Basic Mathematics and Statistics

CHAPTER 4: COORDINATE GEOMETRY

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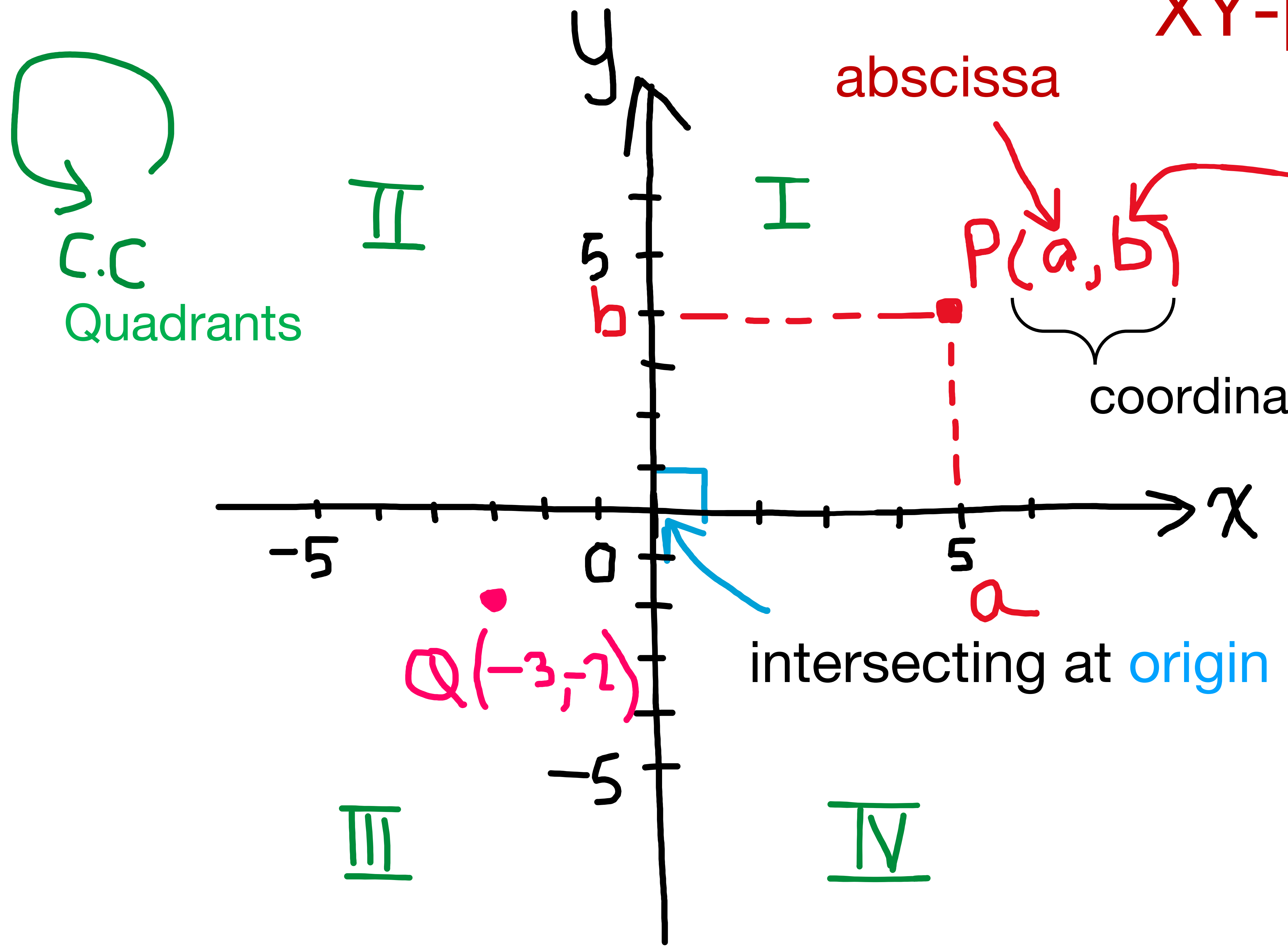
4.1 Cartesian Coordinate System

Cartesian (Rectangular) Coordinate System

Vertical axis (Y-axis)

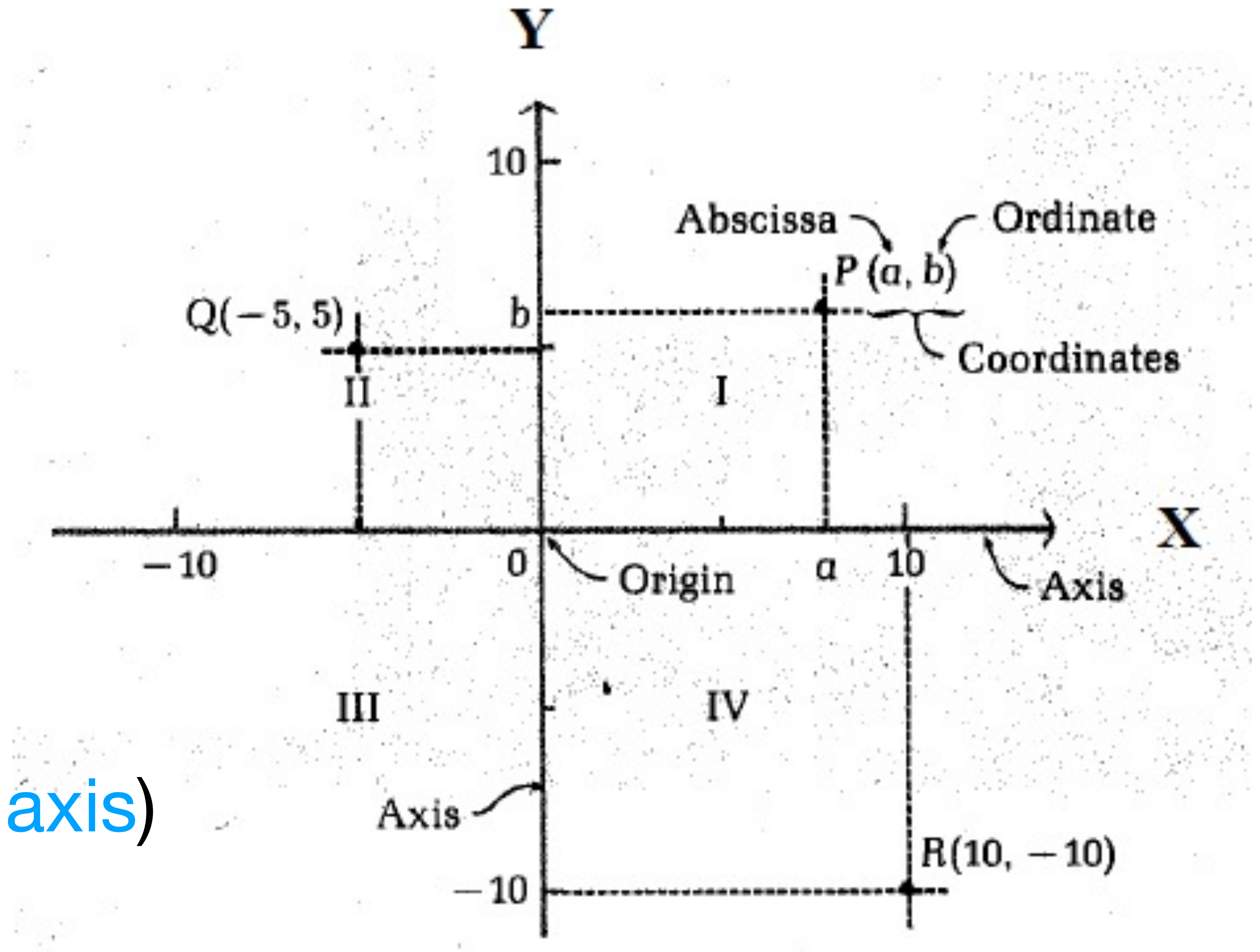
real number line (Coordinate axis)

XY-plane



real number line (Coordinate axis)

Horizontal axis (X-axis)



4.2 Graphing an Equation in Two Variables

4.2.1 Graph of a Linear Equation

$$y = \underbrace{a}_{\text{slope, } m}x + \underbrace{b}_{\text{y-intercept}}$$

To graph a linear equation

- plot any two points of solution set
- intercepts - points where the line cross the axis
  - y-intercept - when $x = 0$
  - x-intercept - when $y = 0$
- 3rd point as a check point (optional)

