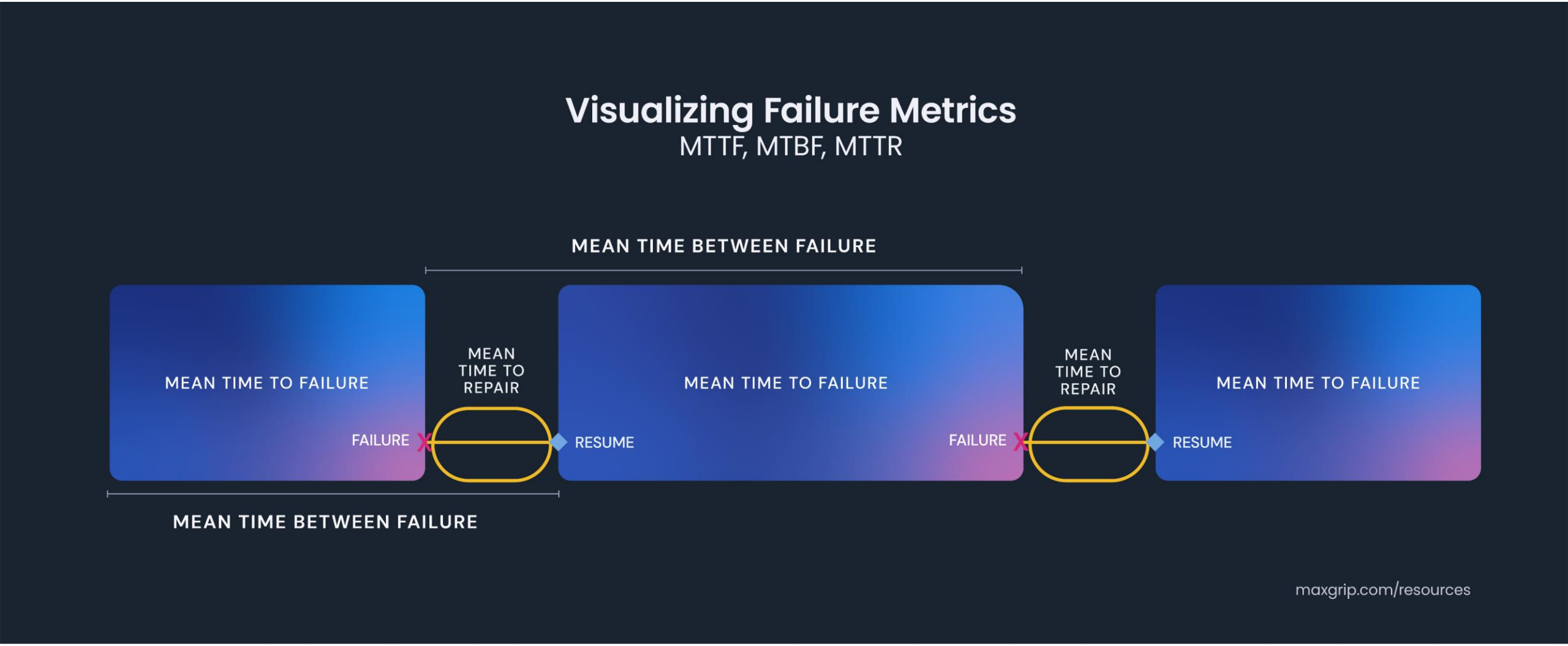
System Reliability Concepts



Mean Time Between Failure (MTBF) is a reliability metric that measures the average time between failures, which helps inform an asset's reliability.

How to calculate MTBF?

$$\theta = T/R.$$

θ = MTBF
T = total time
R = number of failures

Example: Suppose 10 devices are tested for 500 hours. During the test 2 failures occur. The estimate of the MTBF is:

$$\theta = \frac{10*500}{2} = 2{,}500 \text{ hours / failure.}$$

Even though MTBF and reliability are different, you can very easily convert MTBF to reliability by using this equation for exponential distributions:

$$R(t) = e^{-t/MTBF}$$

In this equation:

- e is the mathematical constant approximately equal to 2.71828
- t is the end time, in hours, that you are interested in
- MTBF is expressed in hours

Let's convert our previous MTBF value of 100 hours to reliability as an example. To make it interesting, let's also calculate reliability at 100 hours. This will indicate the probability that a system with an MTBF of 100 hours will still be functioning after 100 hours of operation.

$$R(t) = e^{-\frac{100}{100}}$$

$$= 0.3679 = 36.79\%$$

So, if you have a product with an MTBF of 100 hours, you only have a 36.79% chance that it actually functions for 100 hours!

Service Level Agreement – High Availability

Availability %	Downtime per year	Downtime per quarter	Downtime per month	Downtime per week	Downtime per day (24 hours)
90% ("one nine")	36.53 days	9.13 days	73.05 hours	16.80 hours	2.40 hours
95% ("one nine five")	18.26 days	4.56 days	36.53 hours	8.40 hours	1.20 hours
97% ("one nine seven")	10.96 days	2.74 days	21.92 hours	5.04 hours	43.20 minutes
98% ("one nine eight")	7.31 days	43.86 hours	14.61 hours	3.36 hours	28.80 minutes
99% ("two nines")	3.65 days	21.9 hours	7.31 hours	1.68 hours	14.40 minutes
99.5% ("two nines five")	1.83 days	10.98 hours	3.65 hours	50.40 minutes	7.20 minutes
99.8% ("two nines eight")	17.53 hours	4.38 hours	87.66 minutes	20.16 minutes	2.88 minutes
99.9% ("three nines")	8.77 hours	2.19 hours	43.83 minutes	10.08 minutes	1.44 minutes
99.95% ("three nines five")	4.38 hours	65.7 minutes	21.92 minutes	5.04 minutes	43.20 seconds
99.99% ("four nines")	52.60 minutes	13.15 minutes	4.38 minutes	1.01 minutes	8.64 seconds

