# Basic Mathematics and Statistics

## CHAPTER 1: REAL NUMBER SYSTEM AND SETS

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# 1.1 The Real Number System

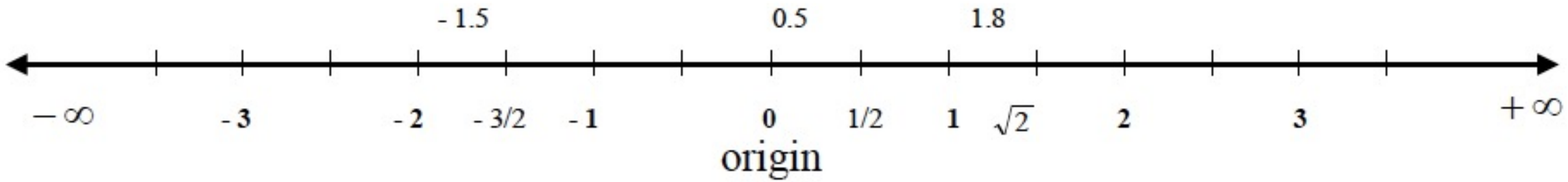
**Number** – is a mathematical object used to count, measure, and label

**Real Numbers System** - describe and calculate the physical measurements such as length, time, temperature, etc.

Thus the Real Numbers are essential for most practical applications of mathematics.