4.2 Graphing an Equation in Two Variables

4.2.1 Graph of a Linear Equation

$3x - 4y = 12$

- y-intercept - when $x = 0$

$3(0) - 4y = 12$

$y = \frac{12}{-4} = -3$

- x-intercept - when $y = 0$

$3x - 0 = 12$

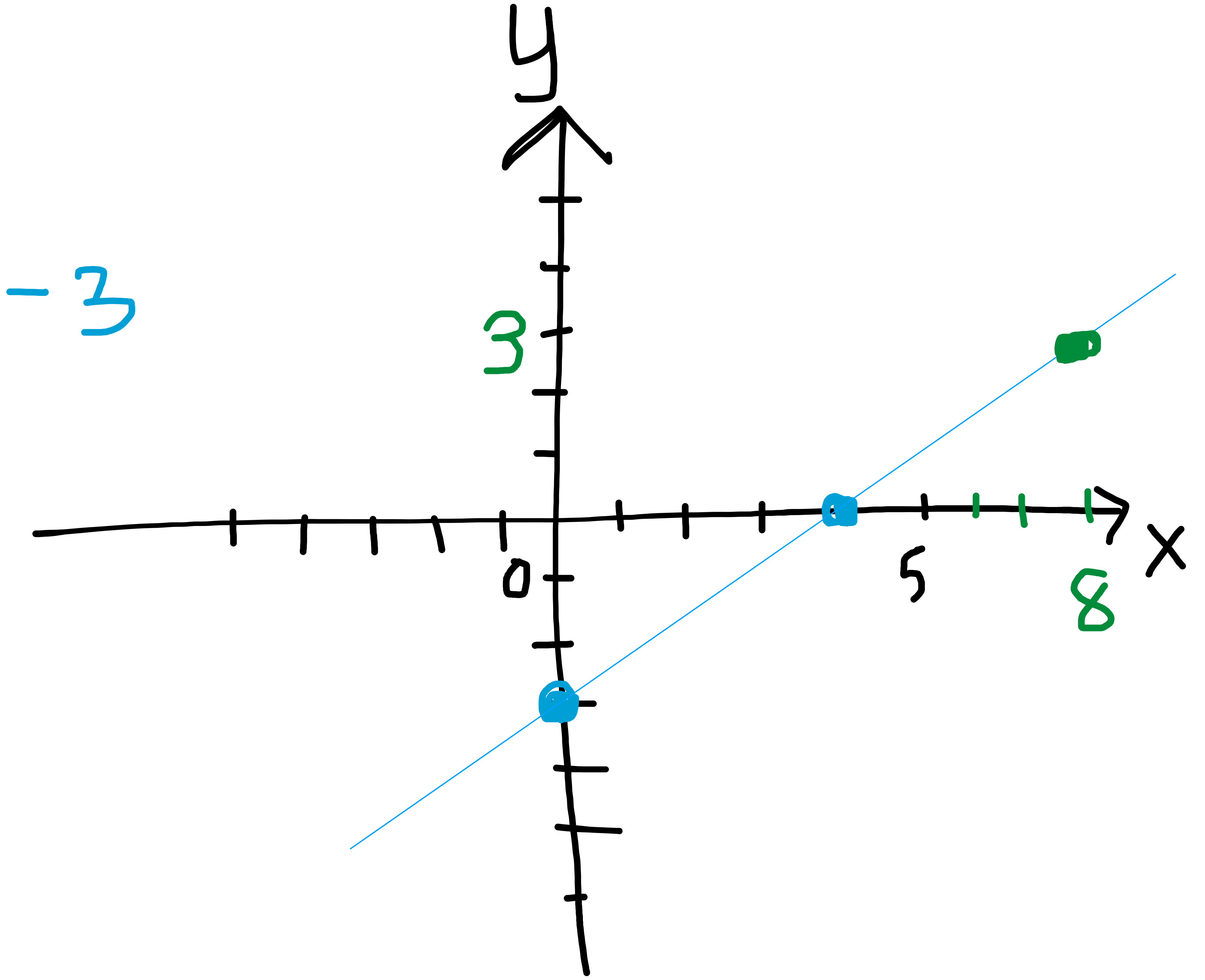
$x = \frac{12}{3} = 4$

- 3rd point as a check point

$x = 8 \Rightarrow 3(8) - 4y = 12$

$\cancel{-}4y = 12 - 24 = \cancel{-}12$

$y = 3$



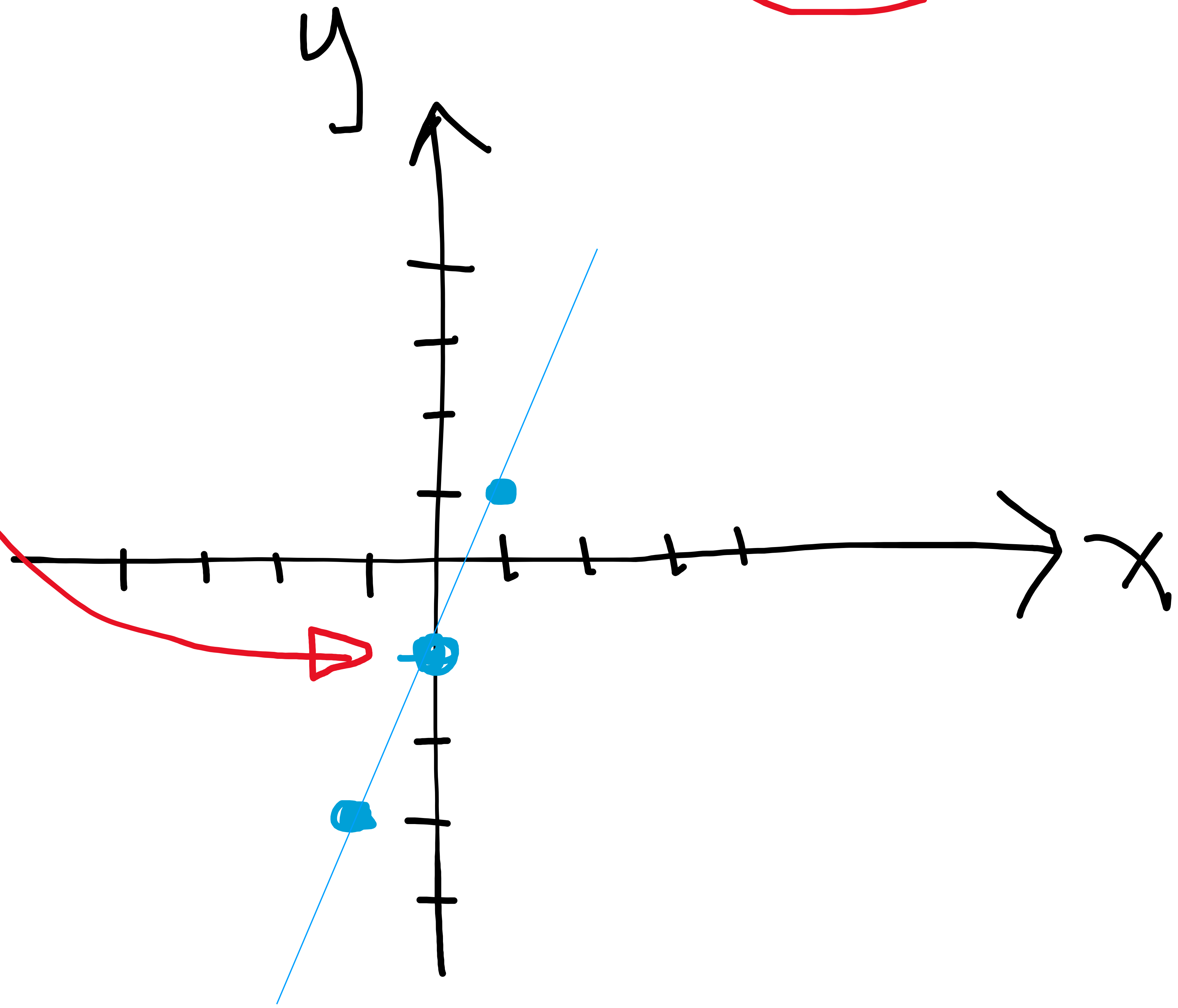
4.2 Graphing an Equation in Two Variables

4.2.1 Graph of a Linear Equation

$y = 2x - 1$

$y = ax + b$ → y-intercept (when x=0)

x	y
0	-1
1	1
-1	-3



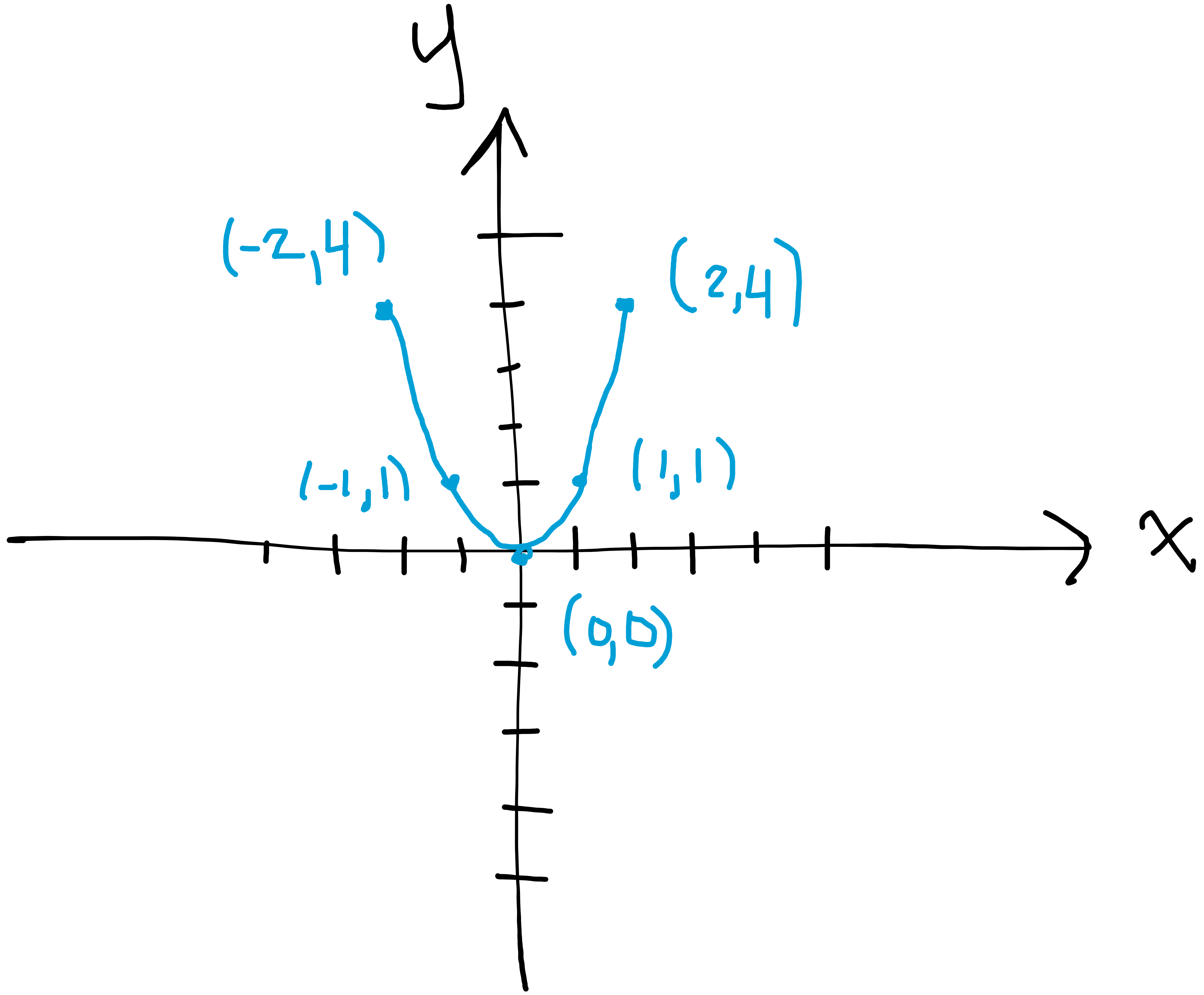
4.2 Graphing an Equation in Two Variables

4.2.2 Graph of a Quadratic Equation

$y = ax^2 + bx + c \qquad (a \neq 0)$

$y = x^2$

x	-2	-1	0	1	2
y	4	1	0	1	4



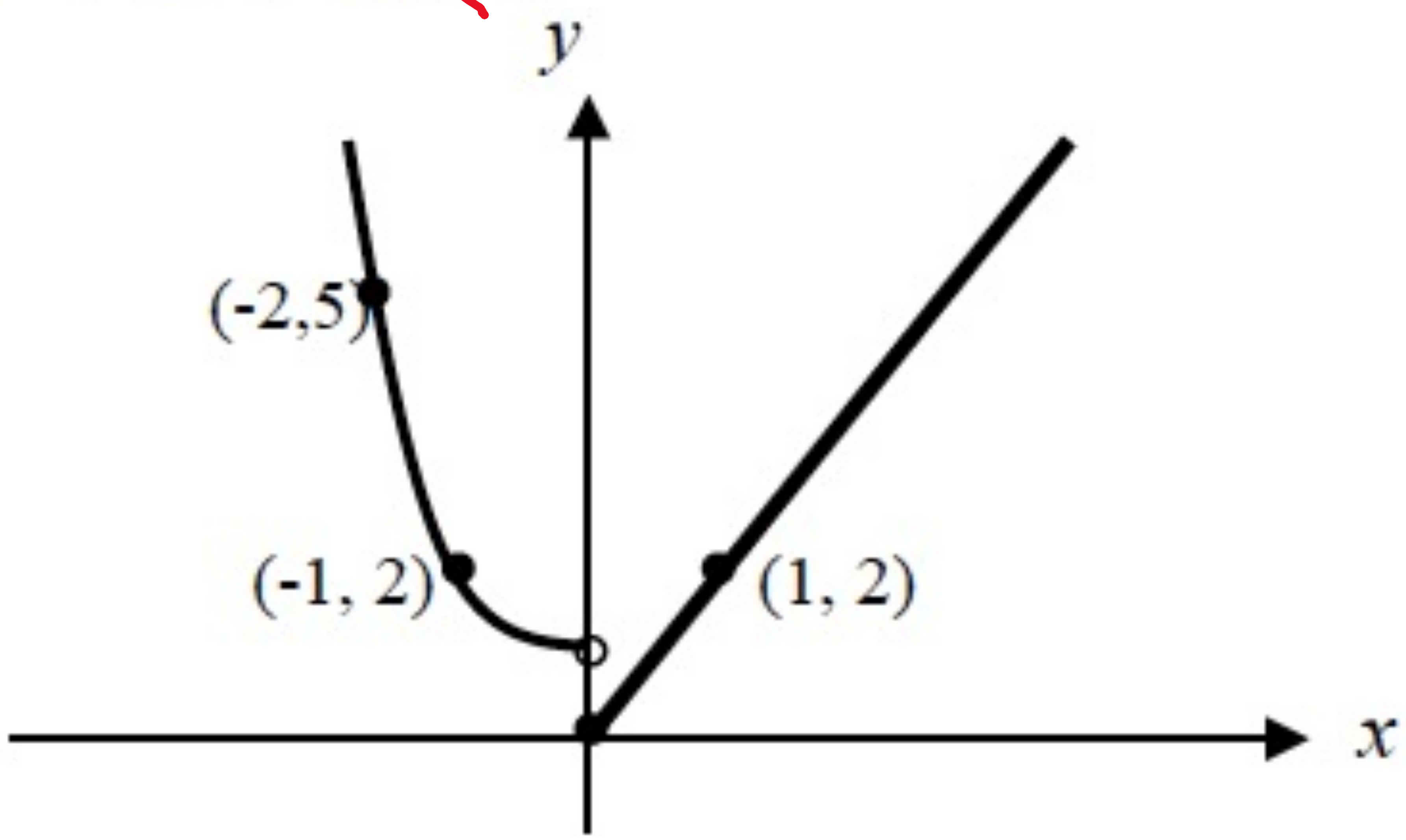
4.2 Graphing an Equation in Two Variables