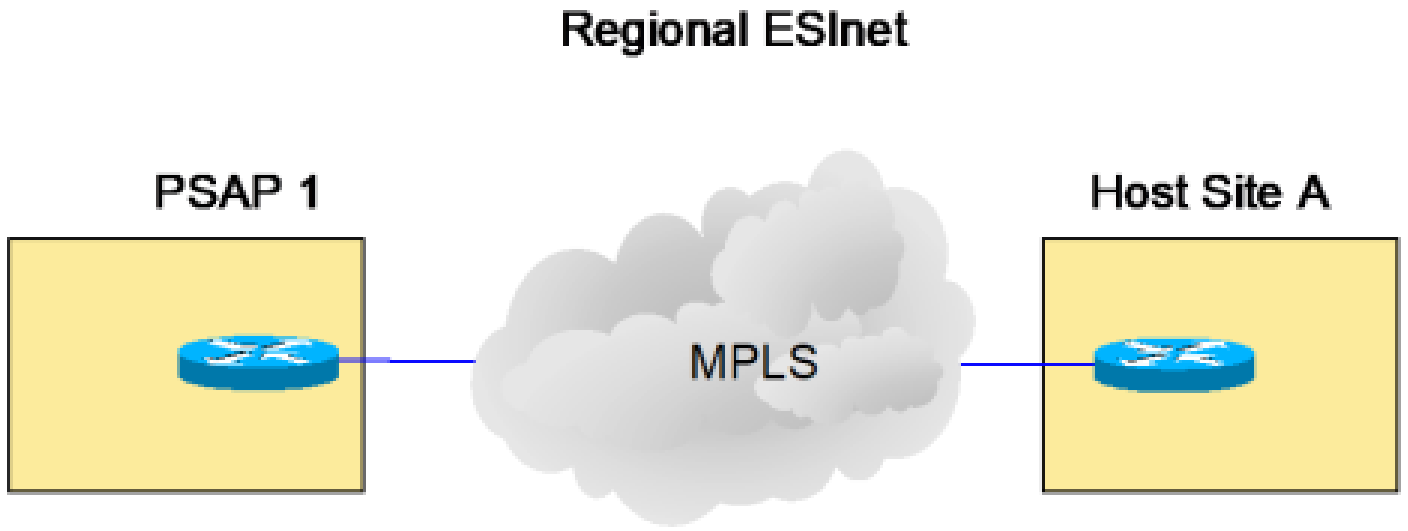
PSAP – Public Safety Answering Point

ESInet – Emergency Service IP Network

$$R_s = R_1 * R_2 * R_3$$

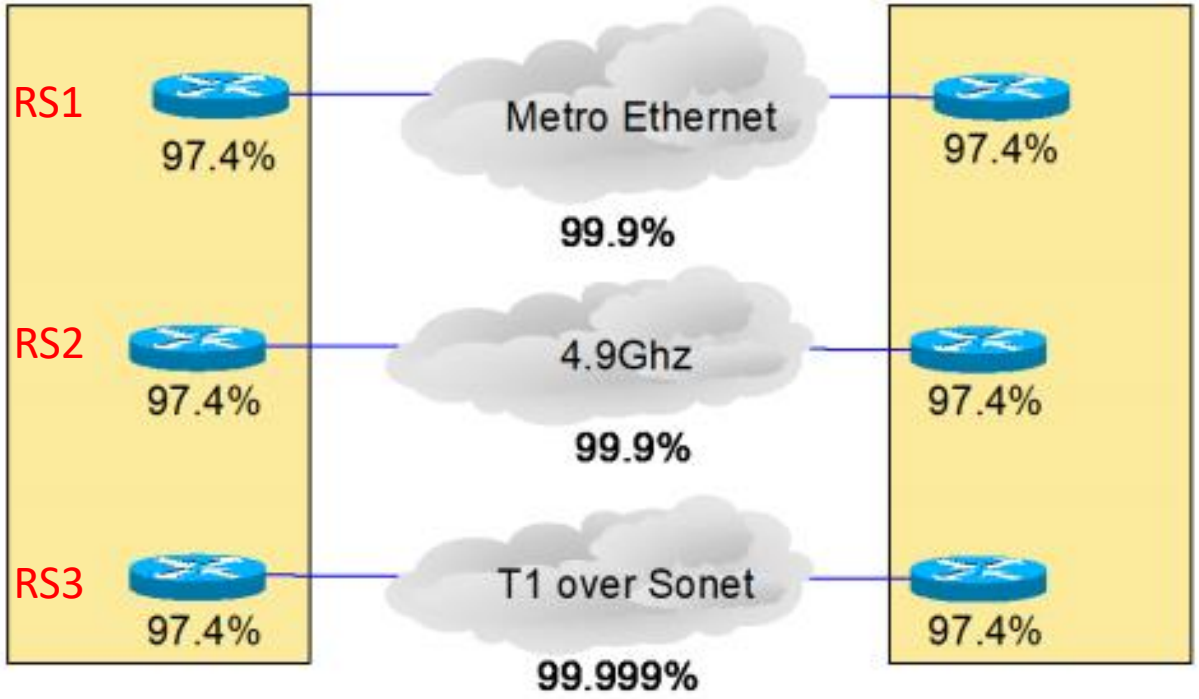
For example, the series reliability of an ESInet shown below is:

.9743 * .999 * .9743 = .948



R1 = 97.4% * R2 = 99.9% * R3 = 97.4% = .948 or 94.8%

Always lower than the lowest reliability value in the series.



$R_P = 1 - ((1-Rs_1) * (1-Rs_2) * (1-Rs_3))$
Where Rp = Parallel Reliability
and Rs1..3 = the series reliability of each independent link

From the configuration shown in the previous slide, it is known that the reliability of the link is 0.948.

We can increase the reliability of the configuration by introducing more links.