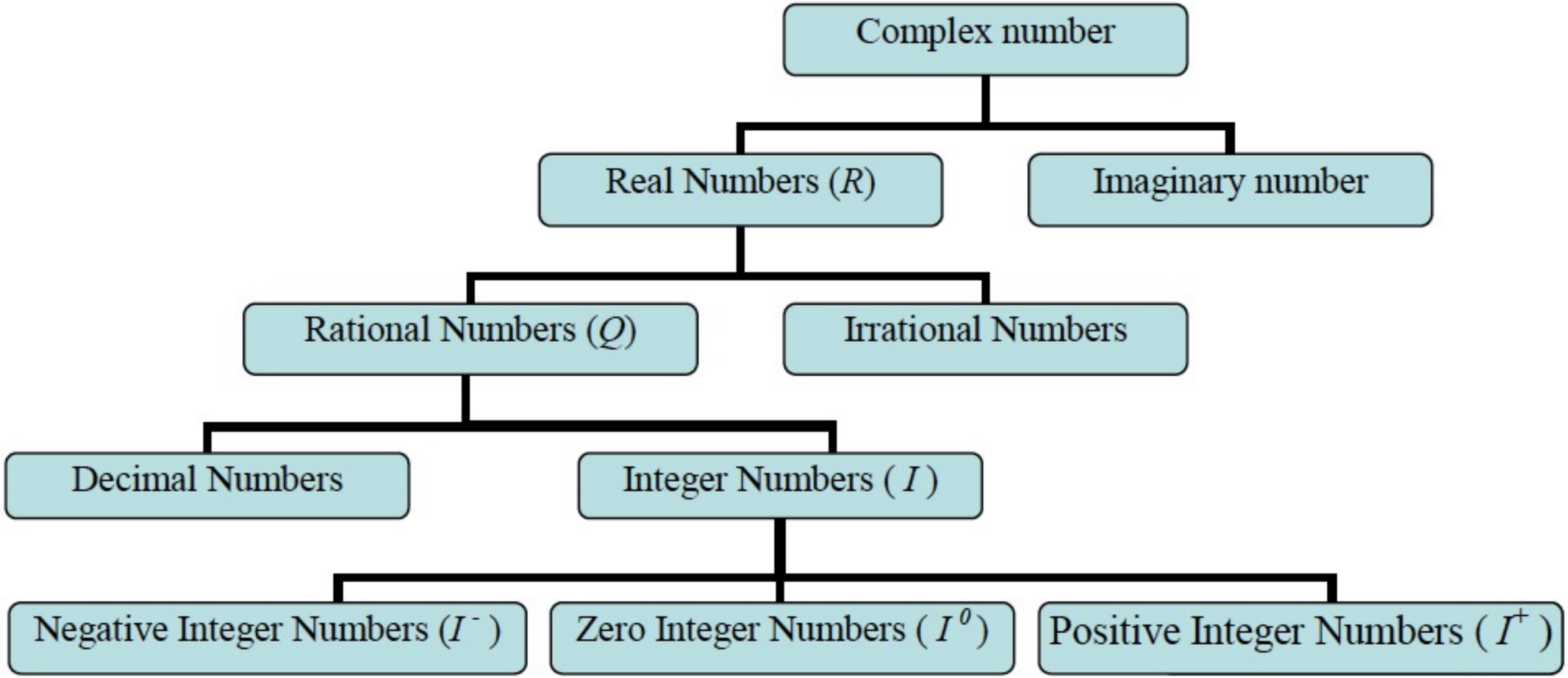
# 1.1 The Real Number System

## 1.1.1 Real Number Line



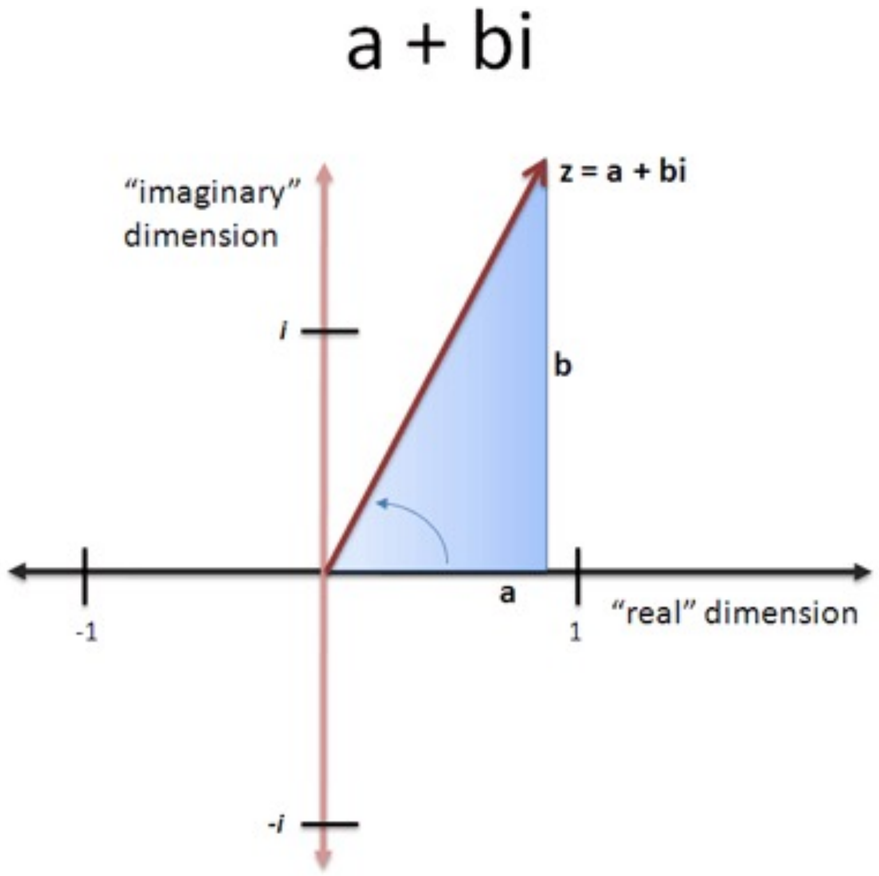
# 1.1 The Real Number System

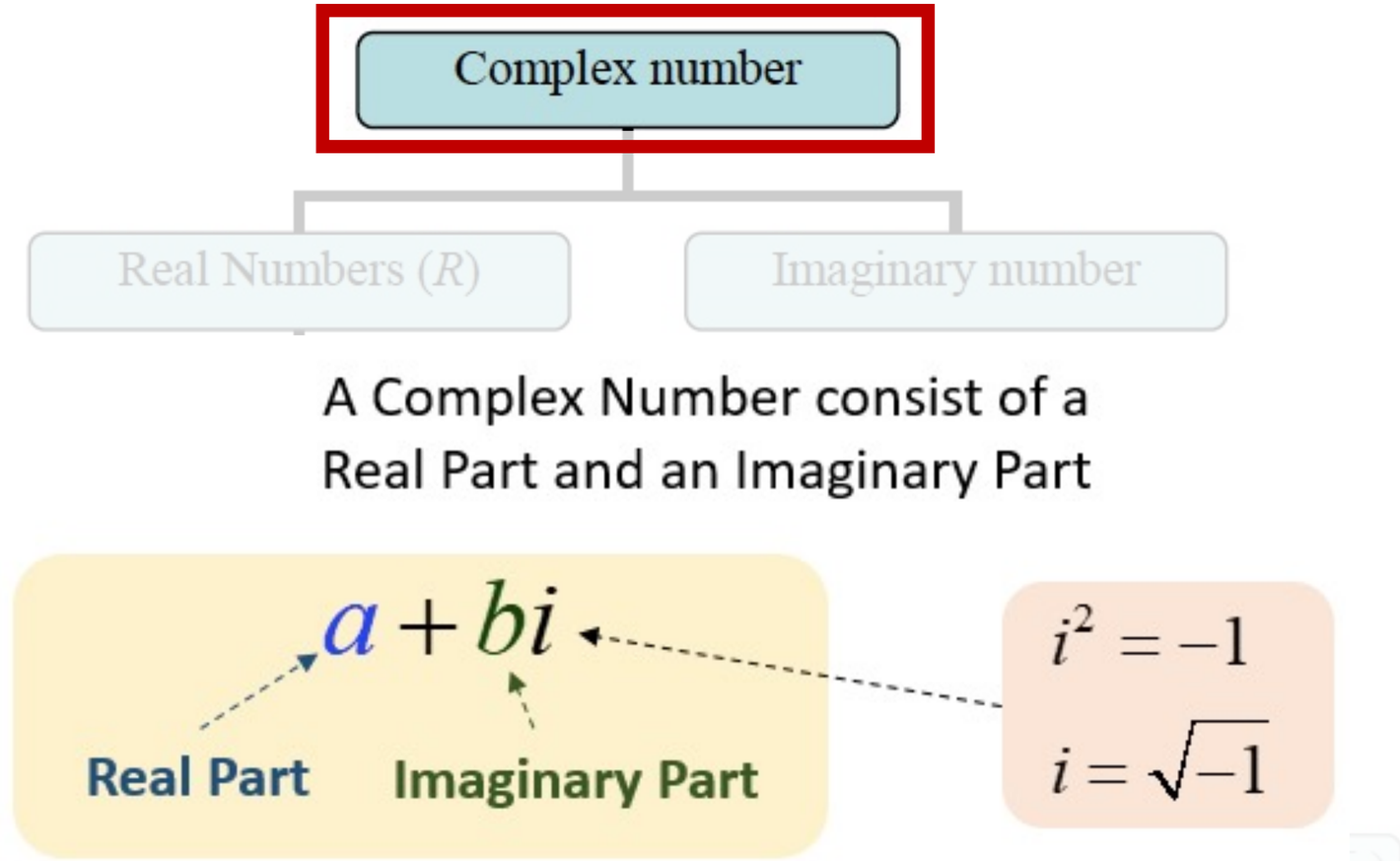
## 1.1.2 Structural Numbers System and Real Numbers System



# 1.1 The Real Number System

## 1.1.2 Structural Numbers System and Real Numbers System

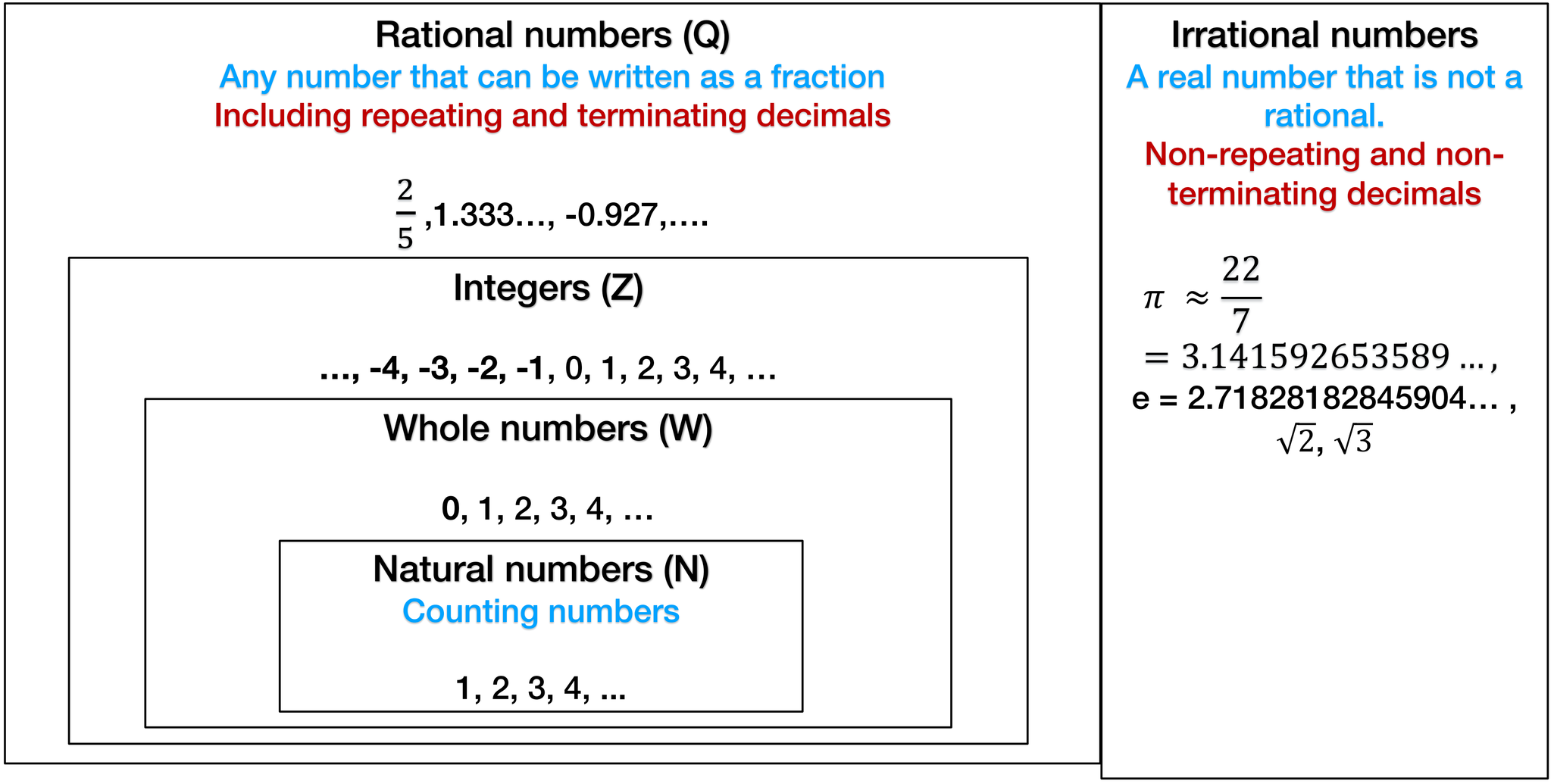



$$\sqrt{-9} = \sqrt{9 * (-1)} = \sqrt{3^2 * i^2} = \pm 3i$$

# 1.1 The Real Number System

## 1.1.2 Structural Numbers System and Real Numbers System



# 1.2 Meaning of a Set

A **set** is a collection of objects.

