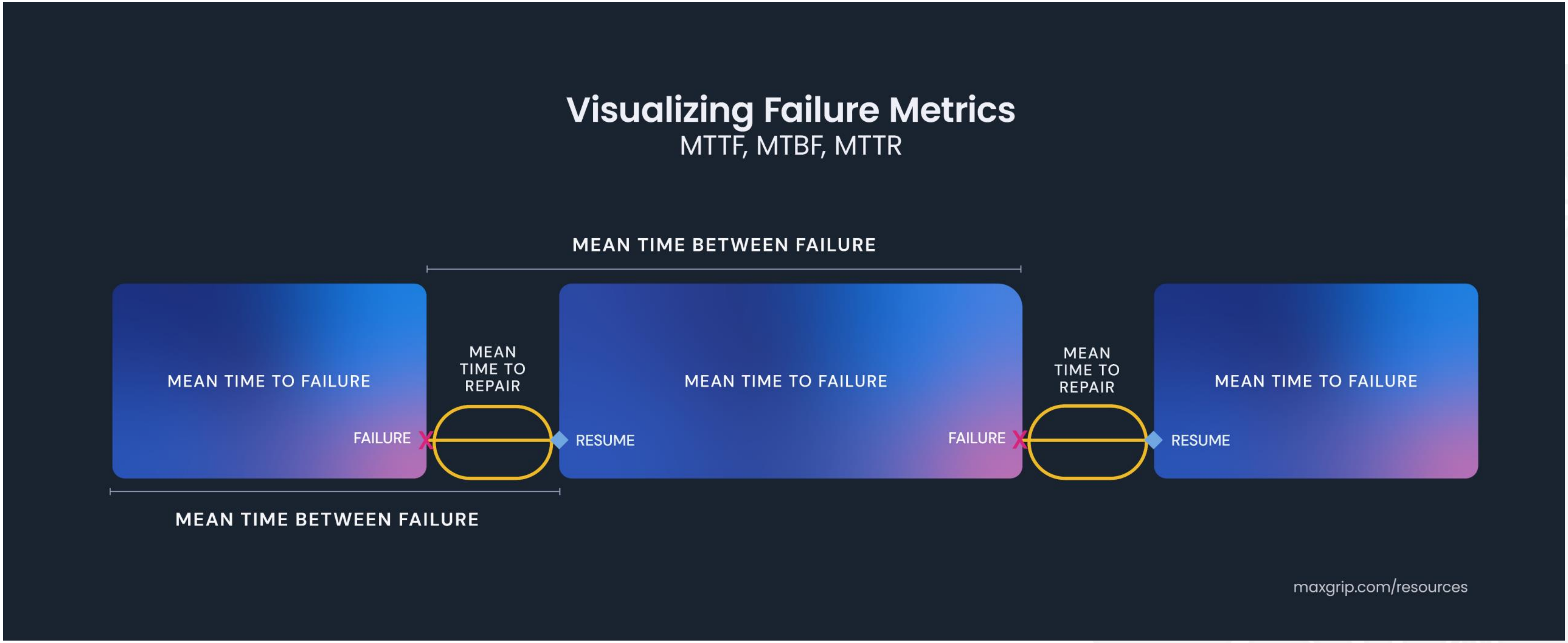
Series, Parallel, and Reliability

Associate Professor Dr. Piyabute Fuangkhon
รองศาสตราจารย์ ดร.ปิยะบุตร เฟื่องคอน

Department of Digital Business Management
Martin de Tours School of Management and Economics


Visualizing Failure Metrics
MTTF, MTBF, MTTR
MEAN TIME BETWEEN FAILURE
MEAN TIME TO FAILURE
FAILURE
MEAN TIME TO REPAIR
RESUME
MEAN TIME TO FAILURE
FAILURE
MEAN TIME TO REPAIR
RESUME
MEAN TIME TO FAILURE
MEAN TIME BETWEEN FAILURE
maxgrip.com/resources

MTBF Example

10 devices are tested for 500 hours. During the test, 2 failures occur.

$$MTBF = \frac{\text{Total time}}{\text{Number of failures}} = \frac{10 \times 500}{2} = 2{,}500 \text{ hours/failure}$$

Associate Professor Dr. Piyabute Fuangkhon
Department of Digital Business Management
Martin de Tours School of Management and Economics

Reliability

- The ability of a system, component, or product to perform its required functions under stated conditions for a specified period of time without failure.

- Use Case: Maintaining uptime and minimizing system outages.

- Formula:

$$R(t) = e^{-\frac{t}{MTBF}}$$

  - e is the mathematical constant approximately equal to 2.71828
  - t is the end time, in hours, that you are interested in.
  - $MTBF$ is expressed in hours.

Associate Professor Dr. Piyabute Fuangkhon
Department of Digital Business Management
Martin de Tours School of Management and Economics

Reliability (Example)

- Let's convert our previous MTBF value of 100 hours to reliability as an example. To make it interesting, let's also calculate reliability at 100 hours. This will indicate the probability that a system with an MTBF of 100 hours will still function after 100 hours of operation.

$$R(t) = e^{-\frac{100}{100}} = 0.3679 = 36.79\%$$

- If you have a product with an MTBF of 100 hours, you only have a 36.79% chance that it actually functions for 100 hours!

Associate Professor Dr. Piyabute Fuangkhon
Department of Digital Business Management
Martin de Tours School of Management and Economics

Service Level Agreement – High Availability

Availability %	Downtime per year	Downtime per quarter	Downtime per month	Downtime per week	Downtime per day (24 hours)
90% ("one nine")	36.53 days	9.13 days	73.05 hours	16.80 hours	2.40 hours
95% ("one nine five")	18.26 days	4.56 days	36.53 hours	8.40 hours	1.20 hours
97% ("one nine seven")	10.96 days	2.74 days	21.92 hours	5.04 hours	43.20 minutes
98% ("one nine eight")	7.31 days	43.86 hours	14.61 hours	3.36 hours	28.80 minutes
99% ("two nines")	3.65 days	21.9 hours	7.31 hours	1.68 hours	14.40 minutes
99.5% ("two nines five")	1.83 days	10.98 hours	3.65 hours	50.40 minutes	7.20 minutes
99.8% ("two nines eight")	17.53 hours	4.38 hours	87.66 minutes	20.16 minutes	2.88 minutes
99.9% ("three nines")	8.77 hours	2.19 hours	43.83 minutes	10.08 minutes	1.44 minutes
99.95% ("three nines five")	4.38 hours	65.7 minutes	21.92 minutes	5.04 minutes	43.20 seconds
99.99% ("four nines")	52.60 minutes	13.15 minutes	4.38 minutes	1.01 minutes	8.64 seconds

Associate Professor Dr. Piyabute Fuangkhon
Department of Digital Business Management
Martin de Tours School of Management and Economics

Series Reliability

- In a series configuration, all components must function for the system to operate. If one component fails, the entire system fails.

- Series systems are less reliable as the number of components increases, because the system depends on all components functioning correctly.

- The overall uptime (reliability) of a series system is the product of the reliabilities of its components:

$$R_{series} = R_1 \times R_2 \times R_3 \times \cdots \times R_n$$

  - R_i = Reliability (uptime probability) of the i^{th} component.
  - n = Number of components in series.

Series Reliability

- Example: Consider three components with uptimes of R_1 = 0.99, R_2 = 0.98, R_3 =0.97

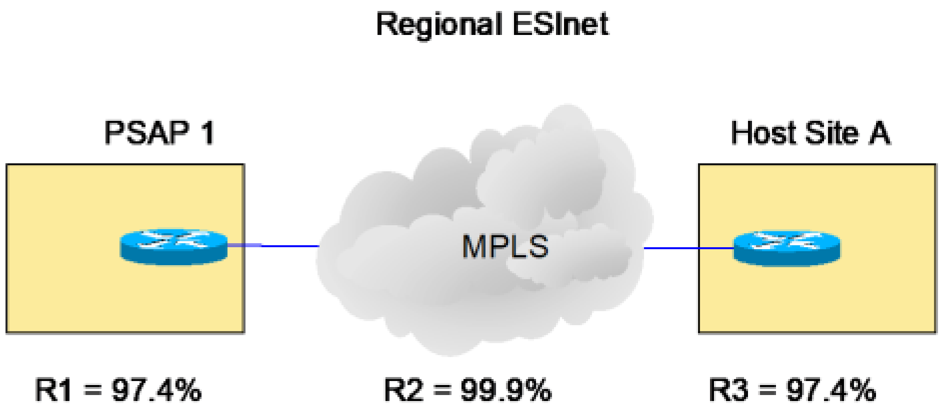
$$R_{series} = 0.99 \times 0.98 \times 0.97 = 0.941$$

- The system uptime is 94.1%, which is lower than the uptime of any individual component.

$R_s = R_1 * R_2 * R_3$

For example, the series reliability of an ESInet shown below is:

.9743 * .999 * .9743 = .948



Associate Professor Dr. Piyabute Fuangkhon
Department of Digital Business Management
Martin de Tours School of Management and Economics

Series Reliability

- In a series configuration, all components must function for the system to operate. If one component fails, the entire system fails.

