College of Engineering

Department of Electronics Engineering

MECHATRONICS ENGINEERING – 3202

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Midterm Project in Robotics 2

Presented to

Engr. Mikko De Torres

Instructor

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MExE 409 – Robotics 2

By:

Alojado, Stephen Gabriel S.

Apostol, Jan Benidict D.

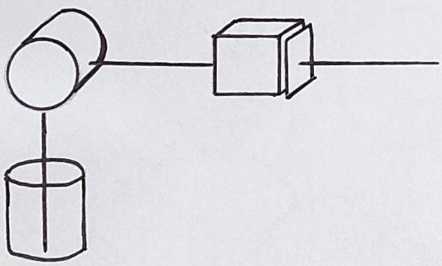
Cardenas, Sofia Bianca J.

Catapang, Jamill Darrius S.

Umali, Ariane Mae D.

SPHERICAL MANIPULATOR

Degrees of Freedom



$m=3 \quad n=3 \quad C_{i_T}=1 \quad C_{i_R}=1 \quad C_{i_P}=1$

$M = 6n \, \cancel{-6n} - \sum_{i=1}^{m}(6-C_i)$

$M = 6(3)-[(6-1)+(6-1)+(6-1)]$

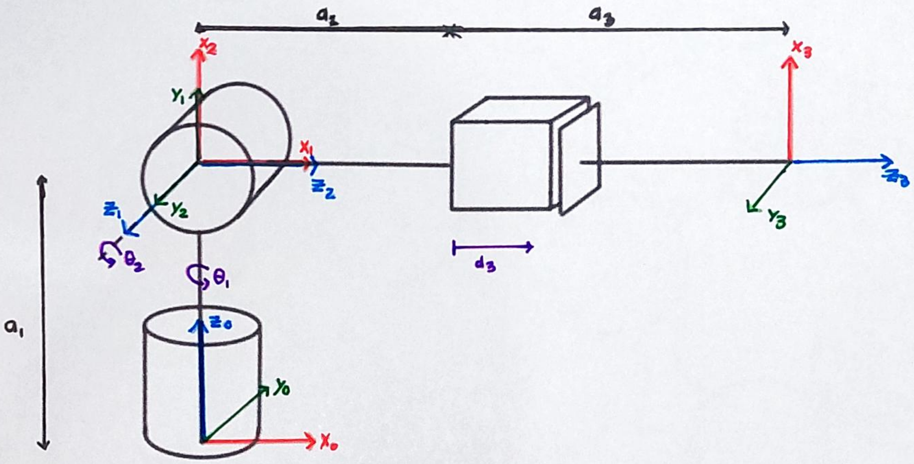
$M = 18 - [5+5+5]$

$M = 3$

Therefore, this is an Under-Actuated Spatial Manipulator with 3 Degrees of Freedom

The illustration above is a simplified diagram of an RRP Spherical Manipulator. It is also shown above the written computation to get the Degrees of Freedom of Spherical Manipulator. Furthermore, a Spherical Manipulator is a robot or manipulator with two rotational joints and one prismatic joint and it is commonly used on industries that involves material handling and welding. Meanwhile, Degrees of Freedom, as discussed in our classes, is the minimum number of independent parameters or variables or coordinates required to fully describe a system. To get the degrees of freedom for spherical manipulator, the Grubler's Criterion for Mobility or DOF of Spatial Manipulator is used as the spherical manipulator is a type of spatial manipulator. Based on the computations above, it can be concluded that this spherical manipulator is an Under-Actuated Spatial Manipulator with three degrees of freedom.

Kinematic Diagram and D-H Frame Assignment



The illustration above is the kinematic diagram of a Spherical Manipulator with D-H frame assignments. To further discuss, a kinematic diagram for a manipulator is a simplified schematic illustration used in robotics. It visually displays the structure of the manipulator by emphasizing the joints and links that connect them. The links are the rigid parts of the manipulator whereas the joints are the connections between links that allow for movement. Moreover, D-H (Denavit-Hartenberg) frame assignment is a systematic method of defining reference frames for each link in a manipulator. These frames play an essential role in determining the kinematic equations of the robot. There are four rules for assigning D-H Frames. The first is that the z-axis must be the axis of rotation for a revolute/twisting joint or the direction of translation for a prismatic joint. As observed in the illustration above, all of the blue arrows, which are the z-axis, point in the direction of the axis of rotation for the two revolute joints and the direction of translation for the single prismatic joint. The next rule is the x axis must be perpendicular both to its own z-axis, and the z-axis of the frame before it hence it is seen in the illustration that all the red arrows, an indicator for the x-axis, are all perpendicular to its own z-axis as well as to the z-axis of the frame prior to it. The next rule states that the x axis must be perpendicular to both its own z-axis and the z-axis of the frame preceding it. As can be seen in the illustration, all of the red arrows, which indicate the x-axis, are perpendicular to both its own z-axis and the z-axis of the frame prior to it. The third rule states that each axis must intersect the z-axis of the frame that preceded it. And the rules for complying with this rule are to rotate or translate the axis until it hits the other. Hence, the frame for the prismatic joint were translated into the joint before it so that it will comply with the third rule. The last rule of the D-H frame assignment is that all the frames must follow the right-hand rule. Thus, all the green arrows which indicate the y-axis are drawn with the right-hand rule in mind.

