信息的表示和处理(2)

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提纲

- 整数
  - 表示：有符号数和无符号数
  - 转换
  - 扩展和截断
  - 计算
  - 总结

无符号数

- 二进制 (物理存储形式)
  - 位向量$[x_{w-1},x_{w-2},x_{w-3},\ldots x_0]$
- 二进制转换成无符号数Unsigned(逻辑表达)

$$B2U(X) = \sum_{i=0}^{w-1} x_i \cdot 2^i$$

例如：N_1=01011，表示无符号数11;

N_2=11011 表示无符号数27;

N_3=00000 表示无符号数0。

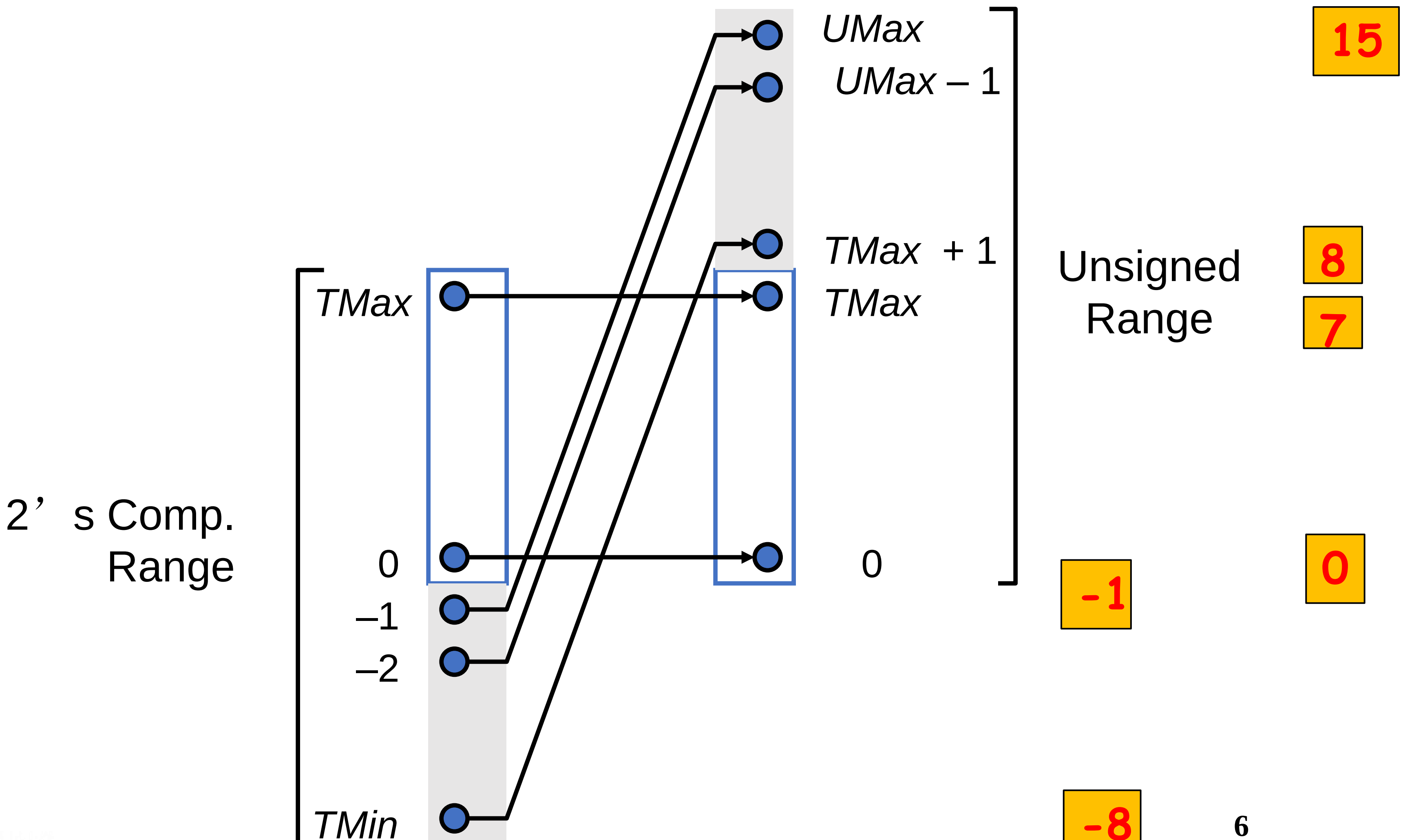
无符号数

- 如果机器字长为n位，则无符号数的表示范围是：
  $0\sim(2^n-1)$。
- 例如字长为8位，则无符号数的表示范围是0～255。
- 计算机的内存地址就是无符号数的例子。

有符号数Signed Representation

- Binary原本的含义与非负整数正好对应