- The overall uptime (reliability) of a series system is the product of the reliabilities of its components:

$$R_{series} = R_1 \times R_2 \times R_3 \times \cdots \times R_n$$

  - R_i = Reliability (uptime probability) of the i^{th} component.
  - n = Number of components in series.

- Series systems are less reliable as the number of components increases, because the system depends on all components functioning correctly.

Series Reliability

- Example: Consider three components with uptimes of R_1 = 0.99, R_2 = 0.98, R_3 =0.97

$$R_{series} = 0.99 \times 0.98 \times 0.97 = 0.941$$

- The system uptime is 94.1%, which is lower than the uptime of any individual component.

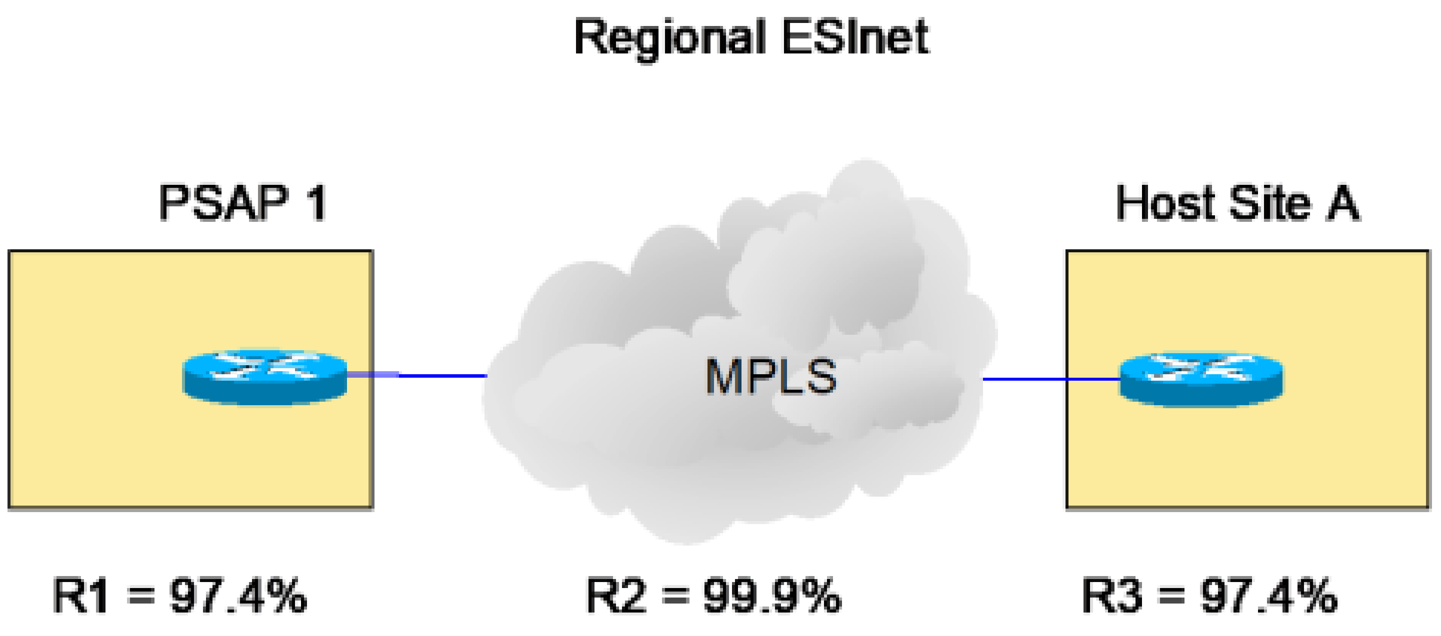
Associate Professor Dr. Piyabute Fuangkhon
Department of Digital Business Management
Martin de Tours School of Management and Economics

Example of Series Reliability

$R_s = R_1 * R_2 * R_3$

For example, the series reliability of an ESInet shown below is:

.9743 * .999 * .9743 = .948



Associate Professor Dr. Piyabute Fuangkhon
Department of Digital Business Management
Martin de Tours School of Management and Economics

Parallel Reliability

- In a parallel configuration, the system can operate as long as at least one component is functioning. Redundancy improves reliability because multiple components can back each other up.

- The overall uptime (reliability) of a parallel system is the product of the reliabilities of its components:

$$R_{parallel} = 1 - \prod_{i=1}^{n}(1 - R_i)$$

  - R_i = Reliability (uptime probability) of the i^{th} component.
  - n = Number of components in parallel.

- Parallel systems are highly reliable, esp. when the individual component uptimes are high.

Associate Professor Dr. Piyabute Fuangkhon
Department of Digital Business Management
Martin de Tours School of Management and Economics

Parallel Reliability

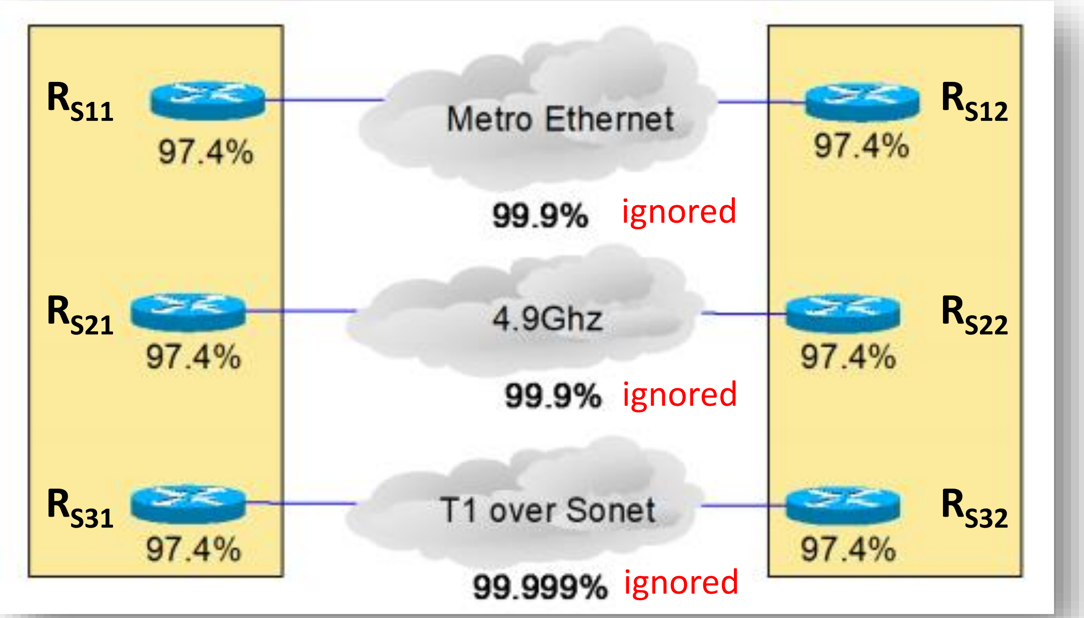
- Example: Consider three components with uptimes of R_1 = 0.99, R_2 = 0.98, R_3 =0.97

$$
\begin{aligned}
R_{parallel} &= 1 - [(1 - 0.99) \times (1 - 0.98) \times (1 - 0.97)] \\
&= 1 - [(0.01 \times 0.02 \times 0.03)] \\
&= 1 - 0.00000 \\
&= 0.999994
\end{aligned}
$$

- The system uptime is 99.9994%, which is **significantly higher** than the uptime of any individual component.

Associate Professor Dr. Piyabute Fuangkhon
Department of Digital Business Management
Martin de Tours School of Management and Economics

