1.

解:

$$设x = (x_1, x_2)^T \ y = (y_1, y_2)^T$$

$$则\langle x, y\rangle = 2x_1y_1 + x_1y_2 + 2x_2y_2$$

$$则\langle x, y\rangle = 2x_1y_1 + x_1y_2 + 2x_2y_2$$

$$而\langle y, x\rangle = 2x_1y_1 + x_2y_1 + 2x_2y_2$$

$$当x_1y_2 \neq x_2y_1时\ \langle x, y\rangle \neq \langle y, x\rangle\ 故\langle ., .\rangle不构成内积$$

2.

解:

(1):

$$d(x, y) = \sqrt{< x - y, x - y >} = \sqrt{\begin{pmatrix} 2 & 3 & 3 \end{pmatrix} \begin{pmatrix} 2 \\ 3 \\ 3 \end{pmatrix}} = \sqrt{22}$$

(2):

$$d(x,y)=\sqrt{<x-y,x-y>}=\sqrt{\begin{pmatrix}2&3&3\end{pmatrix}\begin{pmatrix}2&1&0\\1&3&-1\\0&-1&2\end{pmatrix}\begin{pmatrix}2\\3\\3\end{pmatrix}}=\sqrt{47}$$

3.

解:

(1): 由 U 组成列向量的矩阵 A 为:

$$A=\begin{pmatrix}0&1&-3&-1\\-1&-3&4&-3\\2&1&1&5\\0&-1&2&0\\2&2&1&7\end{pmatrix}$$

化简得:

$$A=\begin{pmatrix}1&0&0&1\\0&1&0&2\\0&0&1&1\\0&0&0&0\\0&0&0&0\end{pmatrix}$$

因此 U 的基组成列向量的矩阵为

$$B=\begin{pmatrix}0&1&-3\\-1&-3&4\\2&1&1\\0&-1&2\\2&2&1\end{pmatrix}$$

因此 B 的转置为:

$$B^T=\begin{pmatrix}0&-1&2&0&2\\1&-3&1&-1&2\\-3&4&1&2&1\end{pmatrix}$$

$$设x_W = B\lambda, \lambda \in R^n$$

$$则由B^T(x - B\lambda) = 0化简可得$$

$$\lambda = (B^TB)^{-1}B^Tx$$

因此，x 在 U 上的正交投影为：

$$\pi_U(x) = B\lambda = B(B^TB)^{-1}B^Tx = \begin{pmatrix} 0 & 1 & -3 \\ -1 & -3 & 4 \\ 2 & 1 & 1 \\ 0 & -1 & 2 \\ 2 & 2 & 1 \end{pmatrix} \begin{pmatrix} \frac{100}{63} & -\frac{31}{21} & -\frac{2}{3} \\ -\frac{31}{21} & \frac{31}{21} & \frac{2}{3} \\ -\frac{2}{3} & \frac{2}{3} & \frac{1}{3} \end{pmatrix} \begin{pmatrix} 0 & -1 & 2 & 0 & 2 \\ 1 & -3 & 1 & -1 & 2 \\ -3 & 4 & 1 & 2 & 1 \end{pmatrix} x$$

$$\pi_U(x) = \begin{pmatrix} 1 \\ -5 \\ -1 \\ -2 \\ 3 \end{pmatrix}$$

(2)

$$d(x, U) = ||x_{W^\perp}|| = ||x - x_W||$$

$$d(x, U) = \sqrt{\begin{pmatrix} -2 & -4 & 0 & 6 & 2 \end{pmatrix} \begin{pmatrix} -2 \\ -4 \\ 0 \\ 6 \\ 2 \end{pmatrix}} = 2\sqrt{15}$$

4.

只需证 $Ax = 0$ 当且仅当 $A^T Ax = 0$。易知 $Ax = 0 \Rightarrow A^T Ax = 0$. 此外，我们有 $A^T Ax = 0 \Rightarrow x^T A^T Ax = 0 \Rightarrow (Ax)^T(Ax) = 0 \Rightarrow Ax = 0$.

5.

数据点位于椭圆上，意味着上述方程满足给定的 x,y 值

$$\text{代入} AX^2 + Y^2 + CXY + DX + EY + F = 0$$

有:

$$0A + 0C + 0D + 2E + F + 4 = 0$$

$$4A + 2C + 2D + 1E + F + 1 = 0$$

$$1A + (-1)C + 1D + (-1)E + F + 1 = 0$$

$$1A + 2C + (-1)D + (-2)E + F + 4 = 0$$

$$9A + (-3)C + (-3)D + 1E + F + 1 = 0$$

$$1A + 1C + (-1)D + (-1)E + F + 1 = 0$$

故有

$$A=\begin{pmatrix}0&0&0&2&1\\4&2&2&1&1\\1&-1&1&-1&1\\1&2&-1&-2&1\\9&-3&-3&1&1\\1&1&-1&-1&1\end{pmatrix} x=\begin{pmatrix}A\\C\\D\\E\\F\end{pmatrix} b=\begin{pmatrix}-4\\-1\\-1\\-4\\-1\\-1\end{pmatrix}$$

解方程$Ax=b$ 的最小二乘解为：$\overline{x}=(A^TA)^{-1}A^Tb=\begin{pmatrix}0.342\\-0.339\\0.628\\-0.450\\-2.645\end{pmatrix}=\begin{pmatrix}\frac{6213}{18190}\\-\frac{3087}{9095}\\\frac{11421}{18190}\\-\frac{4092}{9095}\\-\frac{24056}{9095}\end{pmatrix}$