These objects are called *members* or *elements* of the set.

The symbol ∈ is used for an **element of set**.

$$K = \{a, b, d, n, y\}$$

$$a \in K$$

$$z \notin K$$

# 1.3 Representation of Sets

## Three ways to represent sets

| Word | Set-builder | Roaster |
|---|---|---|
| V is a set of all vowels in English alphabet | V = {x \| x is a vowel in the English alphabet} | V = {a, e, i, o, u} |
| The set of all days of the week | {x \| x is a day of the week} | {Monday, Tuesday, Wednesday, Thursday, Friday, Saturday, Sunday} |
| The set of all square numbers of whole numbers | {x \| x is the square of a whole number}<br>{x \| x =$n^2$, where n ∈ W} | {0, 1, 4, 9, 16, 25, 36,…}<br>{$0^2$, $1^2$, $2^2$, $4^2$, $5^2$, $6^2$,… } |
| The set of all odd positive integers less than or equal to 15 | {x \| x is an odd number and 0< $x \leq 15$} | {1, 3, 5, 7, 9, 11, 13, 15} |

# Exercise 1.1

***List the elements of each set of question* 1. *to* 6.**

**1.** The set of all natural numbers less than 8 $\{1, 2, 3, \ldots, 7\}$

**2.** The set of seasons $\{\_\_\_, \_\_\_, \_\_\_\}$

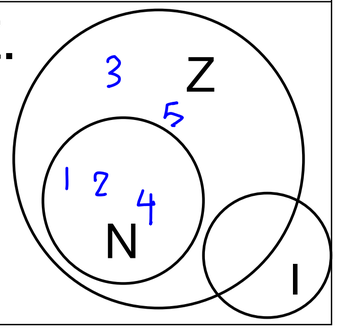
**3.** The set of woman presidents of the United States $\{\quad\}$

**4.** The set of all living humans who are more than 200 years old $\{\quad\}$

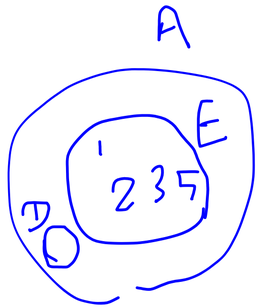
**5.** The set of letters of the alphabet between K and M $\{L\}$

**6.** The set of positive even integers $\{1, \ldots, +\infty, \ldots\}$

# 1.4 Types of Set

| | |
|---|---|
| **Finite Set** | Countable elements in the set |
| **Infinite Set** | Uncountable elements in the set |
| **Empty Set / Null Set** | NO elements<br>**{ }** 'or' ∅ **DON'T WRITE** {∅} |
| **Equal Set** | Exact same elements regardless of order<br>{a, e, i, o, u} = {i, e, u, a, o} |
| **Subset** | N is a subset of Z if and only if every elements of N is in Z.<br>$N \subset Z$<br>$I \not\subset Z$  |

# 1.4 Types of Set




A = {1, 2, 3, 5}

B = {1, 2, 4, 6, 8}

C = {0, 1, 2, 3, 4, …}

D = { }

E = {1, 3, 5, 2}

- Equal set
  - A = E
- Empty set
  - D
- **Finite set**
  - Except C (**Note:** Empty set is a finite set) and its cardinal number is 0.
- Infinite set
  - C
- **Subset**
  - A ⊄ B, A ⊂ C, A ⊂ E ↔ E ⊂ A, A ⊂ A;
  - B ⊂ C, B ⊂ B; C ⊄ A, C ⊄ B, C ⊂ C, C ⊄ D;
  - D ⊂ A, D ⊂ B, D ⊂ C, D ⊂ E

**NOTE:**
- Empty set is a subset of every set. (∅ ⊂ S)
- Every set is a subset of itself. (S ⊂ S)
- Equal sets are subset of each other. ( A = E, A ⊂ E ↔ E ⊂ A )