4.2.3 Graph of a Piecewise-defined Function

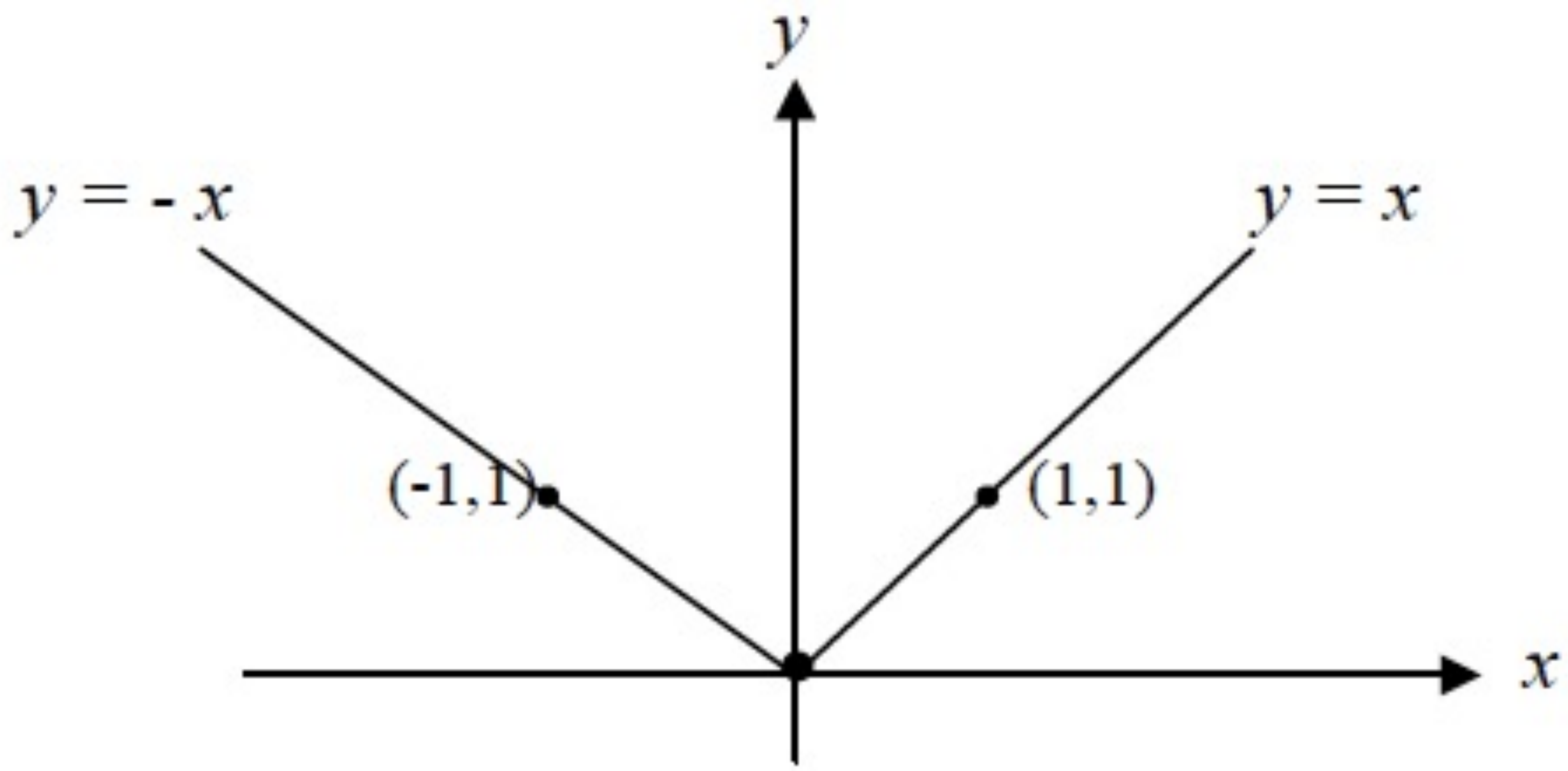
piecewise-defined function - function with two definitions

- involving both a linear and a quadratic rule of assignment

$$y = \begin{cases} 2x & ; \quad x \geq 0 \\ x^2 + 1 & ; \quad x \cancel{\geq} < 0 \end{cases}$$



