

# CAN LANGUAGE MODELS DISCOVER SCALING LAWS?

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## ABSTRACT

Discovering scaling laws for predicting model performance at scale is a fundamental and open-ended challenge, mostly reliant on slow, case specific human experimentation. To investigate the potential for LLMs to automate this process, we collect over 5,000 experiments from existing literature and curate seven diverse scaling law discovery tasks. While existing agents struggle to produce accurate law formulas, this paper introduces SLDAgent, an evolution-based agent that co-optimize the scaling law model and the parameters, enabling it to autonomously explore complex relationships between variables. For the first time, we demonstrates that SLDAgent can automatically discover laws that exhibit *consistently more accurate extrapolation* than their established, human-derived counterparts across all tasks. Through comprehensive analysis, we elucidate why these discovered laws are superior and verify their practical utility in both pretraining and fine-tuning applications. This work establishes a new paradigm for agentic scientific discovery, showing that AI systems can understand their own scaling behavior, and can contribute novel and practical knowledge back to the research community.

## 1 INTRODUCTION

Scaling laws are fundamental to the development of foundation models, including Large Language Models (LLMs) (Achiam et al., 2023; Bi et al., 2024), Vision-Language Models (VLMs) (Achiam et al., 2023), and far beyond. A core objective is to predict performance for models training at large scales, based on experiments at smaller scales (Kaplan et al., 2020). This predictive capability is crucial for determining optimal model configurations (Hoffmann et al., 2022), selecting the best pre-trained models for fine-tuning (Lin et al., 2024), and identifying ideal hyperparameters such as batch size and learning rate (Li et al., 2025). The scope of modern scaling laws has expanded significantly beyond the classical Chinchilla Law (Hoffmann et al., 2022), which expresses pre-training loss  $L$  as a function of model size  $N$  and dataset size  $D$  using a power-law relationship ( $L = \theta_0 + \theta_1/N^{\theta_2} + \theta_3/D^{\theta_4}$ ). More diverse and emerging scenarios, including scaling behaviors for supervised fine-tuning (Lin et al., 2024; Zeng et al., 2025), Mixture-of-Experts (MoE) models (Krajewski et al., 2024), vocabulary size (Tao et al., 2024), domain mixture (Ye et al., 2024), and inference parallelism (Chen et al., 2025a), have emerged rapidly with the development of LLMs.

One of the fundamental challenges in scaling law discovery (SLD) lies in the combination of symbolism and open-endedness. A symbolic and mathematical law formula is required due to the universality and generalizability, yet the underlying formulation space is infinite and a priori knowledge of the problem is necessary. Consequently, most scaling law discoveries in new scenarios are manual and case-specific, where human experts propose mathematical models to characterize the relationship between scaling variables within the system based on intuition and experience. However, this human-centric approach is often inefficient, labor-intensive, and sub-optimal, requiring numerous cycles of hypothesis generation and experimentation. Furthermore, it is constrained by humans' understanding and capacities in analyzing complex relationships that involve multiple variables.

Recent progress in LLMs has enabled a variety of practical agentic workflows (Yang et al., 2025; Soni et al., 2025; Zhang et al., 2025). Building on this, the notion of an “AI scientist” has emerged: systems that generate hypotheses, run analyses, and iterate with minimal human input (e.g., discovering new drugs, developing machine learning models, drafting papers) (Chan et al., 2024b; Lu

\*Equal contribution. Code is available at <https://github.com/linhaowei1/SLD>.

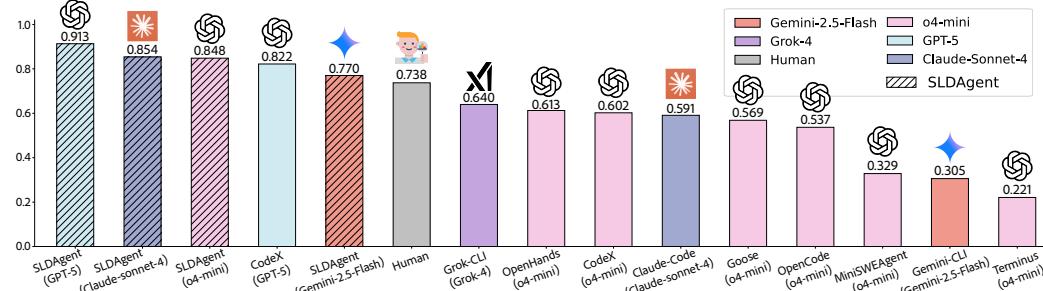


Figure 1: Overview of the scaling law discovery performance on SLDBench of various AI agents, our proposed SLDAgent, and human baselines. Performance is measured by the average prediction score ( $R^2$ ) across all tasks, where higher values indicate a better fit on unseen test sets. Our SLDAgent (hatched bars) consistently enhances the performance of its underlying LLMs, achieving top ranks and outperforming all other baseline agents and human experts. Breakdown of scores on individual tasks can be found in Sec. 4.

et al., 2024; Swanson et al., 2024). Yet these systems struggle with open-ended problem formulation, principled experiment design, and robust long-horizon execution (Xie et al., 2025). Scaling law discovery is precisely such an open-ended scientific discovery task, and as we empirically demonstrate below, existing agentic systems fall short in designing scaling laws that are superior to human-derived counterparts. This motivates us to ask a pivotal question: *can LLM-based agents discover the scaling laws that govern their own behavior more efficiently and accurately than humans?*

To rigorously answer the question, we curate **SLDBench**, a comprehensive scaling law discovery testbed constructed from over 5,000 LLM training experiments from existing scaling law discovery literature. SLDBench contains seven distinct tasks, where each task requires identifying a symbolic expression that maps input variables to a target prediction, such that its *extrapolation accuracy* on an unseen, scaling test set is accurate. Unlike conventional benchmarks with binary success criteria, SLDBench emphasizes the open-ended nature: the objective is clear, continuous, and unbiased (e.g.,  $R^2$  on extrapolation set), but the ground-truth solution remains unknown to human experts, and a minor gain could result in substantial benefits in practical model developments. As existing laws are designed based on a series of continuous and iterative human research efforts, existing agentic systems lag behind these laws on difficult SLD tasks, and the human-derived laws could also give negative-correlated predictions under challenging scenarios.

To fully uncover the capabilities of LLMs, inspired by the human research process and recent studies on evolution-based agents (Gao et al., 2025b), we design an LLM-based agent named **SLDAgent**. Intuitively, SLDAgent keeps pushing the boundary of its solution evolutionarily by co-optimizing two functions based on previously proposed solutions: (1) a generator that proposes a law expression and (2) an optimizer that fits the coefficients on seen data efficiently and robustly. This co-optimization process will be iterated for certain steps and the optimal proposal will be returned.

Our comprehensive evaluation reveals that SLDAgent achieves SOTA performance across all SLDBench tasks, consistently outperforming human experts and existing agent baselines, and can boost the capacities of all LLMs it is paired with. This superiority stems from the agent’s ability to evolve and optimize the law formulations. As case studies, its discovered laws introduce more conceptually sound principles, such as unifying data sources under a single scaling exponent, and ensuring principled asymptotic behavior to be accurate. The practical utility of these laws is demonstrated in two critical applications: enabling direct, analytical optimization of pre-training hyperparameters to a near-optimal state, and successfully identifying the best fine-tuning checkpoints.

In summary, this work establishes a new paradigm for agentic scientific discovery, demonstrating for the first time that an AI agent can automate the discovery of scaling laws superior to human-derived counterparts. We introduce SLDAgent, a novel evolution-based agent that achieves state-of-the-art, superhuman performance, alongside SLDBench, the first comprehensive benchmark for this task. Through rigorous analysis, we show that the laws discovered by SLDAgent are not just more accurate but fundamentally more principled, and we validate their practical utility in various applications. We have open-sourced our entire framework, providing a tangible path toward AI systems that can autonomously understand and accelerate their own development.

## 2 SLDBENCH

**General formulation.** Scaling law discovery (SLD) requires the prediction of a scaling law, including its mathematical formula and instantiated parameters, based on a given set of observed trails.

Specifically, the input of the problem is a dataset

$$\mathcal{D} = \{(\mathbf{x}_i, j_i, y_i)\}_{i=1}^m,$$

where each of the  $m$  data contains:

- (1) a vector of  $n$  **feature variables**  $\mathbf{x}_i = (x_i^{(1)}, \dots, x_i^{(n)})$  that vary across trials (e.g., parameter count, batch size, training token length);
- (2) a vector (usually 1-dimension) of **target**  $y_i$  to be predicted (e.g., loss, perplexity, accuracy), this is the value of interest that the proposed scaling law should output;
- (3) a **control index**  $j_i \in \mathcal{C}$  identifying the experimental setting (e.g., a particular model). All settings are required to fit the same scaling law model, but the parameters for different settings can be different.

The output of SLD should be: (1) a symbolic expression  $f_{\theta} : \mathbf{x} \mapsto \hat{\mathbf{y}}$ , mapping input features  $\mathbf{x}$  to a target prediction  $\hat{\mathbf{y}}$  using parameters  $\theta$ , and (2) a group of parameters  $\{\theta_j\}_{j \in \mathcal{C}}$  that are fitted specifically for each control setting. The objective of SLD is that for any setting  $j \in \mathcal{C}$ , the proposed law parameterized by  $\theta_j$  can predict the target  $\mathbf{y} = f_{\theta_j}(\mathbf{x})$  accurately in unseen, extrapolated  $\mathbf{x}$ .

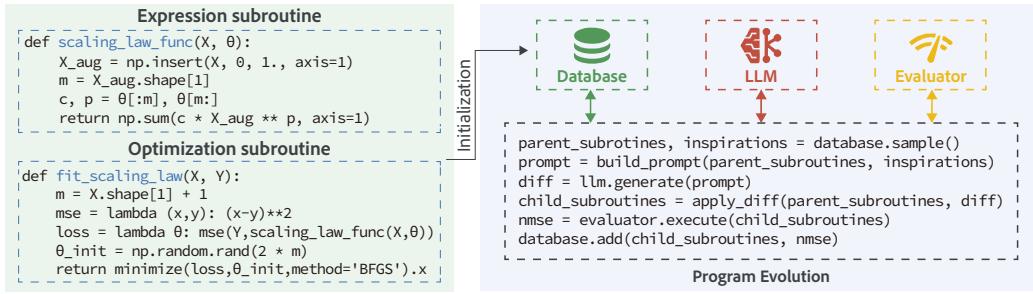
**Example.** Consider classical pre-training scaling laws, where  $\mathbf{x} = (N, D)$  denotes model size and dataset size, and the target  $\mathbf{y}$  is the training loss  $L$ . Different control settings correspond to different corpora (e.g., books, code, news) or architectures (e.g., decoder-only, encoder-decoder, RNN). Human experts have proposed the formula  $L_{\theta} = \theta_0 + \theta_1/N^{\theta_2} + \theta_3/D^{\theta_4}$ , where  $N, D$  is the input features, and  $\theta = \{\theta_0, \theta_1, \theta_2, \theta_3, \theta_4\}$  are coefficients that can be fitted using standard optimization methods (e.g., BFGS (Fletcher, 2000), SGD (Amari, 1993)) for each  $j$ .

**Datasets.** Following the above paradigm, we curate concrete scaling law tasks from comprehensive surveys and literature such as Sengupta et al. (2025) and filter the ones with available open-sourced data. A total of seven tasks are included in SLDBench (we will continuously expand our dataset), and detailed statistics are presented in Table 1: (1) *Parallel Scaling Law* (`parallel`) examines the effect of parallelism  $P$  and model size  $N$  on the language modeling loss. This method involves creating  $P$  augmentations of an input and aggregating the corresponding outputs to generate a final response, conceptually similar to the Best-of-N approach (Chen et al., 2025a). (2) *Vocabulary Scaling Law* (`vocab_size`) models the unigram-normalized loss as a function of the non-vocabulary model size  $N$ , vocabulary size  $V$ , and dataset size  $D$  (Tao et al., 2024). (3) *Supervised Finetuning Scaling Law* (`sft`) models the finetuning loss based on the dataset size  $D$  (Lin et al., 2024). (4) *Domain Mixture Scaling Law* (`domain_mix`) models the pre-training loss for each domain based on its proportion in the training mixture (Ye et al., 2024). (5) *Mixture of Experts Scaling Law* (`moe`) models the loss in relation to the network size  $N$  and the number of experts  $E$  (Krajewski et al., 2024). (6) *Data Constrained Scaling Law* (`d constrain`) models the pre-training loss as a function of network size  $N$ , dataset size  $D$ , and the number of unique tokens  $U$  in the dataset. (7) *Learning Rate and Batch Size Scaling Law* (`lr_and_bsz`) models the pre-training loss based on learning rate  $l$ , batch size  $b$ , dataset size  $D$ , and network size  $N$ . This law is adapted from the Step Law (Li et al., 2025) that predicts the optimal learning rate  $lr^*$  and batch size  $bsz^*$  given  $D$  and  $N$ .

Table 1: Tasks of SLDBench and the number of (un)seen datapoints.  $L$ : loss;  $N$ : #parameters;  $D$ : dataset size;  $\mathbf{r}$ : domain proportions;  $E$ : #experts;  $U$ : #unique tokens;  $l$ : learning rate;  $b$ : batch size.