Example of Series and Parallel Reliability

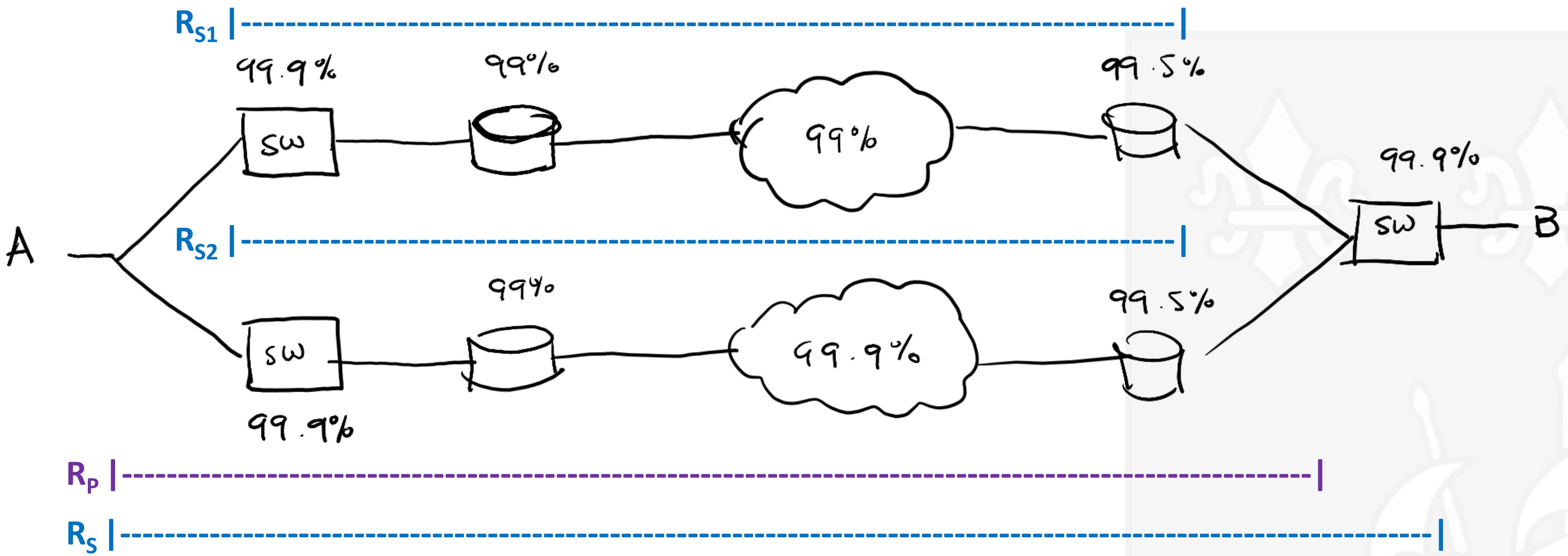


$$
\begin{aligned}
R_{S1} &= R_{S11} \times R_{S12} = 0.974 \times 0.974 \\
&= 0.948 \\
R_{S2} &= R_{S21} \times R_{S22} = 0.974 \times 0.974 \\
&= 0.948 \\
R_{S3} &= R_{S31} \times R_{S32} = 0.974 \times 0.974 \\
&= 0.948
\end{aligned}
$$

$$
\begin{aligned}
R_P &= 1 - [\ (1-R_{S1}) \times (1-R_{S2}) \times (1-R_{S3})\ ] \\
&= 1 - [\ (1-0.948) \times (1-0.948) \times (1-0.948)\ ] \\
&= 1 - [\ 0.052 \times 0.052 \times 0.052\ ] \\
&= 1 - 0.000140608 \\
&= 0.99986 \\
&= 99.986\ \%
\end{aligned}
$$

Associate Professor Dr. Piyabute Fuangkhon
Department of Digital Business Management
Martin de Tours School of Management and Economics

Sample 1 - Series --> Parallels --> Series



R_{S1} = 0.999 × 0.99 × 0.99 × 0.995 = 0.9742243005 = 0.974

R_{S2} = 0.999 × 0.99 × 0.999 × 0.995 = 0.983080885 = 0.983

R_P = { 1 - [(1 - R_{S1}) × (1 - R_{S2})] }

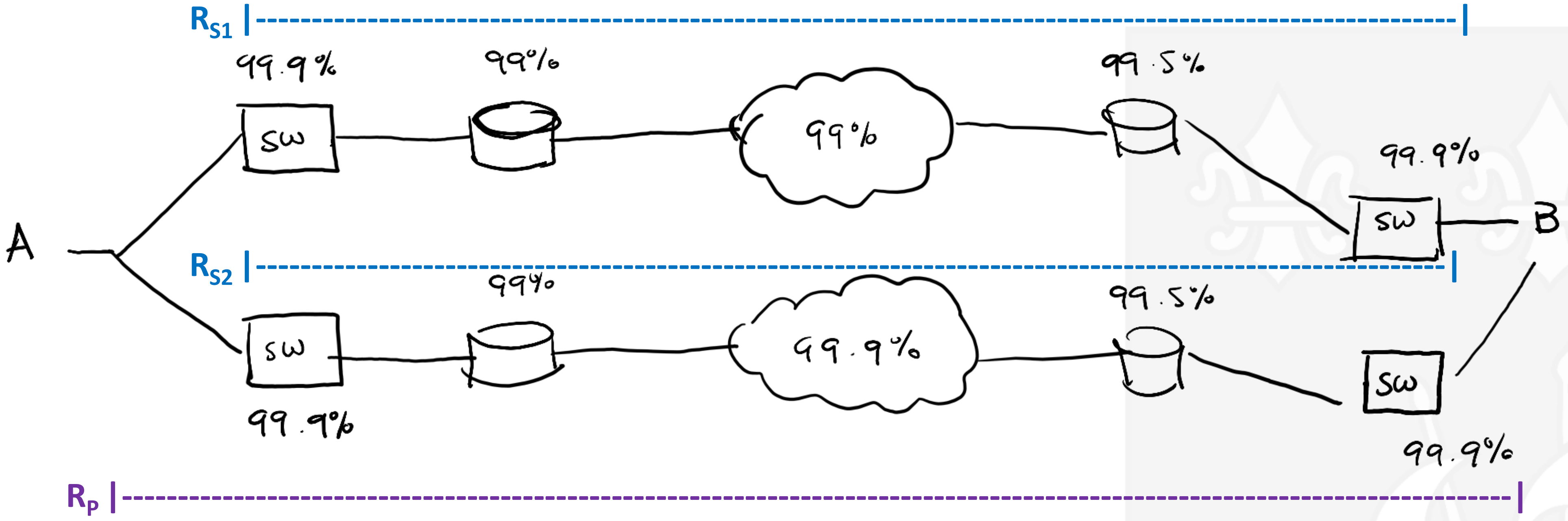
= { 1 - [(1 - 0.974) × (1 - 0.983)] } = { 1 - [0.026 × 0.017] }

= 1 - 0.000442 = 0.999558

R_S = 0.999558 ×0. 999 = 0.998558442 = 0.998

Associate Professor Dr. Piyabute Fuangkhon
Department of Digital Business Management
Martin de Tours School of Management and Economics

Sample 2 - Series --> Parallels



R_{S1} = 0.999 × 0.99 × 0.99 × 0.995 × 0.999 = 0.973250076 = 0.97325

R_{S2} = 0.999 × 0.99 × 0.999 × 0.995 × 0.999 = 0.98210 = 0.98210

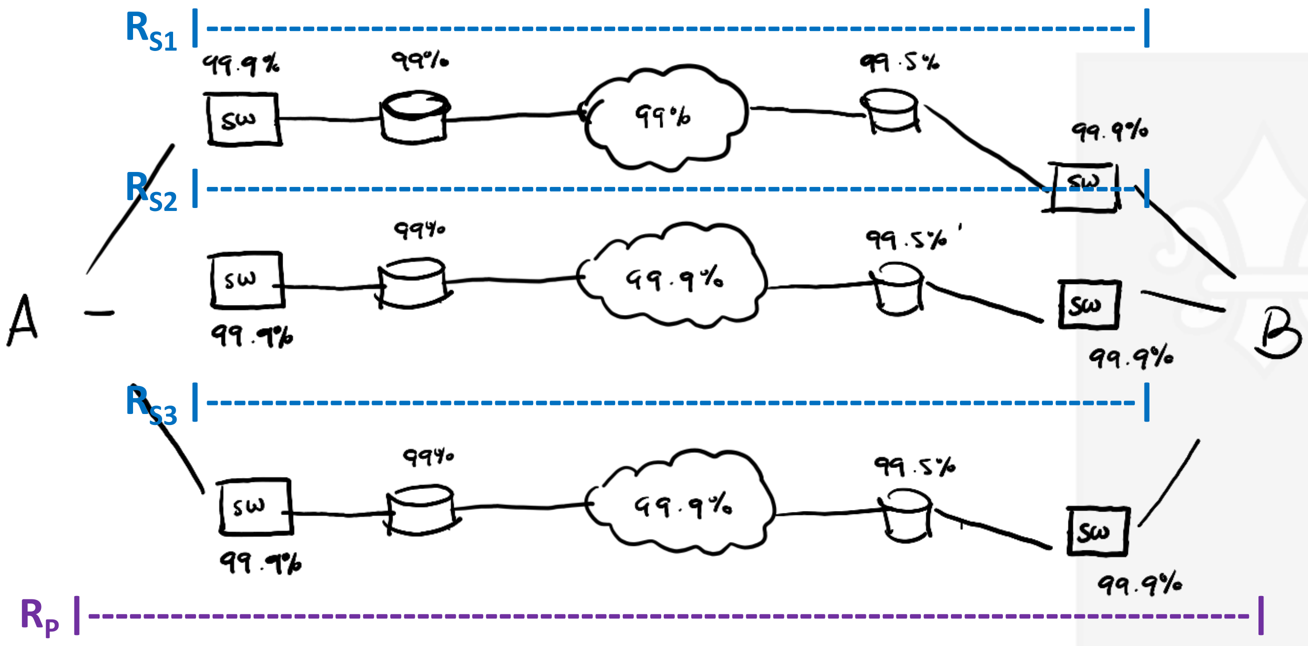
R_P = { 1 - [(1 - R_{S1}) × (1 - R_{S2})] }

