{Y_t}{T\times C}$ $\quad T\times C\times I$ $\quad T_t = 5.10 + 0.15t$

t	Year	Quarter	Sales (1000s). Y_t	MA (4)	CMA (4)	S_t x I_t	S_t	Deseasonalize	T_t	C_t x I_t	C_t	I_t
1	2012	1	4.8				0.93	5.15	5.25	0.98	-	-
2		2	4.1				0.84	4.89	5.39	0.91	0.96	0.9454
3		3	6	5.4	5.5	1.10	1.09	5.49	5.54	0.99	0.9654	1.0255
4		4	6.5	5.6	5.7	1.13	1.15	5.66	5.69	0.999	1.0185	0.9811
5	2013	1	5.8	5.9	6.0	0.97	0.93	6.22	5.83	1.07	1.0343	1.0307
6		2	5.2	6.1	6.2	0.84	0.84	6.21	5.98	1.04	1.0394	0.9981
7		3	6.8	6.3	6.3	1.08	1.09	6.23	6.12	1.01	1.0277	0.9873
8		4	7.4	6.4	6.4	1.16	1.15	6.44	6.27	1.03	1.016	1.015
9	2014	1	6	6.5	6.5	0.92	0.93	6.44	6.42	1.00	1.0169	0.9854
10		2	5.6	6.6	6.7	0.84	0.84	6.68	6.56	1.02	1.0136	1.0037
11		3	7.5	6.7	6.8	1.11	1.09	6.87	6.71	1.02	1.0109	1.0102
12		4	7.8	6.8	6.8	1.14	1.15	6.79	6.86	0.99	0.9931	1.0009
13	2015	1	6.3	6.9	6.9	0.91	0.93	6.76	7.00	0.96	0.9807	0.9831
14		2	5.9	7.0	7.1	0.83	0.84	7.04	7.15	0.98	0.9833	1.0007
15		3	8	7.2			1.09	7.33	7.30	1.00	0.9906	1.0112
16		4	8.4				1.15	7.31	7.44	0.99		

Multiplicative Decomposition: Separating of time series component *(example)*

$$\hat{Y}_t = \hat{T}_t \times \hat{S}_t$$

t	Year	Quarter	Sales (1000s). Y_t	MA (4)	CMA (4)	S_t, I_t	S_t	Deseasonalize	T_t	Forecast
1	2012	1	4.8				0.93	5.15	5.25	4.89
2		2	4.1				0.84	4.89	5.39	4.52
3		3	6	5.4	5.5	1.10	1.09	5.49	5.54	6.05
4		4	6.5	5.6	5.7	1.13	1.15	5.66	5.69	6.53
5	2013	1	5.8	5.9	6.0	0.97	0.93	6.22	5.83	5.44
6		2	5.2	6.1	6.2	0.84	0.84	6.21	5.98	5.01
7		3	6.8	6.3	6.3	1.08	1.09	6.23	6.12	6.69
8		4	7.4	6.4	6.4	1.16	1.15	6.44	6.27	7.20
9	2014	1	6	6.5	6.5	0.92	0.93	6.44	6.42	5.98
10		2	5.6	6.6	6.7	0.84	0.84	6.68	6.56	5.50
11		3	7.5	6.7	6.8	1.11	1.09	6.87	6.71	7.33
12		4	7.8	6.8	6.8	1.14	1.15	6.79	6.86	7.88
13	2015	1	6.3	6.9	6.9	0.91	0.93	6.76	7.00	6.53
14		2	5.9	7.0	7.1	0.83	0.84	7.04	7.15	5.99
15		3	8	7.2			1.09	7.33	7.30	7.97
16		4	8.4				1.15	7.31	7.44	8.55