Task Scenario	# Seen	# Unseen	Target
parallel	36	12	$L(N, P)$
vocab_size	1,080	120	$L(N, V, D)$
sft	504	42	$L(D)$
domain_mix	80	24	$\{L_i(\mathbf{r})\}_{i=1}^5$
moe	193	28	$L(N, E)$
d constrain	161	21	$L(N, D, U)$
lr_and_bsz	2,702	117	$L(l, b, D, N)$

**Execution environment and metrics.** For each task, we hold out an extrapolation test set from the dataset by selecting data corresponding to the *largest model or dataset sizes*. This closely imitates the practical scaling law use cases. The evaluation environment is a sandbox terminal with a minimal set of pre-installed Python packages (`scikit-learn`, `pandas`, and `datasets`). The agent’s goal is to produce a Python script named `law.py`, which must contain the discovered scaling law defined within a function with a specified signature. Upon the agent’s completion, our evaluation script automatically computes three primary metrics on the test set: the coefficient of determination ( $R^2$ ), Normalized Mean Squared Error (NMSE), and Normalized Mean Absolute Error (NMAE).



**Figure 2: The SLDAgent System.** *Left:* Each candidate program consists of two subroutines: an Expression that defines the symbolic model  $f_\theta(\mathbf{x})$ , and an Optimization routine that fits its parameters  $\theta$  to data. *Right:* The evolutionary loop. An LLM mutates a parent program sampled from a database. The resulting child is evaluated and inserted back into the database, continually improving the population.

A perfect law should give  $R^2$  close to 1 and NMSE, NMAE close to 0. Several tasks involve multiple control indices  $j$ , requiring the prediction of a scaling law with a distinct set of coefficients for each experimental setting  $j \in C$ . These include parallel (across two pre-training datasets), sft (across three SFT datasets and 14 pre-trained models), and domain\_mix (across four different models). Details can be found in Sec. A.2 and B.1.

**Specialty of SLDBench.** The symbolism and open-endedness of SLDBench make it distinct from existing benchmarks in various ways. Traditional symbolic regression tasks (Udrescu & Tegmark, 2020; Shojaee et al., 2025) focus on rediscover a pre-known formula, thus they are most synthetic and not truly open-ended; Conversely, while agentic benchmarks like MLE-Bench (Chan et al., 2024b) involve open-ended problem-solving, their goal is to automate engineering workflows, not to produce a generalizable symbolic discovery. SLDBench provides a rigorous testbed for a deeper form of agentic scientific intelligence by uniquely combining the challenge of symbolic reasoning with a demand for abstract, multi-context generalization.

### 3 SLDAgent: EVOLUTIONARY SOLVERS FOR SCALING LAW DISCOVERY

Intuitively, SLDBench features a series of open-ended scientific discovery problems where the objective is clear, computable, continuous, and unbiased (improving  $R^2$  to 1 in our case), while the perfect solution remains unknown even to human experts. Similar to other problems that fall into the same category, such as optimizing matrix multiplication efficiency (Fawzi et al., 2022) or improving the Ramsey number bound (Campos et al., 2023), the solution of these problems requires generations of efforts that continuously improving upon existing proposals. Consequently, a single round of proposal, no matter how impressive it is, is insufficient to push the limits toward the optimal solution, and an evolution-based systems specifically for SLD should be adopted. This motivates us to design SLDAgent, an evolutionary coding agent (Gao et al., 2025a) that discovers scaling laws by searching over the space of executable programs.

**Overview.** As illustrated in Figure 2, the core design of SLDAgent is the evolution of *a pair of subroutines*, (Expression, Optimization), which together define the model and the fitting function. We adopt this co-evolution design due to the nature of the SLD problem and that a general, problem-agnostic evolution (Novikov et al., 2025) faces challenges in fully exploiting the LLM capabilities (see ablations in Sec. B.7). Both functions are implemented by SLDAgent. The Expression( $\mathbf{x}, \theta$ ) implements a function  $f_\theta : \mathbf{x} \mapsto \hat{y}$ , mapping input features  $\mathbf{x}$  to a prediction  $\hat{y}$  using parameters  $\theta$ , while the Optimization routine finds the best-fit parameters  $\hat{\theta} = \arg \min_{\theta} \mathcal{L}(\mathbf{y}, f_\theta(\mathbf{x}))$  for a given loss function  $\mathcal{L}$ , producing a parameterized scaling law ready for extrapolation.

**Evolution.** The SLDAgent is provided with the training set in its working directory, and it begins with a baseline program pair, typically a power-law function for Expression and a standard BFGS optimizer for Optimization. This program is executed once and stored into the evolutionary database (initially empty), together with its fitness score. At each evolution step, a parent program is selected from the database via a probabilistic mixture that prioritizes high-scoring programs (exploitation, 70%), diversity (20%), and the top performers (elitism, 10%). This parent, along with

Table 2: Performance on SLDBench for agents using o4-mini. Scores are  $R^2$  averaged over three runs. The **best** and **second-best** scores for each task are highlighted. “NA” indicates no valid output.

	parallel↑	vocab_size↑	SFT↑	domain-mix↑	moe↑	d_constraint↑	lr_and_bsz↑	<b>Avg. <math>R^2</math>↑</b>
Aider (o4-mini)	-11.614	NA	0.787	NA	NA	-23.268	NA	-0.404
Terminus (o4-mini)	0.304	-586.428	0.926	0.817	0.468	0.806	-0.774	0.221
Goose (o4-mini)	<b>1.000</b>	0.801	0.822	0.859	0.468	0.805	-0.774	0.569
Mini-SWE-Agent (o4-mini)	<b>1.000</b>	0.816	<u>0.960</u>	0.830	0.468	-119.772	-0.774	0.329
OpenCode (o4-mini)	<b>1.000</b>	0.861	0.893	0.505	0.468	0.805	-0.772	0.537
CodeX (o4-mini)	<b>1.000</b>	0.861	0.921	0.936	0.468	0.805	-0.774	0.602
OpenHands (o4-mini)	<b>1.000</b>	0.801	0.893	<u>0.965</u>	0.589	0.805	-0.763	0.613
SLDAgent (o4-mini)	<b>1.000</b>	0.964	<b>0.994</b>	<b>0.976</b>	<b>0.851</b>	<b>0.961</b>	<b>0.194</b>	<b>0.848</b>
Human	<b>1.000</b>	<b>0.966</b>	0.957	0.701	<u>0.703</u>	<u>0.911</u>	-0.076	0.738

several selected high-performing “inspiration” programs, is contextualized into a structured prompt (see Sec. C.1 for details). The LLM takes in the input and proposes a *modification* of parent’s code, for instance, by altering the law form, changing the optimizer, or tuning global variables. The resulting child program is executed: its Optimization routine fits the Expression on the given seen dataset, and the  $R^2$  score on training dataset will be computed. Finally, the new child and its score are added to the database for subsequent generations. The evolutionary loop terminates after a fixed budget of iterations. The program with the highest score in the final database is returned as the proposed law. The test set is left untouched throughout the whole evolution.

**Implementation remarks.** To guide the LLM toward effective laws, the prompt includes task context, selected inspiration programs, and data statistics such as value ranges, mean, and variance. Following AlphaEvolve (Novikov et al., 2025), we increase sample diversity and prevent premature convergence by using a multi-island strategy (five islands) and MAP-Elites, which structures the population by fitting score, complexity, and novelty. The system is built on the OpenEvolve framework (Sharma, 2025); see Sec. C.2 and D for full prompts and configurations.

## 4 EXPERIMENTS AND ANALYSIS

### 4.1 EXPERIMENTAL SETUP

We benchmark SLDAgent again human and existing agents on SLDBench. Our comparisons are along two perspectives: we first fix the LLM (o4-mini) and compare SLDAgent against seven agentic systems, and then we compare SLDAgent with different LLMs against their native CLI systems.

**Agent baselines.** We use o4-mini as the default LLM for all folowing baseline agents: (1) **Aider** (Aider-AI, 2025), a CLI pair programmer that edits files in a local Git repository, shows diffs, and auto-commits changes; (2) **Terminus** (Terminal-Bench Team, 2025), a native Python agent that controls a dockerized environment solely via an interactive tmux session by sending keystrokes, enabling fine-grained terminal control; (3) **OpenHands** (Wang et al., 2025), a general-purpose developer agent that can edit code, run Bash, and perform file operations in a full agentic development environment (v0.55.0); (4) **Codex** (OpenAI, 2025), OpenAI’s cloud software-engineering agent that executes parallel tasks in isolated sandboxes preloaded with the target repository; (5) **OpenCode** (sst, 2025), an open-source, terminal-based assistant oriented toward local coding workflows; (6) **Goose** (Block, 2025), a local, extensible engineering agent that writes and runs code, orchestrates workflows, and integrates with external tools/APIs; (7) **Mini-SWE-Agent** (SWE-agent Team, 2025), a ~100-line agent tailored to repository-level edits and SWE-bench-style tasks (Jimenez et al., 2024a); and finally, **Human**, the law discovered in the original literature, serving as a strong reference baseline. We refit some laws on our seen data split and recruit a human expert to propose a law for `lr_and_bsz`, for which no law exists. Further details are presented in Sec. A.3.

**LLMs baselines.** We compare SLDAgent equipped with different LLMs against the following models equipped with native command-line code agents: Gemini-2.5-Flash with **Gemini-CLI** (Google, 2025) (with Gemini Code Assist for long-context, tool-calling workflows), Claude-Sonnet-4 with **Claude Code** (Anthropic, 2025) (with structured tool-use such as sandbox environ-

Table 3: SLDBench performance of provider-specific agents together with reference rows for SLDAgent paired with the corresponding provider models. Grok-4 was not evaluated due to cost. Scores report the coefficient of determination ( $R^2$ ), averaged over three runs. Bold denotes the best value within each model family.

	parallel↑	vocab_size↑	SFT↑	domain-mix↑	moe↑	d_constraint↑	lr_and_bsz↑	<b>Avg. <math>R^2</math>↑</b>
Grok-CLI (Grok-4)	1.000	0.959	0.845	0.959	0.874	0.843	-4.992	0.640
Gemini-CLI (Gemini-2.5-Flash)	1.000	0.861	0.787	-0.927	0.590	0.830	-157.608	0.306
SLDAgent (Gemini-2.5-Flash)	1.000	<b>0.900</b>	<b>0.967</b>	<b>0.987</b>	<b>0.836</b>	<b>0.892</b>	<b>-0.188</b>	<b>0.771</b>
Claude Code (Claude-Sonnet-4)	1.000	0.931	0.895	0.907	0.468	0.779	-0.841	0.591
SLDAgent (Claude-Sonnet-4)	1.000	<b>0.979</b>	<b>0.973</b>	<b>0.983</b>	<b>0.902</b>	<b>0.969</b>	<b>0.171</b>	<b>0.854</b>
CodeX (o4-mini)	1.000	0.861	0.921	0.936	0.468	0.805	-0.774	0.602
SLDAgent (o4-mini)	1.000	<b>0.964</b>	<b>0.994</b>	<b>0.976</b>	<b>0.851</b>	<b>0.961</b>	<b>0.194</b>	<b>0.848</b>
CodeX (GPT-5)	1.000	0.912	0.866	0.962	0.833	0.915	0.269	0.822
SLDAgent (GPT-5)	1.000	<b>0.987</b>	<b>0.997</b>	<b>0.962</b>	<b>0.870</b>	<b>0.968</b>	<b>0.610</b>	<b>0.913</b>
Human	1.000	0.966	0.957	0.701	0.703	0.911	-0.076	0.738

ment and file I/O), and o4-mini and GPT-5 (both with **Codex**). We additionally evaluate **Grok-CLI** ([Superagent AI, 2025](#)) with Grok-4, but we do not run SLDAgent on it due to the high cost.

**Evaluation metrics.** We report the  $R^2$  scores for each method on the test set of each SLD task in the main paper, and postpone the results for NMSE and NMAE in Sec. B.2. Since  $R^2$  is normalized and has an upper bound 1, the values are also comparable across tasks. We clip the scores between  $[-1, 1]$  when averaging across tasks to avoid significant influence from single task failure.

## 4.2 RESULTS ANALYSIS

**Superiority of SLDAgent against baseline agents.** As shown in Table 2, when benchmarked on the o4-mini model, SLDAgent achieves an average  $R^2$  score of 0.85, significantly outperforming all competing architectures, including the next-best OpenHands (0.61) and CodeX (0.60). Notably, even when constrained to o4-mini, SLDAgent surpasses the strong human baseline on six out of seven tasks, achieving the top average  $R^2$  score. This result underscores that LLM-based agent with evolution designs is a decisive factor, as other agents with the identical LLM struggles to outperform human experts. This demonstrate the potential of AI agent to completely achieve superhuman performance in open-ended scientific discovery.

**Pushing the performance to superhuman levels.** To further push the limit of SLDAgent and explore its upper boundary, we paired SLDAgent with a range of powerful models, as shown in Table 3. The results show that SLDAgent is not just a high-performing agent, but a universally effective framework that successfully enhances the capabilities of *any* underlying LLM on *any* SLD tasks. Equipped with GPT-5, SLDAgent *for the first time outperforms human experts on all seven tasks*. In terms of the Average  $R^2$ , SLDAgent can consistently boost the value by  $0.09 \sim 0.47$ , making it a strong approach to exploit LLM capacities in SLD.

**Task difficulty analysis.** The tasks in SLDBench span a wide spectrum of difficulty, as shown in Table 2. The parallel task, for example, is easy and most agents achieve near-perfect  $R^2$  scores. In contrast, tasks like lr\_and\_bsz, d\_constraint, and moe are significantly more challenging. The highly unstable and often negative  $R^2$  values on these tasks differentiate agent capacities effectively. For most tasks in SLDBench, it remains unclear what the ground-truth scaling laws are.

