# Assumption University of Thailand Vincent Mary School of Science and Technology

### Final Examination (online) Semester 1/2022

Subject: ITX 2006/ CSX 2006 - Mathematics and Statistics for Data Science

Date: Wednesday, October 5, 2022

Time: 13:00 - 16:00

Lecturer: Dr. Khaing Sandar Htun (Full-time Lecturer) Section 541

#### **Instructions:**

- 1. Read the questions carefully and answer each question completely, legibly, and concisely.
- 2. Show detail of your calculation.
- 3. Submit your answer in one single PDF file and name it as "YourName-YourID"
- 4. This examination is **open-book** and the use of books and lecture notes is allowed.

# **Marking Scale:**

The total number of marks for the 6 questions on the exam paper is 100 marks. The total of 100 marks for this examination corresponds to 40% of the final score.

3.6.1	1	2	3	4	5	6	Total
Marks Awarded							

Total: 1 Page (excluding this page)

#### There are 6 questions for the total of 100 marks.

- 1. **(20 marks)** Using your admission number (ABCDEFG) creates the elements for the vector  $\mathbf{u}$  and  $\mathbf{v}$  in  $\mathbf{R}^5$ . Let vector  $\mathbf{u} = (\mathbf{A} \mathbf{B}, 2\mathbf{C}, \mathbf{D} + \mathbf{E}, -\mathbf{F}, 5\mathbf{G})$  and vector  $\mathbf{v} = (-\mathbf{F}, -\mathbf{G}, \mathbf{A} + \mathbf{B}, -(\mathbf{C} + \mathbf{D}), \mathbf{E} + \mathbf{F})$ . For each pair of vector  $\mathbf{u}$  and  $\mathbf{v}$  determine the following values.
  - a) (3 marks) Dot product of vector **u** and **v**.
  - b) (4 marks) Norm of the vector **u** and normalized vector.
  - c) (4 marks) Norm of the vector v and normalized vector.
  - d) (4 marks) Angle between the vector **u** and **v**.
  - e) (5 marks) Are the vector  $\mathbf{u}$  and  $\mathbf{v}$  orthogonal vectors in  $\mathbf{R}^5$ ? Use the <u>Pythagorean</u> <u>Theorem</u>.
- 2. (20 marks) Given the system of linear equations

$$x + 2y + 3z = 2$$
$$3x + 5y - 4z = 0$$
$$-2x - 3y + 2z = 2$$

- a) (10 marks) Solve the system by using the Cramer's Rule.
- b) (10 marks) Solve the system by using the method of Gauss-Jordan elimination.
- 3. **(20 marks)** Given matrix  $\mathbf{A} = \begin{bmatrix} 1 & -2 & 2 \\ -2 & 1 & 2 \\ -2 & 0 & 3 \end{bmatrix}$ ,
  - a) (15 marks) Find the eigenvalues and corresponding eigenvectors of the matrix A.
  - b) (5 marks) Find a diagonal matrix D that is similar to matrix A.
- 4. (10 marks) Determine whether the matrix  $\begin{bmatrix} 5 & 7 \\ 5 & -10 \end{bmatrix}$  is a linear combination of the matrices  $\begin{bmatrix} 1 & 2 \\ 3 & -4 \end{bmatrix}$ ,  $\begin{bmatrix} 0 & 3 \\ 1 & 2 \end{bmatrix}$ , and  $\begin{bmatrix} 1 & 2 \\ 0 & 0 \end{bmatrix}$ .
- 5. (10 marks) Prove that the set  $\{(1, 2, 0), (0, 1, -1), (1, 1, 2)\}$  is linearly independent in  $\mathbb{R}^3$ .
- 6. **(20 marks)** Given matrix  $\mathbf{B} = \begin{bmatrix} -7 & 10 \\ -5 & 8 \end{bmatrix}$ 
  - a) (10 marks) Show that the given matrix B is diagonalizable.
  - b) (2 marks) Find a diagonal matrix **D** that is similar to **B**.
  - c) (8 marks) Determine the similarity transformation that diagonalizes B.

## End of Examination Paper