0.94866

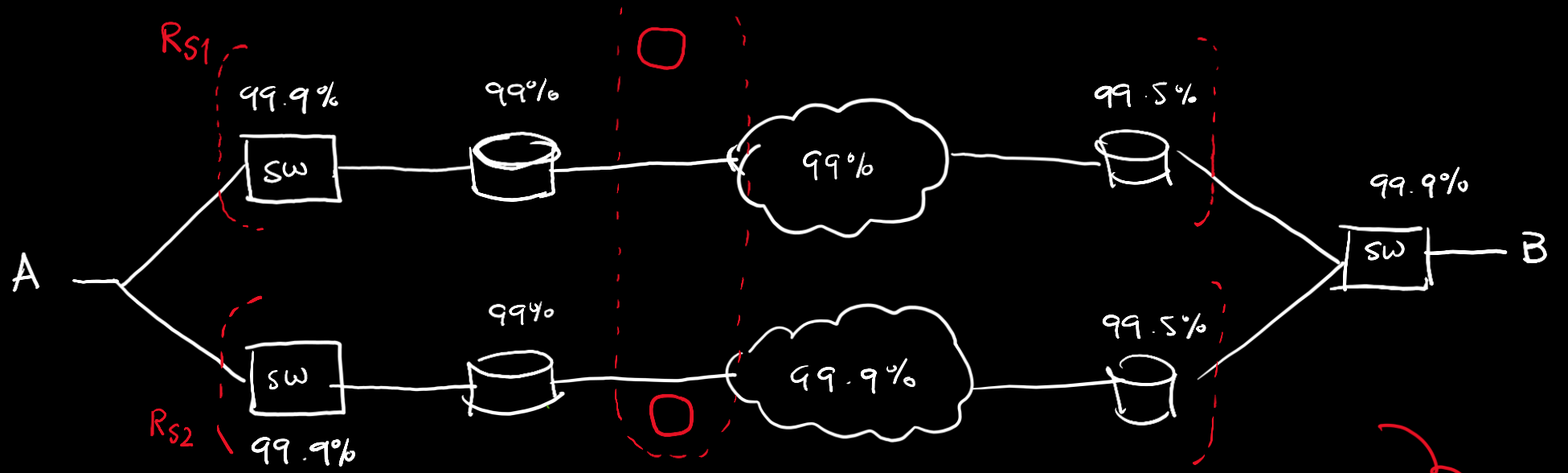
Reliability of the above network configuration

$$R_P = 1-((1-.948) * (1-.948) * (1-.948)) = 1 - (0.052 * 0.052 * 0.052) = 0.99985$$

RS1 RS2 RS3

What if we increase one more link to 4 links?

$$R_P = 1 - ((1-.948) * (1-.948) * (1-.948) * (1-.948))$$
$$= 1 - (0.052 * 0.052 * 0.052 * 0.052)$$
$$= 0.9999927$$



Calculation Steps.

1. Determine all possible series connections.
2. Calculate the reliability for each series connection.
3. Calculate the reliability of the parallel connection(s).

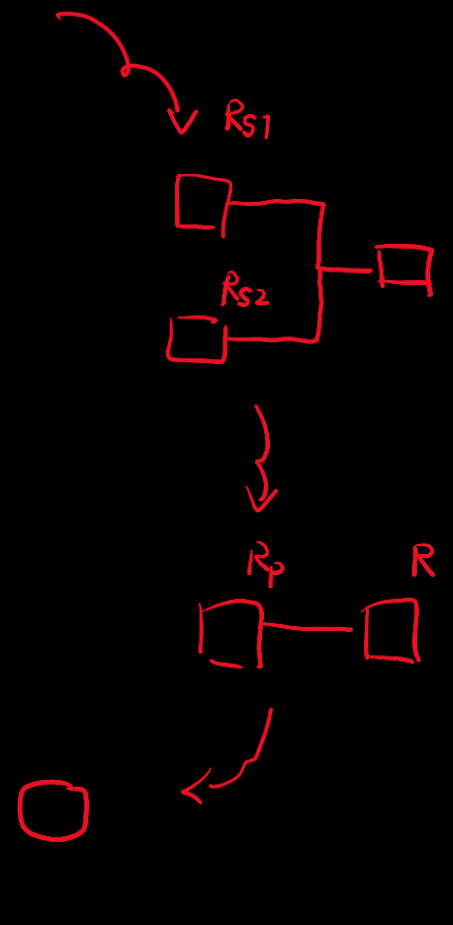
Let’s define the above and below connectionsas RS1 and RS2, respectively.

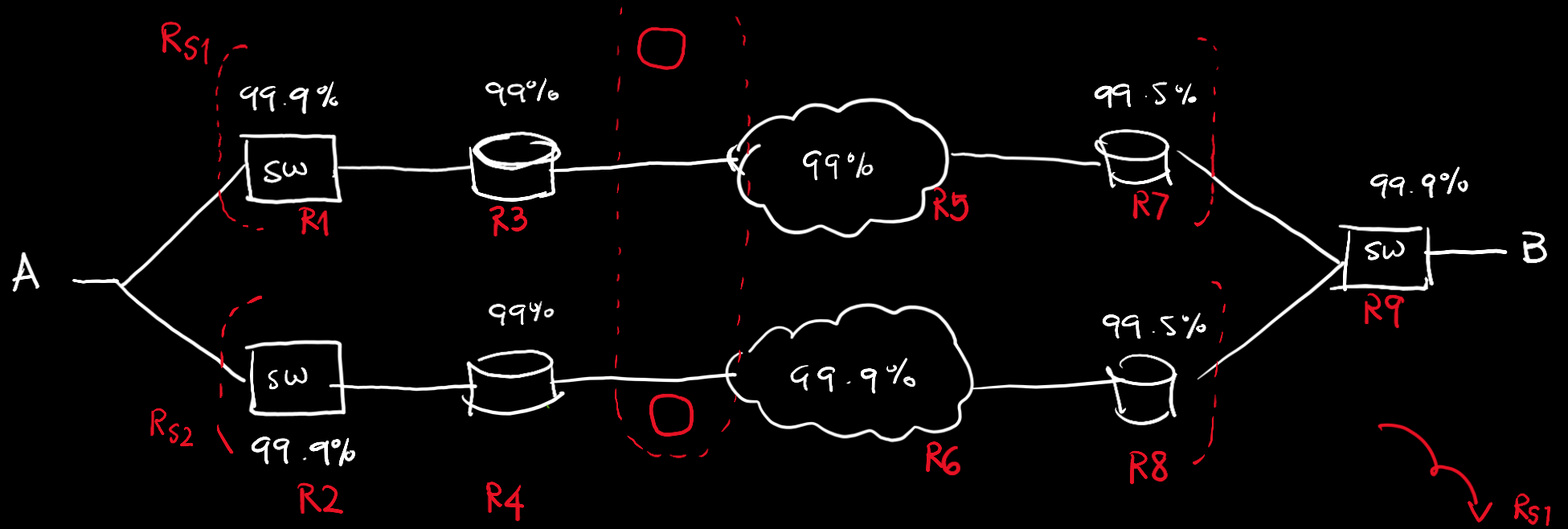
Find

RS1 =

RS2 =

RP =





$R_{S1} = .974$

$R_{S2} = .983$

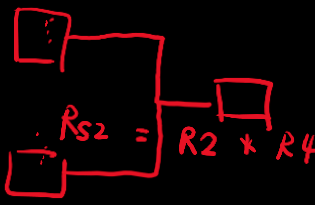
$R_{P1} = 1 - ((1 - R_{S1}) * (1 - R_{S2}))$