## 4.3 LAW FORMULATION ANALYSIS

An intriguing study is how SLDAgent-proposed laws is different from human-proposed laws, and why this leads to better performance. In the easy tasks like parallel where a simple Chinchilla law is sufficient, the slight performance difference is mostly due to optimization choices. While in other tasks, the scaling law formulas are different. Below, we choose the sft and moe tasks to analyze their differences carefully. We postpone the analysis of improved optimization in Sec. B.7. The SLDAgent-discovered laws analyzed below are the ones with the highest observed data fit across all LLMs and random seeds.

**The sft task.** The SFT law models the loss  $L$  as a function of the SFT dataset size ( $D$ ). Previous work (Lin et al., 2024) and SLDAgent propose

$$\text{Human law: } L = \theta_2 + \frac{\theta_0}{D^{\theta_1} + \theta_3}, \quad \text{SLDAgent law: } L = \theta_2 + \frac{\theta_0}{(D + \theta_3)^{\theta_1}},$$

While they are similar, the key distinction lies in the treatment of “effective data size” ( $\theta_3$ ) which captures how much data from the pretraining process is converted to the SFT stage. The human law calculates the power of the SFT data size only ( $D^{\theta_3}$ ), keeping the pre-training knowledge *outside* the exponential scaling term. In contrast, the SLDAgent-proposed law combines the SFT data size ( $D$ ) with an offset representing pre-training knowledge ( $\theta_3$ ) *before* applying the scaling exponent ( $(D + \theta_3)^{\theta_1}$ ). This latter approach is more conceptually sound, as both  $D$  and  $\theta_3$  represent data magnitude and should logically be unified under a single power law that governs diminishing returns. As comprehensively analyzed below in Sec. 4.5, the proposed law more accurately models their relationship, resulting in a superior downstream usability.

**The moe task.** The moe law describes the loss  $L$  as a function of the number of experts ( $e$ ) and the number of parameters ( $N$ ). Krajewski et al. (2024) proposes to predict the loss by

$$\text{Human law: } L = \exp(\theta_0 \log N + \theta_1 \log \hat{e} + \theta_2 \log N \log \hat{e} + \theta_3),$$

where  $\hat{e}$  is a parameterized transformation of  $e$  for stability. SLDAgent proposes the following:

$$\text{SLDAgent law: } L = \theta_0 + \frac{\theta_1}{(N^\alpha + \theta_2 \max(e^{\theta_3} - 1, 0))^{\theta_4}},$$

An essential difference is that the proposed law contains an irreducible loss floor ( $\theta_0$ ), ensuring that as the number of experts or parameters approaches infinity, the predicted loss gracefully converges to a realistic, non-zero value. This reflects the inherent entropy and unpredictability in a dataset. The human-proposed law, however, is formulated as an exponential of log-linear terms, including a positive interaction term ( $\theta_2 \cdot \log(N) \cdot \log(\hat{e})$ ). A critical flaw is that at the limit of infinite parameters, the predicted loss will explode towards infinity, causing a theoretically unsound outcome. The SLDAgent law’s more principled asymptotic behavior demonstrates a superior structural understanding, allowing it to maintain predictive accuracy at scales far beyond the training region.

#### 4.4 APPLICATION I: HYPERPARAMETER OPTIMIZATION FOR PRE-TRAINING

A critical challenge in pre-training LLMs is identifying the optimal learning rate ( $lr$ ) and batch size ( $bsz$ ) for a given model and dataset. Conventional methods often involve extensive hyperparameter sweeps on smaller scales to fit a predictive scaling law. In this process, only the single best-performing configuration from each sweep is typically used to fit a law for the optimal hyperparameters ( $lr^*, bsz^*$ ) as a function of model size ( $N$ ) and dataset size ( $D$ ). For instance, the original StepLaw (Li et al., 2025) ran approximately 3,000 experiments but used only 17 optimal points to fit its law ( $lr^* = cN^\alpha D^\beta$ ,  $bsz^* = dD^\gamma$ ), discarding the vast majority of experimental data.

In contrast, the SLDAgent discovers a comprehensive law that models the final loss  $L$  as a function of  $N$ ,  $D$ ,  $lr$ , and  $bsz$ . This explicit functional form allows us to find the optimal hyperparameters analytically. By treating the loss as a surface over the hyperparameter space, we locate the minimum by setting the partial derivatives  $\partial L / \partial lr$ ,  $\partial L / \partial bsz$  to zero, and solving the resulting system of equations. This calculus-based approach yields a closed-form solution for the optimal  $lr^*$  and  $bsz^*$  as functions of  $N$  and  $D$ . The full derivation is detailed in Sec. B.5. We then apply the derived formula on the extrapolation setting following StepLaw: a 1B-parameter model trained on 100B tokens, which is at least twice as large as all observed

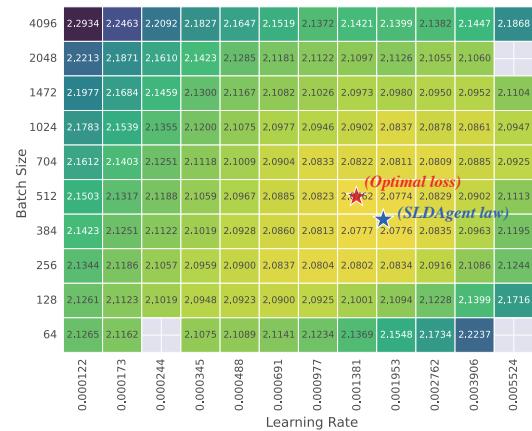


Figure 3: Ground-truth validation-loss heatmap for a 1B-parameter LLM trained on 100B tokens.

Table 4: LLM selection results (PearCorr, RelAcc) on three datasets in percentage. The best result within the same dataset is in **bold** font, and the second best result is underlined.

Dataset	Metric	ModelSize	ZeroShot	RectifiedLaw	NLPMetrics	SubTuning	SLDAgentLaw
FLAN	PearCorr↑	-29.8	-12.8	60.1	76.1	<u>79.4</u>	<b>84.6</b>
	RelAcc↑	75.6	82.3	<b>100.0</b>	84.7	71.8	<b>100.0</b>
Wikitext	PearCorr↑	38.0	6.1	76.4	78.9	<b>90.3</b>	<u>83.6</u>
	RelAcc↑	68.5	84.4	44.0	87.5	<b>100.0</b>	<b>100.0</b>
Gigaword	PearCorr↑	-27.4	-53.2	66.7	80.1	<u>86.3</u>	<b>94.8</b>
	RelAcc↑	77.3	83.3	78.3	86.8	<b>100.0</b>	<b>100.0</b>

experiments. As shown in fig. 3, the analytically derived optimum is marked by the blue star ( $lr = 1.77 \times 10^{-3}$ ,  $bsz = 402$ ). Training at the nearest available grid point ( $lr = 1.95 \times 10^{-3}$ ,  $bsz = 384$ ) yields an actual validation loss of 2.0776, which is remarkably close to the true optimal loss of 2.0762 marked by the red star (the relative error is only 0.067%). This demonstrates that the discovered law is accurate enough to analytically derive near-optimal hyperparameters for a significantly larger scenario, validating its predictive power in practice.

#### 4.5 APPLICATION II: LLM SELECTION FOR FINE-TUNING

Another challenge for LLM practitioners as discussed in Lin et al. (2024) is to select the optimal pre-trained model from model hubs like HuggingFace to fine-tune for a specific downstream task. While fine-tuning all candidates on the full dataset and select the best in hindsight is optimal, it is often prohibitively expensive. A more practical strategy is to predict final performance based on small-scale experiments. Following the methodology of Lin et al. (2024), we fit the sft law on a small data subset (6.25% of the full dataset) and use it to predict performance on the full dataset, and select the best model based on the prediction.

We evaluate the law discovered by SLDAgent (as presented in Sec. 4.3) against five established baseline laws from Zeng et al. (2025): (1) RectifiedLaw (Lin et al., 2024) uses the sft scaling law discovered by human; (2) NLPMetrics (Yang et al., 2022) uses proxy generalization metrics for selection; (3) SubTuning, uses the performance of subset fine-tuning as selection score; (4) ModelSize uses logarithm of the number of model parameters for selection; (5) ZeroShot uses the zero-shot performance on the task data as predictor. LLM selection performance is measured using two metrics from Lin et al. (2024): the **Pearson correlation coefficient (PearCorr)**, which assesses a method’s ability to rank models correctly, and **Relative Accuracy (RelAcc)**, which measures the performance of the selected model relative to the best and worst possible choices.

Table 4 shows the results of selecting the optimal LLM from 14 candidates on 3 downstream datasets. On average, SLDAgentLaw achieves a perfect RelAcc of 100.0%, indicating it consistently identifies the best-performing LLM across all three datasets, while other methods cannot. Furthermore, it obtains the highest average PearCorr of 87.7%. This highlights its superior ability to accurately rank the candidate models, and show that SLDAgent can discover robust laws for computationally efficient model selection.

## 5 RELATED WORKS AND DISCUSSIONS

**Coding agents and benchmarks.** Terminal-centric coding agents combine program synthesis with execution, editing, package management, and verification loops (Yang et al., 2024; Wang et al., 2024), enabling end-to-end automation of development workflows and research pipelines. Recent systems (e.g., Codex (OpenAI, 2025), OpenHands (Wang et al., 2025), Claude Code (Anthropic, 2025), Gemini CLI (Google, 2025)) have shifted from single-shot code generation to iterative, tool-augmented planning and self-repair (Shinn et al., 2023), handling tasks such as dependency resolution, unit-test authoring, and regression debugging (Google, 2025; Yang et al., 2024; Hu et al., 2025). In parallel, agentic benchmarks have moved beyond toy problems toward realistic, repo-level software engineering (e.g., SWE-bench and variants) (Jimenez et al., 2024a; OpenAI, 2024; Jimenez et al., 2024b; Rashid et al., 2025), ML engineering and MLOps tasks (e.g., MLE-oriented suites) (Chan et al., 2025; Huang et al., 2024), and algorithm-optimization settings that require measurement-driven iteration (Romera-Paredes et al., 2024; Mankowitz et al., 2023; Sun et al., 2025). Our benchmark follows this trend: it evaluates agents that plan, execute, and reflect inside a

real development environment (All Hands AI, 2025; Jimenez et al., 2025) and targets problems of interest to both industry practitioners and the scientific community rather than synthetic puzzles.

**Scientific discovery AI.** A growing body of work positions AI agents as assistants—or even autonomous actors—in the scientific discovery loop, as surveyed by recent overviews on scientific AI and self-evolving agents (Gridach et al., 2025; Gao et al., 2025b). Early benchmarks in this area largely emphasized Kaggle-style machine learning engineering (e.g., ScienceAgentBench, MLE-bench) (Chen et al., 2025b; Chan et al., 2024a), while another line of work assessed reproducibility and research implementation fidelity (e.g., PaperBench, nanoGPT reproduction tasks) (Starace et al., 2025; Zhao et al., 2025). Closest to our setting is LLM-SRBench, which probes symbolic regression for disciplines such as physics, materials, and biology, typically using synthetic or augmented datasets (Shojaee et al., 2025). Our work differs in both settings and domain: we introduce SLD with explicit control groups, emphasize open-ended, real-world experimental traces over synthetic datasets with pre-known laws, and firstly unlock symbolic regression in the domain of AI research.

**Neural scaling law.** Scaling laws have been documented across multiple axes such as: (i) *model-size scaling*, where pretraining loss and downstream accuracy follow smooth power-law trends as parameters increase (Kaplan et al., 2020; Hestness et al., 2017; Henighan et al., 2020); (ii) *data scaling*, in which performance improves predictably with the number and diversity of tokens (Kaplan et al., 2020; Hestness et al., 2017); (iii) *transfer scaling*, where downstream performance scales with pretraining compute and data even with modest fine-tuning (Hernandez et al., 2021); (iv) *in-context learning scaling*, where few-shot and reasoning capabilities strengthen with both parameter count and pretraining tokens (Brown et al., 2020); (v) *emergent or phase-transition effects*, in which discrete capabilities appear once scale thresholds are crossed (Wei et al., 2022); (vi) *alignment and preference-model scaling*, tying instruction-following and RLHF quality to the volume and quality of preference data (Ouyang et al., 2022; Bai et al., 2022); (vii) *multimodal scaling*, extending power-law trends to vision–language models and revealing data-mixture sensitivity (Radford et al., 2021; Jia et al., 2021; Alayrac et al., 2022); (viii) *architecture-aware scaling*, including sparse MoE regimes that increase effective capacity at similar training FLOPs (Fedus et al., 2021); and (ix) *data quality scaling*, which quantifies how higher-quality or more task-aligned tokens substitute for raw volume in determining performance (Penedo et al., 2023; Gadre et al., 2023). This is an actively grown up area and SLDBench will continue to collect more tasks in the future.

## 6 DISCUSSION, LIMITATIONS, AND FUTURE WORK

Our work presents a significant step toward agent-based scientific discovery, but it has limitations that chart the course for future research: (1) SLDBench currently comprises seven scenarios; while these cover varied difficulties, we will expand the benchmark with additional tasks to probe a broader spectrum of scientific challenges; we will also adjust the seen/unseen split for existing scenarios to make more challenging extrapolation settings for SLDBench. (2) The present iteration emphasizes rediscovering known scaling laws, serving primarily as capability verification; a pivotal next step is applying our framework to real-world, unexplored datasets to facilitate genuine novel discoveries. (3) While SLDAgent’s specialized design gives it a clear advantage on SLDBench, this raises an important question for future work: can general-purpose agents learn to specialize? An exciting research direction would be to investigate methods for these agents to autonomously adapt or learn from the trajectories of successful specialized agents like SLDAgent.