# Exercise 1.1

***Use these sets to tell whether each statement in question* 7. *to* 14. *is true* (T) *or false* (F).**

Let $A = \{1,3,4,5,7,8\}$

$B = \{2,4,6,8\}$

$C = \{1,3,5,7\}$

$D = \{1,2,3\}$

$E = \{3,7\}$

$U = \{1,2,3,4,5,6,7,8,9,10\}$

**7.** $5 \in \{1,2,5,8\}$

**8.** $2 \in \{1,3,5,7,9\}$

**9.** $7 \notin \{2,4,6,8\}$

**10.** $\{2,4,9,12,13\} = \{13,12,9,4,2\}$

**11.** $A \subset U$

**12.** $\phi \subset A$

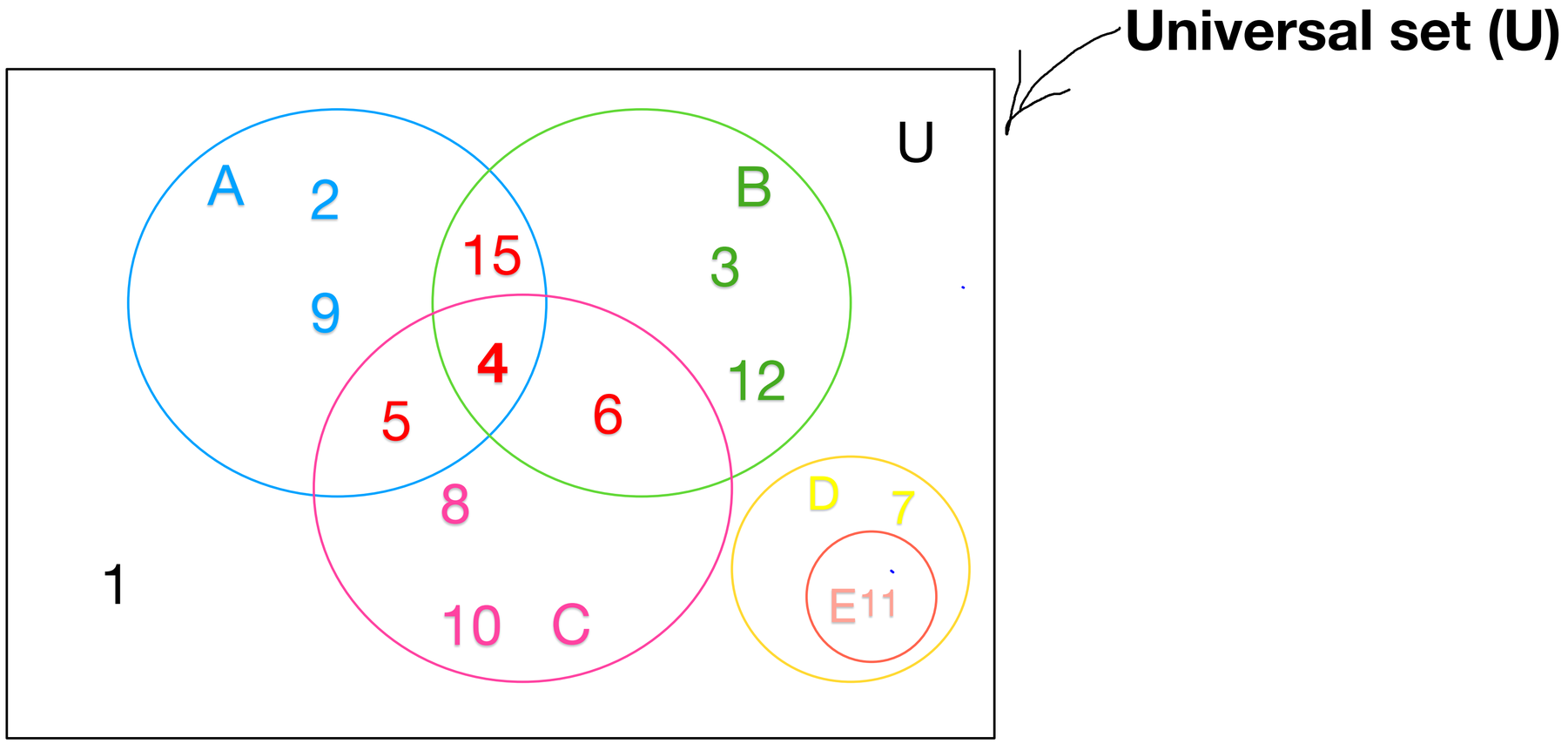
**13.** $C \subset A$

**14.** $D \subset B$

# 1.4 Types of Set

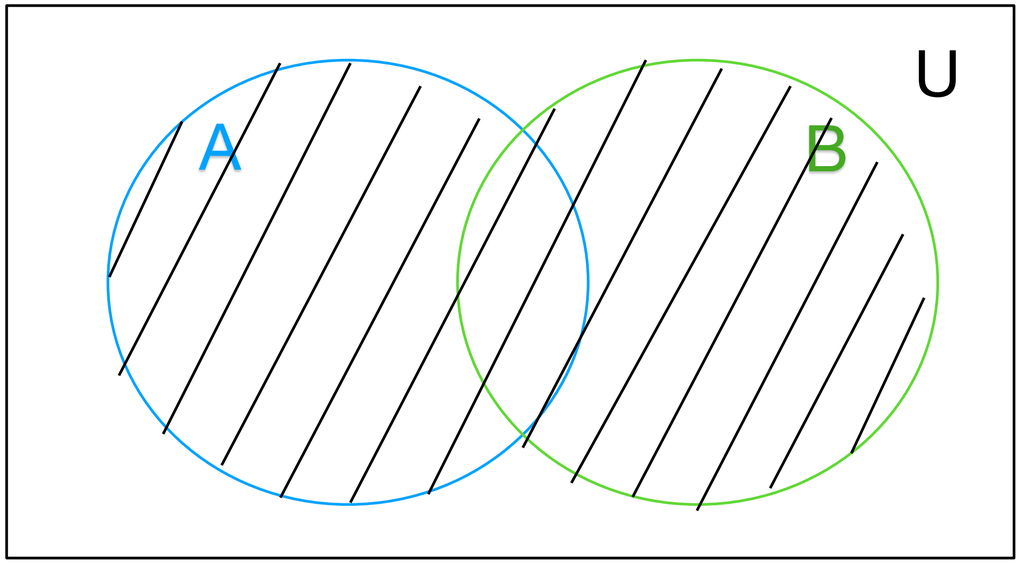
## 1.4.6 Venn Diagrams

A Venn Diagrams shows all the relations between sets.



# 1.5 Set Operations

## 1.5.1 Union (∪)



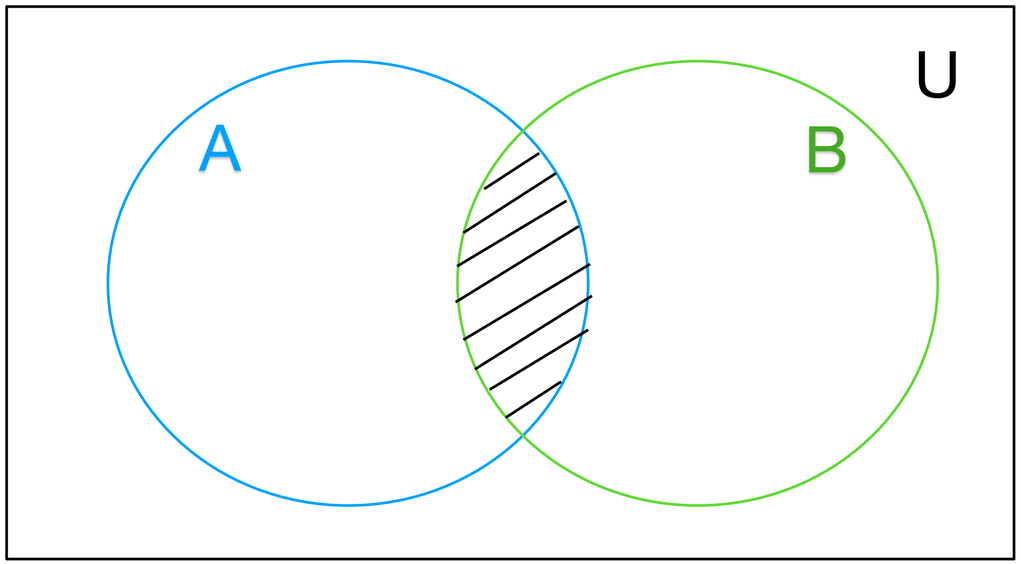

the set which consists of all the elements in A or in B or in both A and B.

(no elements is repeated)

$$\boldsymbol{A} \cup \boldsymbol{B} = \{x \mid x \in \text{A or } x \in \text{B}\}$$

# 1.5 Set Operations

## 1.5.2 Intersection (∩)

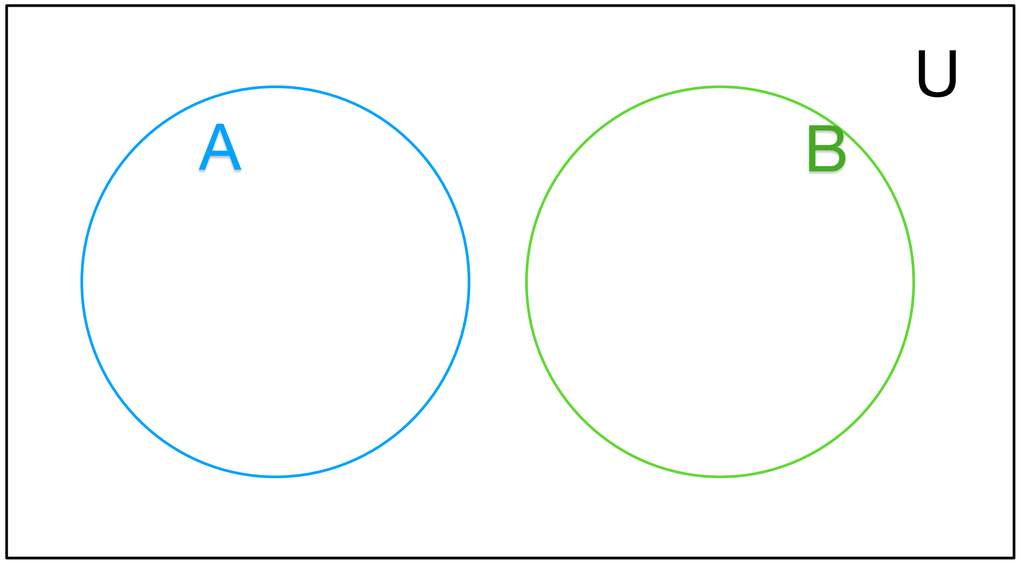



The set which consists of all the elements that members of both A and B.