- 负数如何用binary表达？
  - 解决思路：**映射**(mapping)
  - 将负数段映射到一个不使用的非负数段
    - 对有限范围的整数成立
    - 最好不影响正数和0
  - 且最好负数和映射到的整数在某种意义上是“**等价**”的，可以直接进行运算

补码的映射关系



UMax
UMax – 1
TMax + 1
TMax
0
Unsigned
Range
15
8
7
0
-1
-8
2’ s Comp.
Range
TMax
0
–1
–2
TMin

Two-complement补码

- Binary (physical)
  - Bit vector $[x_{w-1},x_{w-2},x_{w-3},\ldots x_0]$
- Binary to Signed (logical)

w位，有2^w个状态，
所以**模就是2^w**；
增加或减少2^w，值不变

Sign Bit

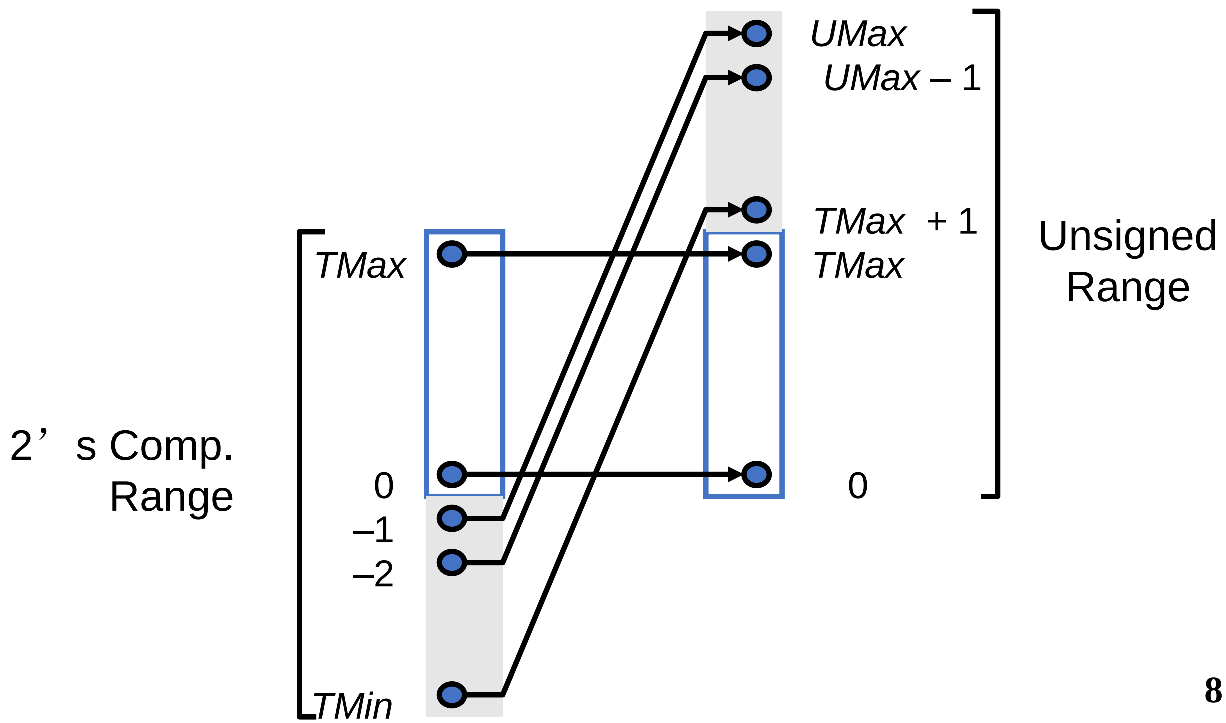
$$B2T(X) = -x_{w-1} \cdot 2^{w-1} + \sum_{i=0}^{w-2} x_i \cdot 2^i$$