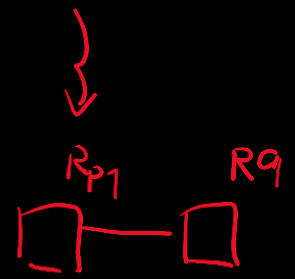
$= .9995$

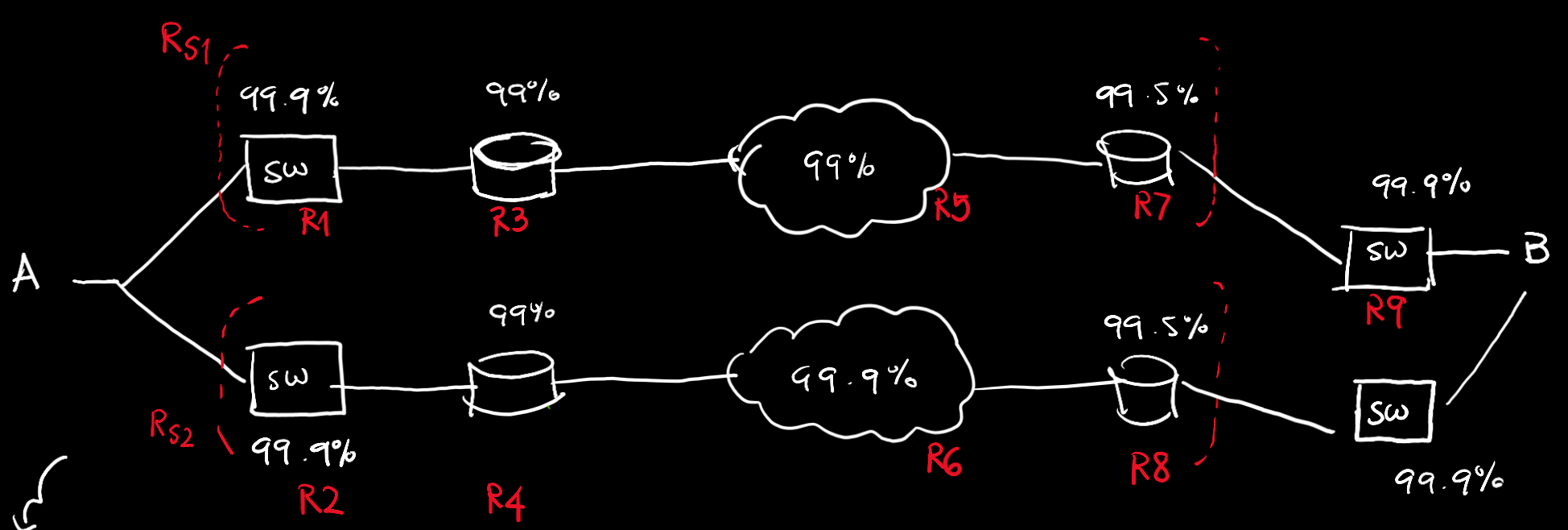
$R = R_{P1} * R9 = .9985$ or 99.85%.