$$y = |x| = \begin{cases} -x & ; x < 0 \\ x & ; x \geq 0 \end{cases}$$



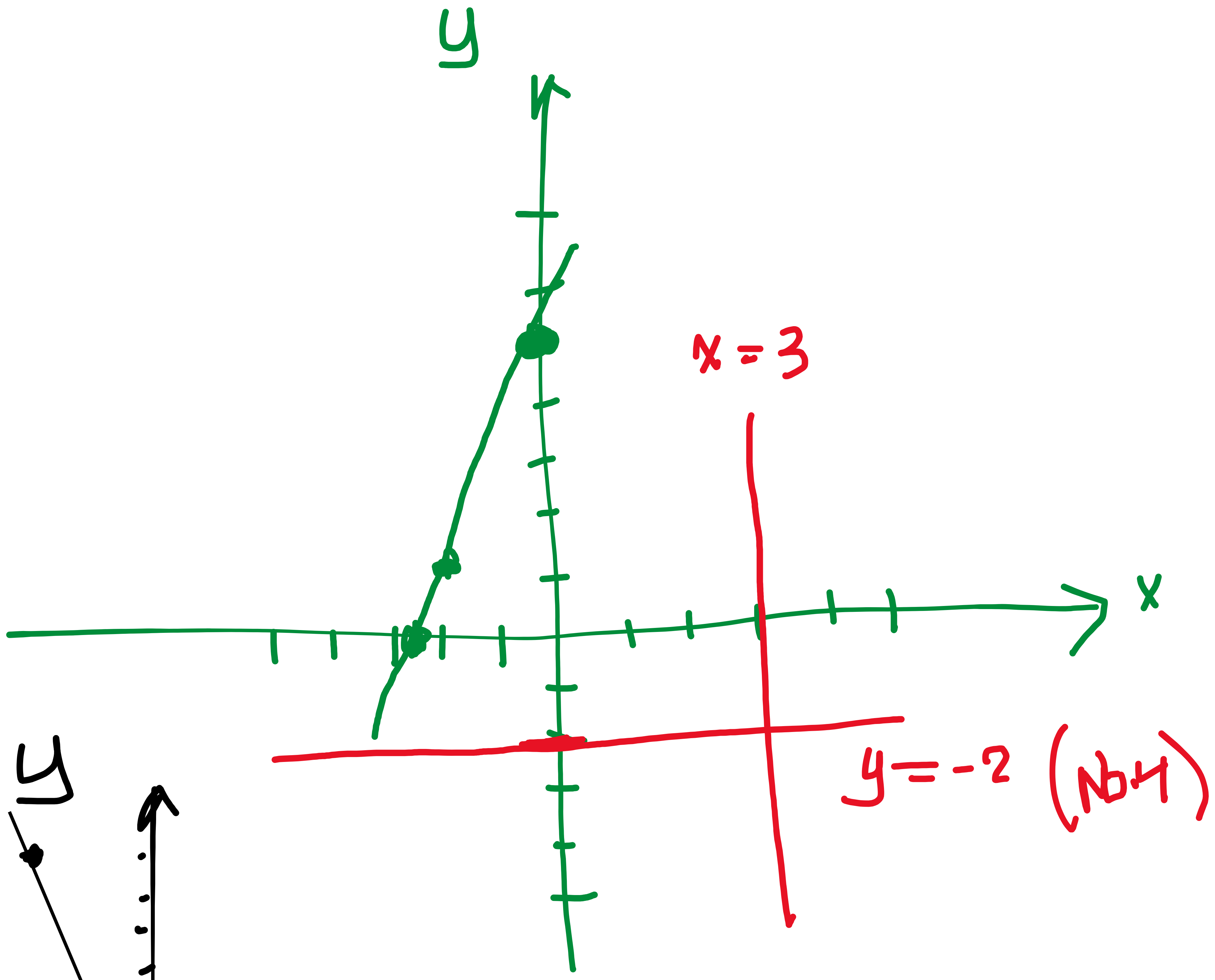
Exercise

Exercise 4.1

Exercise 4.1

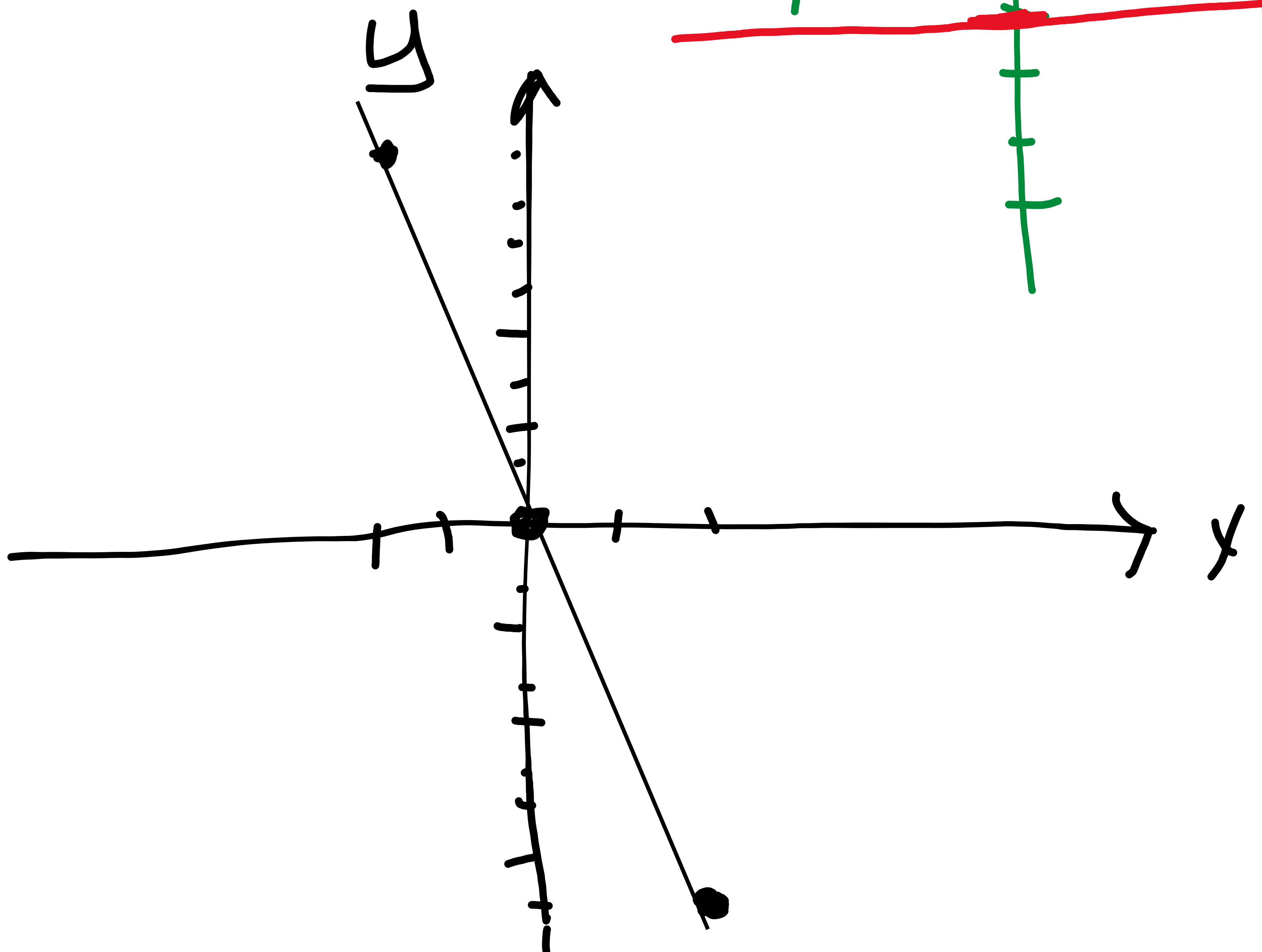
$y = 2x + 5$

x	0	$-\frac{5}{2}$	-2
y	5	0	1



$y = -4x + 0$

x	0	2	-2
y	0	-8	8

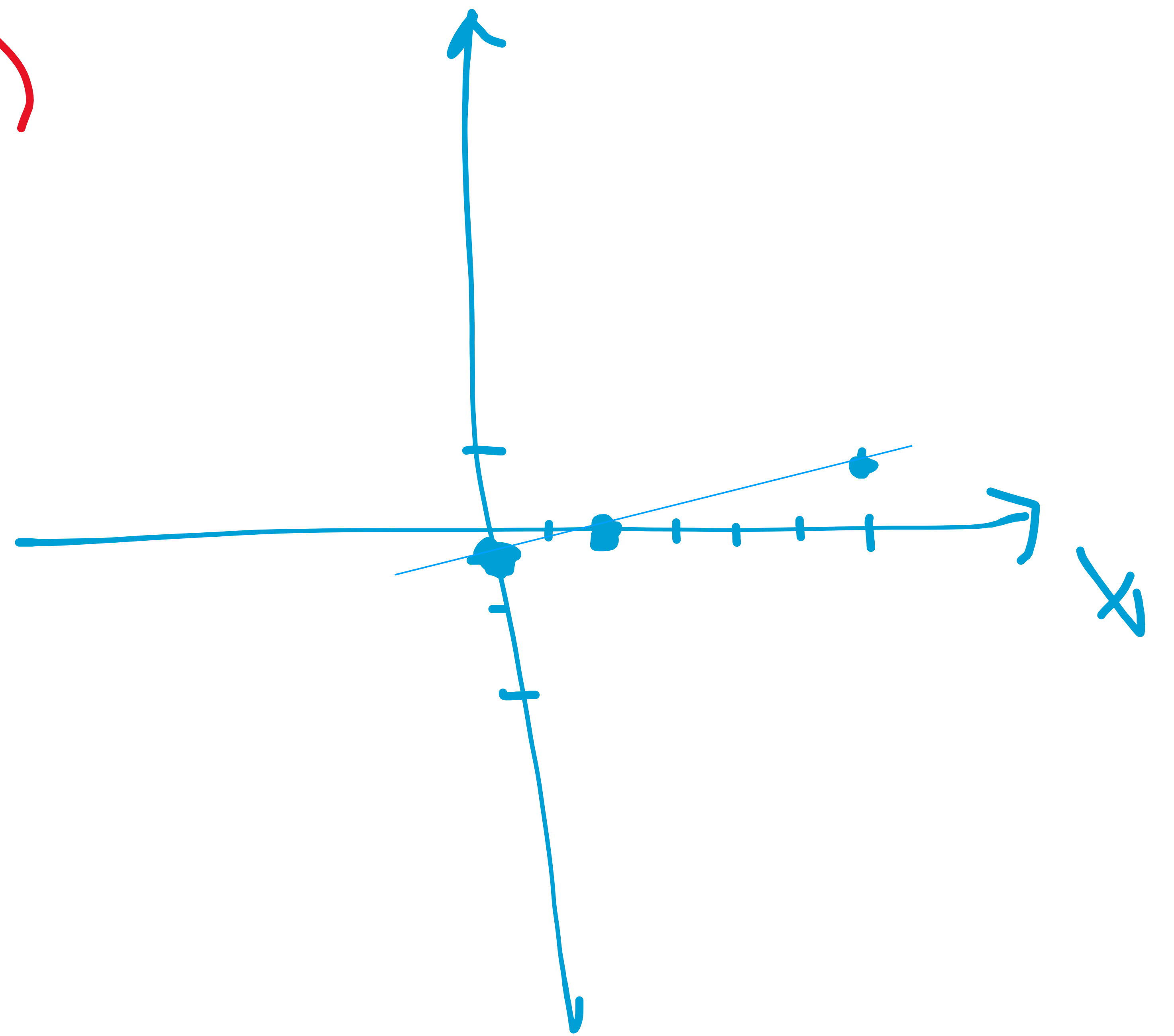


$y = \frac{1}{6}x - \frac{1}{3}$

x	0	6	2
y	$-\frac{1}{3}$	$\frac{2}{3}$	0

$y = 0 \Rightarrow \frac{1}{6}x = \frac{1}{3}$

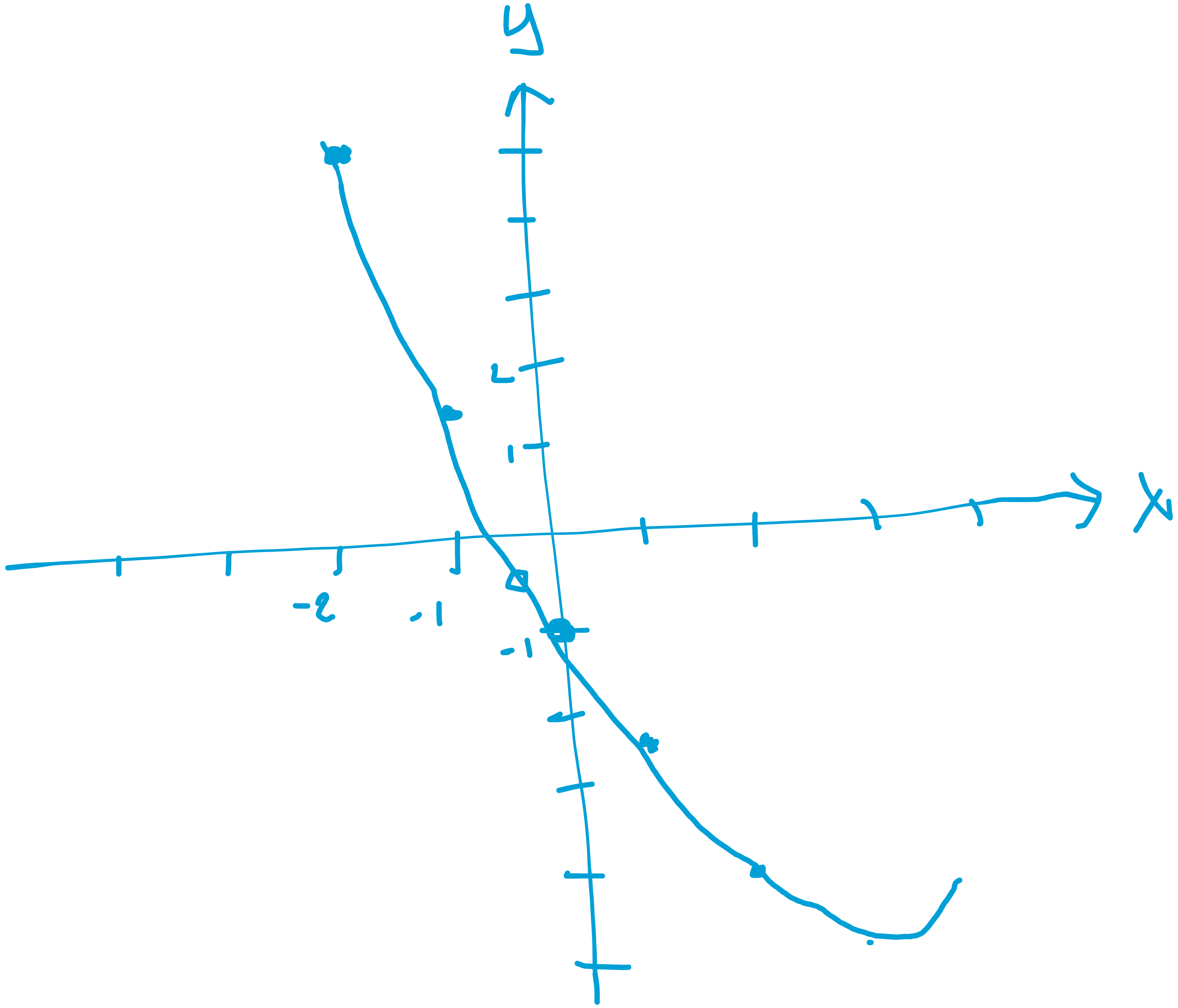
$x = \frac{6}{3} = 2$



12) $y = \frac{1}{2}(x-2)^2 - 3 = \frac{1}{2}(x^2 - 4x + 4) - 3 = \frac{x^2}{2} - 2x + 2 - 3$

$$\boxed{y = \frac{1}{2}x^2 - 2x - 1}$$

x	-2	-1	0	1	2	3
y	5	$1\frac{1}{2}$	-1	$-2\frac{1}{2}$	-3	



$y = \left(\frac{1}{2}\right)(-2)^2 - 2(-2) - 1$

$= \frac{4}{2} + 4 - 1$

$= 2 + 4 - 1 = 5$

$y = \frac{1}{2} + 2 - 1 = 1\frac{1}{2}$

$y = \frac{1}{2} - 2 - 1$

$= -2\frac{1}{2}$

$y = 2 - 4 - 1$

$= -3$

$y = \frac{9}{2} - 6 - 1 = -3\frac{1}{2}$

21)

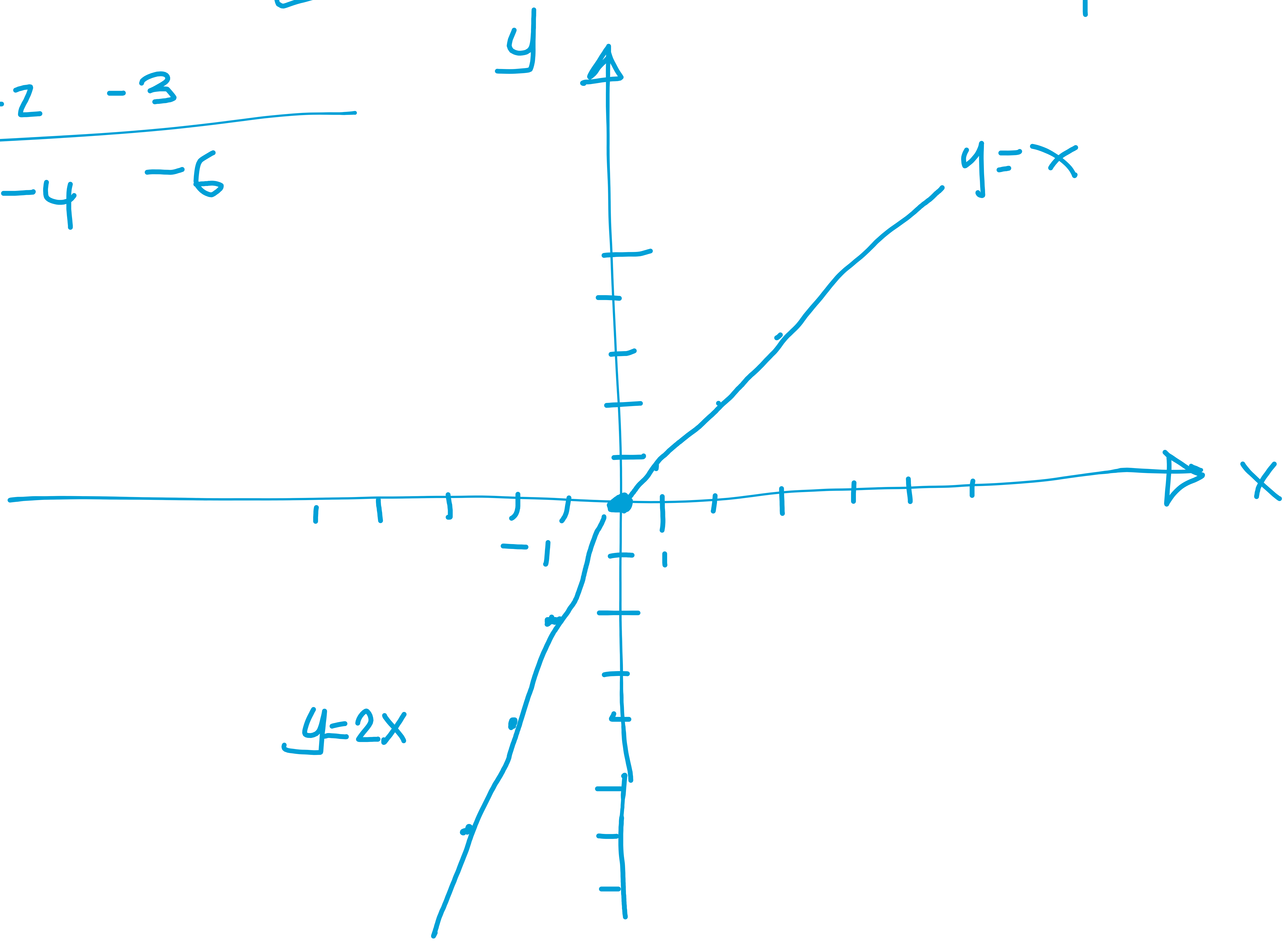
$$y=\begin{cases} x, & \boxed{x\geq 0} \\ 2x, & x<0 \end{cases}$$