In summary, this paper addresses the challenge of scaling law discovery, a traditionally slow and human-driven process critical for AI development, by investigating whether an AI agent can automate this open-ended scientific task. We introduce SLDBench, the first comprehensive benchmark for this problem, and propose SLDAgent, a novel, evolution-based agent designed to navigate the vast symbolic search space of potential formulas. Our extensive experiments demonstrate that SLDAgent achieves state-of-the-art, superhuman performance, discovering laws that are not only more accurate in extrapolation but also more conceptually sound than established human-derived formulas. By validating the practical utility of these discoveries in real-world applications and open-sourcing our framework, this work establishes a new paradigm where AI systems can autonomously understand and accelerate their own development.

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# Appendix

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## A ADDITIONAL DETAILS FOR SLDBENCH

### A.1 WHY IS SCALING LAW DISCOVERY A GOOD AGENTIC CHALLENGE?

As a novel AI-for-research task, SLD presents a compelling agentic challenge because it mirrors the open-ended process of scientific discovery. It is not merely curve fitting or black-box optimization. Instead, it demands the formation and testing of scientific hypotheses to uncover concise, mechanistic regularities that explain behavior across diverse regimes (Schmidt & Lipson, 2009).

**Vast and unstructured search space** A core difficulty lies in identifying the *unknown functional form* of the law. Unlike standard regression where a model family (e.g., linear, polynomial) is pre-specified, symbolic discovery requires searching over a space of expression trees whose size and structure are unknown *a priori*. Even with a modest set of operators like  $\{+, -, \times, \div, \text{pow}, \log, \exp\}$  and a few variables, the number of candidate expressions grows combinatorially, rendering exhaustive search computationally infeasible. This intractability motivates the use of heuristic, evolutionary, or sparsity-driven methods (Koza, 1992; La Cava et al., 2021; Brunton et al., 2016). Consequently, the problem becomes one of *guided exploration* rather than passive estimation.

**Abstraction across diverse contexts** SLDBench elevates this challenge beyond typical symbolic regression (Udrescu & Tegmark, 2020) by requiring the discovery of a *universal* law that generalizes across multiple experimental contexts, each defined by a set of control variables (Sec. 2). In the `sft` task, for example, the goal is not to produce 42 independent scaling laws for the 42 experimental groups, but to discover a single, unifying functional form whose parameters  $\theta_j$  adapt to each specific context  $j$ . Achieving such invariance requires a high level of abstraction (Russeil et al., 2024).

**Emulation of the scientific method** The discovery process is inherently iterative, mirroring the scientific method’s cycle of hypothesize → test → revise. Different workflows are possible (SLDAgent in Sec. 3 is just a proof-of-concept instantiation instead of an necessarily nor sufficiently optimal workflow); an effective agent can involve exploratory data visualization, symbolic analysis (e.g., examining derivatives and asymptotic behavior) to prune the search space, and rigorous validation (e.g., held-out tests or cross-validation) to prevent overfitting (Udrescu & Tegmark, 2020; Brunton et al., 2016). LLMs can act as *judges* to evaluate the plausibility of candidate expressions with high fidelity to human preferences when properly prompted and calibrated (Zheng et al., 2023; Liu et al., 2023). Altogether, scaling law discovery demands agentic strategies that integrate search, reasoning, experimental design, and self-evaluation, much like a real scientist.

## A.2 SEEN AND UNSEEN DATASET SPLITS

We partition each SLDBench dataset into **seen** (training) and **unseen** (test) splits. The unseen split is designed to probe **extrapolation**—generalization beyond the training distribution—typically toward larger models, bigger datasets, or more extreme hyperparameters. Below we detail the split protocol for each of the seven tasks in a unified format.

### A.2.1 D\_CONSTRAIN: DATA-CONSTRAINED SCALING LAW

- **Split:** 161 seen / 21 unseen.
- **Dimensions:** model parameters  $N$ , training tokens  $D$ , unique tokens  $U$ .
- **Unseen selection rule:** include all points with the largest and second-largest values of either  $N$  or  $D$ ; remaining coordinates follow the original dataset specification.

### A.2.2 DOMAIN\_MIX: DOMAIN MIXTURE SCALING LAW

- **Split:** 80 seen / 24 unseen.
- **Dimensions:** five domain proportions; four model sizes (70M, 160M, 305M, 410M).
- **Unseen selection rule:** we follow the original repository for the seen/unseen split.

### A.2.3 LR\_AND\_BSZ: LEARNING RATE & BATCH SIZE SCALING LAW

- **Split:** 2,702 seen / 117 unseen.
- **Dimensions:** learning rate  $l$ , batch size  $b$ ; model size  $N$ ; data size  $D$ .
- **Unseen selection rule:** restrict to experiments using the largest model (1B parameters) and largest dataset (100B tokens).

### A.2.4 MOE: MIXTURE-OF-EXPERTS SCALING LAW

- **Split:** 193 seen / 28 unseen.
- **Dimensions:** dense (non-expert) parameters; number of experts; routing-related settings.
- **Unseen selection rule:** include models with  $1.31 \times 10^9$  dense parameters, exceeding the training maximum of  $3.68 \times 10^8$  ( $\approx 3.6 \times$  larger); other MoE factors match seen ranges.

### A.2.5 SFT: SUPERVISED FINE-TUNING SCALING LAW

- **Split:** 504 seen / 42 unseen (42 base-model / dataset pairs).
- **Dimensions:** SFT data size per base-model/dataset pair.
- **Unseen selection rule:** for each pair, unseen uses a fixed larger size of  $8.19 \times 10^5$  tokens; seen covers up to  $4.10 \times 10^5$  tokens. This yields a clean  $\approx 2 \times$  data-scale extrapolation.

### A.2.6 VOCAB\_SIZE: VOCABULARY SCALING LAW

- **Split:** 1,080 seen / 120 unseen.
- **Dimensions:** vocabulary size  $V$ ; model size.
- **Unseen selection rule:** fix  $V = 96,300$ , outside the seen range [4,100, 64,500] (a  $\approx 1.5 \times$  increase over the seen maximum); model sizes follow the seen set.

### A.2.7 PARALLEL: PARALLEL SCALING LAW

- **Split:** 36 seen / 12 unseen (two data groups: `pile`, `stack`).
- **Dimensions:** model size; degree of parallelism.
- **Unseen selection rule:** use only 8-way parallelism (seen covers 1–4 way), i.e., a  $2 \times$  extrapolation in parallel degree; combine with larger model sizes to assess joint scaling.

### A.3 HUMAN DISCOVERED LAWS IN PRIOR LITERATURE

1. **Parallel Scaling Law** (Chen et al., 2025a):

$$L(N, P) = E + \frac{A}{(N(1 + \kappa \log P))^\alpha}$$

*Explanation.*

- *Variables.*  $N$  is the non-embedding parameter count;  $P$  is the number of parallel streams (independent input transformations run in parallel and then aggregated);  $A, E, \alpha, \kappa > 0$  are fit parameters.
- *Intuition.* Increasing  $P$  reuses the same backbone while adding learnable, diverse prefixes and a dynamic aggregator over the  $P$  outputs, yielding diminishing-returns gains that are *logarithmic* in  $P$ . Empirically, this behaves like scaling the effective parameters by  $O(\log P)$  at much lower memory-latency growth than classic parameter scaling; larger  $N$  benefits more from parallelism. Reported fits show a positive “parallel benefit” coefficient on Stack-V2 (Python) vs. Pile, consistent with stronger improvements on reasoning-heavy data.

2. **Vocabulary Scaling Law** (Tao et al., 2024)<sup>1</sup>:

$$L(N, V, D) = \frac{A}{N^\alpha} + \frac{B}{V^\beta} + \frac{C}{D^\gamma} + E$$

*Explanation.*

- *Variables.*  $N$  is the non-vocabulary (non-embedding) parameter count;  $V$  is the tokenizer vocabulary size;  $D$  is the training budget (characters/tokens, depending on the normalization in use);  $A, B, C, E$  and exponents  $\alpha, \beta, \gamma > 0$  are fit parameters.
- *Intuition.* This extends the standard Chinchilla-style decomposition by adding a vocabulary term: larger  $V$  reduces tokenization loss (fewer characters per token / better segmentation) but increases embedding difficulty; the fitted  $B/V^\beta$  term captures the net effect under a fixed compute budget. To compare across different  $V$ , they propose a unigram-normalized loss that removes the trivial dependence of cross-entropy on  $V$ .

3. **SFT Scaling Law** (Lin et al., 2024; Zeng et al., 2025):

$$L(D) = \frac{A}{D^\alpha + B} + C$$

*Explanation.*

- *Variables.*  $D$  is the fine-tuning set size;  $A, \alpha, B, C$  are fit from a few size-loss pairs.
- *Intuition.* Fine-tuning exhibits a *pre-power* phase at small  $D$  and a *power* phase at larger  $D$ . The offset  $B$  (often interpreted as a “pre-learned data size”) rectifies the classic power law so one function captures both phases and saturates to  $C$  as  $D \rightarrow \infty$ .

4. **Domain Mixture Scaling Law** (Ye et al., 2024):

$$L_i(\mathbf{r}) = c_i + k_i \exp\left(\sum_{j=1}^M t_{ij} r_j\right)$$

*Explanation.*

- *Variables.*  $\mathbf{r} = (r_1, \dots, r_M)$  are training mixture proportions over  $M$  domains (sum to 1);  $L_i$  is loss on validation domain  $i$ ;  $c_i$  is irreducible loss;  $k_i$  scales the reducible part;  $t_{ij}$  encodes how training domain  $j$  transfers to validation domain  $i$ .
- *Intuition.* Loss varies smoothly and predictably with mixture shares; an exponential of a linear form balances synergy ( $t_{ij} > 0$ ), neutrality ( $\approx 0$ ), or conflict ( $< 0$ ). Because  $r_j \in [0, 1]$  and mixtures interpolate observed settings, simple forms fit well and allow mixture optimization without running all mixtures.

<sup>1</sup>**Remark:** Models trained with larger  $V$  naturally have a higher loss. Unigram-normalized loss (Tao et al., 2024) is thus designed to normalize the loss w.r.t.  $V$  to ensure a fair comparison across different  $V$ .

**5. Mixture-of-Experts (MoE) Scaling Law (Krajewski et al., 2024):**

$$\log L(N, E) = a \log N + b \log \hat{E} + c \log N \log \hat{E} + d, \quad \text{where } \frac{1}{\hat{E}} = \frac{1}{E-1+\left(\frac{1}{E_{\text{start}}}-\frac{1}{E_{\text{max}}}\right)^{-1}} + \frac{1}{E_{\text{max}}}$$

*Explanation.*

- *Variables.*  $N$  is non-embedding parameter count;  $E$  is the number of experts (or expansion rate);  $\hat{E}$  is an *effective* experts transform that captures diminishing returns at very small/very large  $E$ ;  $a, b, c, d$  are fit coefficients.
- *Intuition.* In MoE, increasing  $E$  at fixed active parameters improves specialization/routing up to saturation. The interaction  $c \log N \log \hat{E}$  models that the marginal value of more experts depends on  $N$  (and vice versa). Related analyses additionally introduce *granularity* to control expert size and show consistent power-law behavior with respect to it.

**6. Data-Constrained Scaling Law (Muennighoff et al., 2023):**

$$L(N, D, U) = E + \frac{A}{\left(U_N + U_N R_N^* \left(1 - \exp\left(-\frac{U_N^N - 1}{R_N^*}\right)\right)\right)^\alpha} + \frac{B}{\left(U + U R_D^* \left(1 - \exp\left(-\frac{D - 1}{R_D^*}\right)\right)\right)^\beta}$$

*Explanation.*

- *Variables.*  $U$  is the budget of *unique* tokens (data constraint);  $D$  is total seen tokens (epochs  $\times U$ );  $N$  is parameters.  $U_N$  denotes the compute-optimal parameter count for  $U$  unique tokens (the “base” capacity).  $R_D^*$  and  $R_N^*$  are learned “repeat” hyperparameters controlling how quickly returns decay when repeating data or over-sizing the model, respectively;  $A, B, E, \alpha, \beta$  are fit constants.
- *Intuition.* Replace raw  $D$  and  $N$  with *effective* data/parameters using an exponential half-life: repeated tokens become progressively less valuable, and parameters far beyond the data budget have diminishing returns. When  $D=U$  and  $N=U_N$ , this reduces to the standard Chinchilla-style form.

We refit these laws using their expressions and optimization methods on the seen split of data to produce the results in Tables 2 and 3.

#### A.4 HUMAN DISCOVERY OF LEARNING-RATE/BATCH-SIZE SCALING LAW

For the `lr_and_bsz` task—predicting pre-training loss as a function of learning rate, batch size, dataset size, and network size—we are not aware of an established scaling law in the literature. To construct a strong human baseline, we asked two human experts to derive a predictive law from the same background information and the same training split as our agents.

## Participants.

- **Expert A:** junior master's student in economics with experience in econometrics.
  - **Expert B:** senior robotics master's student with a strong background in machine learning and Python programming.

We evaluated both laws on an unseen test set. Expert A obtained  $R^2 = -1.1019$ , while Expert B obtained  $R^2 = -0.0756$ . (Recall that  $R^2 < 0$  indicates worse-than-mean prediction on the test set.) We therefore adopt Expert B's formulation as the *canonical Human baseline* reported in Tables 2 and 3.

