

# CSCI104 Written Homework 1

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## 1. Problem 1

- (a) Observe that after  $k$  iterations of the loop we have  $i = 2^{2^k}$ . Since the body of the loop only runs when  $i < n$ , we have that for some  $n$ , the loop runs

$$\lfloor \log_2(\log_2(n-1)) \rfloor$$

times. Therefore the runtime is  $\lfloor \log_2(\log_2(n-1)) \rfloor \cdot \Theta(1) = \Theta(\log(\log(n)))$ .

- (b) Observe that for some  $n$ , the inner for loop runs  $n/\lfloor \sqrt{n} \rfloor$  times. Since the inner loop scales at a factor of  $n^3$ , the runtime of the inner loop is  $\Theta(n^3)$ . All together, the runtime is

$$n/\lfloor \sqrt{n} \rfloor \cdot \Theta(n^3) = \Theta(n^{3.5})$$

- (c) Observe that the innermost for loop will run at most  $n$  times because the array  $A$  is never changed. Since the inner loop runs at  $\log_2(n)$ , we have that the upper bound runtime is  $O(n^2 + n \log(n)) = O(n^2)$ . On the other hand, the lower bound runtime is also  $\Omega(n^2)$  since the comparison has to run at least  $n^2$  times, even if the inner loop never runs.
- (d) Observe that the body of the if statement will run only if  $n > 10$ . Since the smaller numbers of  $n$  do not matter when considering runtime, we can assume  $n > 10$ . Now observe that the body of the if statement will run

$$\lfloor \log_{\frac{3}{2}}\left(\frac{n}{10}\right) + 1 \rfloor$$

times, and each time the body is run the number of operations is the current length of the array. Furthermore, the total number of operations coming from the *if* body is  $\Theta(n)$ , so we can deduce that the runtime of the total function is  $\Theta(n + n) = \Theta(n)$ .

## 2. Problem 2

- (a) We can list the recursive calls to *llrec* as follows:

- *llrec*(*p1*, *p5*) returns 1,5,2,6,3,4
- *llrec*(*p5*, *p2*) returns 5,2,6,3,4
- *llrec*(*p2*, *p6*) returns 2,6,3,4
- *llrec*(*p6*, *p3*) returns 6,3,4
- *llrec*(*p3*, *NIL*) returns 3,4

- (b) Since *in1* = *nullptr*, there is no recursive call and *in2* = 2 is returned.