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BATANGAS STATE UNIVERSITY
The National Engineering University
Alangilan Campus

BAGONG PILIPINAS

Propelling Transformations and Accelerating Reforms for National Development

College of Engineering

Department of Electronics Engineering

MECHATRONICS ENGINEERING – 3202

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Laboratory Project in Robotics 2

Presented to

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Instructor

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MExE 409 – Robotics 2

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JACOBIAN MATRIX

	PRISMATIC	REVOLUTE
LINEAR	$R^0_{i-1}\begin{bmatrix}0\\0\\1\end{bmatrix}$	$R^0_{i-1}\begin{bmatrix}0\\0\\1\end{bmatrix} \times (d^0_n - d^0_{i-1})$
ROTATIONAL	$\begin{bmatrix}0\\0\\1\end{bmatrix}$	$R^0_{i-1}\begin{bmatrix}0\\0\\1\end{bmatrix}$

$$\overset{R_0^0}{\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}} \times \begin{bmatrix}0\\0\\1\end{bmatrix} = \begin{bmatrix}0\\0\\1\end{bmatrix} \times \left[\overset{d_3^0}{\begin{bmatrix}C\theta_1C\theta_2(a_2+a_3+d_3)\\S\theta_1C\theta_2(a_2+a_3+d_3)\\a_1+S\theta_2(a_2+a_3+d_3)\end{bmatrix}} - \overset{d_0^0}{\begin{bmatrix}0\\0\\0\end{bmatrix}}\right]$$

$$= \begin{bmatrix}0\\0\\1\end{bmatrix} \times \begin{bmatrix}C\theta_1C\theta_2(a_2+a_3+d_3)\\S\theta_1C\theta_2(a_2+a_3+d_3)\\a_1+S\theta_2(a_2+a_3+d_3)\end{bmatrix} = \overset{R_0^0}{\begin{bmatrix}-S\theta_1C\theta_2(a_2+a_3+d_3)\\C\theta_1C\theta_2(a_2+a_3+d_3)\\0\end{bmatrix}}$$

$$\overset{R_1^0}{\begin{bmatrix}C\theta_1&0&S\theta_1\\S\theta_1&0&-C\theta_1\\0&1&1\end{bmatrix}} \times \begin{bmatrix}0\\0\\1\end{bmatrix} = \begin{bmatrix}S\theta_1\\-C\theta_1\\0\end{bmatrix} \times \left[\overset{d_3^0}{\begin{bmatrix}C\theta_1C\theta_2(a_2+a_3+d_3)\\S\theta_1C\theta_2(a_2+a_3+d_3)\\a_1+S\theta_2(a_2+a_3+d_3)\end{bmatrix}} - \overset{d_1}{\begin{bmatrix}0\\0\\0\end{bmatrix}}\right]$$

$$= \begin{bmatrix}S\theta_1\\-C\theta_1\\0\end{bmatrix} \times \begin{bmatrix}C\theta_1C\theta_2(a_2+a_3+d_3)\\S\theta_1C\theta_2(a_2+a_3+d_3)\\a_1+S\theta_2(a_2+a_3+d_3)\end{bmatrix} = \overset{R_1^0}{\begin{bmatrix}-C\theta_1S\theta_2(a_2+a_3+d_3)\\-S\theta_1C\theta_2(a_2+a_3+d_3)\\C\theta_2(a_2+a_3+d_3)\end{bmatrix}}$$

$$\overset{R_1^0}{\begin{bmatrix}C\theta_1&0&S\theta_1\\S\theta_1&0&-C\theta_1\\0&1&1\end{bmatrix}} \times \overset{R_2^1}{\begin{bmatrix}-S\theta_2&0&C\theta_2\\C\theta_2&0&S\theta_2\\0&1&0\end{bmatrix}} = \overset{R_2^0}{\begin{bmatrix}-C\theta_1S\theta_2&S\theta_1&C\theta_1C\theta_2\\-S\theta_1S\theta_2&-C\theta_2&S\theta_1C\theta_2\\C\theta_2&0&S\theta_2\end{bmatrix}}$$

$$\overset{R_2^0}{\begin{bmatrix}-C\theta_1S\theta_2&S\theta_1&C\theta_1C\theta_2\\-S\theta_1S\theta_2&-C\theta_2&S\theta_1C\theta_2\\C\theta_2&0&S\theta_2\end{bmatrix}} \times \begin{bmatrix}0\\0\\1\end{bmatrix} = \overset{R_2^0}{\begin{bmatrix}C\theta_1S\theta_2\\S\theta_1C\theta_2\\S\theta_2\end{bmatrix}}$$