= { 1 - [(1 - 0.97325) × (1 - 0.98210)] }

= 0.9995

Associate Professor Dr. Piyabute Fuangkhon
Department of Digital Business Management
Martin de Tours School of Management and Economics

Sample 3 - Series --> Parallels



R_{S1} = 0.999 × 0.99 × 0.99 × 0.995 × 0.999 = 0.97325

R_{S2} = 0.999 × 0.99 × 0.999 × 0.995 × 0.999 = 0.98210

R_{S3} = 0.999 × 0.99 × 0.999 × 0.995 × 0.999 = 0.98210

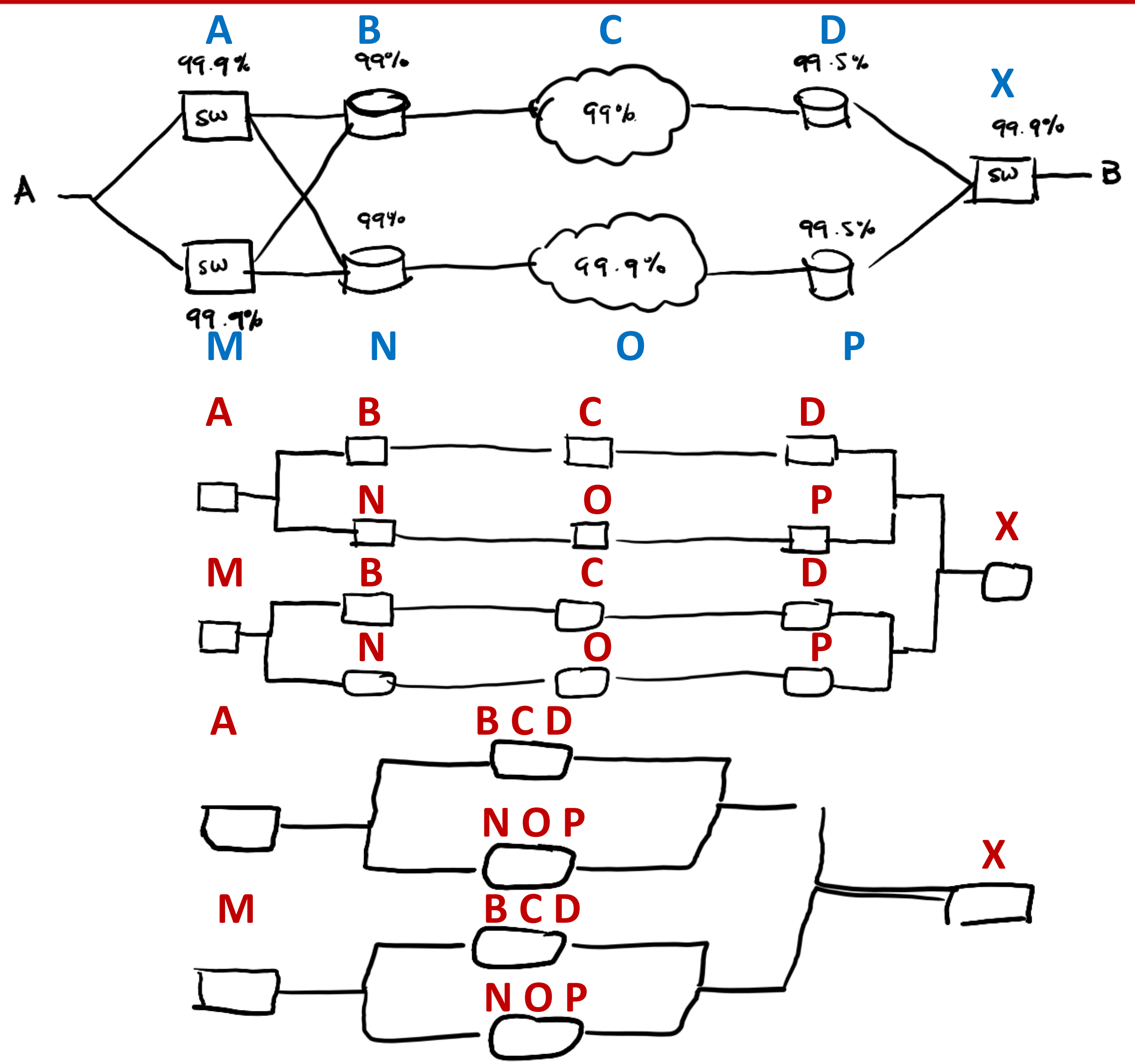
R_P = { 1 - [(1 - R_{S1}) × (1 - R_{S2}) × (1 - R_{S3})] }

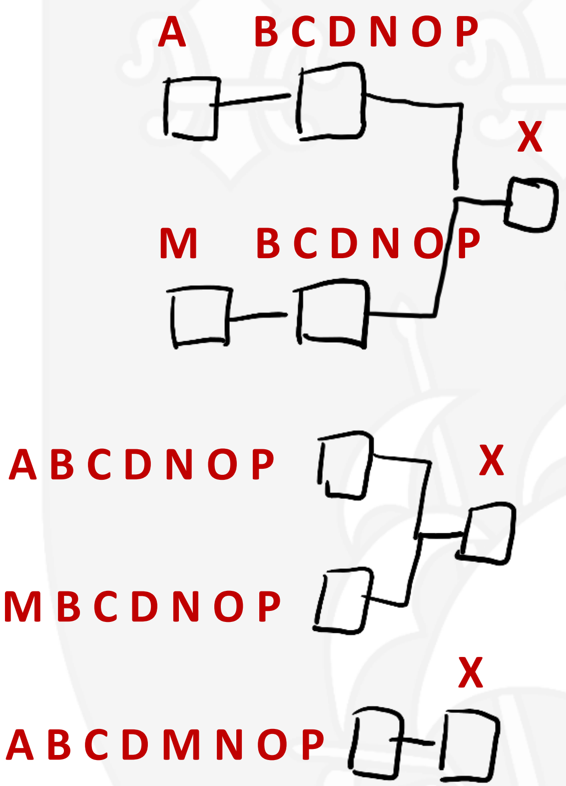
= { 1 - [(1 - 0.97325) × (1 - 0.98210) × (1 - 0.98210)] }

= 0.9999914

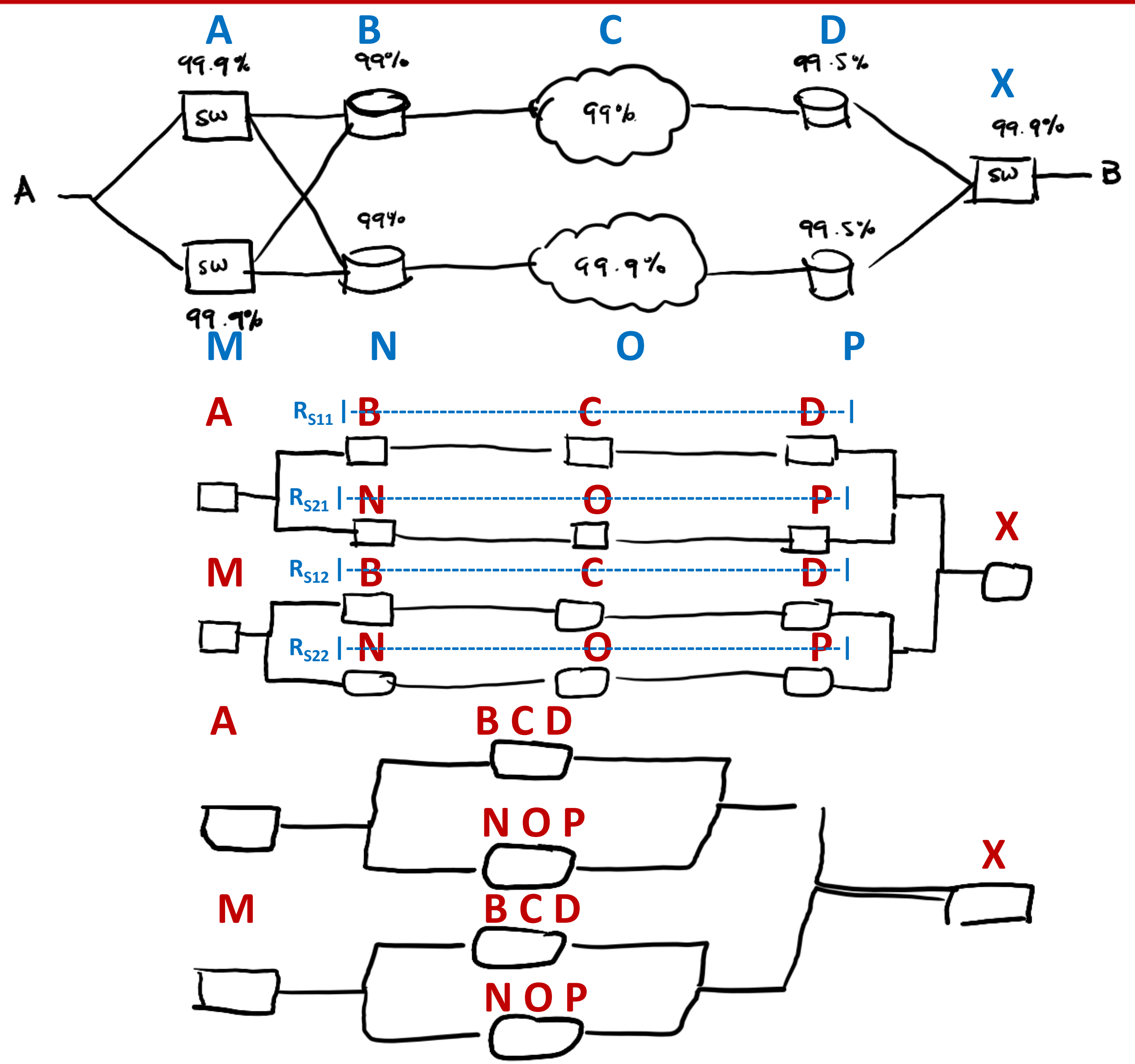
Associate Professor Dr. Piyabute Fuangkhon
Department of Digital Business Management
Martin de Tours School of Management and Economics