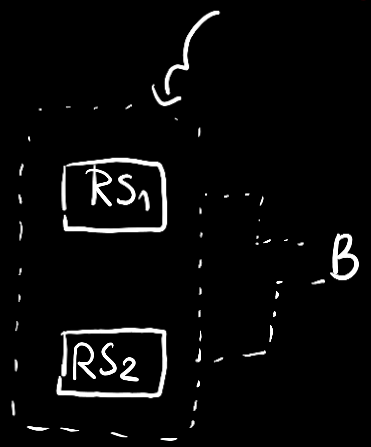
$R_{S1} = R1 * R3 * R5 * R7$

$R_{S2} = R2 * R4 * R6 * R8$







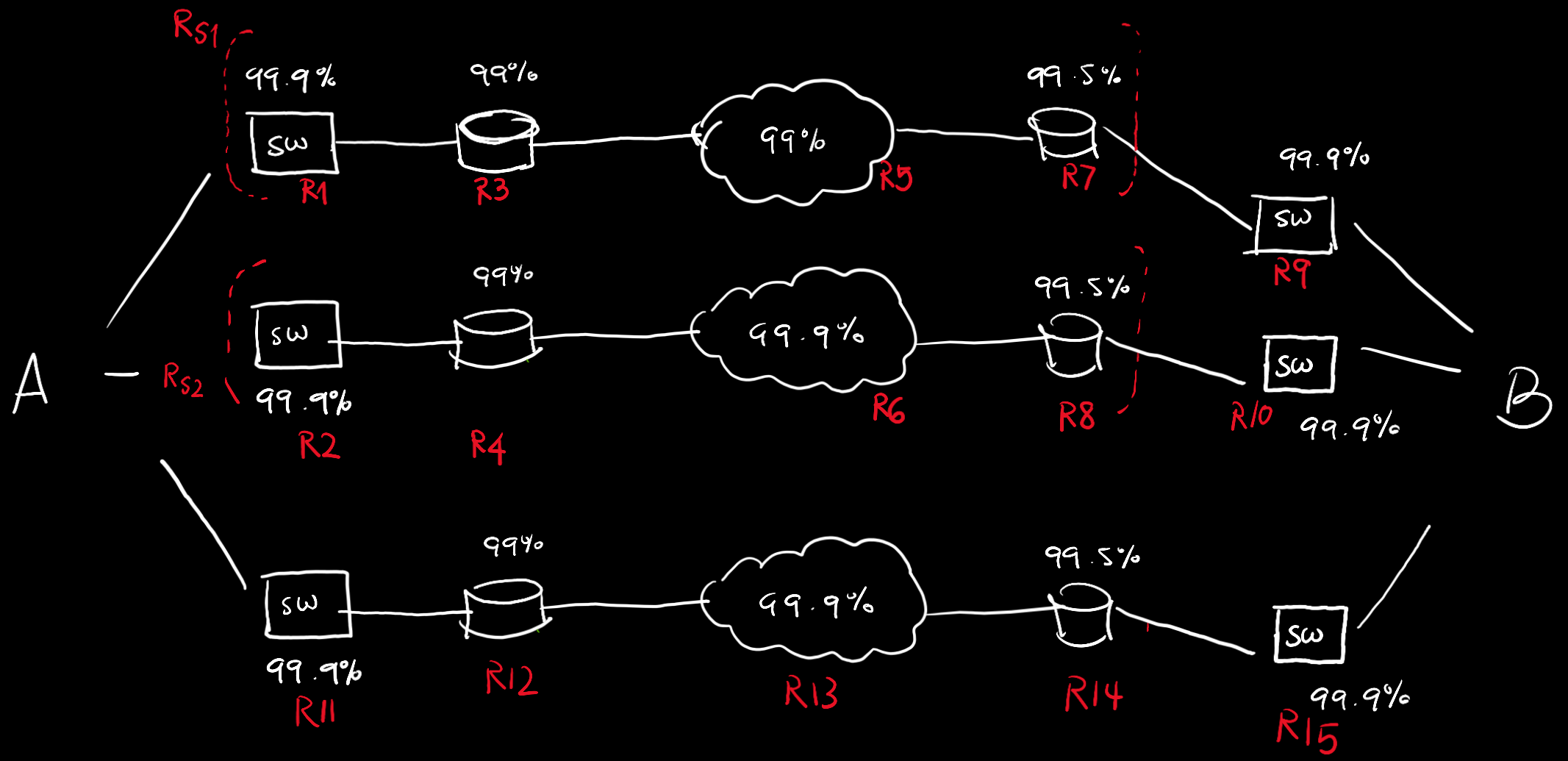


$$R_{S1} = R_1 * R_3 * R_5 * R_7 * R_9 = .97325 \rightarrow 97.325\%$$

$$R_{S2} = \qquad = .98209 \rightarrow 98.209\%$$

$$R_p = 1 - ((1 - .97325) * (1 - .98209))$$

$$= .9995 \rightarrow 99.95\%$$



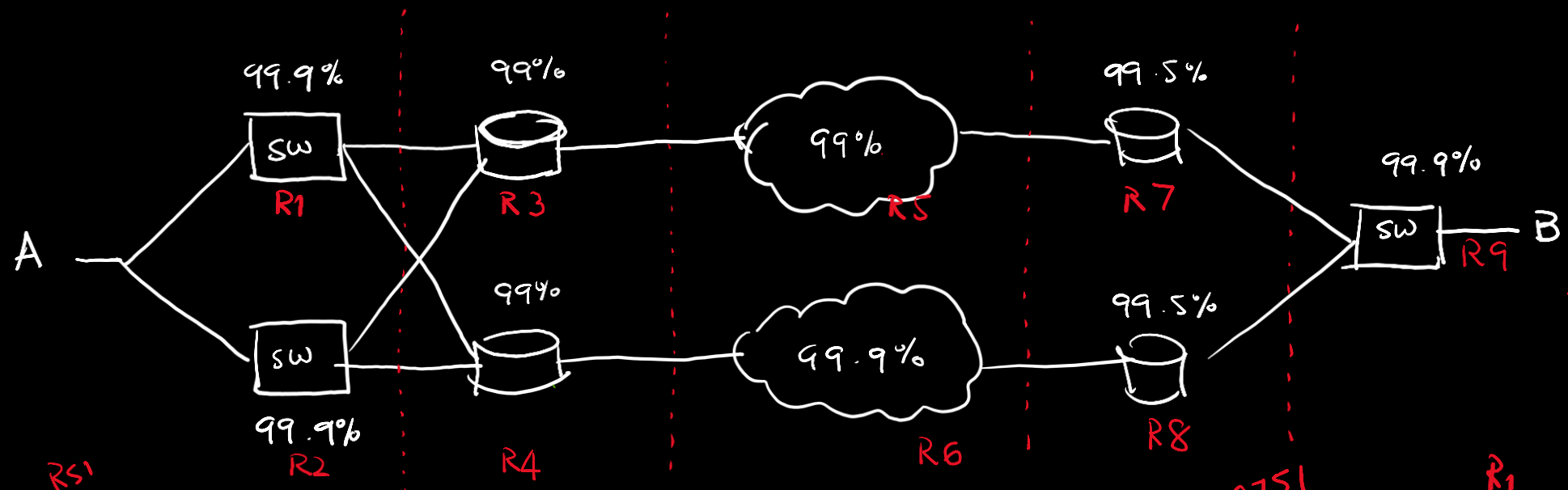
$R_{S1} = .97325$

$R_{S2} = .98029$

$R_{S3} = .98029$

$R = 1 - ((1-.97325) * (1-.98025) * (1-.98025))$