D-H Parametric Table

	n	θ	α	r	d
${}^0_1H \rightarrow$	1	$0° + \theta_1$	90°	0	a_1
${}^1_2H \rightarrow$	2	$90° + \theta_2$	90°	0	0
${}^2_3H \rightarrow$	3	0°	0°	0	$a_2 + a_3 + d_3$

The figure above is the D-H Parametric table for a Spherical Manipulator. Moreover, a D-H (Denavit-Hartenberg) Parametric Table is a method for organizing the four parameters generated by using the D-H frame assignment rules. These parameters are useful in determining the kinematic equations of a robotic manipulator. The four parameters are: theta, alpha, r, and d. Theta represents the angle of the joint between the current and next link. For revolute joints, this is the angle of rotation around the frame's z-axis. It is the rotation around z sub n-1 that is required to get x sub n-1 to match x sub n, with the joint variable theta if the joint is a revolute or twisting joint. The alpha represents the offset angle between the z-axis of frame (n-1) and the z-axis of frame (n). For easier calculations, it is commonly set to 0 or 90 degrees. The r represents the distance from the origin of n-1 and n frames along the x-sub-n direction. And lastly, The d refers to the distance from the origin of n-1 and n frames along the z-sub-n-1 direction, with the joint variable if the joint is prismatic. By filling up this table for each link in the manipulator, we will obtain a concise representation of the manipulator's geometry based on these four parameters. These parameters are then employed in formulae to compute the homogeneous transformation matrices, which ultimately represent the position and orientation of the manipulator's end-effector based on the joint variables.

Inverse Kinematics

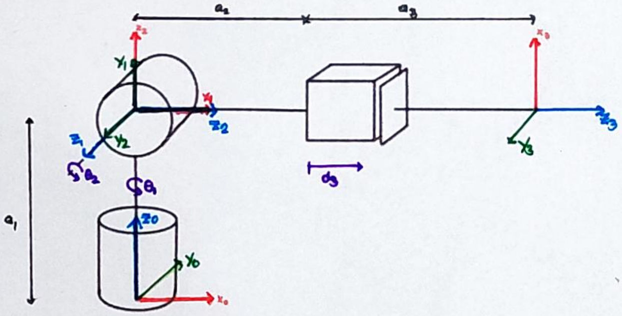
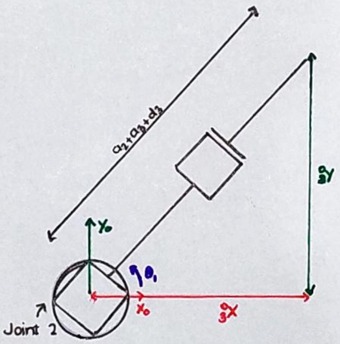


Figure: Kinematic Diagram of a Spherical Manipulator

Inverse Kinematics using Graphical Method

Top View



To solve for θ_1, use sohcahtoa:

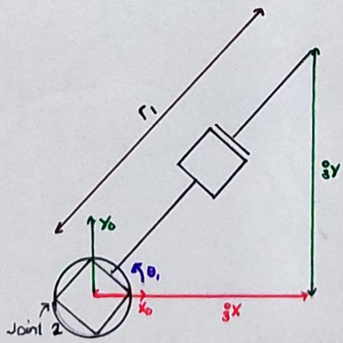
$\tan\theta_1$ = opposite/adjacent

opposite = ${}^0_3Y$

adjacent = ${}^0_3X$

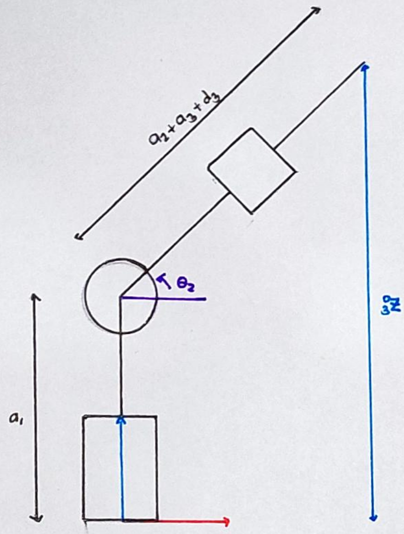
$$\boxed{\theta_1 = \tan^{-1}\left(\frac{{}^0_3Y}{{}^0_3X}\right)} \quad \text{eq. 1}$$