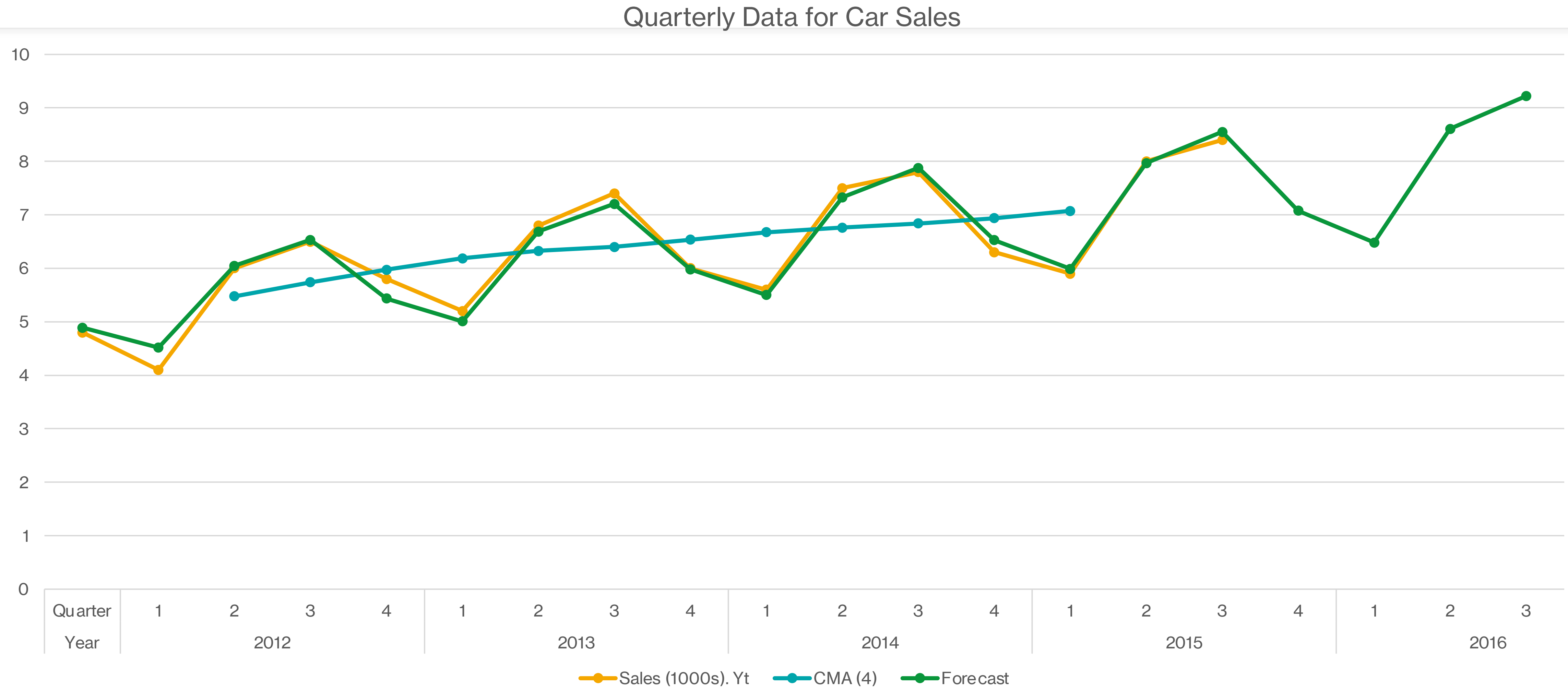
Multiplicative Decomposition: Separating of time series component *(example)*

			$\hat{T}_t$	$\hat{S}_t$	$\hat{Y}_t = \hat{T}_t \times \hat{S}_t$
Year	Quarter	t	Trend forecast (1,000 units) $T = 5.111 + 0.147t$	Seasonal Indexes	Quarterly forecast (1,000 units)
2016	1	17	$5.111 + (0.147 \times 17) = 7.610$	0.9306	$7.610 \times 0.9307 = 7.0819$
	2	18	$5.111 + (0.147 \times 18) = 7.757$	0.8364	$7.757 \times 0.8364 = 6.4879$
	3	19	$5.111 + (0.147 \times 19) = 7.904$	1.0916	$7.904 \times 1.0916 = 8.6280$
	4	20	$5.111 + (0.147 \times 20) = 8.051$	1.1414	$8.051 \times 1.1414 = 9.1894$

Multiplicative Decomposition: Separating of time series component *(example)*

t	Year	Quarter	Sales (1000s). Yt	MA (4)	CMA (4)	St x It	St	Deseasonalize	Tt	Forecast
1	2012	1	4.8				0.93	5.15	5.25	4.89
2		2	4.1				0.84	4.89	5.39	4.52
3		3	6	5.4	5.5	1.10	1.09	5.49	5.54	6.05
4		4	6.5	5.6	5.7	1.13	1.15	5.66	5.69	6.53
5	2013	1	5.8	5.9	6.0	0.97	0.93	6.22	5.83	5.44
6		2	5.2	6.1	6.2	0.84	0.84	6.21	5.98	5.01
7		3	6.8	6.3	6.3	1.08	1.09	6.23	6.12	6.69
8		4	7.4	6.4	6.4	1.16	1.15	6.44	6.27	7.20
9	2014	1	6	6.5	6.5	0.92	0.93	6.44	6.42	5.98
10		2	5.6	6.6	6.7	0.84	0.84	6.68	6.56	5.50
11		3	7.5	6.7	6.8	1.11	1.09	6.87	6.71	7.33
12		4	7.8	6.8	6.8	1.14	1.15	6.79	6.86	7.88
13	2015	1	6.3	6.9	6.9	0.91	0.93	6.76	7.00	6.53
14		2	5.9	7.0	7.1	0.83	0.84	7.04	7.15	5.99
15		3	8	7.2			1.09	7.33	7.30	7.97
16		4	8.4				1.15	7.31	7.44	8.55
17	**2016**	**1**					**0.93**		**7.59**	**7.08**
18		**2**					**0.84**		**7.74**	**6.48**
19		**3**					**1.09**		**7.88**	**8.61**
20		**4**					**1.15**		**8.03**	**9.22**

Multiplicative Decomposition: Separating of time series component *(example)*



Assignment

The quarterly sales data (number of copies sold) for a college textbook over the past three years follow.

a. Construct a time series plot. What type of pattern exists in the data?

b. Show the four-quarter (MA4) and centered moving average (CMA4) values for this time series.

c. Compute the seasonal and adjusted seasonal indexes for the four quarters.

d. When does the publisher have the largest seasonal index? Does this result appear reasonable? Explain.

e. Compute the deseasonalized time series.

f. Using excel or any other softwares, compute the linear trend equation for the deseasonalized data and compute the deseasonalized quarterly trend forecast for year 4.

g. Compute the quarterly forecasts for year 4 based upon both trend and seasonal effects.

Quarter	Year 1	Year 2	Year 3
1	1690	1800	1850
2	940	900	1100
3	2625	2900	2930
4	2500	2360	2615