Sample 4 - Cross-link --> Series --> Parallels





Associate Professor Dr. Piyabute Fuangkhon
Department of Digital Business Management
Martin de Tours School of Management and Economics



R_{S11} = 0.99 × 0.99 × 0.995 = 0.9752

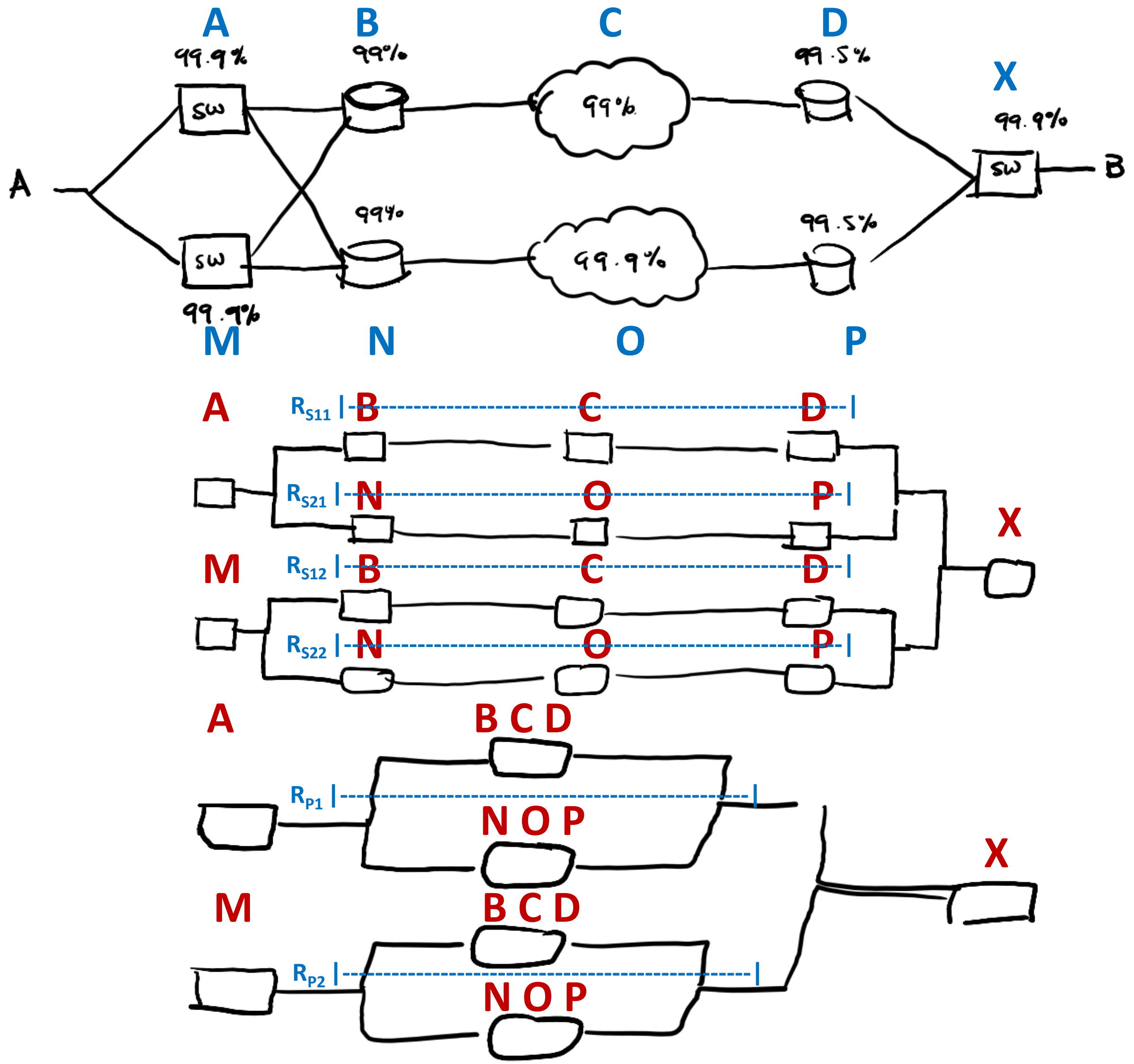
R_{S21} = 0.99 × 0.999 × 0.995 = 0.9841

R_{S12} = 0.99 × 0.99 × 0.995 = 0.9752

R_{S22} = 0.99 × 0.999 × 0.995 = 0.9841

Associate Professor Dr. Piyabute Fuangkhon
Department of Digital Business Management
Martin de Tours School of Management and Economics

Sample 4 - Cross-link --> Series --> Parallels (2)



R_{S11} = 0.99 × 0.99 × 0.995 = 0.9752

R_{S21} = 0.99 × 0.999 × 0.995 = 0.9841

R_{S12} = 0.99 × 0.99 × 0.995 = 0.9752

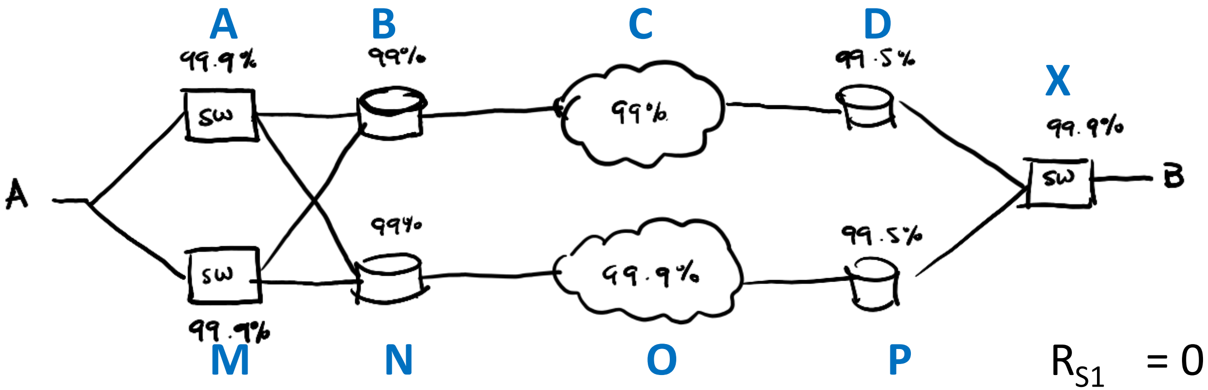
R_{S22} = 0.99 × 0.999 × 0.995 = 0.9841

R_{P1} = { 1 - [(1 - R_{S11}) × (1 - R_{S21})] }
= { 1 - [(1 - 0.9752) × (1 - 0.9841)] }
= 0.99960568

R_{P2} = 0.99960568

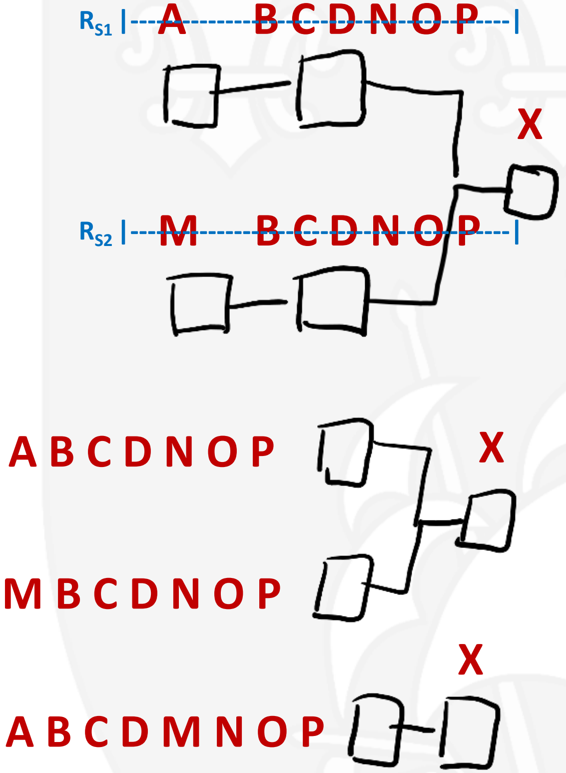
Associate Professor Dr. Piyabute Fuangkhon
Department of Digital Business Management
Martin de Tours School of Management and Economics