**Selected Law (Expert B).** Expert B proposes a hierarchical additive model

$$L(D, N, \ell, b) = \frac{A}{D^\alpha} + \frac{B}{N^\beta} + C + K_\ell (\ell - \ell_0)^2 + E \left( \log b + \frac{b_0}{b} \right), \quad (1)$$

with scale-dependent optima

$$\ell_0 = F N^\gamma D^\zeta, \quad b_0 = G D^\eta. \quad (2)$$

Here  $D$  is dataset size,  $N$  is non-embedding parameter count,  $\ell$  is the learning rate, and  $b$  is the batch size. (To avoid a symbol clash with the dataset size  $D$ , we write  $K_\ell$  for the code parameter `D_1r`.)

### *Why this form is sensible.*

- **Aligned with Step Law.** The optima Eq. (2) is exactly in the same form as Step Law (Li et al., 2025), which is proved to be good in optima prediction.
  - **Power-law decay in scale.** The terms  $A/D^\alpha$  and  $B/N^\beta$  capture the empirically observed diminishing-returns regime as data and model scale increase.
  - **Quadratic well in learning rate.** The term  $K_\ell(\ell - \ell_0)^2$  yields a convex (in  $\ell$ ) basin with a unique minimizer at  $\ell^* = \ell_0$ . Because  $\ell_0$  depends on  $(D, N)$  via equation 2, the preferred step size adapts smoothly with scale; signs of  $(\gamma, \zeta)$  determine whether  $\ell_0$  increases/decreases with  $N$  and  $D$ .
  - **Batch-size with diminishing returns and small-b penalty.** The function  $\log b + b_0/b$  rises for very small  $b$  (variance-dominated regime), enjoys a unique stationary point at  $b^* = b_0$  (since  $\partial/\partial b = 1/b - b_0/b^2 = 0$ ), and then grows only logarithmically for large  $b$  (diminishing returns). The local curvature at the optimum is  $E/b_0^2 > 0$ , ensuring a well-conditioned minimum if  $E > 0$ .
  - **Interpretability and separability.** Additivity isolates contributions from data scale, model scale, step size choice, and batch sizing, making the fitted coefficients interpretable and reducing overfitting risk from high-order interactions.

**Result.** This law is our human baseline and achieved  $R^2 = -0.0756$  on the held-out test set.

```
import numpy as np

def law(input_data: list[dict[str, float]], group: str) -> list[dict[str,
←   float]]:
    """
    Predicts lm_loss using a hierarchical model.
    The loss L is a sum of terms for data size, param size, learning rate, and
    ←   batch size.
    The optimal learning rate (lr_0) and a batch size parameter (bsz_0) are
    ←   themselves
```

```

modeled as functions of data and param size.
"""
PARAMS_BY_GROUP = {
    # Parameters: [A, alpha, B, beta, C, D, E_new, F, gamma, zeta, G, eta]
    "all_data": [
        262.1391, 0.2675, 7.0285, 0.0746, 0.0000136, 1278.595,
        0.0493, 0.3242, -1.0580, 0.6498, 0.0302, 0.3503
    ],
}

params = PARAMS_BY_GROUP.get(group, PARAMS_BY_GROUP["all_data"])
A, alpha, B, beta, C, D_lr, E, F, gamma, zeta, G, eta = params
eps = 1e-12
predictions = []

for pt in input_data:
    # Input features
    l = float(pt["lr"])
    b = float(pt["bsz"])
    D = float(pt["data_size"])
    N = float(pt["non_embedding_param_size"])

    # Submodels for optimal lr and bsz offset
    l0 = F * (N ** gamma) * (D ** zeta)
    b0 = G * (D ** eta)

    # Loss components
    term_data = A * (D ** (-alpha))
    term_param = B * (N ** (-beta))
    term_lr = D_lr * (max(l, eps) - l0) ** 2
    term_bsz = E * (np.log(max(b, eps)) + b0 / max(b, eps))

    loss = term_data + term_param + C + term_lr + term_bsz
    predictions.append({"lm_loss": float(loss)})

return predictions

```

**Alternative Law (Expert A).** Expert A proposes an additive power-law backbone with (i) an explicit small-batch regularizer and (ii) a quadratic penalty in  $\log$ -learning-rate around a scale-dependent optimum:

$$L(D, N, \ell, b) = \frac{A}{D^\alpha} + \frac{B}{N^\beta} + C + D b^{-\gamma} + U(\log \ell - \log \ell_*)^2, \quad (3)$$

where

$$\ell_* = k b^{-\eta} D^{-\nu} N^{-\mu}. \quad (4)$$

*Why this form is sensible.*

- **Power-law backbone.** As in equation 1,  $A/D^\alpha$  and  $B/N^\beta$  encode diminishing returns with scale.
- **Explicit small-batch penalty.** The  $D b^{-\gamma}$  term discourages extremely small batches by making loss large as  $b \rightarrow 0$ , consistent with noise-dominated updates.
- **Log-quadratic in step size.** Penalizing  $(\log \ell - \log \ell_*)^2$  is scale-invariant: multiplicative misspecification of  $\ell$  incurs symmetric cost (over- vs. under-shooting). The optimum  $\ell^* = \ell_*$  scales as a power of  $(b, D, N)$ ; positive  $(\eta, \nu, \mu)$  would imply that larger problems favor smaller  $\ell$ , a common empirical heuristic.

**Result.** This alternative performed worse on the same test set, achieving  $R^2 = -1.1019$ .

```
import numpy as np
```

```

def law(input_data: list[dict[str, float]], group: str) -> list[dict[str,
    float]]:
    """
    Predicts lm_loss based on a power law with a quadratic term for learning
    rate deviation.
    The optimal learning rate (lr_star) is modeled as a power-law function of
    batch size, data size, and parameter count.
    """
    # Parameters: [A, alpha, B, beta, C, D, gamma, U, k, eta, nu, mu]
    PARAMS_BY_GROUP = {
        'all_data': [
            20.0, 0.1359, 20.0, 0.1573, 0.5665, 6.5206, 1.5611,
            0.0010, 0.0057, 0.00013, 0.0, 0.6780
        ],
    }

    params = PARAMS_BY_GROUP.get(group, PARAMS_BY_GROUP['all_data'])
    A, alpha, B, beta, C, D, gamma, U, k, eta, nu, mu = params
    predictions = []

    for point in input_data:
        # Input features
        l = point['lr']
        b = point['bsz']
        D = point['data_size']
        N = point['non_embedding_param_size']

        # Optimal learning rate submodel
        lr_star = k * (b ** -eta) * (D ** -nu) * (N ** -mu)

        # Loss components
        loss = (A * D**(-alpha) +
                B * N**(-beta) +
                C +
                D * b**(-gamma) +
                U * (np.log(l) - np.log(lr_star))**2)

        predictions.append({"lm_loss": loss})

    return predictions

```

**Remarks.** Both laws decompose improvements from scale (via power laws) and hyperparameter choice (via convex penalties around scale-dependent optima). Expert B's batch-size term has an interior optimum  $b^* = b_0(D)$  and grows only logarithmically for large  $b$ , which may better match the empirically slow gains from very large batches; the explicit log-domain quadratic on  $\ell$  in Expert A is appealingly scale-invariant but, in our data, did not generalize as well as the direct quadratic around  $\ell_0(D, N)$ . Future work could explore interaction terms (e.g.,  $D \times N$ ), heteroskedastic residuals, or robust losses for outlier resilience.

## B ADDITIONAL EXPERIMENTAL INFORMATION

### B.1 METRICS

Let  $y_i$  be the true value and  $\hat{y}_i$  be the predicted value for the  $i$ -th sample, for  $i = 1, \dots, n$ . The mean of the true values is denoted by  $\bar{y} = \frac{1}{n} \sum_{i=1}^n y_i$ .

**R-squared ( $R^2$ )** The coefficient of determination,  $R^2$ , measures the proportion of variance in the dependent variable that is predictable from the model. A value closer to 1 indicates a better fit.

$$R^2 = 1 - \frac{\sum_{i=1}^n (y_i - \hat{y}_i)^2}{\sum_{i=1}^n (y_i - \bar{y})^2} \quad (5)$$

**Normalized Mean Squared Error (NMSE)** This metric normalizes the mean squared error by the variance of the target values. A value closer to 0 indicates a better fit.

$$\text{NMSE} = \frac{\sum_{i=1}^n (y_i - \hat{y}_i)^2}{\sum_{i=1}^n (y_i - \bar{y})^2} \quad (6)$$

**Normalized Mean Absolute Error (NMAE)** This metric is the mean absolute error normalized by the sum of the absolute true values, providing a relative error measurement. A value closer to 0 indicates a better fit.

$$\text{NMAE} = \frac{\sum_{i=1}^n |y_i - \hat{y}_i|}{\sum_{i=1}^n |y_i|} \quad (7)$$

## B.2 NMSE AND NMAE

Table 5: Performance comparison of LLM-based agents and human-discovered laws on SLDBench. Scores represent the NMSE (Normalized Mean Squared Error) value, averaged over three successful runs. The **best** and second-best scores for each task are highlighted. “NA” indicates the agent failed to produce a valid output on all attempts.

	parallel↓	vocab_size↓	SFT↓	domain-mix↓	moe↓	d_constraint↓	lr_and_bsz↓
SLDAgent (GPT-5)	0.0003	<b>0.0134</b>	<b>0.0032</b>	0.0378	0.1302	<u>0.0319</u>	<u>0.3896</u>
SLDAgent (o4-mini)	0.0003	0.0365	<u>0.0064</u>	0.0239	0.1495	0.0386	0.8063
SLDAgent (claude-sonnet-4)	<u>0.0002</u>	<u>0.0207</u>	0.0266	<u>0.0171</u>	<u>0.0985</u>	<b>0.0308</b>	0.8287
SLDAgent (Gemini-2.5-Flash)	0.0003	0.1916	0.0326	<b>0.0130</b>	0.1639	0.1079	1.1883
CodeX (GPT-5)	0.0004	0.1541	0.0246	0.0400	<b>0.0342</b>	0.0566	<b>0.0597</b>
Human	<b>0.0001</b>	0.0336	0.0429	0.2993	0.2968	0.0894	2.1019
Grok-CLI (Grok-4)	0.0003	0.0414	0.1551	0.0411	0.1259	0.1568	5.9918
OpenHands (o4-mini)	0.0004	0.1991	0.1070	0.0351	0.4106	0.1954	1.7633
CodeX (o4-mini)	0.0004	0.1389	0.0793	0.0638	0.5324	0.1954	1.7735
OpenCode (o4-mini)	0.0004	0.1389	0.1071	0.4955	0.5323	0.1954	1.7717
Claude Code (Claude-Sonnet-4)	0.0004	0.0691	0.1054	0.0933	0.5323	0.2215	1.8409
Mini-SWE-Agent (o4-mini)	0.0004	0.1844	0.0398	0.1697	0.5323	120.7720	1.7736
Goose (o4-mini)	0.0004	0.1990	0.1776	0.1412	0.5324	0.1954	1.7735
Gemini-CLI (Gemini-2.5-Flash)	0.0004	0.1389	0.2129	1.9271	0.4097	0.1699	158.6075
Terminus (o4-mini)	0.6963	587.4283	0.0742	0.1832	0.5324	0.1943	1.7735
Aider (o4-mini)	12.6136	NA	0.2128	NA	NA	24.2679	NA

Table 6: Performance comparison of LLM-based agents and human-discovered laws on SLDBench. Scores represent the NMAE (Normalized Mean Absolute Error) value, averaged over three successful runs. The **best** and second-best scores for each task are highlighted. “NA” indicates the agent failed to produce a valid output on all attempts.

	parallel↓	vocab_size↓	SFT↓	domain-mix↓	moe↓	d_constraint↓	lr_and_bsz↓
SLDAgent (GPT-5)	0.0130	<b>0.1127</b>	<b>0.0496</b>	0.1472	<u>0.3461</u>	<b>0.2078</b>	<b>0.6471</b>
SLDAgent (claude-sonnet-4)	0.0109	<u>0.1448</u>	0.1361	<u>0.1144</u>	<b>0.3084</b>	<u>0.2160</u>	<u>0.8643</u>
SLDAgent (o4-mini)	0.0142	0.1809	<u>0.0710</u>	0.1239	0.3764	0.2583	0.9410
SLDAgent (Gemini-2.5-Flash)	0.0150	0.3570	0.1403	<b>0.0822</b>	0.3919	0.4349	1.1635
Human	<b>0.0085</b>	<u>0.1494</u>	0.1373	0.5076	0.5268	0.4420	1.5039
CodeX (GPT-5)	<u>0.0101</u>	0.2388	0.3029	0.1769	0.3955	0.4439	0.8882
Grok-CLI (Grok-4)	0.0142	0.1952	0.3283	0.1433	0.3490	0.5968	2.6987
OpenHands (o4-mini)	0.0178	0.4552	0.2936	<u>0.1223</u>	0.5654	0.4842	1.3630
Claude Code (Claude-Sonnet-4)	0.0154	0.2253	0.2681	0.2616	0.6508	0.5679	1.4144
CodeX (o4-mini)	0.0180	0.3587	0.2458	0.1999	0.6508	0.4842	1.3577
OpenCode (o4-mini)	0.0178	0.3587	0.2935	0.4102	0.6508	0.4842	1.3584
Goose (o4-mini)	0.0178	0.4554	0.3699	0.3725	0.6508	0.4842	1.3577
Mini-SWE-Agent (o4-mini)	0.0178	0.4310	0.1687	0.4125	0.6508	11.3928	1.3577
Terminus (o4-mini)	0.4886	18.3605	0.2341	0.4324	0.6508	0.4816	1.3577
Gemini-CLI (Gemini-2.5-Flash)	0.0179	0.3587	0.4081	0.4846	<u>0.5525</u>	0.5862	16.7311
Aider (o4-mini)	3.4579	NA	0.4082	NA	NA	7.5139	NA