$$\boldsymbol{A} \cap \boldsymbol{B} = \{x \mid x \in A \text{ and } x \in B\}$$

# 1.5 Set Operations

## 1.5.3 Disjoint




The sets have no element in common

$$A \cap B = \emptyset$$

# Exercise 1.1

***Use these sets to tell whether each statement in question* 15. *to* 19. *is true* (T) *or false* (F).**

Let $A = \{1,3,4,5,7,8\}$

$B = \{2,4,6,8\}$

$C = \{1,3,5,7\}$

$D = \{1,2,3\}$

$E = \{3,7\}$

$U = \{1,2,3,4,5,6,7,8,9,10\}$

**15.** $\{4, 6, 8, 12\} \cup \{6, 8, 14, 17\} = \{6, 8\}$ F

**16.** $\{3, 1, 0\} \cup \{0, 2, 4\} = \{0\}$ F

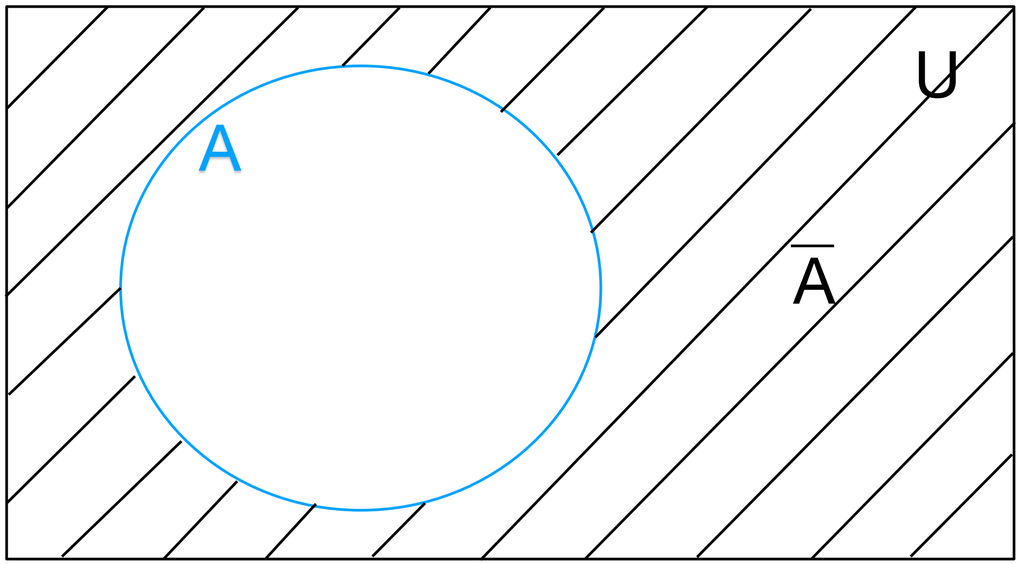
**17.** $\{3, 9, 12\} \cup \phi = \{3, 9, 12\}$ ✓

**18.** $\{4, 9, 11, 7, 3\} \cap \{1, 2, 3, 4, 5\} = \{1, 2, 3, 4, 5, 7, 9, 11\}$ ✗

**19.** $\{3, 5, 7, 9\} \cap \{4, 6, 8\} = \phi$ ✓

# 1.5 Set Operations

## 1.5.4 Complement of set




The **complement of set A** consists of all the **elements** in the universal set U but **not in A**.

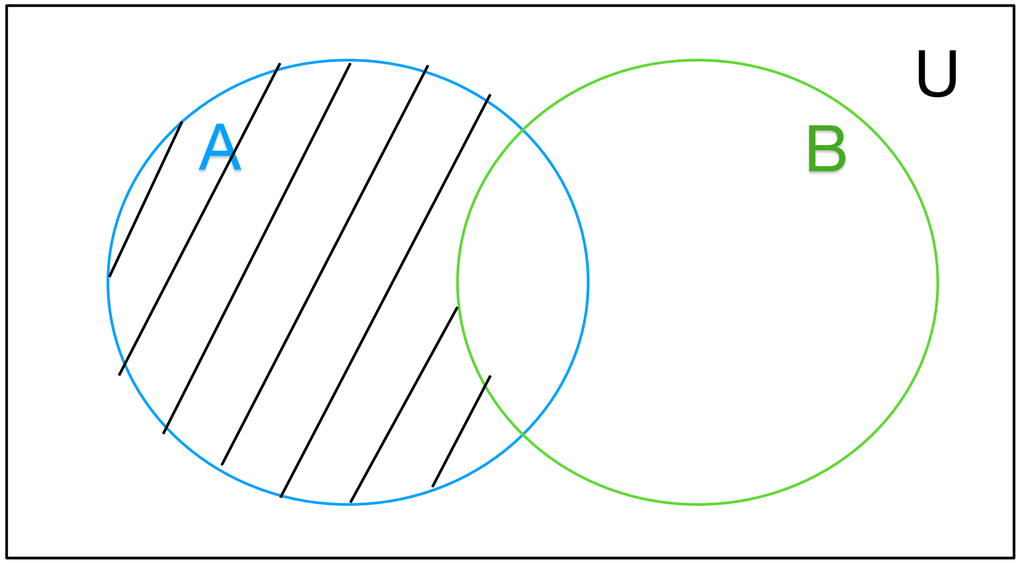
$\overline{A}$ **,** $A^c$**,** $A'$ $= \{x \in U: x \notin A\}$

**De Mogran's Laws:**

1. $\overline{A \cup B} = \overline{A} \cap \overline{B}$
2. $\overline{A \cap B} = \overline{A} \cup \overline{B}$

# 1.5 Set Operations

## 1.5.4 Difference of sets




The difference of two sets (A & B) is the set of elements which belongs to A but B.

