Inner Time and The Inner ear

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Abstract

This document will contain the relevant equations I need to numerically calculate the Paschen-Back for aribitrary I and J.

I. THE WHOLE PAPER

The hyperfine splitting up to magnetic octopole contributions is given by:

$$\begin{aligned} H_{\text{hfs}} &= A_{\text{hfs}} \frac{\mathbf{I} \cdot \mathbf{J}}{\hbar^2} \\ &+ B_{\text{hfs}} \frac{\frac{3}{\hbar^2}(\mathbf{I} \cdot \mathbf{J})^2 + \frac{3}{2\hbar}\mathbf{I} \cdot \mathbf{J} - I(I+1)J(J+1)}{2I(2I-1)J(2J-1)} \\ &+ C_{\text{hfs}} \frac{\frac{10}{\hbar^3}(\mathbf{I} \cdot \mathbf{J})^3 + \frac{20}{\hbar^2}(\mathbf{I} \cdot \mathbf{J})^2 + \frac{2}{\hbar}\mathbf{I} \cdot \mathbf{J}\left[I(I+1) + J(J+1) + 3\right] - 3I(I+1)J(J+1) - 5I(I+1)J(J+1)}{I(I-1)(2I-1)J(J-1)(2J-1)} \end{aligned} \tag{1}$$

For Sodium 23, we have:

$$\begin{array}{lll} \text{Magnetic Dipole Constant, } 3^2S_{1/2} & A_{3^2S_{1/2}} & h \cdot 885.813 \text{ MHz} \\ \text{Magnetic Dipole Constant, } 3^2P_{1/2} & A_{3^2P_{1/2}} & h \cdot 94.44 \text{ MHz} \\ \text{Magnetic Dipole Constant, } 3^2P_{3/2} & A_{3^2P_{3/2}} & h \cdot 18.534 \text{ MHz} \\ \text{Electric Quadrupole Constant, } 3^2P_{3/2} & B_{3^2P_{3/2}} & h \cdot 2.724 \text{ MHz} \end{array} \tag{2}$$

For Lithium 6:

$$\begin{array}{lll} \text{Magnetic Dipole Constant, } 2^2S_{1/2} & A_{3^2S_{1/2}} & h \cdot 152.137 \text{ MHz} \\ \text{Magnetic Dipole Constant, } 2^2P_{1/2} & A_{3^2P_{1/2}} & h \cdot 17.386 \text{ MHz} \\ \text{Magnetic Dipole Constant, } 2^2P_{3/2} & A_{3^2P_{3/2}} & h \cdot -1.155 \text{ MHz} \\ \text{Electric Quadrupole Constant, } 2^2P_{3/2} & B_{3^2P_{3/2}} & h \cdot -0.10 \text{ MHz} \end{array} \tag{3}$$

For the DC Zeeman shift, we want to compute the eigenstaes of

$$H_B^{(\text{fs})} + H_B^{(\text{hfs})} \tag{4}$$

where for a B-field in the z direction, we have

$$H_B^{(\text{fs})} = -\mu_S \cdot \mathbf{B} - \mu_L \cdot \mathbf{B} = \frac{\mu_B}{\hbar}\left(g_s S_z + g_L L_z\right) B \tag{5}$$

and

$$H_B^{(\text{hfs})} = -\mu_I \cdot \mathbf{B} = \frac{\mu_B}{\hbar} g_I I_z B. \tag{6}$$

For now, we will lump together the effect of S and L into J via the Lande g_J factor, and treat only the Paschen-Back effect due to the hyperfine interaction (since this occurs at much lower energy B-fields than the separation of S and L eigenstates). Thus we must diagonalize

$$H_B^{(\mathrm{fs})} + H_B^{(\mathrm{hfs})} = \frac{\mu_B}{\hbar}\left(g_J J_z + g_I I_z\right) B \tag{7}$$

where

$$\begin{aligned} g_J &\equiv g_L + (g_S - g_L)\frac{J(J+1) + S(S+1) - L(L+1)}{2J(J+1)} \\ g_L &\equiv \frac{\text{Reduced Mass}}{m_e} = \frac{1}{1 + m_e/m_n} \\ g_S &= 2.002319. \end{aligned} \tag{8}$$

In order to compute matrix elements of $\mathbf{I}\cdot\mathbf{J}$ in $H_B^{(\mathrm{hfs})}$, we will need to use:

$$\begin{aligned} \mathbf{I}\cdot\mathbf{J} &= I_z J_z + \frac{I_+J_- + I_-J_+}{2} \\ (\mathbf{I}\cdot\mathbf{J})^2 &= (I_z J_z)^2 + \frac{1}{2}\{I_z J_z, I_+J_- + I_-J_+\} + \frac{(I_+J_-)^2 + (I_-J_+)^2}{4} + \frac{I_+I_-J_-J_+ + I_-I_+J_+J_-}{4} \end{aligned} \tag{9}$$

We will calculate the energy splittings ignoring the octopole, C_{hfs} term in eq. (1). For the case of $I = 1/2$ or $J = 1/2$, the B_{hfs} term vanishes (but does it really? The denomintor is certainly zero–what about the numerator?), and the result would be the Breit-Rabi formula. Thus, for a first non-trivial go at numerical calculations, let's try the case of the Lithium 6