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} = \begin{bmatrix} -S\theta_1C\theta_2(a_2+a_3+d_3) & -C\theta_1S\theta_2(a_2+a_3+d_3) & C\theta_1S\theta_2 \\ C\theta_1C\theta_2(a_2+a_3+d_3) & -S\theta_1C\theta_2(a_2+a_3+d_3) & S\theta_1C\theta_2 \\ 0 & C\theta_2(a_2+a_3+d_3) & S\theta_2 \\ 0 & S\theta_1 & 0 \\ 0 & -C\theta_1 & 0 \\ 1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{d}_3 \end{bmatrix}$$

$$x = -S\theta_1C\theta_2\theta_1(a_2+a_3+d_3) - C\theta_1S\theta_2\theta_2(a_2+a_3+d_3) + C\theta_1S\theta_2d_3$$

$$y = C\theta_1C\theta_2\theta_1(a_2+a_3+d_3) - S\theta_1C\theta_2\theta_2(a_2+a_3+d_3) + S\theta_1C\theta_2d_3$$

$$z = C\theta_2\theta_2(a_2+a_3+d_3) + S\theta_2d_3$$

$$\omega_x = S\theta_1\theta_2$$

$$\omega_y = -C\theta_1\theta_2$$

$$\omega_z = \theta_1$$

SINGULARITY

$$\mathrm{DET}(J) = \begin{bmatrix} -S\theta_1C\theta_2(a_2+a_3+d_3) & -C\theta_1S\theta_2(a_2+a_3+d_3) & C\theta_1S\theta_2 \\ C\theta_1C\theta_2(a_2+a_3+d_3) & -S\theta_1C\theta_2(a_2+a_3+d_3) & S\theta_1C\theta_2 \\ 0 & C\theta_2(a_2+a_3+d_3) & S\theta_2 \end{bmatrix}$$

$$\mathrm{DET}(J) = -S^2\theta_1C\theta_2(a_2+a_3+d_3)^2 + C^2\theta_1S^2\theta_2C\theta_2(a_2+a_3+d_3)^2 + C^3\theta_2(a_2+a_3+d_3)$$

INVERSE VELOCITY

GIVEN:

LINK LENGTHS: $a_1 = 50$, $a_2 = 20$, $a_3 = 20$

JOINT VARIABLES: $\theta_1 = 0$, $\theta_2 = 90°$, $d_3 = 30$

FROM JACOBIAN MATRIX,

$$\begin{bmatrix} -S\theta_1 C\theta_2(a_2+a_3+d_3) & -C\theta_1 S\theta_2(a_2+a_3+d_3) & C\theta_1 S\theta_2 \\ C\theta_1 C\theta_2(a_2+a_3+d_3) & -S\theta_1 C\theta_2(a_2+a_3+d_3) & S\theta_1 C\theta_2 \\ 0 & C\theta_2(a_2+a_3+d_3) & S\theta_2 \end{bmatrix} = \begin{bmatrix} 0 & -70 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$C_{ij} = (-1)^{(i+j)} \cdot \det(M_{ij})$

$\det(M_{11}) = 0$

$C_{11} = (-1)^{(1+1)} \cdot 0$

$C_{11} = 0$

$\det(M_{12}) = 0$

$C_{12} = (-1)^{(1+2)} \cdot 0$

$C_{12} = 0$

$\det(M_{13}) = 0$

$C_{13} = (-1)^{(1+3)} \cdot 0$

$C_{13} = 0$

$\det(M_{21}) = -70$

$C_{21} = (-1)^{(2+1)} \cdot -70$

$C_{21} = 70$

$\det(M_{22}) = 0$

$C_{22} = (-1)^{(2+2)} \cdot 0$

$C_{22} = 0$

$\det(M_{23}) = 0$

$C_{23} = (-1)^{(2+3)} \cdot 0$

$C_{23} = 0$

$\det(M_{31}) = 0$

$C_{31} = (-1)^{(3+1)} \cdot 0$

$C_{31} = 0$

$\det(M_{32}) = 0$

$C_{32} = (-1)^{(3+2)} \cdot 0$

$C_{32} = 0$

$\det(M_{33}) = 0$

$C_{33} = (-1)^{(3+3)} \cdot 0$

$C_{33} = 0$

INVERSE VELOCITY

$$\begin{bmatrix} 0 & 0 & 0 \\ 70 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} = \begin{bmatrix} \theta_1 \\ \theta_2 \\ d_3 \end{bmatrix}$$

INVERSE VELOCITIES

$\theta_1 = 0$

$\theta_2 = 70\dot{x}$

$d_3 = 0$