Sample 4 - Cross-link --> Series --> Parallels (3)

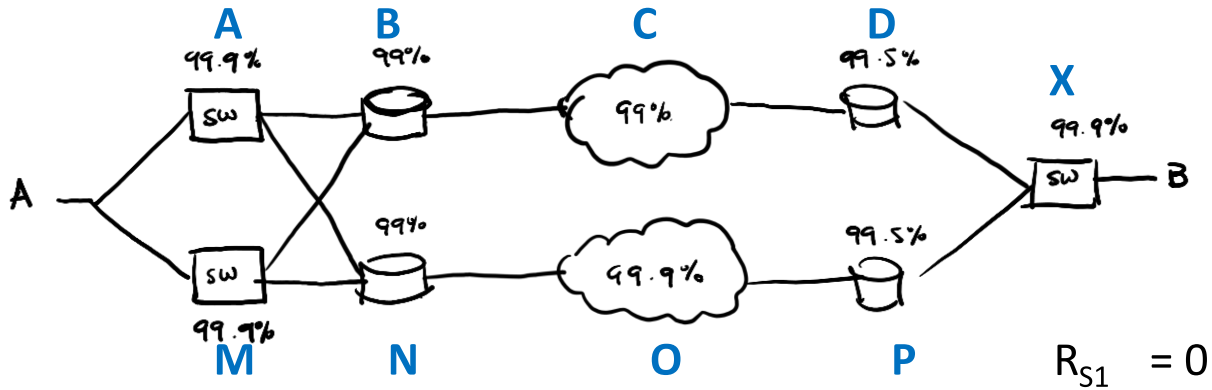


$$R_{S1} = 0.999 \times 0.99960568 = 0.998606074$$

$$R_{S2} = 0.999 \times 0.99960568 = 0.998606074$$



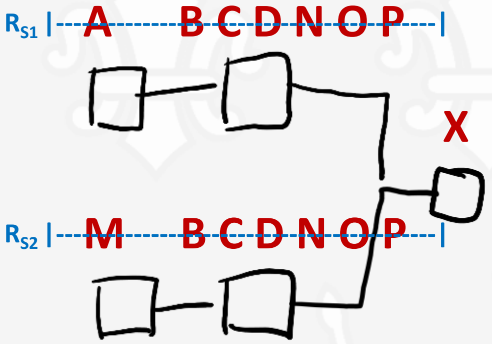
Sample 4 - Cross-link --> Series --> Parallels (4)

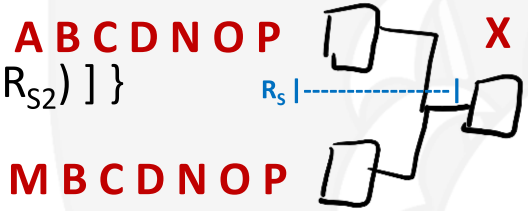


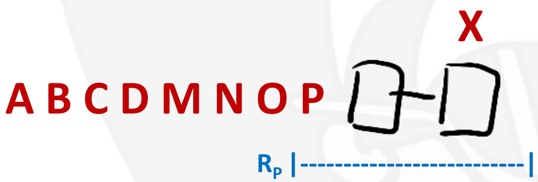
R_{S1} = 0.999 × 0.99960568
= 0.998606074

R_{S2} = 0.999 × 0.99960568
= 0.998606074

R_P = { 1 - [(1 - R_{S1}) × (1 - R_{S2})] }
= 0.999998057

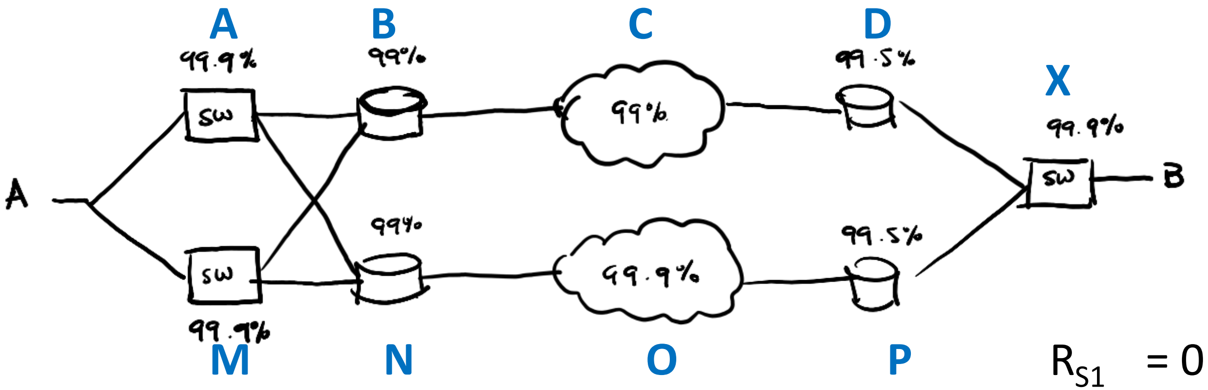






Associate Professor Dr. Piyabute Fuangkhon
Department of Digital Business Management
Martin de Tours School of Management and Economics

Sample 4 - Cross-link --> Series --> Parallels (5)