$y = x$

$y = 2x$

x	0	1	2
y	0	1	2

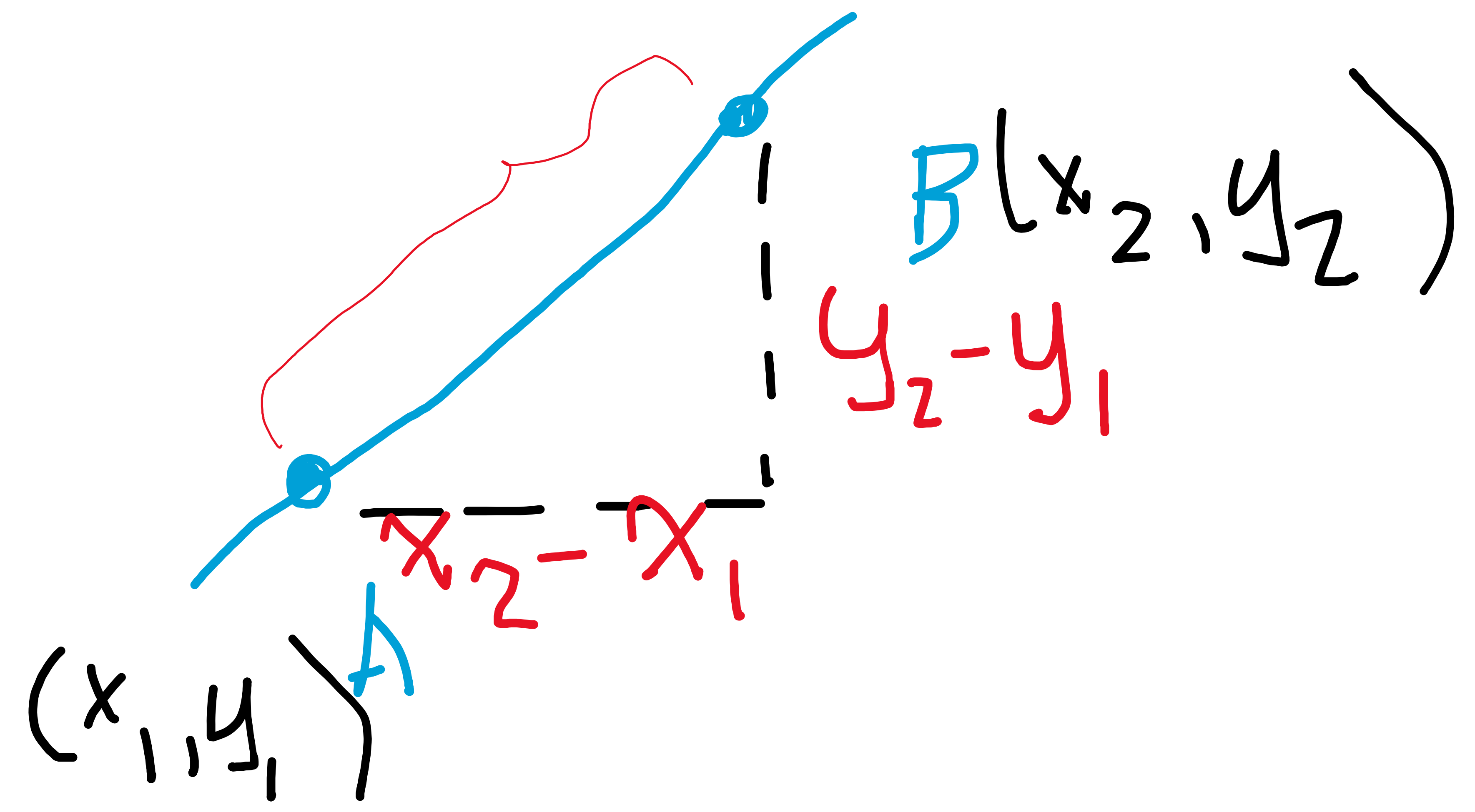
x	-1	-2	-3
y	-2	-4	-6



4.3 Distance between two points

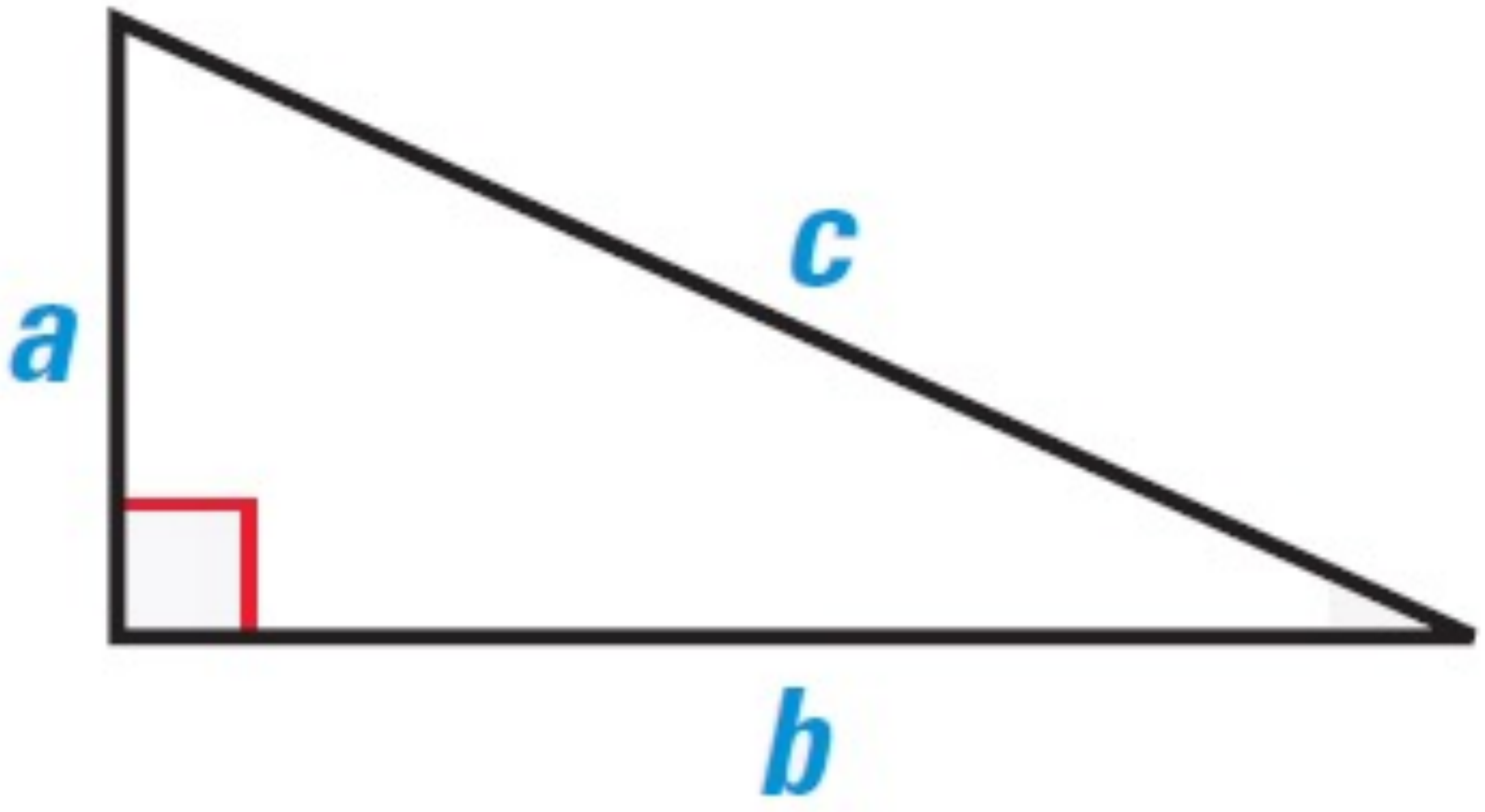
Given two points A (x_1, y_1) and B (x_2, y_2) in a rectangular coordinate system, then the distance between point A and B

$$d\,(A, B) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

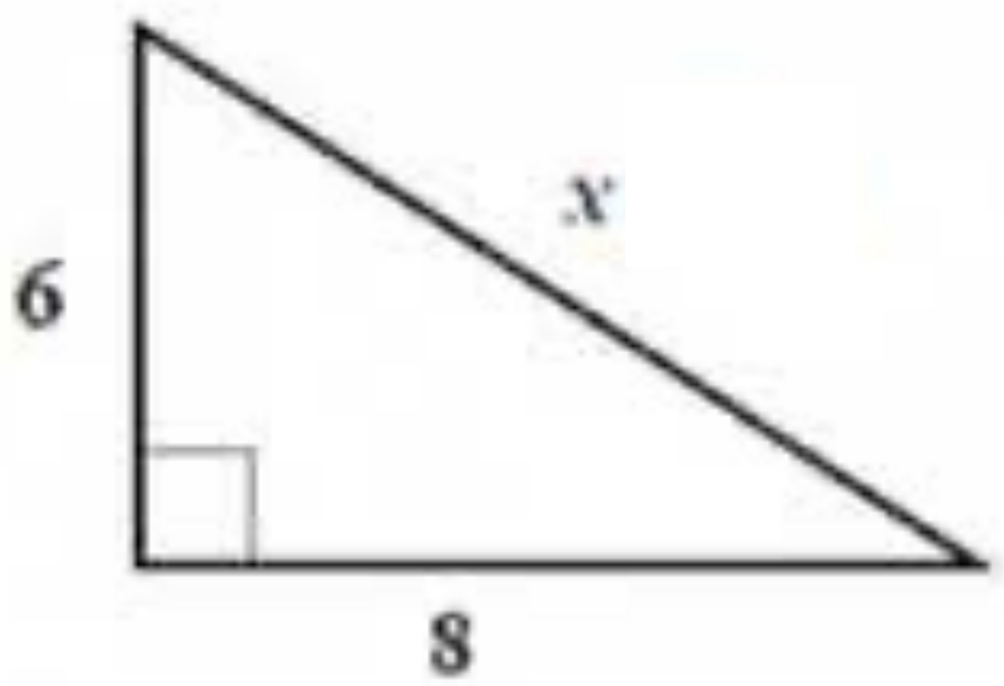


4.3 Distance between two points

Pythagorean theorem



$$c^2 = a^2 + b^2$$



$$a^2 + b^2 = c^2$$

$$6^2 + 8^2 = x^2$$

$$36 + 64 = x^2$$

$$\sqrt{100} = \sqrt{x^2}$$

$$x = \pm 10$$

$$x = 10$$

4.3 Distance between two points

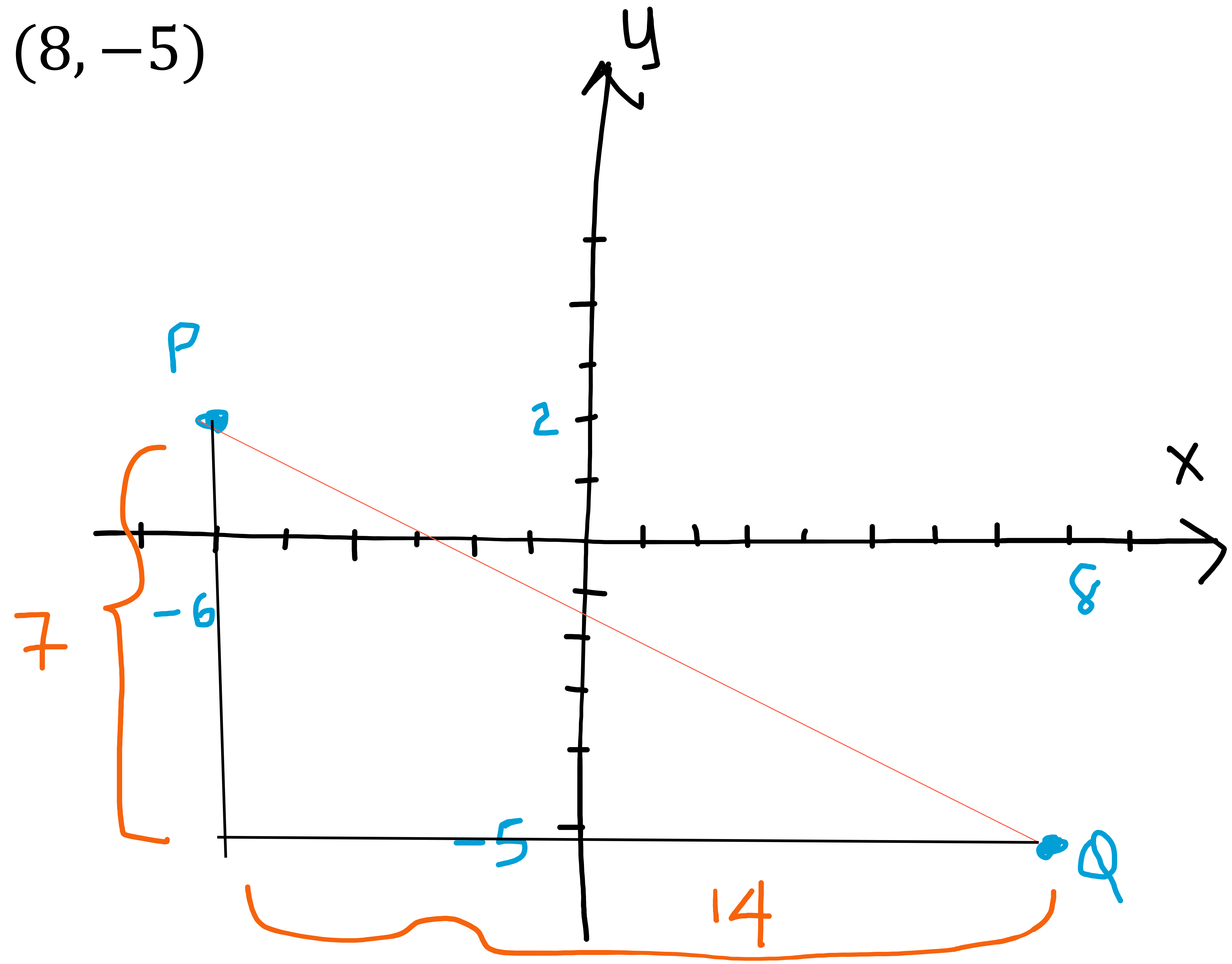
Find the distance between $\mathrm{P}\ (-6,2)$ and $\mathrm{Q}\ (8,-5)$

$$\mathrm{d}\ (P,Q) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(8-(-6))^2 + ((-5)-2)^2}$$

$$= \sqrt{196 + 49}$$

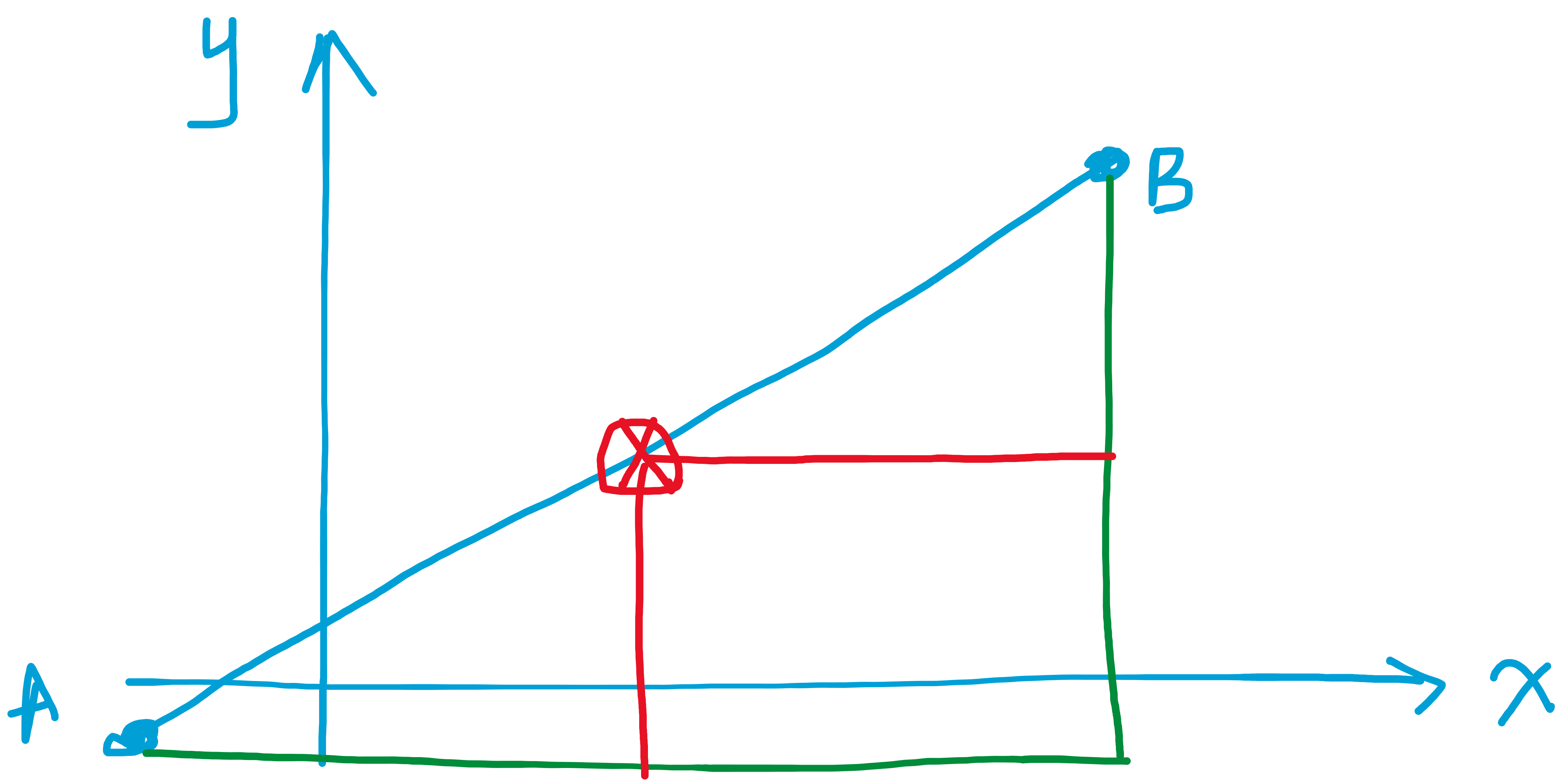
$$= 7\sqrt{5} \approx 15.7$$



4.4 The midpoint of the line segment

If $\mathrm{A}\ (x_1, y_1)$ and $\mathrm{B}\ (x_2, y_2)$ represent the endpoint of a line segment, the midpoint is represented by