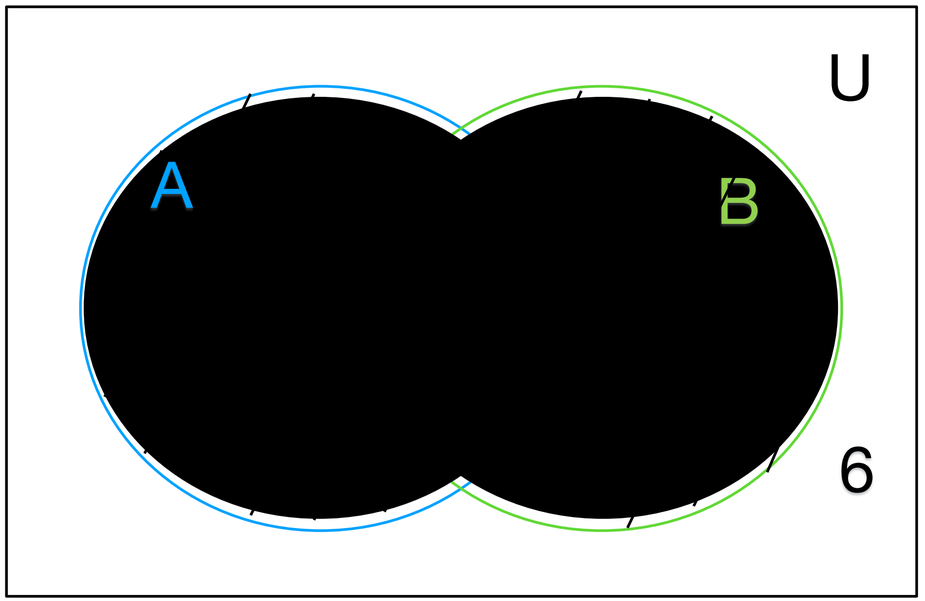
**A - B** = {x ∈ A but x ∉ B}

***Note:*** $A - B = A \cap \overline{B}$

# 1.5 Set Operations

## examples

Let U = {1, 2, 3, 4, 5, 6}, A = {1, 2, 3, 4}, B ={1, 2, 5}




a) U - A **= {5, 6}**

b) A' **= {5, 6}**

c) B' **= {3, 4, 6}**

d) (A ∪ B)' or $\overline{A \cup B}$ **= {6}**

e) (A ∩ B)' or $\overline{A \cap B}$ **= {3, 4, 5, 6}**

f) A – B **= {3, 4}**

g) B - A **= {5}**

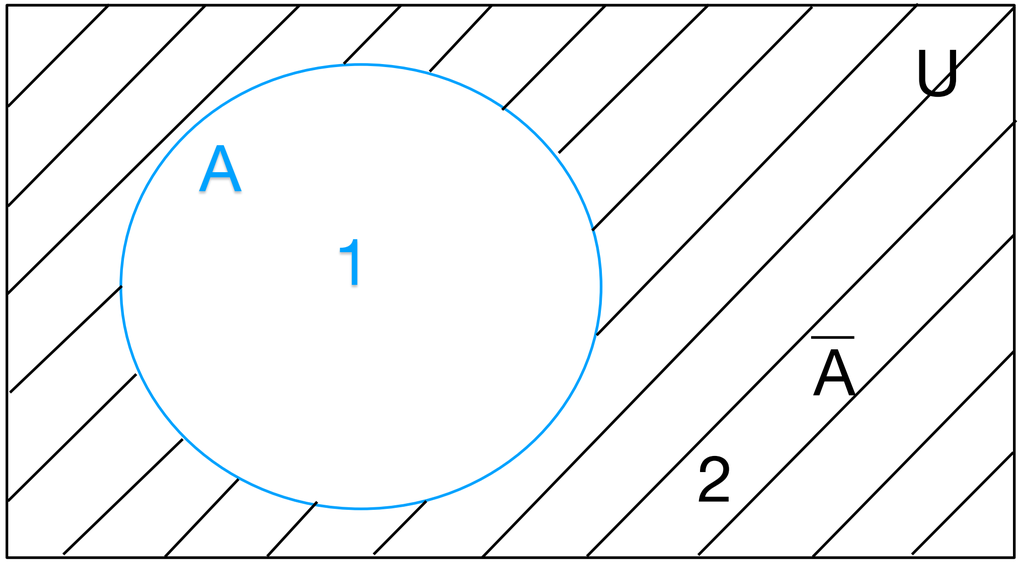
# 1.5 Set Operations

**Set Identities:**

(a) $A \cup \phi =$ (b) $A \cap \phi =$ (c) $A \cup \overline{A} =$ (d) $A \cap \overline{A} =$

(e) $\overline{U} =$ (f) $\overline{\phi} =$ (g) $\overline{\overline{A}} =$ (h) $A \cup U =$

(i) $A \cap U =$



# 1.6 The Number of Elements in a Set

A = {1, 2, 3, 4, 7, 9}

**The number of elements in set A, n(A)** = 6

B = { } or ∅, n(B) = 0

**Empty set has no elements. n(∅) = 0**

T = {x|x is a thai alphabet}; n(T) = ?

# 1.6 The Number of Elements in a Set

**Theorem 1.6.1:**

Let $A$ and $B$ be two finite sets. Then $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

If $A$ and $B$ are **disjoint sets** then $n(A \cup B) = n(A) + n(B)$

**Theorem 1.6.2:** Let $A$, $B$ and $C$ is the finite sets. Then

$$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap \mathrm{B}) - n(A \cap C) - n(B \cap C) + n(A \cap B \cap C)$$

# 1.6 The Number of Elements in a Set

**example**

In a survey of 100 coffee drinkers it was found that 70 take sugar with their coffee, 60 take cream with their coffee, and 50 take both sugar and cream with their coffee. How many coffee drinkers take sugar or cream with their coffee?

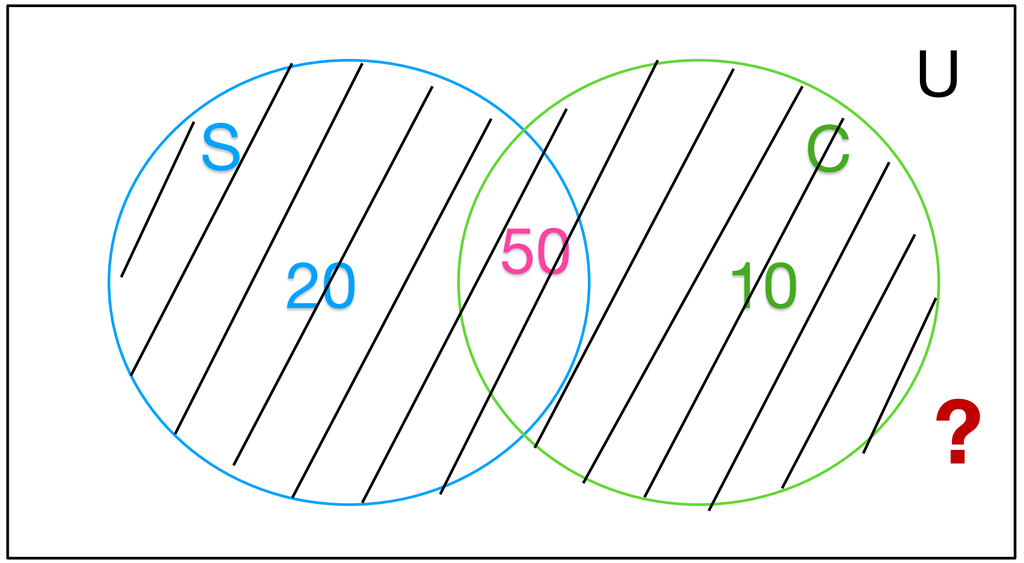
$n(U) = 100$

$n(S) = 70$

$n(C) = 60$

$n(S \cap C) = 50$

$n(S \cup C) = ?$




$n(S \cup C) = n(A) + n(B) - n(S \cap C)$

$= 70 + 60 - 50$

$= \mathbf{80}$

# 1.6 The Number of Elements in a Set

## example

Consumer Survey, in a survey of 75 consumers, 12 indicated they were going to buy a new car, 18 said they were going to buy a new refrigerator, and 24 said they were going to buy a stereo. Of these, 6 were going to buy both a car and a refrigerator, 4 were going to by a car and a stereo, and 10 were going to buy a stereo and refrigerator. One person indicated he was going to buy all three items.

**(a)** How many were going to buy none of these items?

**(b)** How many were going to buy only a car?

**(c)** How many were going to buy only a stereo?

**(d)** How many were going to buy only a refrigerator?

**(e)** How many were going to buy a car and a stereo but not a refrigerator?

# 1.6 The Number of Elements in a Set

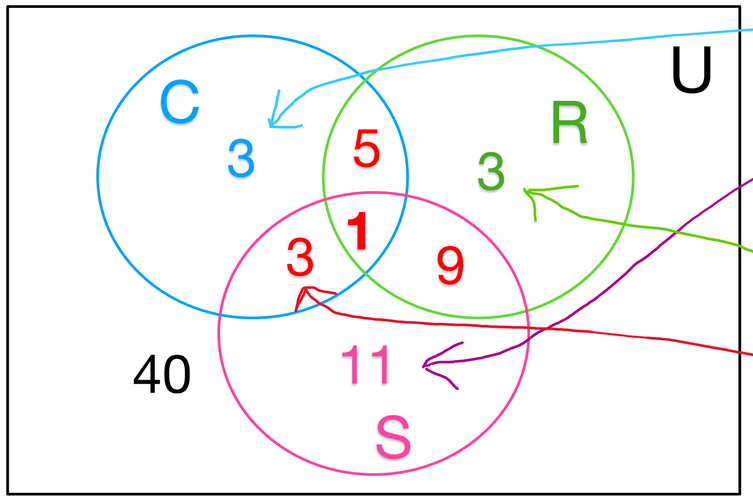
## example

n(U) = 75 n(C ∩ R) = 6

n(C) = 12 n(C ∩ S) = 4

n(R) = 18 n(S ∩ R) = 10

n(S) = 24 n(C ∩ S ∩ R) = 1




**(a) How many were going to buy none of these items?** $n(\overline{C \cup R \cup S})$

n(C ∪ S ∪ R) = n(C) + n(R) + n(S) - n(C ∩ R) - n(C ∩ S) - n(S ∩ R) + n(C ∩ S ∩ R)

