Домашнее задание по Математическому анализу 1

1. $\int \frac{x-1}{x^2-x-1}dx$

$\int \frac{x-1}{x^2-x-1}dx = \int (\frac{2x-1}{x^2-x-1})dx - \frac{1}{2}\int \frac{dx}{x^2-x-1}$

$\int \frac{2x-1}{x^2-x-1}dx = \begin{vmatrix} t = x^2-x-1 \\ dt = (2x-1)dx \end{vmatrix} = \int \frac{dt}{t} = \ln t = \ln(x^2-x-1)$

$\int \frac{dx}{x^2-x-1} = \int \frac{4}{(2x-\sqrt{5}-1)(2x+\sqrt{5}-1)}dx = \int (\frac{2}{\sqrt{5}(2x-\sqrt{5}-1)} - \frac{2}{\sqrt{5}(2x+\sqrt{5}-1)})dx =$
$\frac{1}{\sqrt{5}}\ln(2x-\sqrt{5}-1) - \frac{1}{\sqrt{5}}\ln(2x+\sqrt{5}-1)$

Итого: $\int \frac{x-1}{x^2-x-1}dx = \frac{\ln|x^2-x-1|}{2} + \frac{\ln|2x+\sqrt{5}-1|}{2\sqrt{5}} - \frac{\ln|2x-\sqrt{5}-1|}{2\sqrt{5}} + C,\ C \in \mathbb{R}$

2. $\int \frac{3x-6}{\sqrt{x^2-4x+5}}dx$

$\int \frac{3x-6}{\sqrt{x^2-4x+5}}dx = \begin{vmatrix} t = \sqrt{x^2-4x+5} \\ dt = \frac{x-2}{\sqrt{x^2-4x+5}}dx \end{vmatrix} = \int 3dt =$
$= 3\sqrt{x^2-4x+5} + C,\ C \in \mathbb{R}$

3. $\int \frac{x^2-1}{x^4+1}dx$

$\int \frac{x^2-1}{x^4+1}dx = \int \frac{1-\frac{1}{x^2}}{x^2+\frac{1}{x^2}}dx = \int \frac{1-\frac{1}{x^2}}{(1+\frac{1}{x})^2-2}dx = \begin{vmatrix} t = x+\frac{1}{x} \\ dt = (1-\frac{1}{x^2})dx \end{vmatrix} = \int \frac{dt}{t^2-2} =$
$= \frac{1}{2\sqrt{2}}\ln|\frac{x+\frac{1}{x}-\sqrt{2}}{x+\frac{1}{x}+\sqrt{2}}| + C,\ C \in \mathbb{R}$

4. $\int \frac{e^x dx}{\sqrt{4-e^{2x}}}$

$\int \frac{e^x dx}{\sqrt{4-e^{2x}}} = \begin{vmatrix} e^x = t \\ e^x dx = dt \end{vmatrix} = \int \frac{dt}{\sqrt{4-t^2}} = \arcsin(\frac{e^x}{2}) + C,\ C \in \mathbb{R}$

5. $\int \frac{dx}{x\ln x \ln(\ln x)}dx$

$\int \frac{dx}{x\ln x \ln(\ln x)}dx = \begin{vmatrix} \ln(\ln x) = t \\ \frac{dx}{\ln x \cdot x} = dt \end{vmatrix} = \int \frac{dt}{t} = \ln(|\ln(\ln x)|) + C,\ C \in \mathbb{R}$

6. $\int \frac{\sin x dx}{\sqrt{1+2\cos x}}dx$

$\int \frac{\sin x dx}{\sqrt{1+2\cos x}}dx = \begin{vmatrix} t = \sqrt{1+2\cos x} \\ dt = -\sqrt{\sin x}\sqrt{1+2\cos x} \end{vmatrix} = -\int -1dt =$
$= -\sqrt{1+2\cos x} + C,\ C \in \mathbb{R}$

7. $\int \frac{\ln\tan x}{\sin 2x}dx$

$\int \frac{\ln\tan x}{\sin 2x}dx = \begin{vmatrix} t = \ln(\tan x) \\ dt = \frac{dx}{\sin x \cos x} \end{vmatrix} = \int \frac{t}{2}dt = \frac{(\ln(\tan x))^2}{4} + C,\ C \in \mathbb{R}$

8. $\int \frac{\arctan\sqrt{x}}{(1+x)\sqrt{x}}dx$

$\int \frac{\arctan\sqrt{x}}{(1+x)\sqrt{x}}dx = \begin{vmatrix} t = \arctan\sqrt{x} \\ dt = \frac{dx}{2\sqrt{x}+2x\sqrt{x}} \end{vmatrix} = \int 2tdt = (\arctan\sqrt{x})^2 + C,\ C \in \mathbb{R}$

9. $\int \frac{\arccos^2 2x}{\sqrt{1-4x^2}}dx$

$$\int \frac{\arccos^2 2x}{\sqrt{1-4x^2}}dx = \begin{vmatrix} t = \arccos 2x \\ dt = -\frac{2}{\sqrt{1-4x^2}} \end{vmatrix} = \int -\frac{t^2}{2} = \frac{(\arccos 2x)^3}{6} + C,\ C \in \mathbb{R}$$