$$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$



4.4 The midpoint of the line segment

Find the midpoint of the line segment jointing points A $(-3, 2)$ and B $(4, -5)$

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

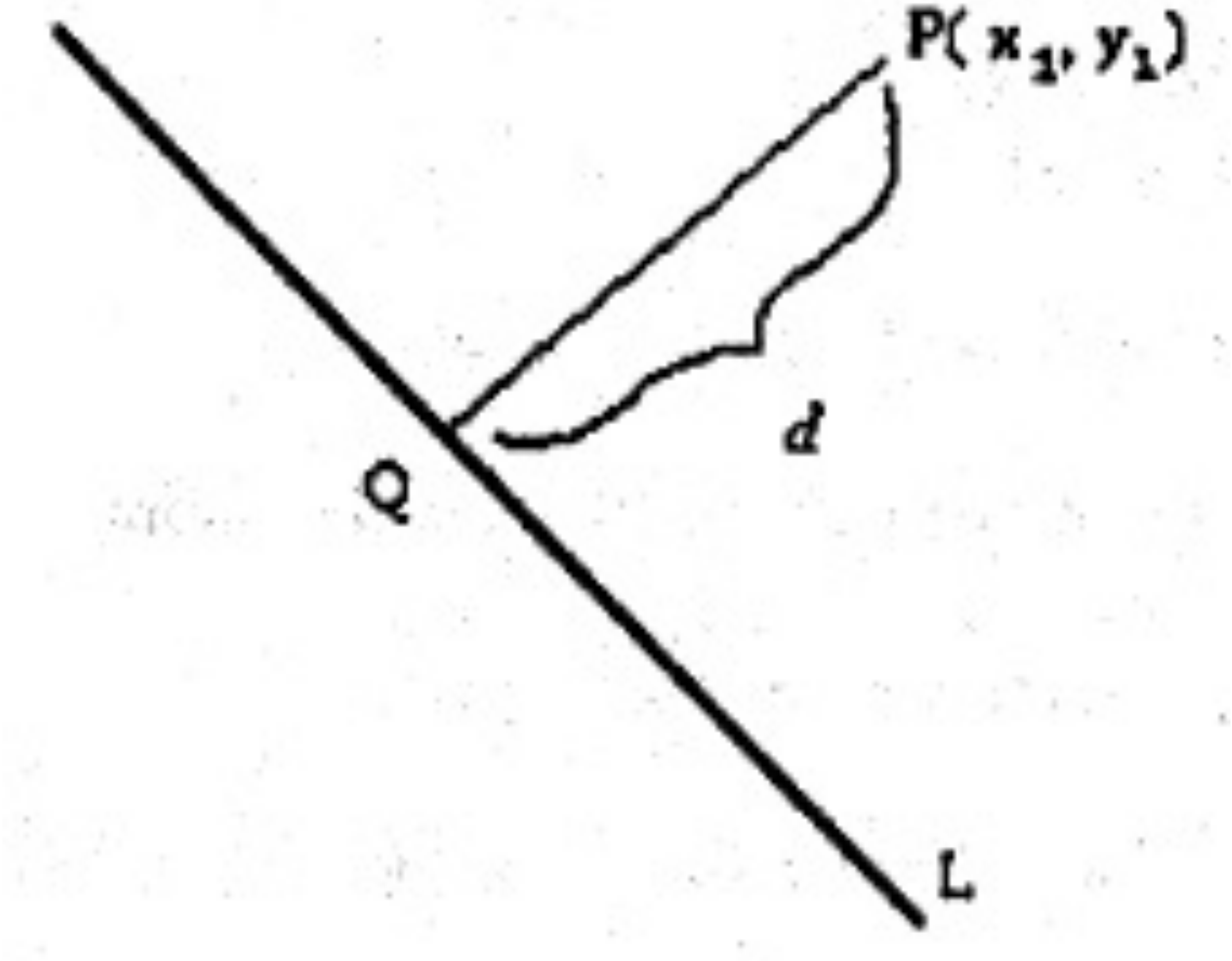
$$\left(\frac{-3 + 4}{2}, \frac{2 + (-5)}{2}\right)$$

$$\left(\frac{1}{2}, \frac{-3}{2}\right)$$

4.5 Distance between point and line

Given $\mathrm{P}\,(x_1, y_1)$ is any fixed point not in the line L: A$x + By + C = 0$ and ***d*** is the shortest distance that is perpendicular to the line L at the point Q. The distance ***d*** between point P and the line L is given by

$$d = \frac{|\,Ax_1 + By_1 + C\,|}{\sqrt{A^2 + B^2}}$$



4.5 Distance between point and line

Find the distance between line $6x - 8y + 4 = 0$ and the point $\mathrm{P}\ (2, -3)$

$$d = \frac{|\ Ax_1 + By_1 + C\ |}{\sqrt{A^2 + B^2}}$$

$$= \frac{|\ 6(2) + (-8)(-3) + 4\ |}{\sqrt{6^2 + (-8)^2}}$$

$$= \frac{|\ 40\ |}{\sqrt{100}}$$

$$= \frac{|\ 40\ |}{\sqrt{100}} = \frac{40}{10} = 4$$

4.6 Slope of a line

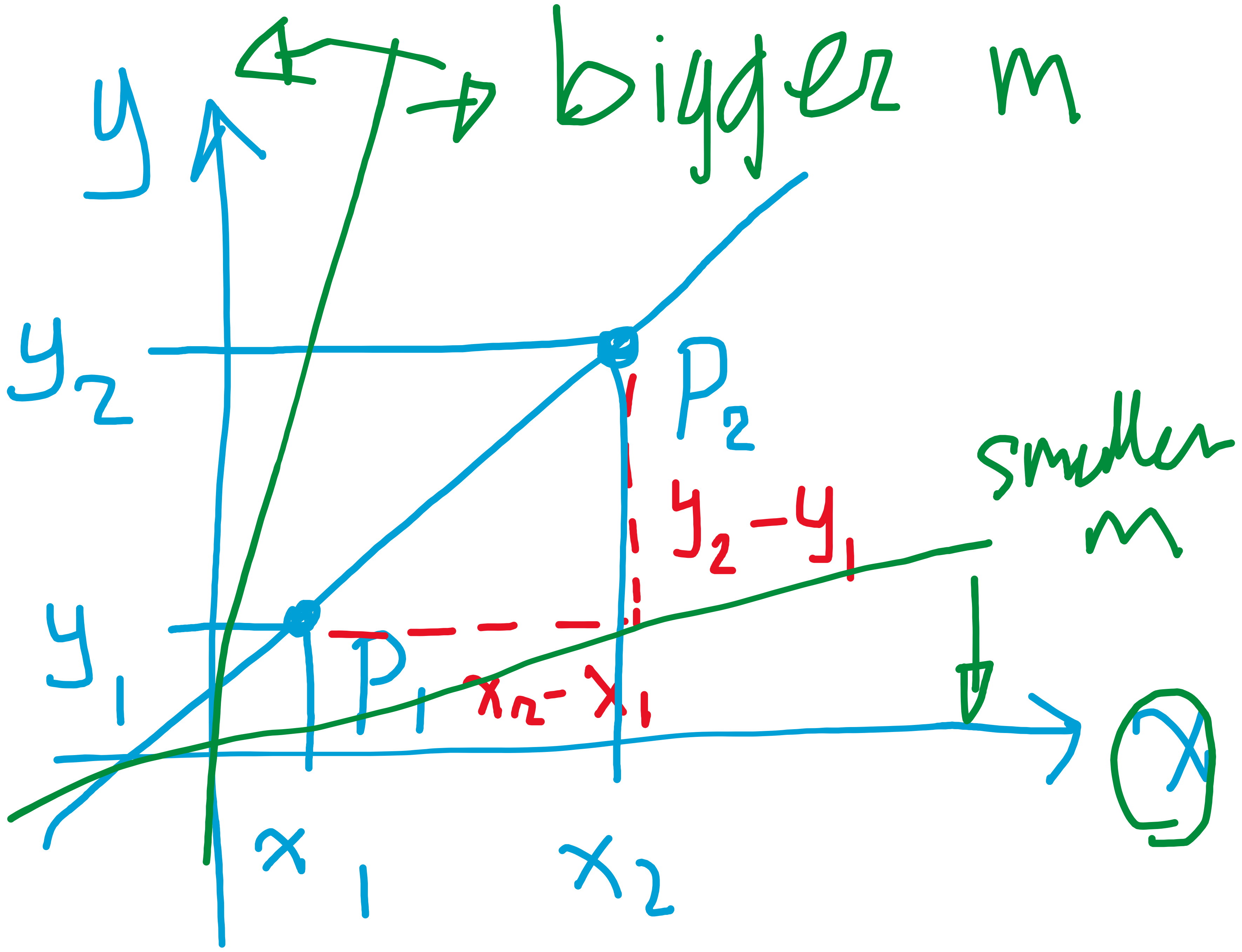
- Numerical measure of the "steepness"
- ratio of the change in *y* to the change in *x* as we move from point $P_1(x_1, y_1)$ to point $P_2\,(x_2, y_2)$

If a line passes through two distinct points

$P_1(x_1, y_1)$ and $P_2\,(x_2, y_2)$, then it's slope **m** of the line

Is given by the formula

$$m = \frac{y_2 - y_1}{x_2 - x_1};\ x_1 \neq x_2$$



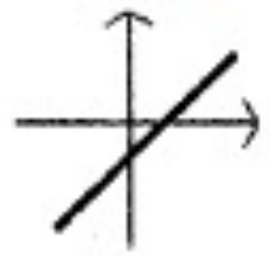
4.6 Slope of a line

For **a horizontal line,** *y* doesn’t change as *x* changes, so its slope is zero.

For **a vertical line**, *x* doesn’t change as *y* changes, so its slope is not defined.

In general, the slope of a line may be **positive**, **negative**, **0**, or **not defined**.

