Практика по математическому анализу занятие 1

7) $\int \frac{dx}{7x^2-5} = \frac{1}{7}\int \frac{dx}{x^2-\frac{5}{7}} = \frac{\sqrt{7}}{14\sqrt{5}}\ln|\frac{x-\sqrt{\frac{5}{7}}}{x+\sqrt{\frac{5}{7}}}| + C,\ C\in\mathbb{R}$

8) $\int \frac{dx}{3x^2-x^4} = \int \frac{dx}{x^2(3-x^2)} = \frac{1}{3}\int(\frac{1}{x^2}+\frac{1}{3-x^2}) = \frac{1}{3}(-\frac{1}{x}-\frac{1}{2\sqrt{3}}\ln|\frac{x-\sqrt{3}}{x+\sqrt{3}}|) + C,\ C\in\mathbb{R}$

9) $\int(2x+5)^{10}dx = \begin{vmatrix} 2x+5=t \\ 2dx=dt \end{vmatrix} = \frac{1}{2}\int t^{10}dt = \frac{(2x+5)^{11}}{22} + C,\ C\in\mathbb{R}$

10) $\int \sin(1-4x)dx = -\frac{1}{4}\int\sin(1-4x)d(-4x) = -\frac{1}{4}\int\sin(1-4x)d(1-4x) = \frac{\cos(1-4x)}{4} + C,\ C\in\mathbb{R}$

11) $\int \frac{dx}{2x^2-5x+7} = \frac{1}{2}\int\frac{dx}{(x-\frac{5}{4})^2+\frac{31}{16}} = \begin{vmatrix} x-\frac{5}{4}=t \\ dx=dt \end{vmatrix} = \frac{1}{2}\int\frac{dt}{t^2+\frac{31}{16}} = \frac{1}{2\sqrt{\frac{31}{16}}}\arctan\frac{x-\frac{5}{4}}{\sqrt{\frac{31}{16}}} + C,\ C\in\mathbb{R}$

12) $\int 5^{4-7x}dx = -\frac{1}{7}\int 5^{4-7x}d(4-7x) = -\frac{1}{7}\frac{5^{4-7x}}{\ln 5} + C,\ C\in\mathbb{R}$

13) $\int\frac{dx}{\sqrt{x+x^2}} = \int\frac{dx}{\sqrt{(x+\frac{1}{2})^2-\frac{1}{4}}} = \ln|\frac{x}{x+1}| + C,\ C\in\mathbb{R}$

14) $\int\frac{6x-7}{3x^2-7x+1}dx = \begin{vmatrix} 3x^2+7x=t \\ (6x+7)dx=dt \end{vmatrix} = \int\frac{dt}{t+1} = \ln|3x^2+7x+1| + C,\ C\in\mathbb{R}$

15) $\int\left(\frac{x}{x^5+2}\right)^2dx = \int\frac{x^2}{(x^5+2)^2}dx = \begin{vmatrix} x^5=t \\ (5x^4)dx=dt \end{vmatrix} = \frac{1}{5}\int\frac{dt}{(t+2)^4} = \frac{1}{5}(-\frac{1}{3(x^5+2)^3}) + C,\ C\in\mathbb{R}$

16) $\int\frac{xdx}{x^4+6x^2+5} = \frac{1}{2}\int\frac{dx^2}{x^4+6x^2+5} = \frac{1}{2}\int\frac{d(x^2+3)}{(x^2+3)^2-4} = \frac{1}{8}\ln|\frac{x^2+1}{x^2+5}| + C,\ C\in\mathbb{R}$

17) $\int x^2\sqrt{x^3+1}dx = \begin{vmatrix} x^3=t \\ (3x^2)dx=dt \end{vmatrix} = \frac{1}{3}\int\sqrt{t+1}d(t+1) = \frac{2(x^3+1)\sqrt{x^3+1}}{9} + C,\ C\int\mathbb{R}$

18) $\int x\sqrt{1+x}dx = \begin{vmatrix} \sqrt{1+x}=t \\ \frac{dx}{2\sqrt{1+x}}=dt \end{vmatrix} = \int(2t^4-2t^2)dt = \frac{2(\sqrt{x+1})^5}{5} - \frac{2(\sqrt{x+1})^3}{3} + C,\ C\in\mathbb{R}$

19) $\int\frac{xdx}{(1-x)^{12}} = \begin{vmatrix} (1-x)=t \\ -dx=dt \end{vmatrix} = \int\frac{t-1}{t^{12}}dt = -\frac{1}{10(1-x)^{10}} + \frac{1}{11(1-x)^{11}} + C,\ C\in\mathbb{R}$

20) $\int xe^{-x^2}dx = \begin{vmatrix} e^{-x^2}=t \\ (-e^{-x^2}2x)dx=dt \end{vmatrix} = -\int\frac{dt}{2} = -\frac{e^{-x^2}}{2} + C,\ C\in\mathbb{R}$

21) $\int e^{\sqrt{x}}\frac{dx}{\sqrt{x}} = \begin{vmatrix} \sqrt{x}=t \\ \frac{dx}{2\sqrt{x}}=dt \end{vmatrix} = \int 2e^tdt = 2e^{\sqrt{x}} + C,\ C\in\mathbb{R}$

22) $\int\frac{2^xdx}{\sqrt{1-4^x}} = \begin{vmatrix} 2^x=t \\ (\ln(2)\cdot 2^x)dx=dt \end{vmatrix} = \frac{1}{\ln(2)}\int\frac{dt}{\sqrt{1-t^2}} = \frac{1}{\ln(2)}\arcsin(2^x) + C,\ C\int\mathbb{R}$