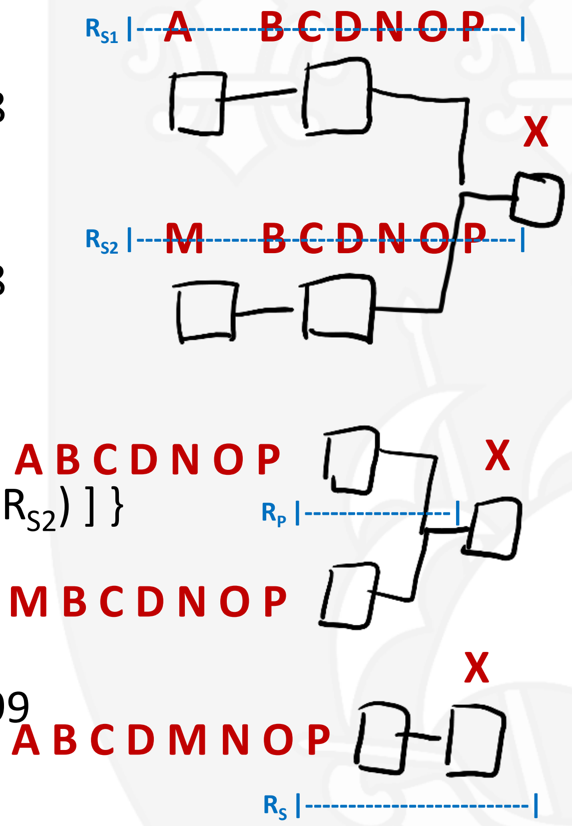


$$R_{S1} = 0.999 \times 0.99960568 = 0.998606074$$

$$R_{S2} = 0.999 \times 0.99960568 = 0.998606074$$

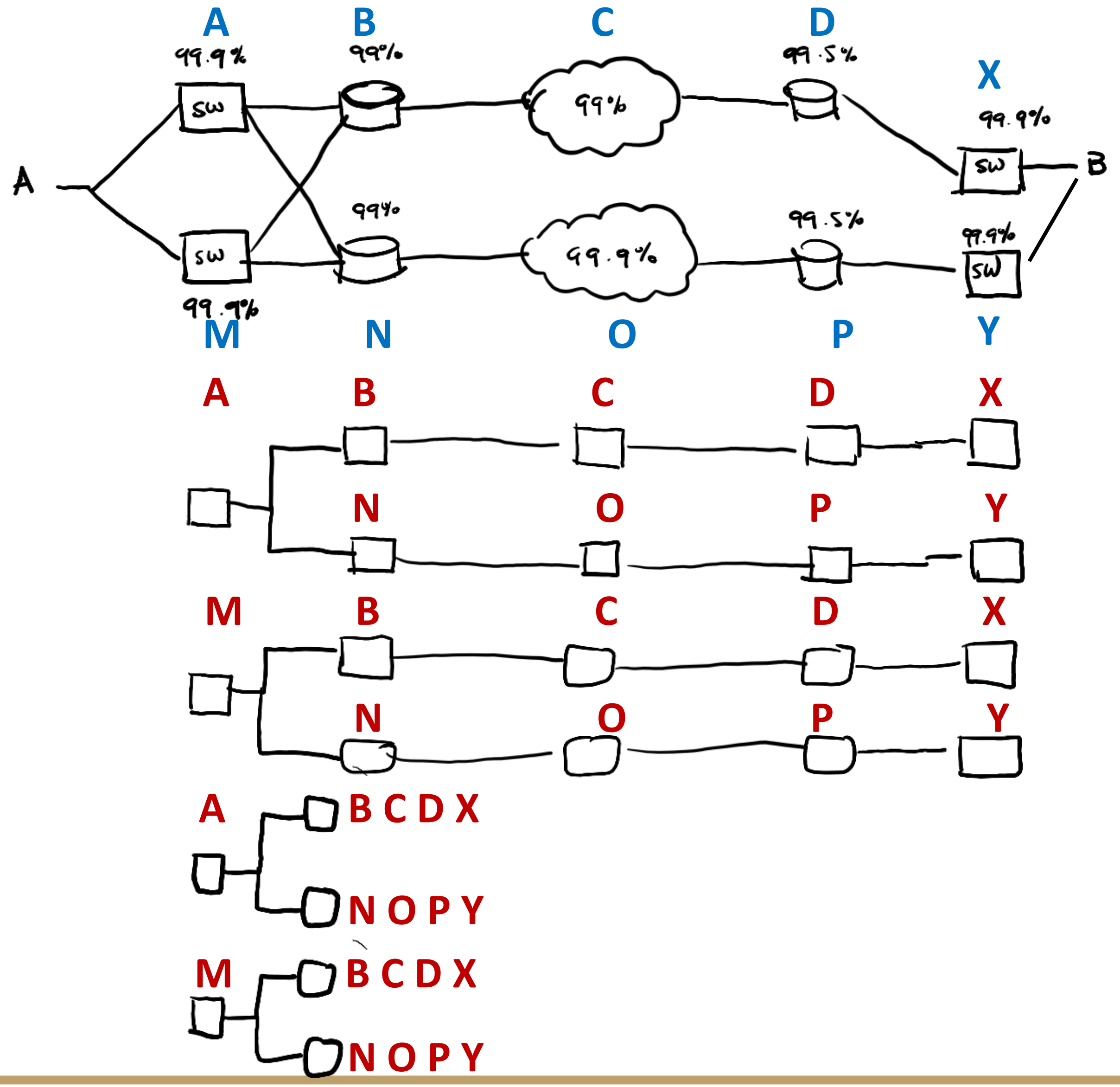
$$R_P = \{ 1 - [(1 - R_{S1}) \times (1 - R_{S2})] \} = 0.999998057$$

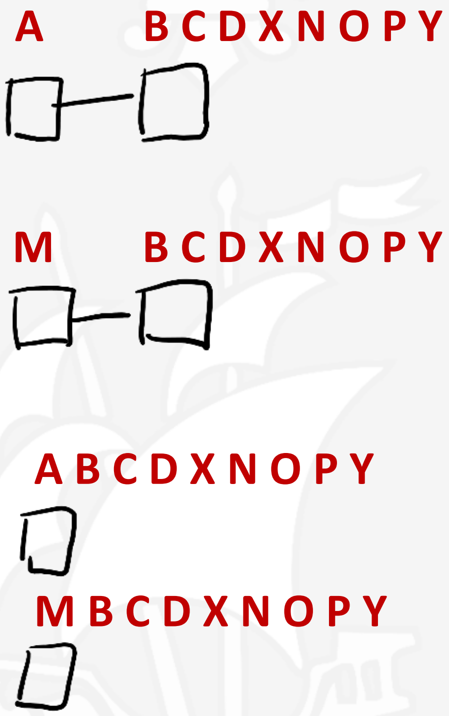
$$R_S = 0.999998057 \times 0.999 = 0.998998059$$



Associate Professor Dr. Piyabute Fuangkhon
Department of Digital Business Management
Martin de Tours School of Management and Economics

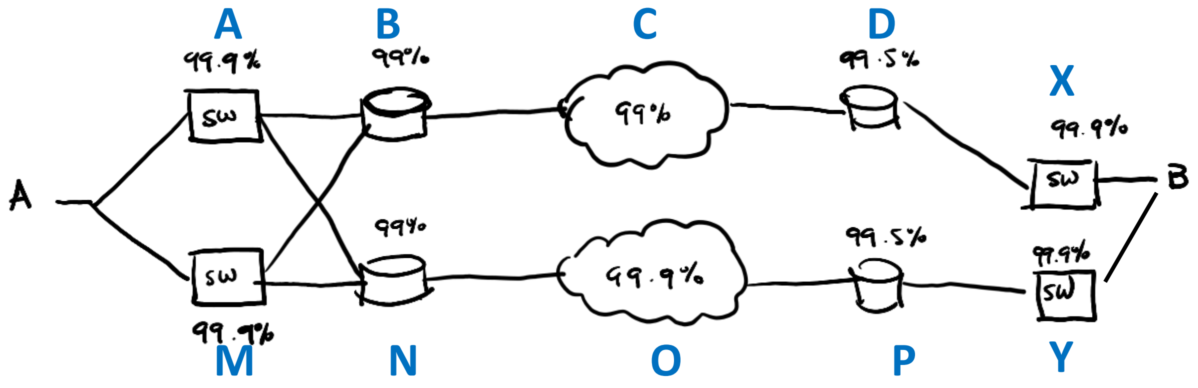
Sample 5 - Cross-link --> Series --> Parallels

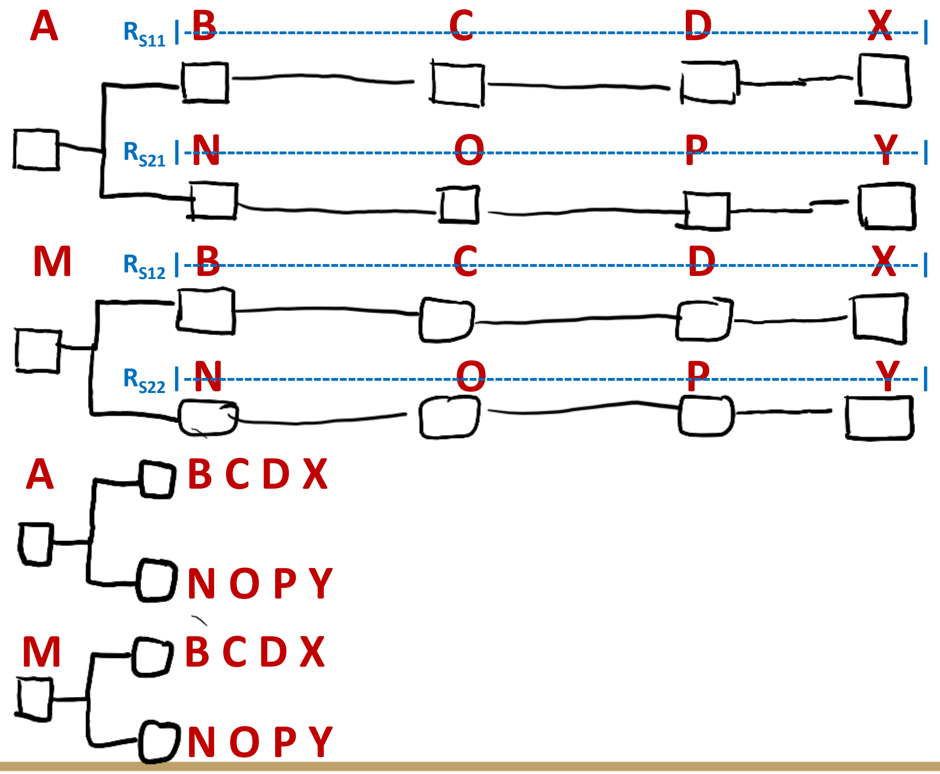




Associate Professor Dr. Piyabute Fuangkhon
Department of Digital Business Management
Martin de Tours School of Management and Economics

Sample 5 - Cross-link --> Series --> Parallels





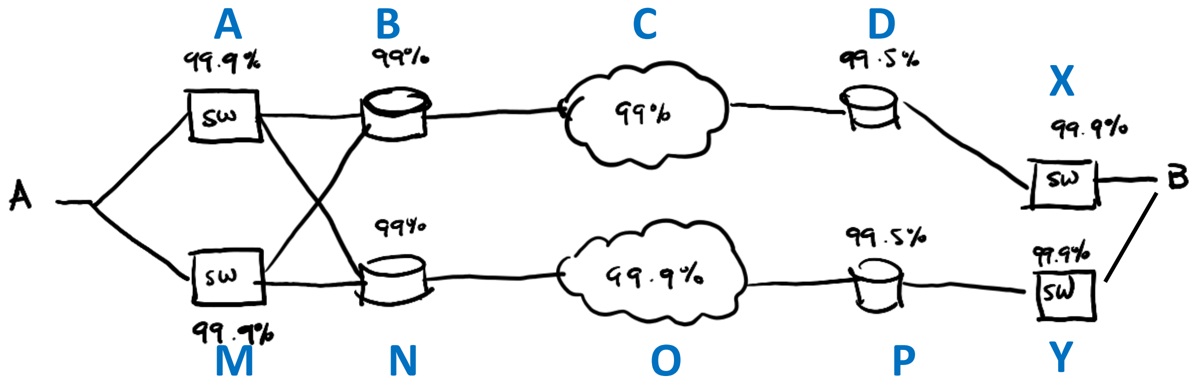
R_{S11} = 0.99 × 0.99 × 0.995 × 99.9 = 0.974224301