### B.3 STANDARD DEVIATION FOR SLDAgent

Table 7: Standard deviation of NMSE on SLDBench for SLDAgent variants across seven tasks. Lower values indicate more stable performance.

	parallel	vocab_size	SFT	domain-mix	moe	d constrain	lr_and_bsz
SLDAgent (claude-sonnet-4)	0.0001	0.0065	0.0036	0.0033	0.0076	0.0289	0.0716
SLDAgent (GPT-5)	0.0001	0.0039	0.0013	0.0403	0.0381	0.0175	0.1310
SLDAgent (o4-mini)	0.0000	0.0197	0.0015	0.0150	0.0331	0.0263	0.3455
SLDAgent (Gemini-2.5-Flash)	0.0001	0.2942	0.0184	0.0002	0.0107	0.0782	0.3619

Table 8: Standard deviation of NMAE on SLDBench for SLDAgent variants across seven tasks. Lower values indicate more stable performance.

	parallel	vocab_size	SFT	domain-mix	moe	d constrain	lr_and_bsz
SLDAgent (claude-sonnet-4)	0.0026	0.0249	0.0149	0.0199	0.0170	0.1305	0.0945
SLDAgent (GPT-5)	0.0029	0.0091	0.0077	0.0860	0.0379	0.0856	0.1437
SLDAgent (Gemini-2.5-Flash)	0.0020	0.2200	0.0335	0.0001	0.0086	0.1997	0.1723
SLDAgent (o4-mini)	0.0000	0.0561	0.0095	0.0513	0.0390	0.1169	0.2637

Table 9: Standard deviation of  $R^2$  on SLDBench for SLDAgent variants across seven tasks. Lower values indicate more stable performance.

	parallel	vocab_size	SFT	domain-mix	moe	d constrain	lr_and_bsz
SLDAgent (claude-sonnet-4)	0.0001	0.0065	0.0036	0.0033	0.0076	0.0289	0.0716
SLDAgent (GPT-5)	0.0001	0.0039	0.0013	0.0403	0.0381	0.0175	0.1310
SLDAgent (o4-mini)	0.0000	0.0197	0.0015	0.0150	0.0331	0.0263	0.3455
SLDAgent (Gemini-2.5-Flash)	0.0001	0.0631	0.0184	0.0002	0.0107	0.0782	0.3619

## B.4 SLDAGENT DISCOVERED LAWS

### B.4.1 THE LAW USED FOR HYPERPARAMETER OPTIMIZATION

```

import numpy as np
def scaling_law_func(data_points, params):
    """
    Predict LM loss via a richer log-linear scaling law with quadratic and
    interaction terms.

    Inputs:
        data_points: array of shape (N,4) with strictly positive columns
                     [lr, bsz, data_size, non_embedding_param_size]
        params:      length-11 array

    Returns:
        preds: array of shape (N,) of predicted lm_loss values
    """
    X = np.asarray(data_points, dtype=float)
    if X.ndim != 2 or X.shape[1] != 4:
        raise ValueError(f"Expected data_points of shape (N,4), got {X.shape}")
    if np.any(X <= 0):
        raise ValueError("All input features must be strictly positive for log
                         ← transforms.")

    # unpack and log-transform
    lr   = X[:, 0]
    bsz  = X[:, 1]
    D    = X[:, 2]
    P    = X[:, 3]
    l_lr = np.log(lr)
    l_b  = np.log(bsz)
    l_D  = np.log(D)
    l_P  = np.log(P)

    beta = np.asarray(params, dtype=float).ravel()
    if beta.shape[0] != Z.shape[1]:
        raise ValueError(f"Expected {Z.shape[1]} params, got {beta.shape[0]}")

    log_pred = Z.dot(beta)           # (N,)
    return np.exp(log_pred)

def fit_scaling_law(data_points, loss_values):
    X = np.asarray(data_points, dtype=float)
    y = np.asarray(loss_values, dtype=float).ravel()

    if X.ndim != 2 or X.shape[1] != 4:
        raise ValueError(f"Expected data_points of shape (N,4), got {X.shape}")
    if y.ndim != 1 or X.shape[0] != y.shape[0]:
        raise ValueError("Number of loss values must match number of data
                         ← points.")
    if np.any(X <= 0) or np.any(y <= 0):
        raise ValueError("All inputs and losses must be strictly positive for
                         ← log transforms.")

    # compute logs
    l_lr = np.log(X[:, 0])
    l_b  = np.log(X[:, 1])
    l_D  = np.log(X[:, 2])
    l_P  = np.log(X[:, 3])
    l_y  = np.log(y)

    # assemble design matrix
    Z = np.column_stack([

```

```

        np.ones_like(l_lr),
        l_P,
        l_D,
        l_b,
        l_lr,
        l_lr**2,
        l_b**2,
        l_lr * l_b,
        l_P * l_D,
        l_P * l_b,
        l_D * l_lr
    ]) # shape (N,11)

# ridge least-squares
P = Z.shape[1]
lambda_reg = 1e-6
A = Z.T.dot(Z) + lambda_reg * np.eye(P)
b = Z.T.dot(l_y)
beta = np.linalg.solve(A, b) # (11,)

return beta

```

#### B.4.2 THE LAW USED FOR LLM SELECTION

```

import numpy
def scaling_law_func(data_points, params):
    """
    Predicts fine-tuning loss L from data size D using a 4-parameter shifted
    ← power law:
    L(D) = B + A * (D + D0)^(-alpha)

    params = [logA, log_alpha, logD0, B]
    - A      = exp(logA)      > 0
    - alpha = exp(log_alpha) > 0
    - D0    = exp(logD0)     >= 0
    - B      = 0               (irreducible plateau)
    """
    D = np.asarray(data_points, dtype=float).ravel()
    p = np.asarray(params, dtype=float).ravel()
    if p.size != 4:
        raise ValueError(f"Expected 4 parameters, got {p.size}")
    logA, log_alpha, logD0, B = p
    A = np.exp(logA)
    alpha = np.exp(log_alpha)
    D0 = np.exp(logD0)
    return B + A * np.power(D + D0, -alpha)

def fit_scaling_law(data_points, loss_values):
    """
    Fits the 4-parameter shifted power law using optimization.
    """
    D = np.asarray(data_points, dtype=float).ravel()
    y = np.asarray(loss_values, dtype=float).ravel()

    # Numeric safeguards
    eps = 1e-8
    y = np.maximum(y, eps)
    D = np.maximum(D, eps)

    # Initialize parameters
    y_min, y_max = np.min(y), np.max(y)
    y_range = max(y_max - y_min, eps)

    # Initialize plateau B0 just below minimum observed loss

```

```

B0 = max(0.9 * y_min, eps)

# Small horizontal shift for stability
D0_0 = max(0.1 * np.min(D), eps)

# Estimate A and alpha via log-log fit
y_shift = y - B0
y_shift = np.clip(y_shift, eps, None)
valid = (D > 0) & (y_shift > 0)

if np.sum(valid) >= 2:
    logD = np.log(D[valid] + D0_0)
    logY = np.log(y_shift[valid])
    slope, intercept = np.polyfit(logD, logY, 1)
    alpha0 = max(-slope, eps)
    A0      = max(np.exp(intercept), eps)
else: # Fallback guesses
    alpha0 = 0.5
    A0      = max(y_max - B0, eps)

p0 = np.array([np.log(A0), np.log(alpha0), np.log(D0_0), B0], dtype=float)

def objective(p):
    pred = scaling_law_func(D, p)
    if np.any(pred <= 0):
        return np.inf
    r_lin = (pred - y) / y_range
    r_log = np.log(pred + eps) - np.log(y + eps)
    err = r_lin**2 + r_log**2
    return np.mean(err)

bounds = [(-20.0, 20.0), (-10.0, 10.0), (-20.0, 20.0), (0.0, y_min)]

res = minimize(
    objective, p0, method="L-BFGS-B", bounds=bounds,
    options={"ftol":1e-9, "gtol":1e-9}
)

return res.x if res.success else p0

```

## B.5 CLOSED-FORM OPTIMIZER FOR $lr$ AND $bsz$

**Model.** Let  $N$  be the non-embedding parameter count and  $D$  the token count. For  $x = \ln(lr)$  and  $y = \ln(bsz)$ , the SLDAgent law for the *log loss* can be written (constants w.r.t.  $(x, y)$  are grouped into  $C(N, D)$ ):

$$\log(\text{loss}) = C(N, D) + \beta_3 y + \beta_4 x + \beta_5 x^2 + \beta_6 y^2 + \beta_7 xy + \beta_9 (\ln N) y + \beta_{10} (\ln D) x. \quad (8)$$

Minimizing loss is equivalent to minimizing the quadratic form above in  $(x, y)$ .

**First-order optimality.** Setting partial derivatives to zero gives a  $2 \times 2$  linear system:

$$\frac{\partial}{\partial x} : \quad \beta_4 + 2\beta_5 x + \beta_7 y + \beta_{10} \ln D = 0, \quad (9)$$

$$\frac{\partial}{\partial y} : \quad \beta_3 + \beta_7 x + 2\beta_6 y + \beta_9 \ln N = 0. \quad (10)$$

In matrix form,

$$\underbrace{\begin{bmatrix} 2\beta_5 & \beta_7 \\ \beta_7 & 2\beta_6 \end{bmatrix}}_H \begin{bmatrix} x \\ y \end{bmatrix} = - \begin{bmatrix} \beta_4 + \beta_{10} \ln D \\ \beta_3 + \beta_9 \ln N \end{bmatrix}.$$

When  $\Delta = \det(H) = 4\beta_5\beta_6 - \beta_7^2 > 0$  (true for our fitted model),  $H$  is positive-definite and the minimizer is unique.

**Closed-form solution.** Solving the system yields

$$x^* = \frac{-2\beta_6(\beta_4 + \beta_{10} \ln D) + \beta_7(\beta_3 + \beta_9 \ln N)}{4\beta_5\beta_6 - \beta_7^2}, \quad (11)$$

$$y^* = \frac{\beta_7(\beta_4 + \beta_{10} \ln D) - 2\beta_5(\beta_3 + \beta_9 \ln N)}{4\beta_5\beta_6 - \beta_7^2}. \quad (12)$$

Therefore,

$$\boxed{lr^* = f_1(N, D) = \exp(x^*)}, \quad \boxed{bsz^* = f_2(N, D) = \exp(y^*)}.$$

**Instantiation with fitted coefficients.** The fitted parameters are

$\beta_3 = 0.0595$ ,  $\beta_4 = 0.1906$ ,  $\beta_5 = 0.0098$ ,  $\beta_6 = 0.0073$ ,  $\beta_7 = -0.006$ ,  $\beta_9 = -0.0089$ ,  $\beta_{10} = -0.0012$ , giving  $A = 2\beta_5 = 0.0196$ ,  $B = \beta_7 = -0.006$ ,  $C = 2\beta_6 = 0.0146$ , and  $\Delta = AC - B^2 = 0.00025016$ . Expanding the solution,

$$x^* = \frac{-C\beta_4 + B\beta_3}{\Delta} + \frac{-C\beta_{10}}{\Delta} \ln D + \frac{B\beta_9}{\Delta} \ln N = -12.5510 + 0.070035 \ln D + 0.213463 \ln N, \quad (13)$$

$$y^* = \frac{B\beta_4 - A\beta_3}{\Delta} + \frac{B\beta_{10}}{\Delta} \ln D - \frac{A\beta_9}{\Delta} \ln N = -9.23329 + 0.028782 \ln D + 0.697314 \ln N. \quad (14)$$

**Numeric example.** For  $N = 1,073,741,824$  and  $D = 10^{11}$  tokens,  $\ln N \approx 20.7944$ ,  $\ln D \approx 25.3284$  and thus

$$lr^* = \exp(x^*) \approx 1.7673 \times 10^{-3}, \quad bsz^* = \exp(y^*) \approx 401.79.$$

In practice we pick the nearest admissible grid point for training; in our experiment this is  $(lr, bsz) = (1.953 \times 10^{-3}, 384)$ .

## B.6 BASELINE AGENT CONFIGS

We standardize installation sources and, when available, pin versions to those obtainable on **2025-09-03**. Unless noted, we use upstream defaults and recommended settings.

- **OpenHands** ([Wang et al., 2025](#)): *source* All-Hands-AI/OpenHands; *channel* GitHub Release; *version* 0.55.0 (released 2025-09-02).
- **Claude Code** ([Anthropic, 2025](#)): *package* @anthropic-ai/cl Claude-code; *channel* npm; *version* 1.0.102 (published 2025-09-03).
- **Codex** ([OpenAI, 2025](#)): *source* openai/codex; *channel* GitHub Release (also available via Homebrew/npm); *version* 0.29.0 (released 2025-09-03 08:32).
- **OpenCode** ([sst, 2025](#)): *source* opencode-ai/opencode; *channel* GitHub Release; *version* v0.0.55 (released 2025-07-29).
- **Mini-SWE-Agent** ([SWE-agent Team, 2025](#)): *source* All-Hands-AI/mini-swe-agent; *channel* GitHub Release / PyPI; *version* v1.10.0 (released 2025-08-29).
- **Grok-CLI** ([Superagent AI, 2025](#)): *source* superagent-ai/grok-cli; *channel* GitHub Release / npm scope @vibe-kit/grok-cli; *version* 0.0.19 (released 2025-08-04).
- **Gemini-CLI** ([Google, 2025](#)): *source* google-gemini/gemini-cli; *channel* GitHub Releases (stable); *version* v0.3.0 (stable) as of 2025-09-03 (the stable tag from Tue 2025-09-02).