23) $\int\frac{e^{2x}dx}{\sqrt[4]{1+e^x}} = \begin{vmatrix} 1+e^x=t \\ e^xdx=dt \end{vmatrix} = \int\frac{t-1}{\sqrt[4]{t}}dt = \int(t^{\frac{3}{4}}-t^{-\frac{1}{4}})dt = \frac{4(1+e^x)\sqrt[4]{(1+e^x)^3}}{7} - \frac{4\sqrt[4]{(1+e^x)^3}}{3} + C,\ C\in\mathbb{R}$

24) $\int\frac{\ln^2 x}{x}dx = \begin{vmatrix} \ln x=t \\ \frac{dx}{x}=dt \end{vmatrix} = \int t^2dt = \frac{\ln^3|x|}{3} + C,\ C\in\mathbb{R}$

25) $\int\sin^6 x\cos xdx = \begin{vmatrix} \sin x=t \\ \cos xdx=dt \end{vmatrix} = \int t^6dt = \frac{sin^7x}{7} + C,\ C\in\mathbb{R}$

26) $\int\frac{\sin x}{\sqrt{\cos 2x}}dx = \int\frac{\sin x}{\sqrt{2\cos^2 x-1}}dx = \begin{vmatrix} \cos x=t \\ -\sin xdx=dt \end{vmatrix} = -\frac{1}{\sqrt{2}}\int\frac{dt}{\sqrt{t^2-\frac{1}{2}}} = -\frac{1}{2}\ln|\cos x+\sqrt{\cos^2 x-\frac{1}{2}}| + C,\ C\in\mathbb{R}$

27) $\int\frac{\sqrt[4]{\tan x}}{\sin^2 x}dx = \begin{vmatrix} \cot x=t \\ -\frac{dx}{\sin^2 x}=dt \end{vmatrix} = \int\sqrt[4]{\frac{1}{t}}dt = \frac{4\cot^{\frac{3}{4}}x}{3} + C,\ C\in\mathbb{R}$

28) $\int\frac{\ln\arccos xdx}{\sqrt{1-x^2}\arccos x} = \begin{vmatrix} t=\arccos x \\ dt=-\frac{dx}{\sqrt{1-x^2}} \end{vmatrix} = -\int\frac{\ln t}{t}dt = \begin{vmatrix} \ln t=q \\ \frac{dt}{t}=dq \end{vmatrix} = -\int qdq = -\frac{(\ln(\arccos x))^2}{2} + C,\ C\in\mathbb{R}$

29) $\int\frac{3x-2}{2-3x+x^2}dx = \int(\frac{4}{x-2}-\frac{1}{x-1})dx = 4\ln|x-2| - \ln|x-1| + C,\ C\in\mathbb{R}$

33) $\int xe^{-x}dx = \begin{vmatrix} u=x & dx=du \\ e^{-x}dx=dv & v=-e^{-x} \end{vmatrix} = x\cdot(-e^{-x}) + \int e^{-x}dx = x\cdot(-e^{-x}) - e^{-x} + C,\ C\in\mathbb{R}$

34) $\int x\ln xdx = \begin{vmatrix} \ln x=u & \frac{dx}{x}=du \\ xdx=dv & v=\frac{x^2}{2} \end{vmatrix} = \ln x\frac{x^2}{2} - \int\frac{x}{2}dx = \ln x\frac{x^2}{x} - \frac{x^2}{4} + C,\ C\in\mathbb{R}$

35) $\int\ln xdx = \begin{vmatrix} u=x & dv=dx \\ \ln x=v & \frac{dx}{x}=dv \end{vmatrix} = x\ln x - \int\frac{x}{x}dx = x\ln x - x + C,\ C\in\mathbb{R}$

36) $\int x^2\cos(5x-7)dx = \begin{vmatrix} x^2=u & 2xdx=du \\ \cos(5x-7)dx=dv & \frac{1}{5}\sin(5x-7)=v \end{vmatrix} = \frac{1}{5}\sin(5x-7)x^2 - \frac{2}{5}\int x\sin(5x-7)dx =$
$\begin{vmatrix} x=u & dx=du \\ \sin(5x-7)dx=dv & -\frac{1}{5}\cos(5x-7)=v \end{vmatrix} = \frac{1}{5}\sin(5x-7)x^2 - \frac{2}{5}(x(-\frac{1}{5}\cos(5x-7)) + \frac{1}{5}\int\cos(5x-7)dx) =$
$= \frac{1}{5}x^2\sin(5x-7) - \frac{2}{5}(x(-\frac{1}{5}\cos(5x-7)) + \frac{1}{25}\sin(5x-7)) + C,\ C\in\mathbb{R}$

37) $\int\arctan xdx = \begin{vmatrix} x=u & dx=du \\ v=\arctan x & dv=\frac{1}{1+x^2} \end{vmatrix} = x\arctan x - \int\frac{x}{1+x^2}dx = x\arctan x - \frac{1}{2}\ln|1+x^2| + C,\ C\in\mathbb{R}$

38) $\int x\sin^2 xdx = \int x\frac{1-cos2x}{2}\begin{vmatrix} x=u & dx=du \\ \cos 2xdx=dv & \frac{1}{2}\sin 2x=v \end{vmatrix} = x\frac{1}{2}\sin 2x - \int\cos 2xdx =$
$= \frac{1}{2}(\frac{x^2}{2} + x\frac{1}{2}\sin 2x - \frac{1}{2}\sin 2x) + C,\ C\in\mathbb{R}$