10. $\int \arccos(5x-2)dx$

$$\int \arccos(5x-2)dx = \begin{vmatrix} 5x-2=t & 5dx = dt \\ u = \arccos t & du = -\frac{1}{\sqrt{1-t^2}}dt \\ dv = dt & d = t \end{vmatrix} = \frac{1}{5}(\arccos(t) \cdot$$
$$t - \int t \cdot (-\frac{1}{\sqrt{1-t^2}})dt) = \begin{vmatrix} q = \sqrt{1-t^2} \\ dq = -\frac{t}{\sqrt{1-t^2}} \end{vmatrix} = \frac{1}{5}(\arccos(t) \cdot t + \int(-1)dq) =$$
$$\frac{1}{t}(\arccos(t) \cdot t - q) + C =$$
$$= \frac{1}{5}(\arccos(5x-2)(5x-2) - \sqrt{1-(5x-2)^2}) + C,\ C \in \mathbb{R}$$

11. $\int \frac{\arcsin x}{x^2}dx$ $\int \frac{\arcsin x}{x^2}dx = \begin{vmatrix} u = \arcsin x & du = \frac{1}{\sqrt{1-x^2}}dx \\ dv = \frac{1}{x^2}dx & v = \frac{-}{1}x \end{vmatrix} = \arcsin x(-\frac{1}{x}) -$

$$\int -\frac{1}{x} \cdot \frac{1}{\sqrt{1-x^2}}dx = \begin{vmatrix} t = \sqrt{1-x^2} \\ dt = -\frac{x}{\sqrt{1-x^2}}dx \end{vmatrix} = \arcsin x(-\frac{1}{x}) + \int \frac{dt}{t^2-1} = \arcsin x(\frac{1}{x}) +$$
$$\frac{1}{2}\ln\frac{\sqrt{1-x^2}-1}{\sqrt{1-x^2}+1} + C,\ C \in \mathbb{R}$$

12. $\int \left(\frac{\ln x}{x}\right)^3 dx$

$$\int \left(\frac{\ln x}{x}\right)^3 dx = \begin{vmatrix} u = \ln^3 x & du = 3\ln^2 x \cdot \frac{1}{x}dx \\ dv = \frac{dx}{x^3} & v = -\frac{1}{2x^2} \end{vmatrix} = \ln^3 x \cdot (-\frac{1}{2x^2}) - \int -\frac{1}{2x^2} 3\ln^2 x \cdot$$
$$\frac{1}{x}dx$$

$$\int \frac{\ln^2 x}{x^3}dx = \begin{vmatrix} u = \ln^2 x & du = 2\ln x \frac{1}{x}dx \\ dv = \frac{dx}{x^3} & v = -\frac{1}{2x^2} \end{vmatrix} =$$
$$= \ln^2 x(-\frac{1}{2x^2}) - \int -\frac{1}{2x^2} \cdot 2\ln x \cdot \frac{1}{x}dx$$

$$\int \frac{\ln x}{x^3}dx = \begin{vmatrix} u = \ln x & du = \frac{dx}{x} \\ dv = \frac{dx}{x^3} & v = -\frac{1}{2x^2} \end{vmatrix} = \ln x \cdot (\frac{1}{2x^2}) - \int -\frac{1}{2x^2} \cdot \frac{1}{x}dx$$

Подведем небольшие итоги по подсчетам всех интегралов:

$$\int \left(\frac{\ln x}{x}\right)^3 dx = \ln^3 x \cdot (-\frac{1}{2x^2}) + \frac{3}{2}(\ln^2(x) \cdot (-\frac{2}{3}) + \ln x(-\frac{1}{2x^2}) + \frac{1}{2} \cdot (-\frac{1}{2x^2})) +$$
$$+ C,\ C \in \mathbb{R}$$

15. $\int \cos^5 x dx$

$$\int \cos^5 x dx = \begin{vmatrix} \sin x = t \\ \cos x dx = dt \end{vmatrix} = \int \cos^4 x dt = \int (1 - sin^2 x)^2 dt =$$
$$= \int (1-t^2)^2 dt = \sin x - \frac{2\sin^2 x}{3} + \frac{\sin^5 x}{5} + C,\ C \in \mathbb{R}$$

16. $\int \frac{x^2 e^x}{(x+2)^2}dx$

$$\int \frac{x^2e^x}{(x+2)^2}dx = \begin{vmatrix} u = x^2e^x & du = (2xe^x + x^2e^x)dx \\ dv = \frac{dx}{(x+2)^2} & v = -\frac{1}{x+2} \end{vmatrix} =$$

$$= x^2e^x(-\frac{1}{x+2}) - \int -\frac{1}{x+1}(2xe^x + x^2e^x)dx = -\frac{x^2e^x}{x+2} + \int xe^x dx = \begin{vmatrix} u = x & du = dx \\ dv = e^x dx & v = e^x \end{vmatrix} =$$

$$= -\frac{x^2e^x}{x+2} + xe^x - \int e^x dx = -\frac{x^2e^x}{x+2} + xe^x - e^x + C,\ C \in \mathbb{R}$$

Итог: не справился с номером 13 и 14

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