=12 + 18 + 24 – 6 – 4 – 10 + 1 = 35

**n(C ∪ S ∪ R)'** = n(U) - n(C ∪ S ∪ R) = 75 – 35 = **40**

**(b) How many were going to buy only a car?** n(C ∩ S' ∩ R')

**(c) How many were going to buy only a stereo?** n(C' ∩ S ∩ R')

**(d) How many were going to buy only a refrigerator?** n(C' ∩ S' ∩ R)

**(e) How many were going to buy a car and a stereo but not a refrigerator?**

n(C ∩ S ∩ R')

# 1.6 The Number of Elements in a Set

## example

Use previous example, and let's say the number of persons who buy all of three item is unknown but the number of persons who buy none of three items is 40. How many were going to buy all of these three items? **n(C ∩ S ∩ R) = x**

n(U) = 75

n(C) = 12

n(R) = 18

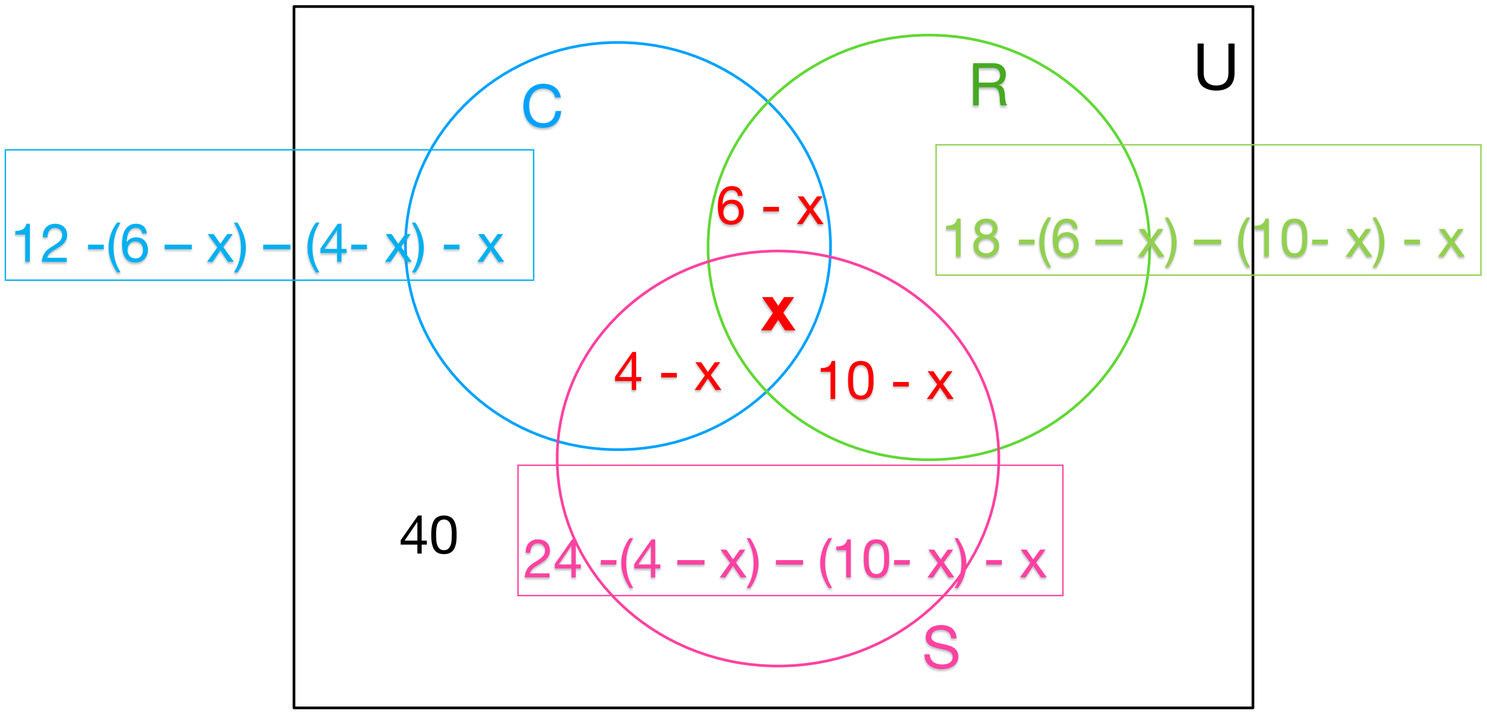
n(S) = 24

n(C ∩ R) = 6

n(C ∩ S) = 4

n(S ∩ R) = 10

**n(C ∪ S ∪ R)' = 40**



# 1.6 The Number of Elements in a Set

## example

$$n(C \cup S \cup R) = n(C) + n(R) + n(S) - n(C \cap R) - n(C \cap S) - n(S \cap R) + \mathbf{n(C \cap S \cap R)}$$

$$n(U) - n(C \cup S \cup R)' = n(C) + n(R) + n(S) - n(C \cap R) - n(C \cap S) - n(S \cap R) + \mathbf{n(C \cap S \cap R)}$$

$$75 - 40 = 12 + 18 + 24 - 6 - 4 - 10 + \mathbf{x}$$

$$35 = 34 + \mathbf{x}$$

$$\mathbf{x} = 1$$

**(a) How many were going to buy all the three items?** $n(C \cap S \cap R)$ $\mathbf{x} = 1$

**(b) How many were going to buy only a car?** $n(C \cap S' \cap R')$ $= 12 - (6 - x) - (4 - x) - x = 12 - 5 - 3 - 1 = \mathbf{3}$

**(c) How many were going to buy only a stereo?** $n(C' \cap S \cap R')$ $= 24 - (4 - x) - (10 - x) - x = 24 - 3 - 9 - 1 = \mathbf{11}$

**(d) How many were going to buy only a refrigerator?** $n(C' \cap S' \cap R)$ $= 18 - (6 - x) - (10 - x) - x = 18 - 5 - 9 - 1 = \mathbf{3}$

**(e) How many were going to buy a car and a stereo but not a refrigerator?** $n(C \cap S \cap R')$ $= 4 - x = 4 - 1 = \mathbf{3}$