$= .999989 \rightarrow 99.9989\%$



0.0249 0.016

$1-((1-.9751)*(1-.9840))$

$R_{P1} = .9996$

$R_{P2} = .9996$

$R_{S1} = .9751$

$R_{S2} = .9840$

R_{S3}

R_{S4}

$R_{S5} = R1 * RP1$
$= .9986$

R_{S6}

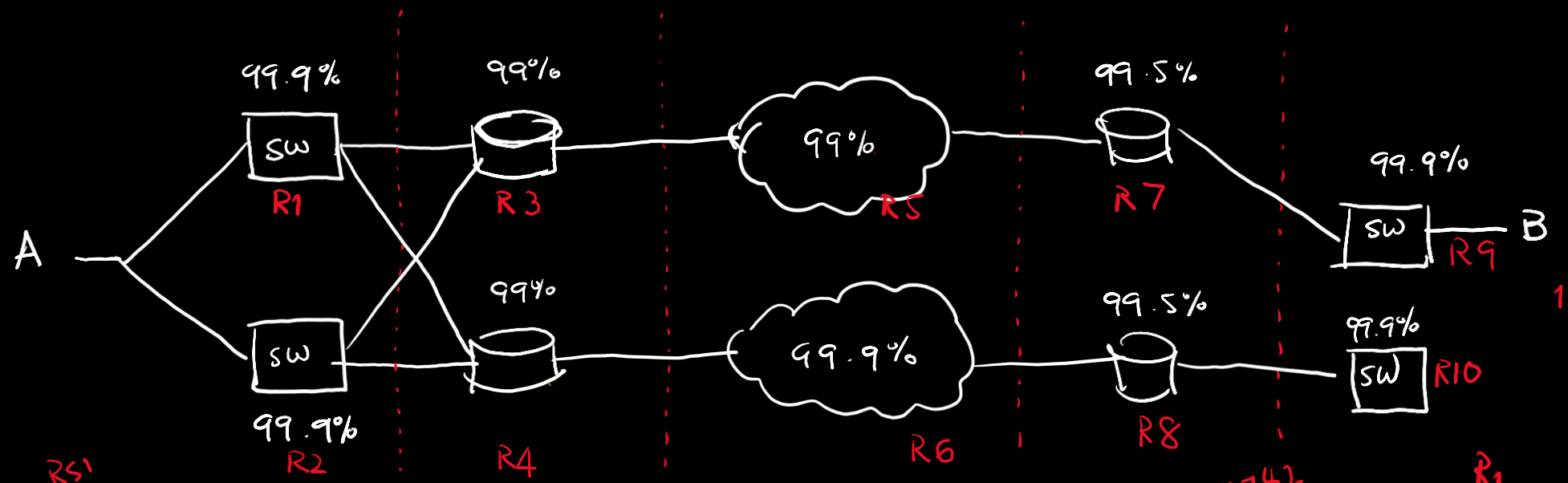
R_{P3} R9

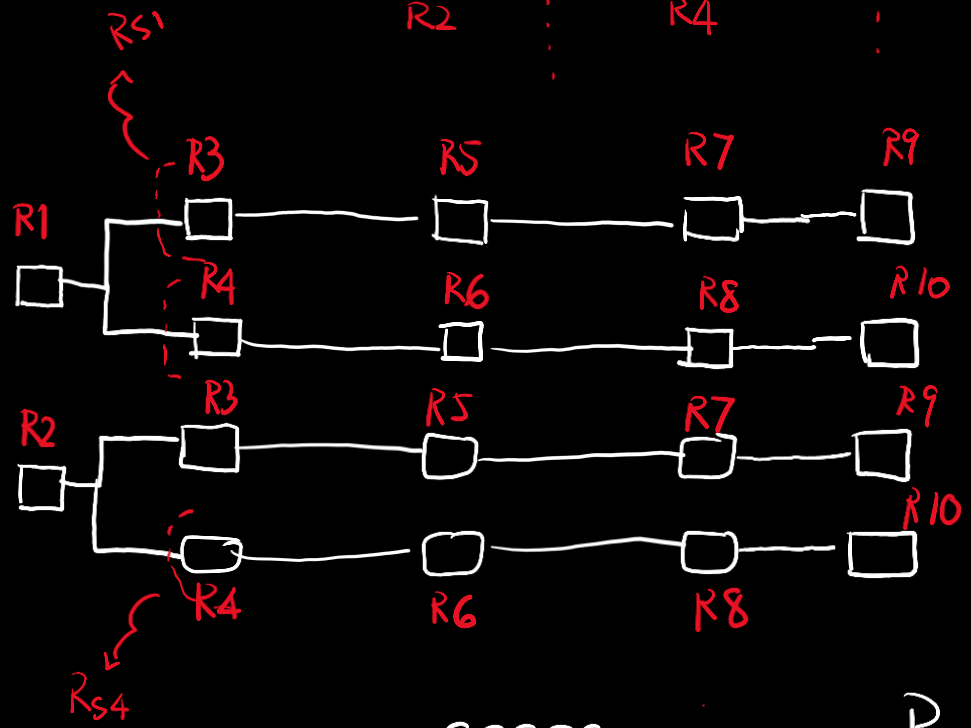
$= .9986$

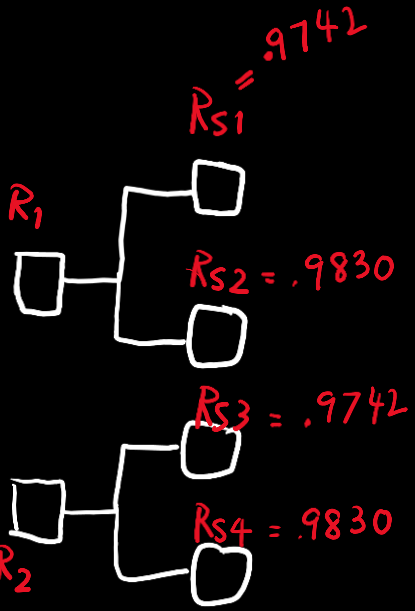
$R = .998998$

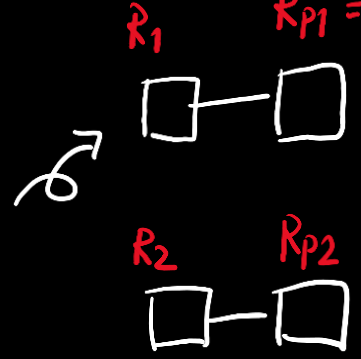
$R_{P3} = 1-((1-.9986)*(1-.9986))$
$= .999998$