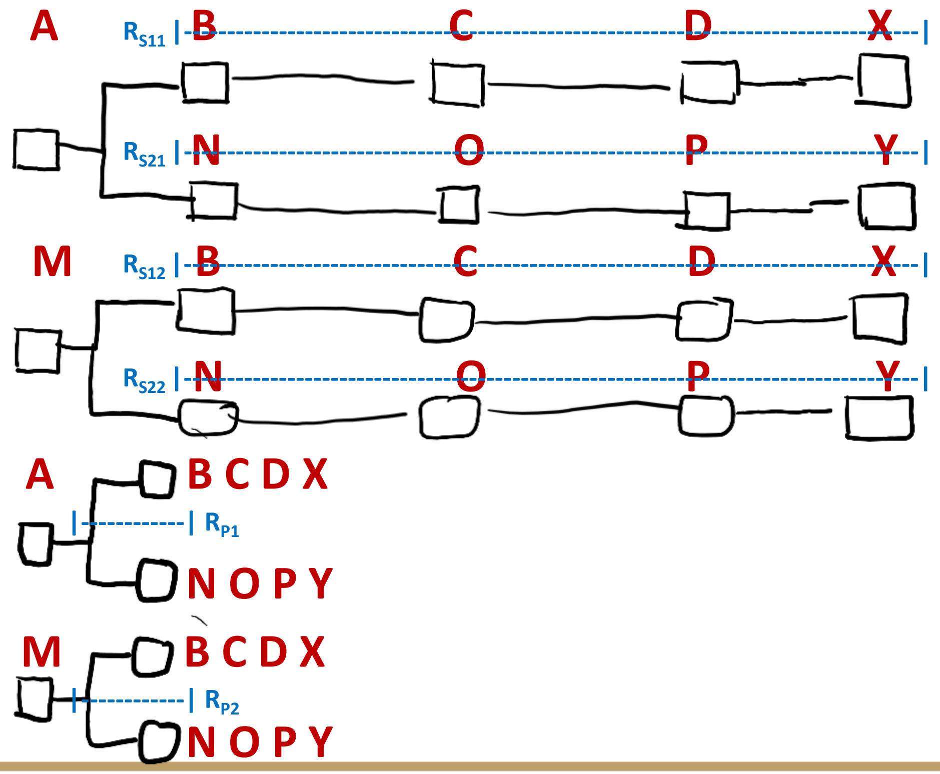
R_{S21} = 0.99 × 0.999 × 0.995 × 99.9 = 0.983080885

R_{S12} = 0.99 × 0.99 × 0.995 × 99.9 = 0.974224301

R_{S22} = 0.99 × 0.999 × 0.995 × 99.9 = 0.983080885

Sample 5 - Cross-link --> Series --> Parallels





R_{S11} = 0.99 × 0.99 × 0.995 × 99.9 = 0.974224301

R_{S21} = 0.99 × 0.999 × 0.995 × 99.9 = 0.983080885

R_{S12} = 0.99 × 0.99 × 0.995 × 99.9 = 0.974224301

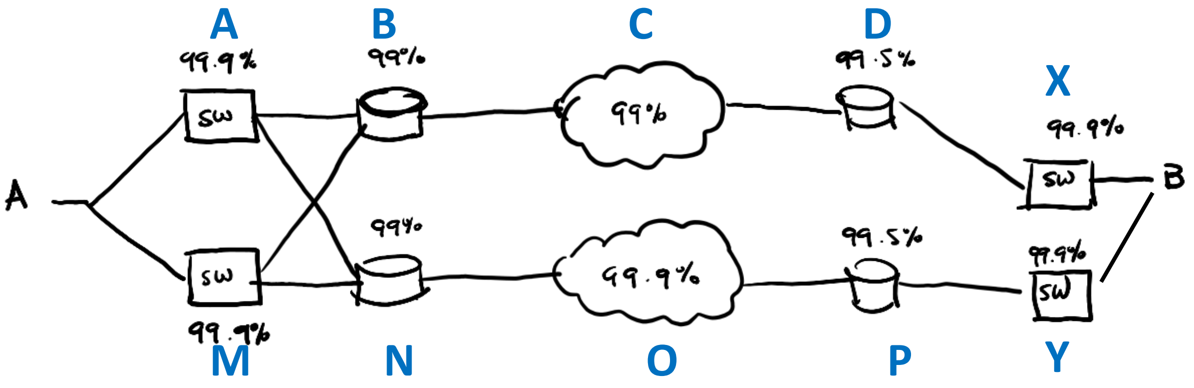
R_{S22} = 0.99 × 0.999 × 0.995 × 99.9 = 0.983080885

R_{P1} = { 1 - [(1 - R_{S11}) × (1 - R_{S21})] }
= { 1 - [(1 - 0.974224301)) × (1 - 0.983080885) }
= 0.999563898

R_{P2} = 0.999563898

Associate Professor Dr. Piyabute Fuangkhon
Department of Digital Business Management
Martin de Tours School of Management and Economics

Sample 5 - Cross-link --> Series --> Parallels



$$R_{S1} = 0.999 \times 0.999563898$$
$$= 0.998564334$$

$$R_{S2} = 0.998564334$$

R_{S1} |---A--- ~~B C D X N O P Y~~ |

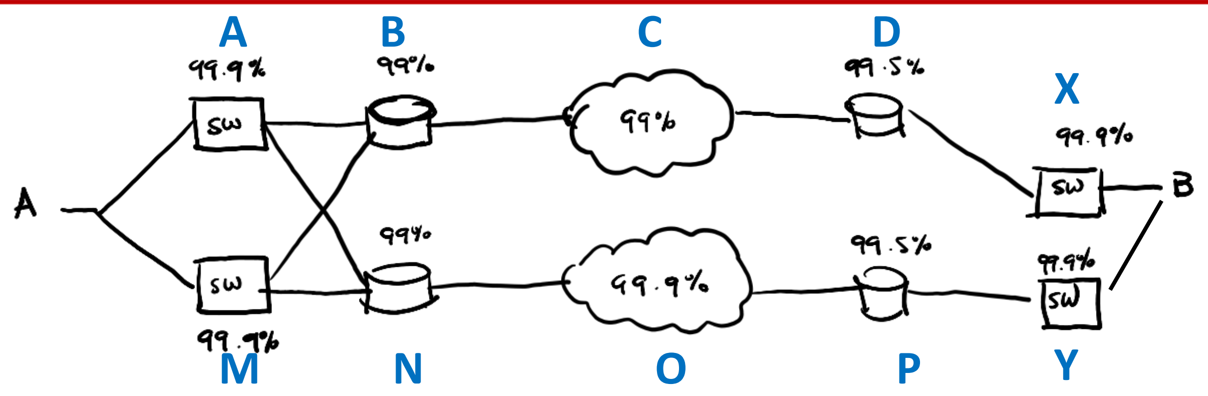
R_{S2} |---~~M~~--- ~~B C D X N O P Y~~ |

A B C D X N O P Y

M B C D X N O P Y

Associate Professor Dr. Piyabute Fuangkhon
Department of Digital Business Management
Martin de Tours School of Management and Economics

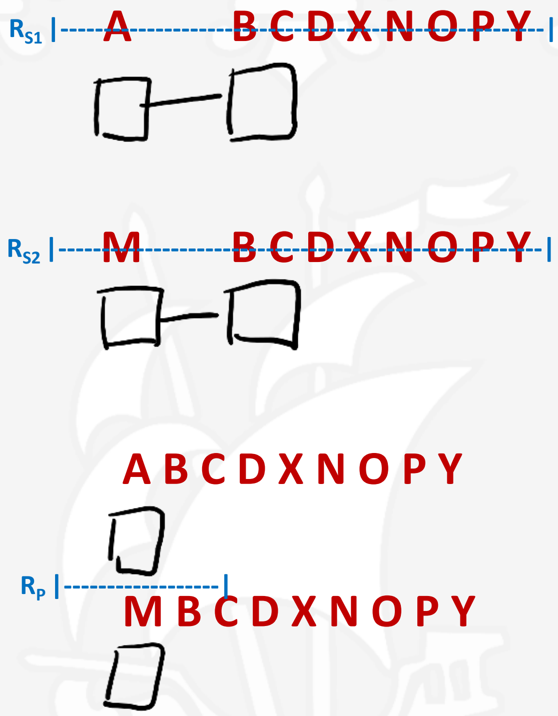
Sample 5 - Cross-link --> Series --> Parallels



R_{S1} $= 0.999 \times 0.999563898$
$= 0.998564334$

R_{S2} $= 0.998564334$

R_P $= \{ 1 - [(1 - R_{S1}) \times (1 - R_{S2})] \}$
$= \{ 1 - [(1 - 0.998564334) \times (1 - 0.998564334) \}$
$= 0.9999979$



Associate Professor Dr. Piyabute Fuangkhon
Department of Digital Business Management
Martin de Tours School of Management and Economics