	-16	8	4	2	1	
10 =	0	1	0	1	0	8+2 = 10

	-16	8	4	2	1	
-10 =	1	0	1	1	0	-16+4+2 = -10

Two's Complement

- 2's complement
  - “模” 就是2^w
  - 因为binary形式的值是2^{w-1}+…，实际值是-2^{w-1}+…，二者相差2^w



补码 - 模运算（modular运算）

重要概念：在一个模运算系统中，一个数与它除以“模”后的余数等价。

时钟是一种模12系统　现实世界中的模运算系统

假定钟表时针指向10点，要将它拨向6点，　则有两种拨法：

① 倒拨4格：10- 4 = 6

② 顺拨8格：10+8 = 18 ≡ 6　　(mod 12)

模12系统中：　10- 4 ≡ 10+8　(mod 12)

- 4 ≡ 8　(mod 12)

则，称8是- 4对模12的补码（即：- 4的模12补码等于8）。

同样有 -3 ≡ 9　（mod 12）

-5 ≡ 7　（mod 12）等

结论1：　一个负数的补码等于模减该负数的绝对值。

结论2：　对于某一确定的模，某数减去小于模的另一数，总可以用该数加上另一数负数的补码来代替。

补码（modular运算）：+ 和– 的统一

Signed Representation

补码表示法

- 只要确定了“模”，就可找到一个与负数等价的正数（该正数即为负数的补数）来代替此负数，而这个正数可以用模加上负数本身求得，这样就可把减法运算用加法实现了。
- 例： 9-**5**=9+(12-5)=9**+7**=4 (mod 12)

  65-**25**=65+(-25)=65+(100-25)=65**+75**=40 (mod 100)

求补码的快捷方式

设 $x = -1010$ 时

$$
\begin{aligned}
\text{则}[x]_{\text{补}} &= 2^{4+1} - 1010 \\
&= 100000 \\
&\quad\; - 1010 \\
&= 1,0110
\end{aligned}
\qquad
\begin{aligned}
&= 11111 + 1 - 1010 \\
&= 11111 + 1 \\
&\quad\; - 1010 \\
&\quad\; 10101 + 1 \\
&= 1,0110
\end{aligned}
$$

当真值为 负 时

每位取反，末位加 1 求得

From a Number to Two’s Complement

- 真值-5
  - y=5
  - 0101 (binary for 5)
  - 1010 (after complement)
  - 1011 (add 1)
  - 补码x为1011

- 补码x=1011
  - 1011 (two’s complement binary)
  - 0100 (after complement)
  - 0101 (add 1)
  - Y=0101, 真值-y=-5

Two's Complement Encoding Examples

Binary/Hexadecimal Representation for -12345

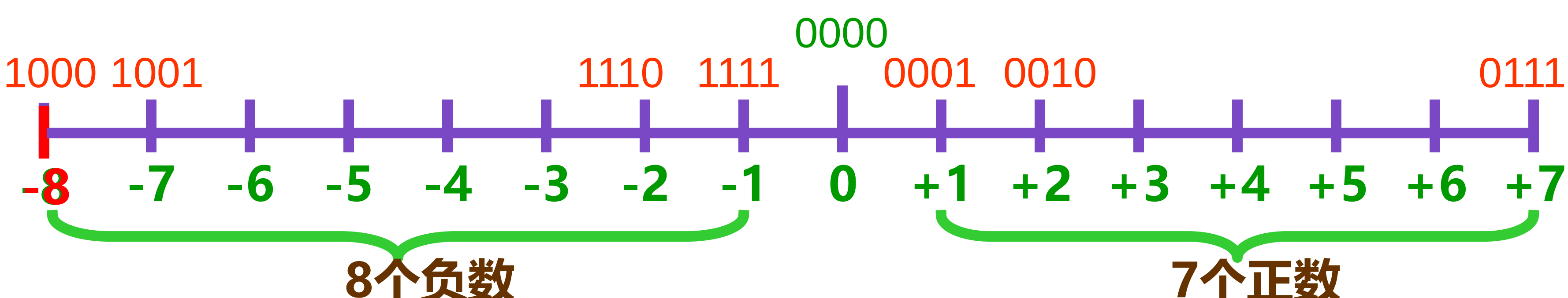
Binary:	0011	0000	0011	1001	(12345)
Hex:	3	0	3	9	
Binary:	1100	1111	1100	0110	(after complement)
Hex:	C	F	C	6	
Binary:	1100	1111	1100	0111	(add 1)
Hex:	C	F	C	7	