.0014 .0014









$1-\left((1-.9742)*(1-.9830)\right)$

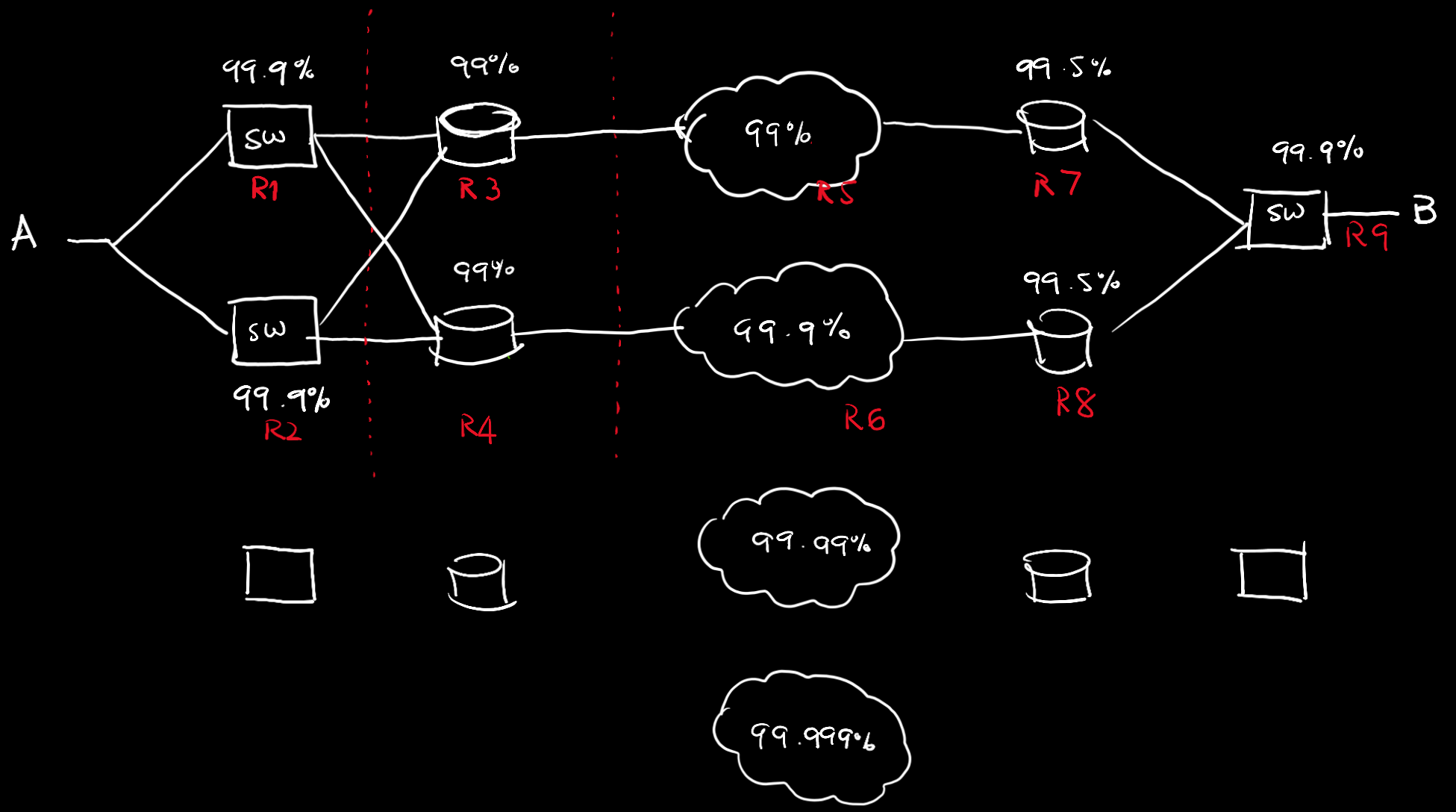
$R_{S5} = R1 * RP1$

$= .9985$

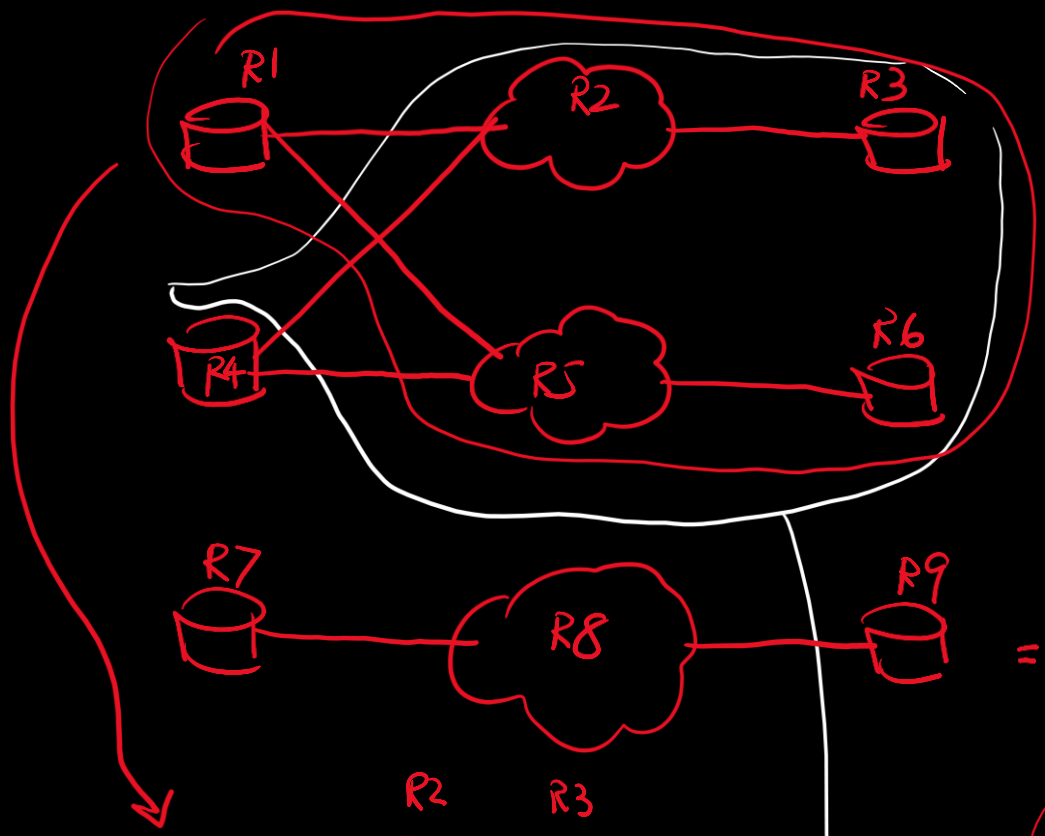
R_{S6}

$= .9985$

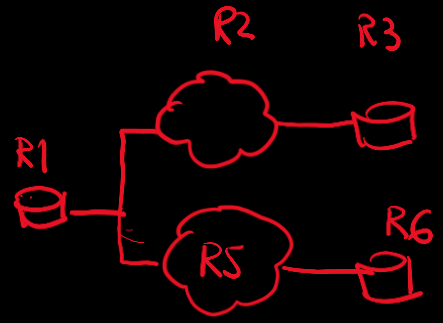
99.999% $.9999977 = R = 1-\left((1-.9985)*(1-.9985)\right)$