# Exercise 1.1

***Use these sets to list the elements of each set in question* 20. *to* 27.**

If $U$ = Universal set = {0, 1, 2, 3, 4 , 5, 6, 7, 8 ,9},

$A$ = { 0, 1, 5, 7}, $B$ = {2, 3, 5, 8}, and $C$ = {5, 6, 9},

Find the elements of the

**20.** $\overline{A} \cap \overline{C}$

**21.** $(A \cup B) \cap C$

**22.** $A \cup (B \cap C)$

**23.** $(A \cup B) \cap (A \cup C)$

**24.** $\overline{A \cap C}$

**25.** $\overline{A \cup B}$

**26.** $\overline{A} \cap \overline{B}$

**27.** $(A \cap B) \cup C$

# Exercise 1.1

***Use these sets to list the elements of each set in question* 28. *to* 35.**

Let $U = \{a, b, c, d, e, f, g, h\}$, $A = \{a, b, c, d, e, f\}$,

$B = \{a, c, e\}$, $C = \{a, f\}$ $D = \{d\}$

**28.** $\overline{A}$

**29.** $\overline{C}$

**30.** $A \cup B$

**31.** $A \cup D$

**32.** $B \cap D$

**33.** $B \cup C$

**34.** $C \cap B$

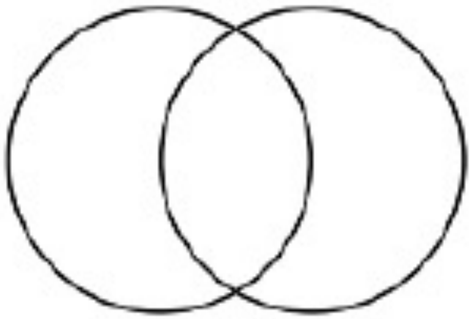
**35.** $A \cup \phi$

**36.** $\overline{A \cap C}$

# Exercise 1.1

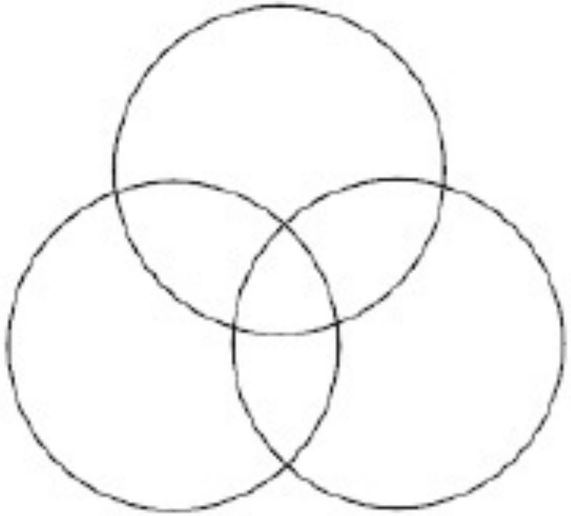
***Use Venn Diagrams to illustrate the following laws in question* 37. *to* 39.**

**37.**



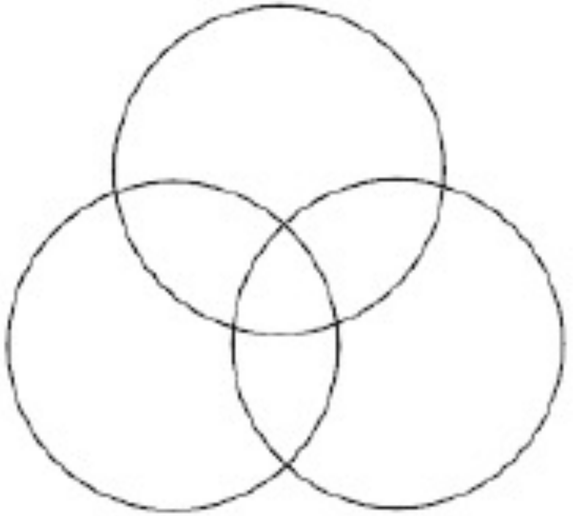
$A \cap (A \cup B) = A$

**38.**



$(\overline{A} \cap \overline{B}) \cup C$

**39.**



$(A \cup B) \cap (A \cup C)$

# Exercise 1.1

**40.** Motors, Inc. manufactured 325 cars with automatic transmissions, 216 with power steering, and 89 with both these options. How many cars were manufactured if every car has at least one option?

# Exercise 1.1

**41. Enrollment Patterns** At a small mid-western college:

31 female seniors were on the dean's list

62 female were on the dean's list who were not seniors

45 male seniors were on the dean's list

87 female seniors were not on the dean's list

96 male seniors were not on the dean's list

275 female were not seniors and were not on the dean's list

88 men were on the dean's list who were not senior

227 men were not on the dean's list who were not senior

**(a)** Draw a Venn diagram for this problem.

**(b)** How many were senior?

**(c)** How many were female?

**(d)** How many were on the dean's list?

**(e)** How many were seniors on the dean's list?

**(f)** How many were female seniors?

**(g)** How many were female on the dean's list?

**(h)** How many were students at the college?

# Exercise 1.1

**42. Purchasing Pattern** Of the cars sold during the month of July, 90 had air conditioning, 100 had automatic transmissions and 75 had power steering. Five cars had all three of extras. Twenty cars had none of these extras. Twenty cars had only air conditioning; 60 cars had only automatic transmissions; and 30 cars had only power steering. Ten cars had both automatic transmission and power steering.

**(a)** Draw a Venn diagram for this problem.

**(b)** How many cars had both power steering and air conditioning?

**(c)** How many had both automatic transmission and air conditioning?

**(d)** How many had neither power steering nor automatic transmission?

**(e)** How many had automatic transmission or air conditioning or both?

n(A)=90

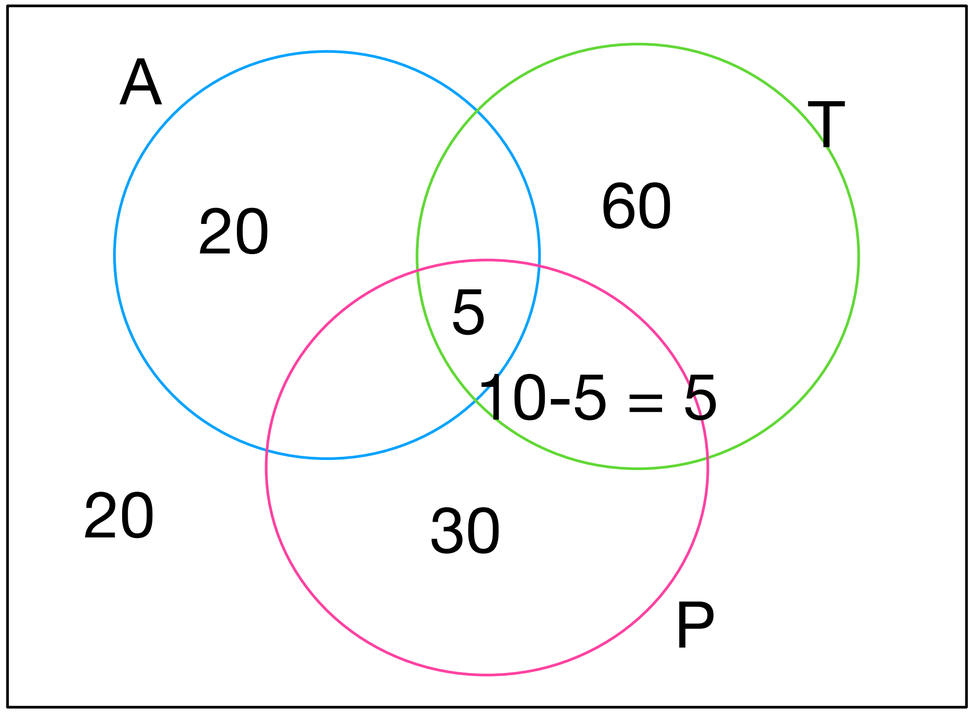
n(T)=100

n(P)=75

n(A ∩ T ∩ P)= 5

n(A ∪ T ∪ P)’= 20

n(T ∩ P)= 10



# Exercise 1.1

**43.** In a survey of 120 consumers conducted in a shopping mall, 80 indicated that they buy Brand A of a certain product, 68 buy Brand B, and 42 buy both brands. Determine the number of consumers participating in the survey who buy

**(a)** at least one of those brands

**(b)** exactly one of those brands

**(c)** only brand A

**(d)** none of those brands

# Exercise 1.1

**44.** A newspaper reports that of 25 senators.

15 voted for subsidies

13 voted for tariff control

9 voted for increased pay

6 voted for subsidies and tariff control

4 voted for subsidies and increased pay

3 voted for tariff control and increased pay

Everyone voted in favor of at least one of the measures.

**(a)** Draw a Venn diagram for this problem.

**(b)** How many voted for three measures

**(c)** How many voted for only tariff control

**(d)** How many voted for at least increased pay

**(e)** How many voted for subsidies or tariff control

# Exercise & Assignment

*Deadline for submission: next week* ***Monday***

- Exercises 1.1, No 40-44
- Please submit in one single PDF file named “you name – your id”
- Send it to  - khainghtun@scitech.au.edu or MS Team chat