Inner Time and The Inner ear

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Abstract

This document will contain the relevant equations I need to numerically calculate the Paschen-Back for aribitrary I and J.

I. THE WHOLE PAPER

The hyperfine splitting up to magnetic octopole contributions is given by:

$$
\begin{aligned}
H_{\text{hfs}} = {} & A_{\text{hfs}}\frac{\mathbf{I}\cdot\mathbf{J}}{\hbar^2} \\
& + B_{\text{hfs}}\frac{\frac{3}{\hbar^2}(\mathbf{I}\cdot\mathbf{J})^2 + \frac{3}{2\hbar}\mathbf{I}\cdot\mathbf{J} - I(I+1)J(J+1)}{2I(2I-1)J(2J-1)} \\
& + C_{\text{hfs}}\frac{\frac{10}{\hbar^3}(\mathbf{I}\cdot\mathbf{J})^3 + \frac{20}{\hbar^2}(\mathbf{I}\cdot\mathbf{J})^2 + \frac{2}{\hbar}\mathbf{I}\cdot\mathbf{J}\,[I(I+1)+J(J+1)+3] - 3I(I+1)J(J+1) - 5I(I+1)J(J+1)}{I(I-1)(2I-1)J(J-1)(2J-1)}
\end{aligned}
\tag{1}
$$

For Sodium 23, we have:

$$
\begin{array}{lll}
\text{Magnetic Dipole Constant, } 3^2S_{1/2} & A_{3^2S_{1/2}} & h\cdot 885.813\text{ MHz} \\
\text{Magnetic Dipole Constant, } 3^2P_{1/2} & A_{3^2P_{1/2}} & h\cdot 94.44\text{ MHz} \\
\text{Magnetic Dipole Constant, } 3^2P_{3/2} & A_{3^2P_{3/2}} & h\cdot 18.534\text{ MHz} \\
\text{Electric Quadrupole Constant, } 3^2P_{3/2} & B_{3^2P_{3/2}} & h\cdot 2.724\text{ MHz}
\end{array}
\tag{2}
$$

For the DC Zeeman shift, we want to compute the eigenstaes of

$$H_B^{(\text{fs})} + H_B^{(\text{hfs})} \tag{3}$$

where for a B-field in the z direction, we have

$$H_B^{(\text{fs})} = -\mu_S\cdot\mathbf{B} - \mu_L\cdot\mathbf{B} = \frac{\mu_B}{\hbar}\left(g_s S_z + g_L L_z\right)B \tag{4}$$

and

$$H_B^{(\text{hfs})} = -\mu_I\cdot\mathbf{B} = \frac{\mu_B}{\hbar}g_I I_z B. \tag{5}$$

For now, we will lump together the effect of S and L into J via the Lande g_J factor, and treat only the Paschen-Back effect due to the hyperfine interaction (since this occurs at much lower energy B-fields than the separation of S and L eigenstates). Thus we must diagonalize

$$H_B^{(\text{fs})} + H_B^{(\text{hfs})} = \frac{\mu_B}{\hbar}\left(g_J J_z + g_I I_z\right)B \tag{6}$$

where

$$
\begin{aligned}
g_J &\equiv g_L + (g_S - g_L)\frac{J(J+1)+S(S+1)-L(L+1)}{2J(J+1)} \\
g_L &\equiv \frac{\text{Reduced Mass}}{m_e} = \frac{1}{1+m_e/m_n} \\
g_S &= 2.002319.
\end{aligned}
\tag{7}
$$

In order to compute matrix elements of $\mathbf{I}\cdot\mathbf{J}$ in $H_B^{(\mathrm{hfs})}$, we will need to use:

$$\begin{aligned}
\mathbf{I}\cdot\mathbf{J} &= I_z J_z + \frac{I_+ J_- + I_- J_+}{2} \\
(\mathbf{I}\cdot\mathbf{J})^2 &= (I_z J_z)^2 + \frac{1}{2}\{I_z J_z, I_+ J_- + I_- J_+\} + \frac{(I_+ J_-)^2 + (I_- J_+)^2}{4} + \frac{I_+ I_- J_- J_+ + I_- I_+ J_+ J_-}{4}
\end{aligned} \tag{8}$$