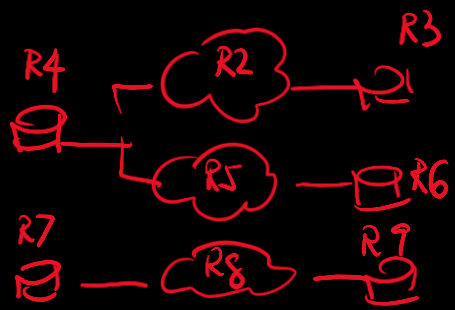


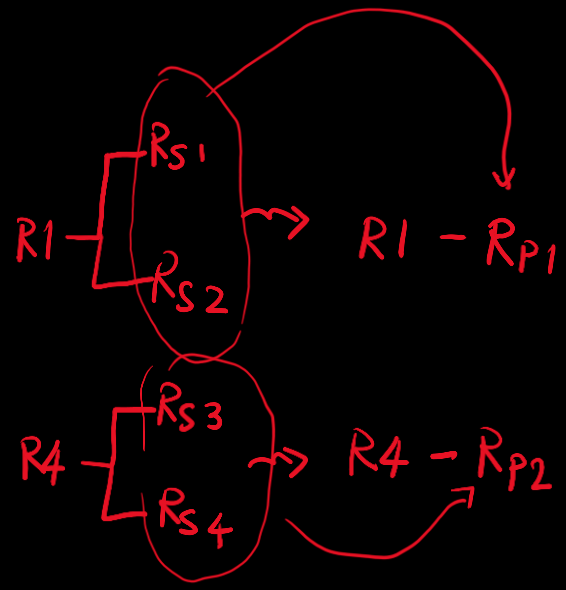
TARGET = 99.999%



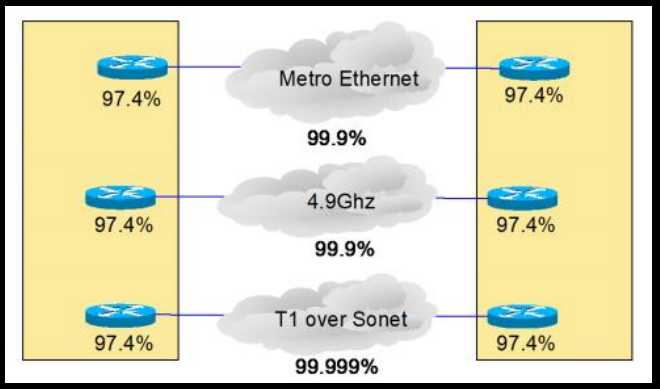
= .948







R7 – R8 – R9 = Rs7



From slide #3, it is known that adding one more series network connection will result in 5 9's

$R_p = 1 - ((1-.948) * (1-.948) * (1-.948) * (1-.948))$
$= 1 - (0.052 * 0.052 * 0.052 * 0.052)$
$= 0.9999927$

$R = 1-((1-R_{S5}) * (1-R_{S6}) * (1-R_{S7}))$

= ??

In-class work is due by the end of the class.

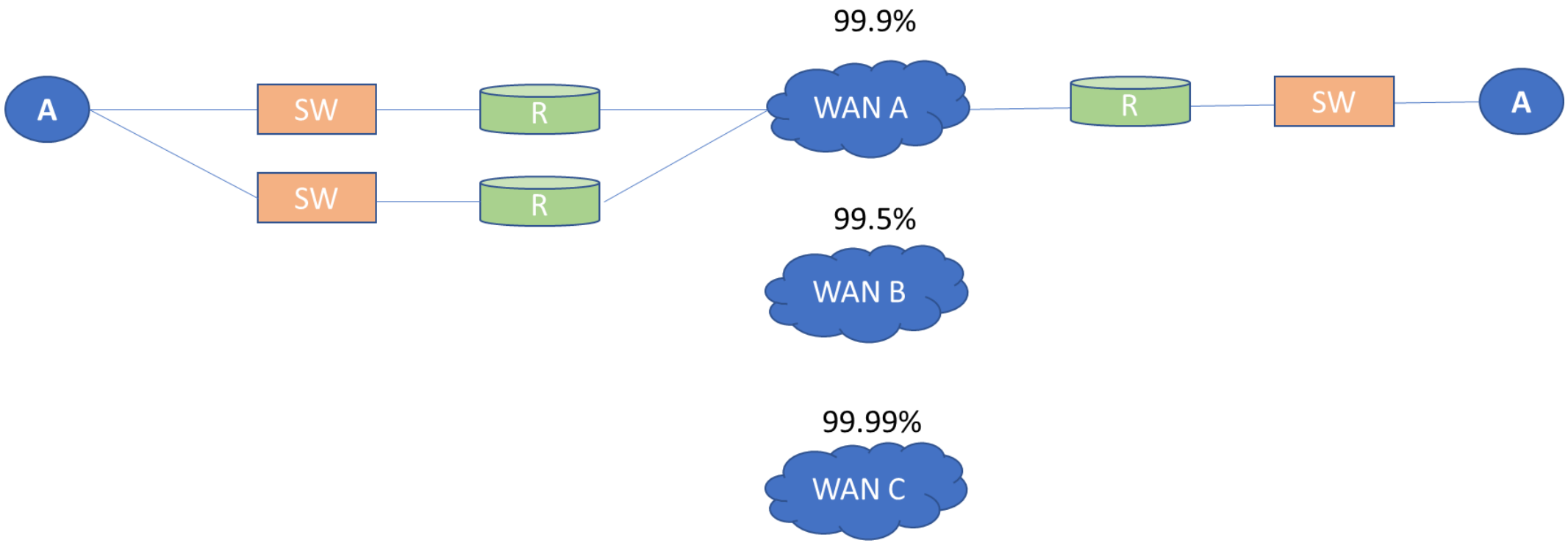
Complete all the tasks on the provided A4 paper and submit your work to the lecturer.

MTBF and Reliability Calculation

Suppose 6 devices are tested for 20,000 hours. During the test 3 failures occur. Determine the estimated Mean-time Between Failure (MTBF) value and the reliability of the system that will still be functioning after 1,000 hours of operation. The reliability equation is given below.

$$R(t) = e^{-t/MTBF}$$

where *e* is 2.7182, and *t* is the operation time.



Task 1: Determine the reliability between Devices A and B shown in Figure 1. Step-by-step calculations must be given.

Task 2: Propose a new configuration that helps to achieve 5-nine reliability (99.999%). Assume that the reliability of switches and routers are 99.5% and 99.9%, respectively. The reliability of WAN A, WAN B, and WAN C is shown in the figure. **Note that your proposed design should use as few new switches, routers, or WAN providers as possible.**