## B.7 ABLATION STUDY

We conduct two key ablation studies to validate the design choices of SLDAgent: one on the co-optimization of the symbolic expression and the fitting procedure, and another on the impact of the number of evolutionary iterations.

**Co-optimization of Expression and Fitting Procedure.** A key novelty of SLDAgent is its co-optimization of two distinct subroutines: the symbolic Expression defining the law and the Optimization routine for fitting its parameters. To assess the importance of this design, we compare it against an ablation, **SLDAgent-law-only**, which only evolves the law’s expression while using a fixed, standard BFGS optimizer. This setup is analogous to recent methods like LLMSR, which also decouples expression generation from a fixed optimization step.

The results in Table 10 clearly demonstrate the superiority of the co-optimization approach. The full SLDAgent achieves a higher average  $R^2$  score (0.848 vs. 0.775) and outperforms the law-only variant on five of the seven tasks. The most significant difference is observed on the challenging `lr_and_bsz` task, where SLDAgent-law-only fails ( $R^2 = -0.224$ ), while the full SLDAgent finds a reasonably predictive law ( $R^2 = 0.194$ ). This highlights that discovering a good symbolic expression is only half the battle; tailoring the optimization procedure to the specific law and data is critical for success.

Table 10: Performance comparison of the full SLDAgent against an ablation that only evolves the law’s expression (“SLDAgent-law-only”), using the o4-mini model. Scores are  $R^2$  averaged over three runs. The **best** scores for each task are bolded.

	parallel↑	vocab_size↑	SFT↑	domain-mix↑	moe↑	d_constraint↑	lr_and_bsz↑	Avg. $R^2$ ↑
SLDAgent-law-only (o4-mini)	<b>1.000</b>	0.947	0.931	0.966	0.835	0.952	-0.224	0.775
SLDAgent (o4-mini)	<b>1.000</b>	<b>0.964</b>	<b>0.994</b>	<b>0.976</b>	<b>0.851</b>	<b>0.961</b>	<b>0.194</b>	<b>0.848</b>

**Effect of Evolutionary Iterations.** We set the number of evolutionary iterations to 50 as a trade-off between computational cost and performance. To analyze the agent’s behavior over time, we visualize the best  $R^2$  score found at each iteration for tasks of varying difficulty: `parallel` (easy), `moe` (medium), and `lr_and_bsz` (hard).

As shown in Figure 4, the performance trends confirm the robustness of our evolutionary approach. For the easy `parallel` task, the agent quickly discovers a near-perfect solution. For the `moe` task, performance improves steadily and stabilizes at a high  $R^2$  value. On the most difficult `lr_and_bsz` task, the score is volatile in early iterations but trends consistently upward as the agent explores the solution space. Crucially, in all cases, performance either improves or plateaus without any signs of degradation, indicating that the evolutionary search does not overfit to the seen data even without a dedicated validation set. This justifies that 50 iterations provide a sufficient budget for the agent to converge toward effective solutions.

## C PROMPTS

### C.1 TASK INSTRUCTION

```
You have to analyze an experimental dataset to discover the underlying
↳ mathematical relationship, or "scaling law," that governs it.
The dataset is located at `/app/data` and can be loaded using
↳ `datasets.load_from_disk()`.

The experimental context: {problem_statement}
You must produce two files:
1. A Python function in `/app/law.py`
This file must contain a single function named `law` with the following
↳ signature. This function should embody your discovered mathematical formula.

```python
def law(input_data: list[dict[str, float]], group: str) -> list[dict[str,
↳ float]]:
    """
    Predicts output variables based on input variables according to a discovered
    ↳ scaling law.

    Args:
        input_data: A list of dictionaries, where each dictionary is a single
        ↳ data
            point containing input variable names as keys and their
            corresponding values.
        group: The name of the experimental group for which to make predictions.
            The functional form of the law must be the same for all groups,
            but the constant parameters/coefficients can differ per group.

    Returns:
        A list of dictionaries, corresponding to the input_data list, with each
        dictionary containing the predicted output variable(s).
    """
    # Your implementation here
    pass
```

2. A detailed explanation in `/app/explain.md`
This Markdown file should explain:
* The mathematical formula you discovered.
* Your reasoning and the methodology used to find the law.
* The fitted values of the parameters/coefficients for each distinct group.

Your submitted `law` function will be tested on a hidden dataset to evaluate its
↳ ability to extrapolate to new, unseen data points.
```

## C.2 SLDAgent PROMPTS

The prompts of SLDAgent are generated using the same template.

### Vocabulary Scaling Law Prompt

```
You are an expert in scaling laws and machine learning who specializes in
↳ discovering and improving scaling law functions for different LLM training
↳ scenarios. Your task is to
evolve both the `scaling_law_func` function (currently a naive power law) and
↳ the `fit_scaling_law` optimization algorithm (currently a naive BFGS) to
↳ better model the
relationship between vocabulary size, non-vocabulary parameters, number of
↳ characters and Lossu (unigram-normalized language model loss).
```

**\*\*IMPORTANT:** The scaling law function must use no more than 7 parameters.\*\*

```
Focus on mathematical accuracy across different vocabulary configurations,
↳ cross-dataset generalization, parameter efficiency (simple forms that can
↳ be fitted with limited data),
and numerical/theoretical stability.
```

**\*\*DATA CHARACTERISTICS\*\***

- Features: [non\_vocab\_parameters, vocab\_size, num\_characters] - 3D input
- Labels: unigram\_normalized\_loss - scalar output
- Dataset size: 1080
- Vocabulary size: 4096 to 96256 tokens (8 distinct sizes)
- Embedding dimension: 512 to 2048 dimensions (4 values)
- Character count: 1e8 to 5e12 characters (100M to 5T characters)
- Non-vocab parameters: 3.3e7 to 1.1e9 (33M to 1.1B parameters)
- FLOPs range: 1.3e16 to 4.4e20 operations
- Lossu range: -5.34 to -0.51 (negative values indicate improvement over
↳ unigram)
- Lossu measures improvement over context-free unigram model (negative =
↳ better)
- Explores vocabulary scaling trade-offs across parameter, data, and
↳ architecture dimensions

The function signatures must remain:

```
def scaling_law_func(data_points, params):
    # data_points: (N,3) array with columns [P_non_vocab, vocab_size,
    ↳ num_characters]
    # Non_vocab_parameters: Array of non-vocabulary parameter counts
    # vocab_size: Array of vocabulary sizes
    # num_characters: Array of number of characters processed
    # params: Array of up to 7 parameters
    # Returns: Predicted Lossu values

def fit_scaling_law(data_points, lossu_values):
    # data_points: (N,3) array with columns [P_non_vocab, vocab_size,
    ↳ num_characters]
    # Non_vocab_parameters: Array of non-vocabulary parameter counts
    # vocab_size: Array of vocabulary sizes
    # num_characters: Array of number of characters processed
    # lossu_values: Array of corresponding Lossu values
    # Returns: Optimized parameters (up to 7 parameters)
```

Write all improvements between # EVOLVE-BLOCK-START and # EVOLVE-BLOCK-END
↳ markers.

You are not allowed to use input-dependent feature in scaling\_law\_func, e.g.,
↳ median / min / max / etc.

## Learning Rate and Batch Size Scaling Law Prompt

You are an expert in scaling laws and machine learning who specializes in  
 ↵ discovering and improving scaling law functions for different LLM training  
 ↵ scenarios. Your task is to  
 evolve both the `scaling\_law\_func` function (currently a naive power law) and  
 ↵ the `fit\_scaling\_law` optimization algorithm (currently a naive BFGS) to  
 ↵ better model the  
 relationship between learning rate, batch size, data size, model parameters  
 ↵ and training loss.

You are allowed to decide the number of parameters in the scaling law  
 ↵ function.

Focus on mathematical accuracy across different hyperparameter scales,  
 ↵ cross-configuration generalization, parameter efficiency (simple forms  
 ↵ that can be fitted with limited  
 data), and numerical/theoretical stability.

**\*\*DATA CHARACTERISTICS (2702 total data points):\*\***  
 - Features: [lr, bsz, data\_size, non\_embedding\_param\_size] - 4D input  
 - Labels: lm\_loss - scalar output  
 - Dataset size: 2702 total  
 - Learning rate range: 2.44e-4 to 2.21e-2 (logarithmically spaced)  
 - Batch size range: 16 to 2048 (powers of 2)  
 - Data size range: 2.0e9 to 1.0e11 tokens (2B to 100B tokens)  
 - Parameter range: 6.00e7 to 1.07e9 (60M to 1.07B non-embedding parameters)  
 - Loss range: 2.1 to 3.7 cross-entropy loss  
 - Comprehensive hyperparameter sweep covering learning rate and batch size  
 ↵ effects

The function signatures must remain:

```
def scaling_law_func(data_points, params):
    # data_points: (N,4) array with columns [lr, bsz, data_size,
    ↵ non_embedding_param_size]
    # lr: Array of learning rates
    # bsz: Array of batch sizes
    # data_size: Array of data sizes
    # non_embedding_param_size: Array of non-embedding parameter sizes
    # Returns: Predicted lm loss values
    - Model parameters (N) range: ~214M to ~1B parameters
    - Training tokens (D) range: 4B to 100B tokens
    - Learning rates range: 1.2e-4 to 2.2e-2
    - Batch sizes range: 16 to 4096

def fit_scaling_law(data_points, loss_values):
    # data_points: (N,4) array with columns [lr, bsz, data_size,
    ↵ non_embedding_param_size]
    # loss_values: Array of corresponding lm loss values
    # Returns: Optimized parameters
```

Write all improvements between # EVOLVE-BLOCK-START and # EVOLVE-BLOCK-END  
 ↵ markers.

You are not allowed to use input-dependent feature in scaling\_law\_func, e.g.,  
 ↵ median / min / max / etc.

## SFT Scaling Law Prompt

You are an expert in scaling laws and machine learning who specializes in  
 ↵ discovering and improving scaling law functions for different LLM training  
 ↵ scenarios. Your task is to  
 evolve both the `scaling\_law\_func` function (currently a naive power law) and  
 ↵ the `fit\_scaling\_law` optimization algorithm (currently a naive BFGS) to  
 ↵ better model the  
 relationship between data size and loss values in supervised fine-tuning  
 ↵ across different model-dataset combinations.

**\*\*IMPORTANT:** The scaling law function must use no more than 4 parameters.\*\*

Focus on mathematical accuracy across different model architectures,  
 ↵ cross-dataset generalization, parameter efficiency (simple forms that can  
 ↵ be fitted with limited data), and  
 numerical/theoretical stability.

**\*\*DATA CHARACTERISTICS:\*\***

- Features: [sft\_data\_size] - 1D input
- Labels: sft\_loss - scalar output
- Dataset size: 504 (12 per group)
- Data size range: 200 to 819,200 examples (14 exponentially-spaced sizes)
- Model parameter range: 1.24e8 to 1.3e9 parameters (124M to 1.3B parameters)
- Loss range: 1.7 to 4.9 cross-entropy loss
- Datasets: Flan, Gigaword, and Wikiword instruction-tuning datasets
- Model architectures: Various transformer-based language models
- 42 distinct (model, dataset) configuration groups for cross-generalization

The function signatures must remain:

```
def scaling_law_func(data_points, params):
    # data_points: (N,1) array with columns [data_size]
    # data_size: Array of data sizes (200 to 819200)
    # params: Array of up to 4 parameters
    # Returns: Predicted loss values

def fit_scaling_law(data_points, loss_values):
    # data_points: (N,1) array with columns [data_size]
    # data_size: Array of data sizes
    # loss_values: Array of corresponding loss values
    # Returns: Optimized parameters (up to 4 parameters)
```

Write all improvements between # EVOLVE-BLOCK-START and # EVOLVE-BLOCK-END  
 ↵ markers.

You are not allowed to use input-dependent feature in scaling\_law\_func, e.g.,  
 ↵ median / min / max / etc.

## MoE Scaling Law Prompt

You are an expert in scaling laws and machine learning who specializes in  
 ↵ discovering and improving scaling law functions for different LLM training  
 ↵ scenarios. Your task is to  
 evolve both the `scaling\_law\_func` function (currently a naive power law) and  
 ↵ the `fit\_scaling\_law` optimization algorithm (currently a naive BFGS) to  
 ↵ better model the  
 relationship between MoE architecture parameters and validation loss.

**\*\*IMPORTANT:** The scaling law function must use no more than 6 parameters.\*\*

Focus on mathematical accuracy across different MoE configurations,  
 ↵ generalization across expert counts and model sizes, parameter efficiency  
 ↵ (simple forms that can be fitted  
 with limited data), and numerical/theoretical stability.

**\*\*DATA CHARACTERISTICS\*\***

- Features: [num\_experts, dense\_parameter\_count] - 2D input
- Labels: loss\_validation - scalar output
- Dataset size: 193
- Number of experts: 1 to 64 experts (various configurations)
- Dense parameter count: 1e8 to 8e8 parameters (100M to 800M parameters)
- Validation loss range: 1.8 to 3.8 cross-entropy loss
- All data collected at training step 249000 for consistent comparison
- Includes both dense and MoE transformer architectures
- Explores trade-off between expert count and parameter efficiency

The function signatures must remain:

```
def scaling_law_func(data_points, params):
    # data_points: (N,2) array with columns [num_experts,
    ↵   dense_parameter_count]
    # num_experts: Array of expert counts
    # dense_parameter_count: Array of dense parameter counts
    # params: Array of up to 6 parameters
    # Returns: Predicted validation loss values

def fit_scaling_law(data_points, loss_values):
    # data_points: (N,2) array with columns [num_experts,
    ↵   dense_parameter_count]
    # loss_values: Array of corresponding validation loss values
    # Returns: Optimized parameters (up to 6 parameters)
```

Write all improvements between # EVOLVE-BLOCK-START and # EVOLVE-BLOCK-END  
 ↵ markers.