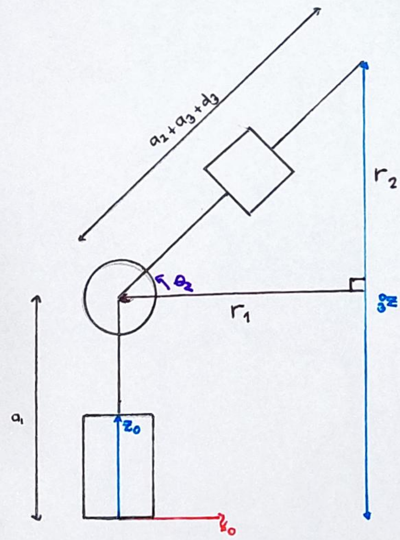
However, the hypotenuse is changing its length due to d_3 since it is a prismatic joint as well as because of the joint 2 being a revolute joint and so, it can change its orientation hence, we will give a new name for our hypotenuse, in this case, r_1. So from the equation 1, we can now derive the formula for r_1.



$$\boxed{r_1 = \sqrt{({}^0_3X)^2 + ({}^0_3Y)^2}} \quad \text{eq. 2}$$

Front View





And now in terms of the front view of the spherical manipulator, the r_1 will now be the adjacent length or side of θ_2. After placing the r_1, it can be noticed that we now have a right triangle with the $a_2+a_3+d_3$ being the hypotenuse. However we can't use the Z as the opposite side of our triangle thus, we will make a another line that would represent the opposite length of θ_2 and we will call it as r_2.

Based on the illustration above, we can derive that,

$Z = a_1 + r_2$ therefore,

$\boxed{r_2 = Z - a_1}$ eq. 3

And so, we can now derive for the formula of θ_2 using again the sohcahtoa;

$\tan\theta_2$ = opposite/adjacent

opposite = r_2

adjacent = r_1

$\boxed{\theta_2 = \tan^{-1}\left(\frac{r_2}{r_1}\right)}$ eq. 4

Finally, to find the joint variable d_3, the Pythagorean theorem is used;

$c^2 = a^2 + b^2$

$(a_2+a_3+d_3)^2 = r_1^2 + r_2^2$

$a_2+a_3+d_3 = \sqrt{r_1^2 + r_2^2}$

$\boxed{d_3 = \sqrt{r_1^2 + r_2^2} - a_2 - a_3}$ eq. 5

Homogeneous Transformation Matrix

To get the Homogeneous Transformation Matrix of the spherical manipulator, we can use the Homogeneous Transformation Matrix formula as well as the D-H Parametric Table that we have already derived.

Homogeneous Transformation Matrix Formula

$$ {}^{n-1}_{n}T = {}^{n-1}_{n}H = \begin{bmatrix} C\theta_n & -S\theta_n C\alpha_n & S\theta_n S\alpha_n & r_n C\theta_n \\ S\theta_n & C\theta_n C\alpha_n & -C\theta_n S\alpha_n & r_n S\theta_n \\ 0 & S\alpha_n & C\alpha_n & d_n \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^{0}_{1}H = \begin{bmatrix} C\theta_1 & 0 & S\theta_1 & 0 \\ S\theta_1 & 0 & -C\theta_1 & 0 \\ 0 & 1 & 0 & a_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^{1}_{2}H = \begin{bmatrix} -S\theta_2 & 0 & C\theta_2 & 0 \\ C\theta_2 & 0 & S\theta_2 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^{2}_{3}H = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & a_2+a_3+d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

Multiplying all of these together we get,

$$ {}^{0}_{3}H = {}^{0}_{1}H \; {}^{1}_{2}H \; {}^{2}_{3}H = \begin{bmatrix} -C\theta_1 S\theta_2 & S\theta_1 & C\theta_1 C\theta_2 & C\theta_1 C\theta_2(a_2+a_3+d_3) \\ -S\theta_1 S\theta_2 & -C\theta_1 & S\theta_1 C\theta_2 & S\theta_1 C\theta_2(a_2+a_3+d_3) \\ -C\theta_2 & 0 & S\theta_2 & a_1+S\theta_1(a_2+a_3+d_3) \\ 0 & 0 & 0 & 1 \end{bmatrix} $$