Table 4.1 Geometric Interpretation of Slope

Line	Slope	Example
Rising as x moves from left to right y values are increasing	Positive	
Falling as x moves from left to right y values are increasing	Negative	
Horizontal y values are constant	0	
Vertical x values are constant	Not defined	

4.6 Slope of a line

Find the slope of the line through each pair of points:

$(-2, 5)$ and $(4, -7)$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-7-5}{4-(-2)} = \frac{-12}{6} = -2$$

$(-3, -1)$ and $(-3,5)$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5-(-1)}{-3-(-3)} = \frac{6}{0} = undefined!$$

4.6 Slope of a line

Sometimes we can find the slope ***m*** and the ***y*-intercept** from the equation

$y = mx + b$

slope

y-intercept (when $x=0$)

- $y = \frac{3}{4}x - 1 \quad \rightarrow \boldsymbol{m} = \frac{3}{4},\ \boldsymbol{y - intercept} = -1$
- $6x - 2y = -10$

$$-2y = -10 - 6x$$

$$y = 5 + 3x \qquad \rightarrow \boldsymbol{m} = 3,\ \boldsymbol{y - intercept} = 5$$

4.7 Determining Special Forms of the Equation of a Line

two points → slope, m

equation → slope, m and y-intercept

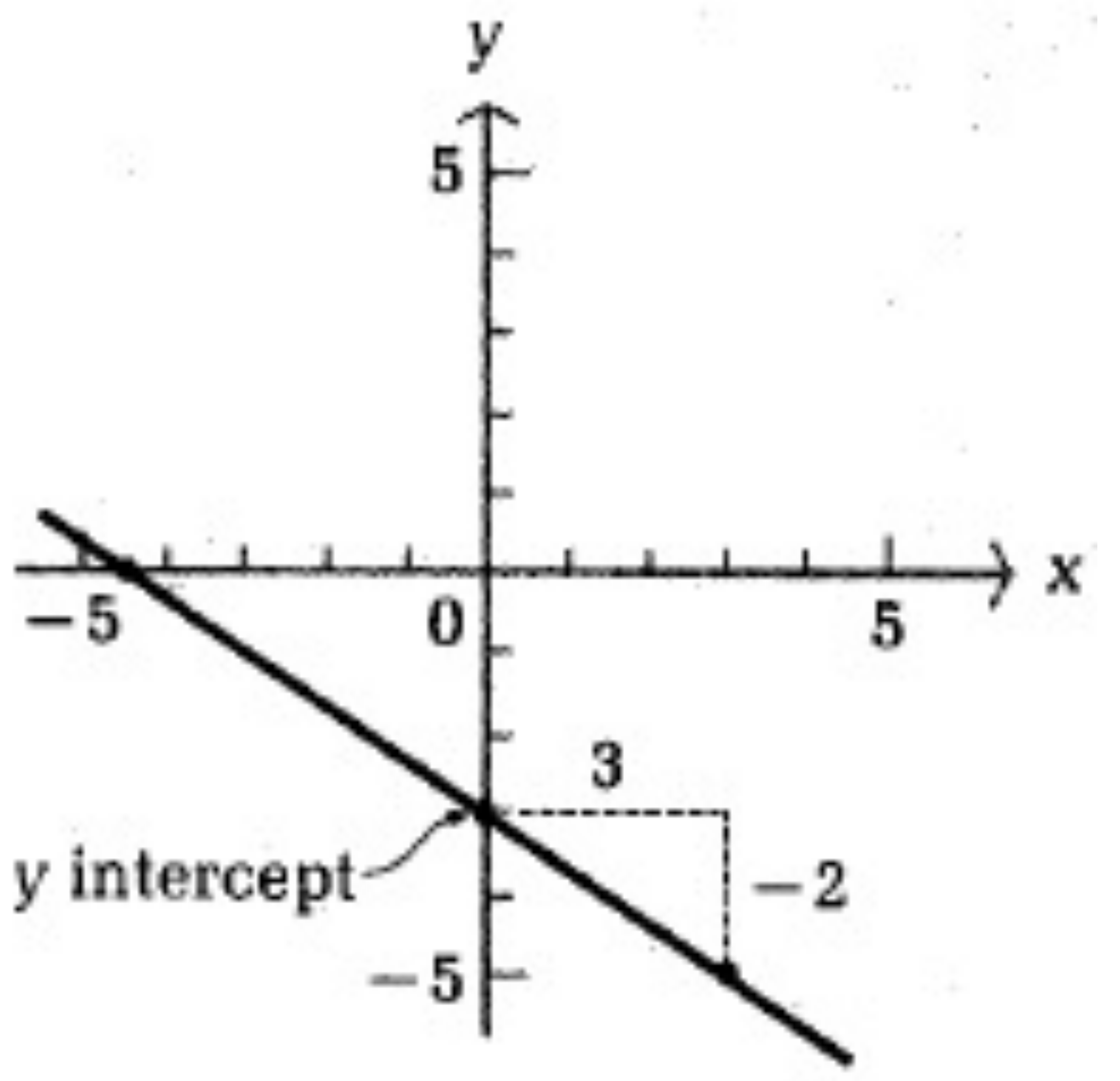
The equation of the line can be determined

- Slope and y-intercept (**slope-intercept form**)
- Slope that passes through a point (x_1, y_1) (**point-slope form**)

4.7 Determining Special Forms of the Equation of a Line

4.7.1 Slope-intercept form:

- Find the slope and y-intercept, and graph $y = -\frac{2}{3}x - 3$



$\rightarrow \boldsymbol{m} = -\frac{2}{3},\ \boldsymbol{y - intercept} = -3$

- Write the equation of the line with slope $\frac{2}{3}$ and y-intercept -2

$m = \frac{2}{3},\ b = -2\ (\,y = mx + b\,)$

$\rightarrow \boldsymbol{y = \frac{2}{3}x - 2}$

4.7 Determining Special Forms of the Equation of a Line

4.7.2 Point-slope form

An equation of the line with slope *m* that passes through the point (x_1, y_1) is

$$\boldsymbol{y - y_1 = m(x - x_1)}$$

- Find an equation of the line with slope 4 and passing through the point (1, 2).

$y - y_1 = m(x - x_1)$

$y - 2 = 4(x - 1)$

$y - 2 = 4x - 4$

$y = 4x - 2$

4.7 Determining Special Forms of the Equation of a Line

4.7.2 Point-slope form

- Find an equation of the line L passing through the point (2,3) and (-4,5).

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5-3}{-4-2} = \frac{\mathbf{-1}}{\mathbf{3}}$$

slope, m ⇒ equation

$$y - y_1 = m(x - x_1)$$

$$y - 3 = -\frac{1}{3}(x - 2)$$

$$3(y - 3) = -1(x - 2)$$

$$3y - 9 = x + 2$$

$$\boldsymbol{x + 3y = 11}$$

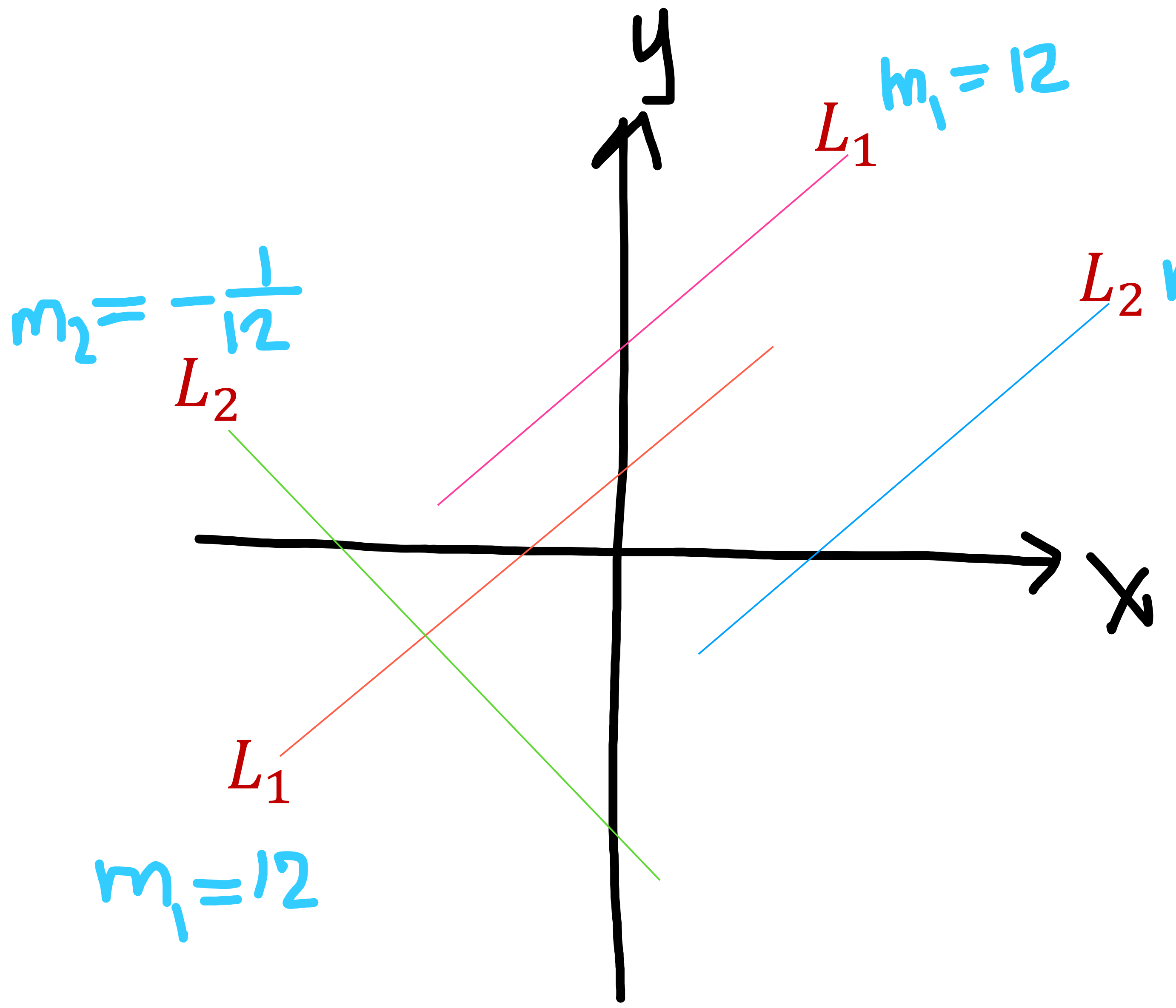
4.8 Parallel and Perpendicular Lines

Nonvertical lines L_1 and L_2 with slope m_1 and m_2

If these two lines are *parallel*, $L_1 \parallel L_2$ if and only if $m_1 = m_2$

If these two lines are *perpendicular*, $L_1 \perp L_2$ if and only if $m_1 m_2 = -1$ $\left(m_1 = -\frac{1}{m_2} \text{ 'or' } m_2 = -\frac{1}{m_1}\right)$

$$\boxed{\begin{array}{c} L_1 \parallel L_2 \ \rightarrow m_1 = m_2 \\ L_1 \perp L_2 \ \rightarrow \ m_1 m_2 = -1 \end{array}}$$



4.8 Parallel and Perpendicular Lines

Show that the lines given by the equations below are parallel $2x + 3y = 6 \quad and \quad 4x + 6y = 0$

$3y = -2x + 6$

$y = -\frac{2}{3}x + 2$

$6y = -4x$

$y = -\frac{2x}{3}$

$m_1 = -\frac{2}{3}$

$m_2 = -\frac{2}{3}$

$m_1 = m_2 \quad \therefore L_1 // L_2$

Show that the lines given by the equations below are perpendicular $x - 2y = 6 \quad and \quad 2x + y = 1$

$-2y = -x + 6$

$y = \frac{1}{2}x - 3$

$y = -2x + 1$

$m_1 = \frac{1}{2}$

$m_2 = -2$

$m_1 m_2 = \frac{1}{2}(-2) = -1$

$\therefore L_1 \perp L_2$

4.8 Parallel and Perpendicular Lines

$x_1 \quad y_1$

Given the line $x - 2y = 4$, find the equation of a line that passes through $(2, -3)$