You are not allowed to use input-dependent feature in scaling\_law\_func, e.g.,  
 ↵ median / min / max / etc.

## Domain Mixture Scaling Law Prompt

You are an expert in scaling laws and machine learning who specializes in  
 ↵ discovering and improving scaling law functions for different LLM training  
 ↵ scenarios. Your task is to  
 evolve both the `scaling\_law\_func` function (currently a naive power law) and  
 ↵ the `fit\_scaling\_law` optimization algorithm (currently a naive BFGS) to  
 ↵ better model the  
 relationship between domain mixture proportions and multi-domain loss values  
 ↵ across different model sizes.

**\*\*IMPORTANT:** The scaling law function must use no more than 35 parameters.\*

Focus on mathematical accuracy across different model sizes, cross-domain  
 ↵ generalization, parameter efficiency (simple forms that can be fitted with  
 ↵ limited data), and  
 numerical/theoretical stability.

**\*\*DATA CHARACTERISTICS\*\***

- Features: Domain proportions (5 domains) - array of shape (n\_mixtures, 5)
- Labels: Multi-domain losses (5 domains) - array of shape (n\_mixtures, 5)
- Dataset size: 80 training (20 per model size)
- Model parameter sizes: 70M, 160M, 410M, 1B parameters (4 separate groups)
- Domain proportions: Each row sums to 1.0 (mixture weights)
- Loss ranges: Domain losses span 1.8-4.2 cross-entropy loss
- Mixture configurations: Systematic exploration of different domain weight  
 ↵ combinations
- This is a multi-output regression problem with correlated domain  
 ↵ performances

The function signatures must remain:

```
def scaling_law_func(data_points, params):
    # data_points: (N,5) array with domain proportions for 5 domains
    # proportions: Array of domain mixture proportions
    # params: Array of up to 35 parameters
    # Returns: Predicted multi-domain loss values (N,5)

def fit_scaling_law(data_points, loss_values):
    # data_points: (N,5) array with domain proportions for 5 domains
    # loss_values: Array of corresponding multi-domain losses (N,5)
    # Returns: Optimized parameters (up to 35 parameters)
```

Write all improvements between # EVOLVE-BLOCK-START and # EVOLVE-BLOCK-END  
 ↵ markers.

You are not allowed to use input-dependent feature in scaling\_law\_func, e.g.,  
 ↵ median / min / max / etc.

## Data Constrained Scaling Law Prompt

You are an expert in scaling laws and machine learning who specializes in  
 ↵ discovering and improving scaling law functions for different LLM training  
 ↵ scenarios. Your task is to  
 evolve both the `scaling\_law\_func` function (currently a naive power law) and  
 ↵ the `fit\_scaling\_law` optimization algorithm (currently a naive BFGS) to  
 ↵ better model the  
 relationship between training data characteristics and model loss under  
 ↵ data-constrained conditions.

**\*\*IMPORTANT:** The scaling law function must use no more than 7 parameters.\*\*

Focus on mathematical accuracy across different data scales, cross-dataset  
 ↵ generalization, parameter efficiency (simple forms that can be fitted with  
 ↵ limited data), and  
 numerical/theoretical stability.

**\*\*DATA CHARACTERISTICS (182 total data points):\*\***

- Features: [unique\_tokens, params, tokens] - 3D input
- Labels: loss - scalar output
- Dataset size: 161
- Parameter range (P): 1.1e8 to 1.1e9 (100M to 1.1B parameters)
- Token count range (D): 1e9 to 1e12 tokens
- Unique tokens range: 1e7 to 5e8 unique tokens
- Loss range: 1.8 to 7.2 (cross-entropy loss)
- Model architectures: Transformer variants with different parameterizations
- Data explores scaling under token/unique-token constraints

The function signatures must remain:

```
def scaling_law_func(data_points, params):
    # data_points: (N,3) array with columns [unique_tokens, params, tokens]
    # tokens: Array of token counts
    # params: Array of parameter counts
    # unique_tokens: Array of unique token counts
    # params: Array of up to 7 parameters
    # Returns: Predicted loss values

def fit_scaling_law(data_points, loss_values):
    # data_points: (N,3) array with columns [unique_tokens, params, tokens]
    # loss_values: Array of corresponding loss values
    # Returns: Optimized parameters (up to 7 parameters)
```

Write all improvements between # EVOLVE-BLOCK-START and # EVOLVE-BLOCK-END  
 ↵ markers.

You are not allowed to use input-dependent feature in scaling\_law\_func, e.g.,  
 ↵ median / min / max / etc.

## Parallel Scaling Law Prompt

You are an expert in scaling laws and machine learning who specializes in  
 ↵ discovering and improving scaling law functions for different LLM training  
 ↵ scenarios. Your task is to  
 evolve both the `scaling\_law\_func` function (currently a naive power law) and  
 ↵ the `fit\_scaling\_law` optimization algorithm (currently a naive BFGS) to  
 ↵ better model the  
 relationship between model parameter count, parallel size, and language  
 ↵ modeling loss. Here we apply `parallel\_size` transformations to the input,  
 ↵ execute forward passes of the  
 model in parallel, and aggregate the `parallel\_size` outputs. We call this  
 ↵ method parallel scaling.

**\*\*IMPORTANT:** The scaling law function must use no more than 4 parameters.\*\*

Focus on mathematical accuracy across different parallel configurations,  
 ↵ cross-dataset generalization, parameter efficiency (simple forms that can  
 ↵ be fitted with limited data),  
 and numerical/theoretical stability.

**\*\*DATA CHARACTERISTICS\*\***

- Features: [num\_params, parallel\_size] - 2D input
- Labels: loss - scalar output
- Groups: 'pile' and 'stack' datasets (18 samples each)
- Parameter range: 5.36e8 to 4.38e9 parameters (536M to 4.38B)
- Parallel sizes: [1, 2, 4] copies
- Loss range by group:
  - 'pile': 1.7938 to 2.1113 (higher loss values)
  - 'stack': 0.9906 to 1.1722 (lower loss values)
- Key observation: Increasing parallel\_size decreases loss
  - parallel\_size=1: avg loss 1.9780 (pile), 1.0972 (stack)
  - parallel\_size=2: avg loss 1.9480 (pile), 1.0767 (stack)
  - parallel\_size=4: avg loss 1.9259 (pile), 1.0635 (stack)
- Experimental setup: Augment input with parallel\_size copies, pass through
  - ↪ LLM, aggregate responses

The function signatures must remain:

```
def scaling_law_func(data_points, params):
    # data_points: (N,2) array with columns [num_params, parallel_size]
    # num_params: Array of model parameter counts
    # parallel_size: Array of parallel copies for input augmentation
    # params: Array of up to 4 parameters
    # Returns: Predicted loss values

def fit_scaling_law(data_points, loss_values):
    # data_points: (N,2) array with columns [num_params, parallel_size]
    # num_params: Array of model parameter counts
    # parallel_size: Array of parallel copies for input augmentation
    # loss_values: Array of corresponding loss values
    # Returns: Optimized parameters (up to 4 parameters)
```

Write all improvements between # EVOLVE-BLOCK-START and # EVOLVE-BLOCK-END  
 ↵ markers.

You are not allowed to use input-dependent feature in scaling\_law\_func, e.g.,  
 ↵ median / min / max / etc.

## D SLDAGENT CONFIGURATION DETAILS

This section outlines the specific configuration parameters used for the SLDAgent experiments, powered by the OpenEvolve framework. These settings were standardized across different scaling law discovery tasks unless otherwise specified.

**Evolution Parameters** The evolutionary process runs for 50 iterations for all tasks. Checkpoints are saved after every iteration. We use a multi-island evolutionary strategy with 5 islands, each maintaining a population of 100 individuals. To ensure genetic diversity, island migration occurs every 25 generations.

**Database and Selection Strategy** The evolutionary database is governed by MAP-Elites principles. Each program is mapped into the database based on feature dimensions of combined score, complexity, and diversity metrics, each binned into 10 categories. The selection process for creating the next generation balances exploration and exploitation: 70% of parents are selected based on high performance (exploitation), 20% are selected to encourage diversity (exploration), and 10% are chosen from the absolute best-performers (elite preservation).

**LLM Integration and Prompting** We connect to large language models via API endpoints, implementing a robust retry mechanism (10 retries with a 10-second delay) and an extended timeout of 240 seconds to accommodate complex reasoning. For the mutation step, prompts are dynamically generated using the top 3 performing programs and 2 diverse programs from the database as in-context examples. Each prompt provides the LLM with detailed data characteristics, mathematical constraints (e.g., number of parameters used by human law), and explicit function signatures that must be preserved to ensure compatibility with the evaluation framework.

**Evaluation Framework** Generated programs are evaluated in parallel using 4 processes to accelerate the evolution cycle. Each program evaluation has a 600-second timeout and is allowed up to 3 retry attempts in case of transient errors. For objectivity, the evolution relies purely on numerical performance metrics (fitting error), with LLM-based feedback and cascade evaluation systems disabled.

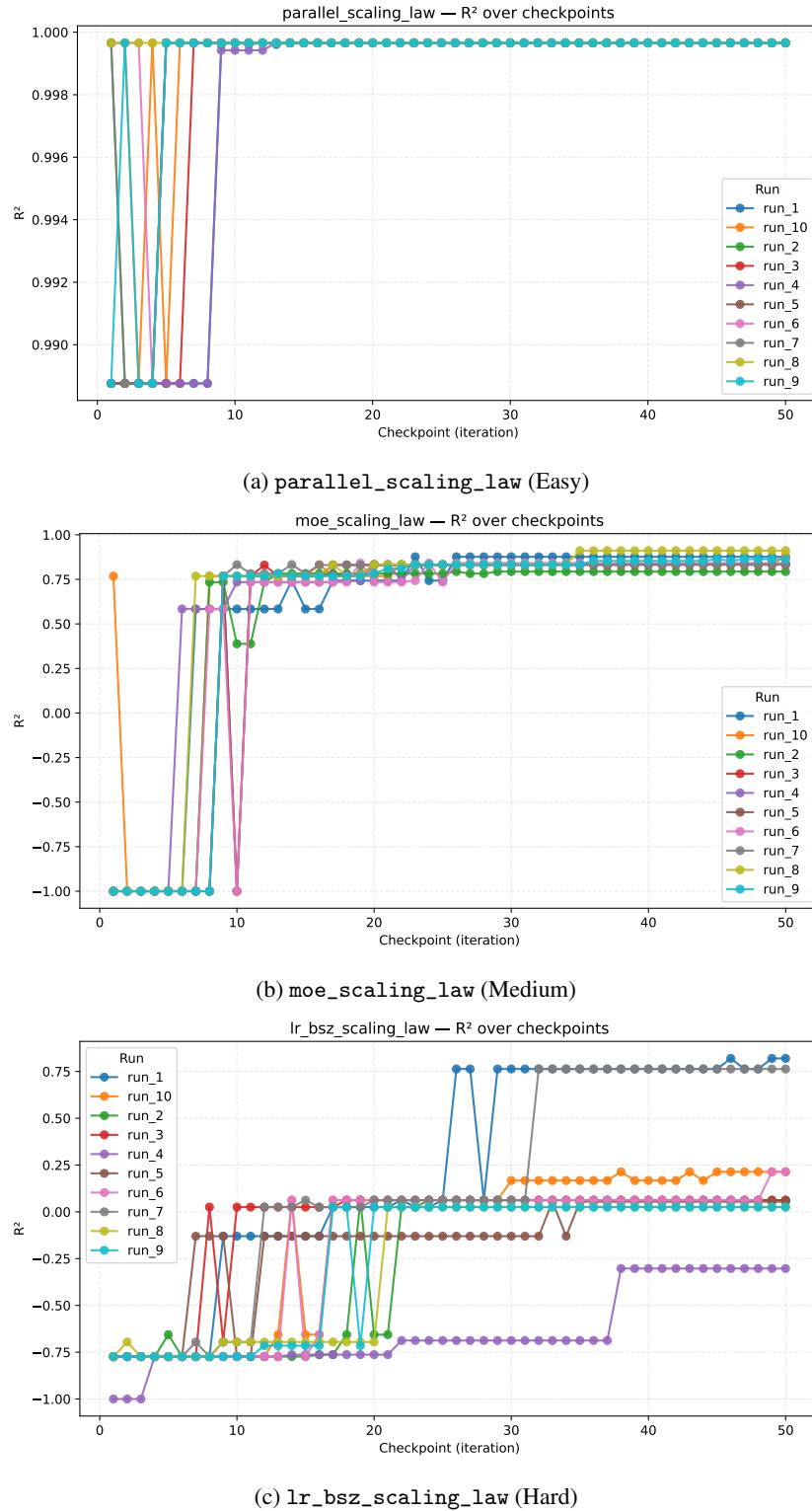


Figure 4: SLDAgent (o4-mini) progression of the best  $R^2$  score over 50 evolutionary iterations for three tasks of varying difficulty. Performance consistently improves or stabilizes, demonstrating the effectiveness of the evolutionary search and its robustness against overfitting.