(a) Parallel to the given line (b) Perpendicular to the given line

Write final equations in the form $y = mx + c$

$x - 2y = 4$

$-2y = -x + 4$

$y = \frac{1}{2}x - 2$

$m_1 = \frac{1}{2}$

(a) $m_2 = m_1 = \frac{1}{2}$ $(\parallel)$

$y - y_1 = m(x - x_1)$

$y + 3 = \frac{1}{2}(x - 2)$

$y = \frac{1}{2}x - 1 - 3$

$y = \frac{1}{2}x - 4$ $\leftarrow$

(b) $\perp \Rightarrow m_2 = -\frac{1}{m_1}$

$= -\frac{1}{1/2} = -2$

$y - y_1 = m(x - x_1)$

$y + 3 = -2(x - 2)$

$y = -2x + 4 - 3$

$y = -2x + 1$ $\leftarrow$

Exercise

Exercise 4.2

$\boxed{4.2}$

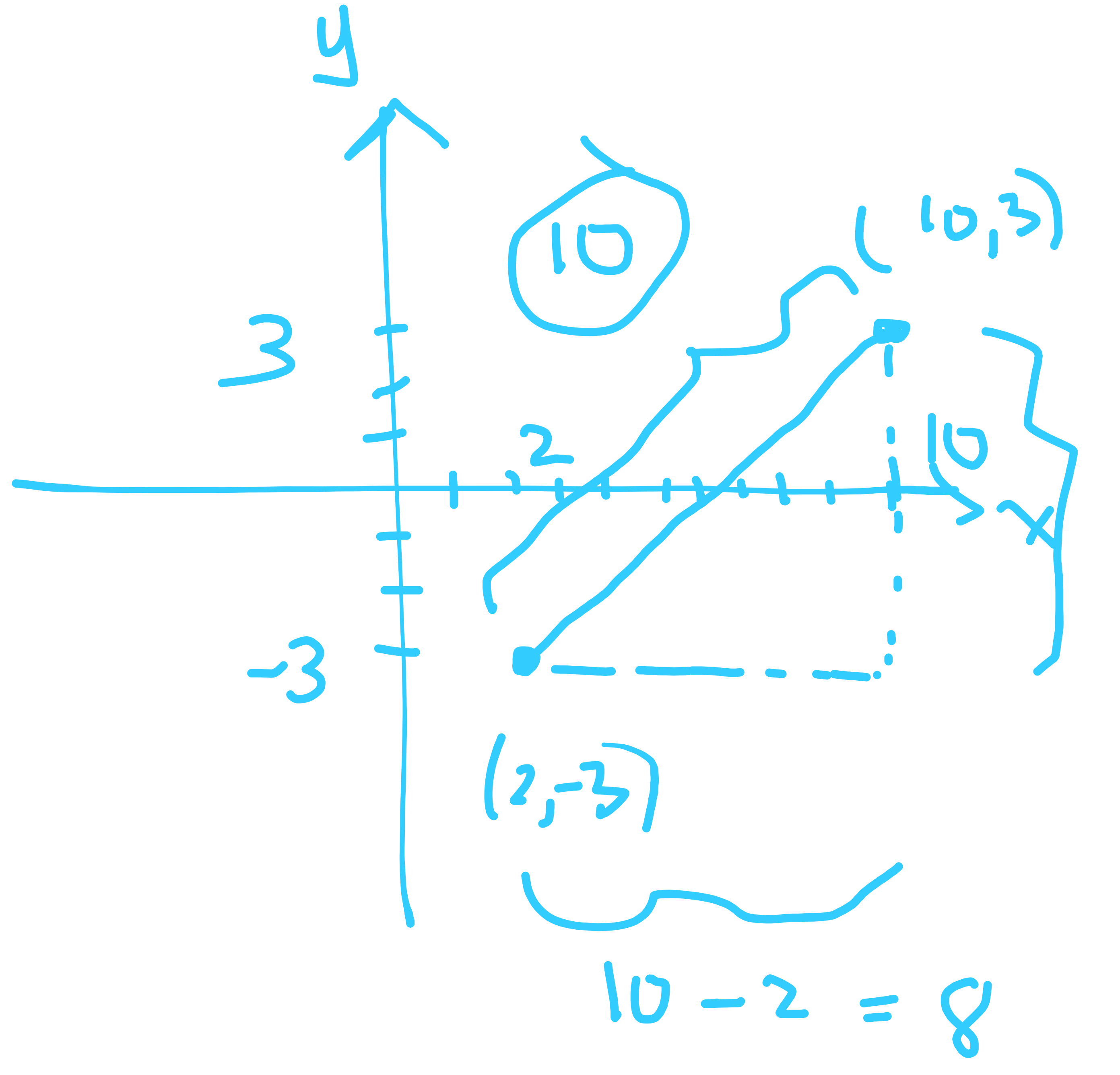
6) A $(2,-3)$ B $(10,3)$

$$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

$-2-(-6)$

$$d_{AB}=\sqrt{(10-2)^2+(3-(-3))^2}$$

$$=\sqrt{8^2+6^2}=\sqrt{64+36}=\sqrt{100}=10$$



$3-(-3)=6$

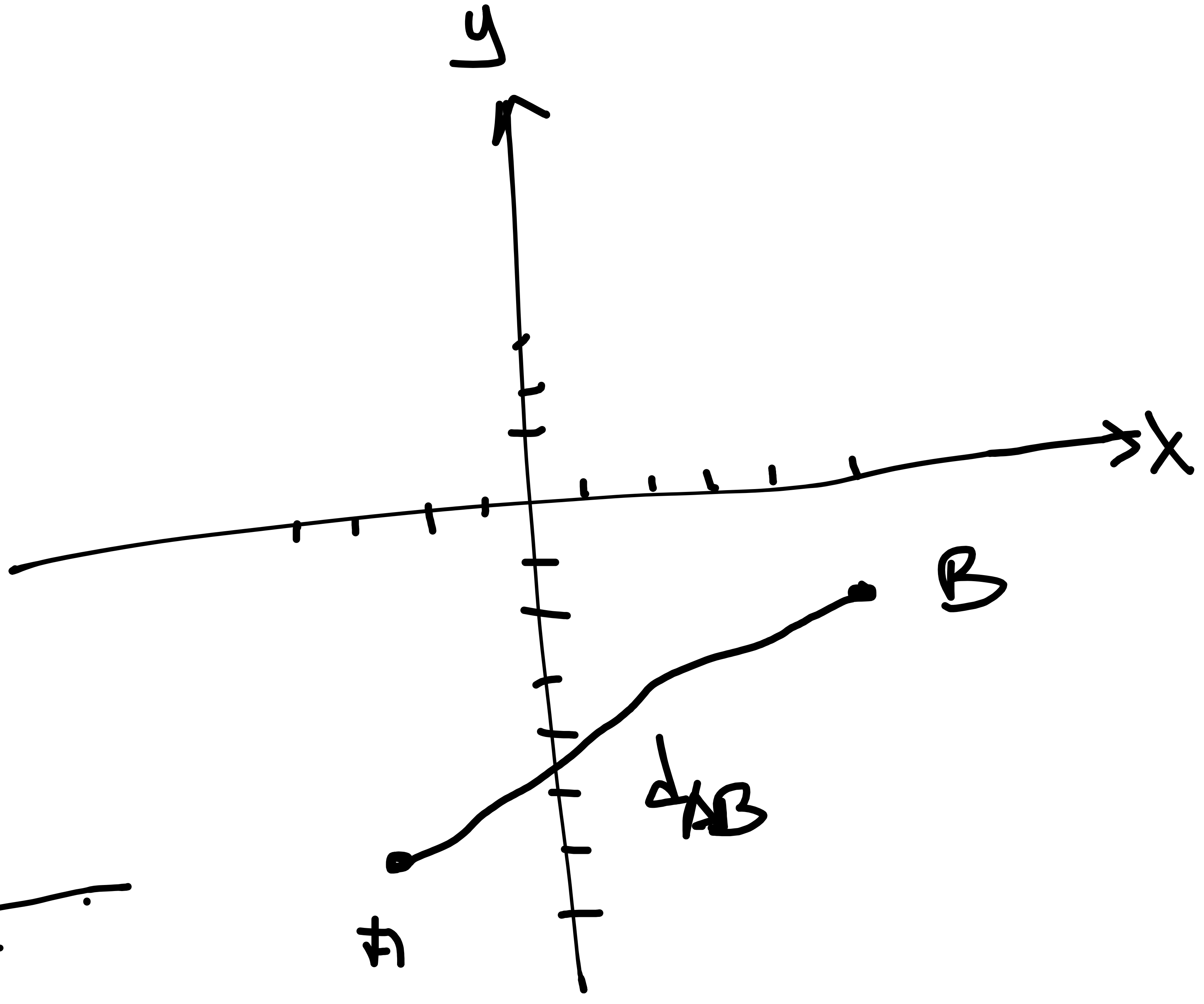
2) 1st A $(-3,-6)$ 2nd B $(5,-2)$

x_1, y_1; x_2, y_2

$$d_{AB}=\sqrt{(5-(-3))^2+(-2-(-6))^2}$$

$$=\sqrt{8^2+(4)^2}$$

$$=\sqrt{80}=8.9$$



Pm = 5-

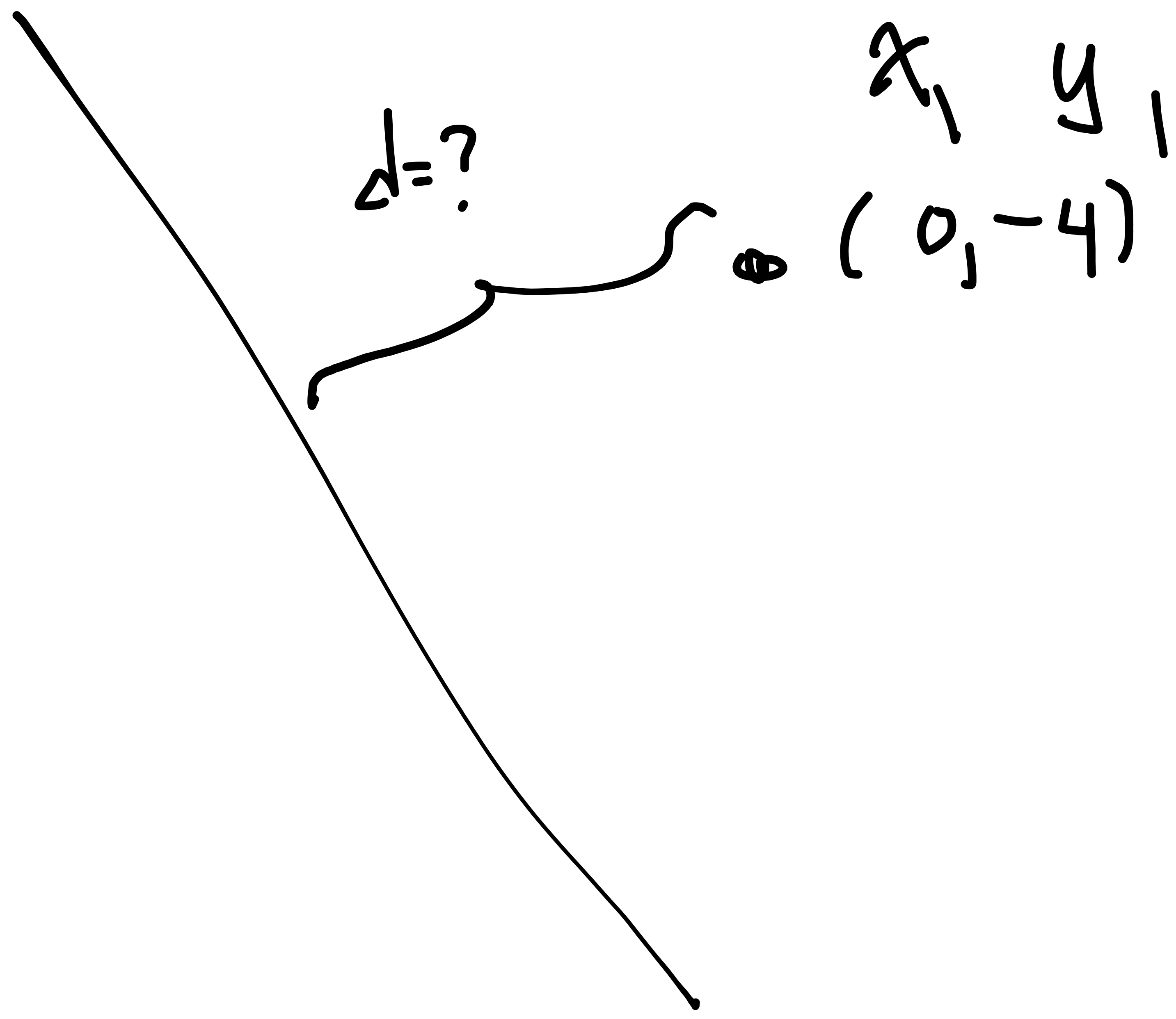
9)

$$d = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2+B^2}}$$

$$= \frac{|1(0) + 3(-4) + (-9)|}{\sqrt{1+9}}$$

$$= \frac{|0-12-9|}{\sqrt{10}}$$

$$= \frac{|-21|}{\sqrt{10}} = \frac{21}{\sqrt{10}}$$



$Ax + By + C = 0$

$x + 3y = 9$

$x + 3y - 9 = 0$

$A = 1$ $B = 3$ $C = -9$

	m	y-intercept
10	2	-3
11	$-2/3$	2
12	-3	5
13	$-2/3$	4
14	-2	0
15	$+\frac{6}{7}$	7
16	$3/2$	$-7/2$
17	$\frac{1}{3}$	$-5/6$
18	-1	2

$6x - 7y =$

$3x - 2y = 7$

$-2y = 7 - 3x$

$$\boxed{y = -\frac{7}{2} + \frac{3}{2}x}$$

$3x + y = 5$

$2x + 3y = 12$

$4x + 2y = 0$

$2y = -4x$

$y = \frac{-4}{2}x = -2x + 0$

$x + y = 2$

$y = 2 - x$

19 $y = mx + b$ $\quad y = -2x + 4$

20 $y = 4x - 10$

21 $6 \times y = \frac{7}{2}_{6\times} x - \frac{1}{3}_{\times 6} \Rightarrow 6y = 21x + 2$

22 $y = -\frac{3}{5}x + 3 \Rightarrow 5y = -3x + 15$

23 $y = 0 + \frac{2}{3} \Rightarrow y = \frac{2}{3}$ $\quad 3y = 2$

24 $y = -\frac{5}{4}x + \frac{11}{5}$

$20y = -25x + 44$

24) $m = -2$; $\overset{x_1}{(0,}\overset{y_1}{3)}$

$y - y_1 = m(x - x_1)$

$y - 3 = -2(x - 0)$

$y - 3 = -2x + 0$

$\boxed{y = -2x + 3}$

25) $y = 3x + \boxed{11}$ -13

26) $y = \frac{2}{3}x - 1$

$y + 5 = \frac{2}{3}(x + 6)$

$y = \frac{2}{3}x + \cancel{4} - 5$

$\boxed{y = \frac{2}{3}x - 1}$

28) $y = 3x - 12$

29) $y = -\frac{4}{5}x - \frac{7}{5}$

$y + 3 = -\frac{4}{5}(x - 2)$

$y = -\frac{4}{5}x + \frac{8}{5} - 3$

$5y = -4x + 8 - 15$

$5y = -4x - 7$

Assignment

Deadline for submission: next week ***Monday***

Exercise 4.1- 6, 17, 22, 25, 27

Exercise 4.2 – 5, 9, 14, 15, 21, 23, 29, 35, 48, 65, 69, 71, 73 – 85(True or False)