Numeric Ranges

- Unsigned Values
  - *UMin* = 0
    000…0
  - *UMax* = $2^w - 1$
    111…1

- Two's Complement Values
  - *TMin* = -2^{w-1}
    100…0
  - *TMax* = $2^{w-1} - 1$
    011…1

E.g., w=4
0000 ~ 0111: 0~7
1000 ~ 1111: -8~-1

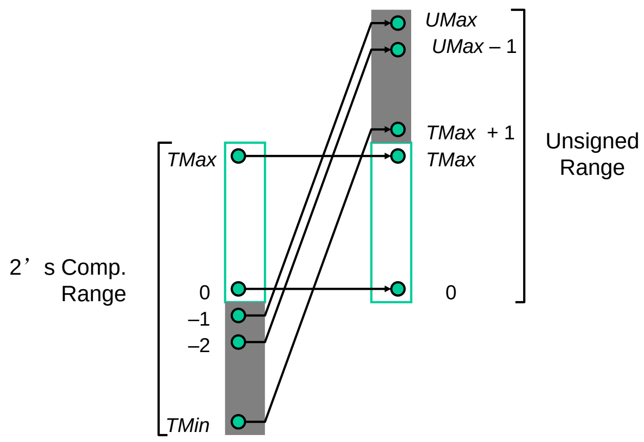


Numeric Ranges

- Unsigned Values
  - *UMin* = 0
    000…0
  - *UMax* = $2^w - 1$
    111…1
- Two's Complement Values
  - *TMin* = -2^{w-1}
    100…0
  - *TMax* = $2^{w-1} - 1$
    011…1
- Other Values
  - Minus 1
    111…1

Values for *W* = 16

	Decimal	Hex	Binary
UMax	65535	FF FF	11111111 11111111
TMax	32767	7F FF	01111111 11111111
TMin	-32768	80 00	10000000 00000000
-1	-1	FF FF	11111111 11111111
0	0	00 00	00000000 00000000



Values for Different Word Sizes

	W			
	8	**16**	**32**	**64**
UMax	255	65,535	4,294,967,295	18,446,744,073,709,551,615
TMax	127	32,767	2,147,483,647	9,223,372,036,854,775,807
TMin	-128	-32,768	-2,147,483,648	-9,223,372,036,854,775,808

- Observations
  - $|TMin| = TMax + 1$
    - Asymmetric range
  - $UMax = 2 * TMax + 1$

- **C Programming**
  - #include <limits.h>
  - Declares constants, e.g.,
    - ULONG_MAX
    - LONG_MAX
    - LONG_MIN
  - Values platform specific

有符号数的多种表示方式

- Ones' Complement (反码)
  - The most significant bit has weight $2^{w-1}-1$
- Sign-Magnitude (原码)
  - The most significant bit is a sign bit
    - that determines whether the remaining bits should be given negative or positive weight

- 对于正数，原码 = 补码 = 反码
- 对于负数，符号位为 1，其 数值部分
  原码除符号位外每位取反末位加 1⟶ 补码
  原码除符号位外每位取反⟶ 反码

填空题 5分

设置

写出下列数字的补码

(1)+0.1101B的补码是 [填空1] B

(2)-0.1101B的补码是[填空2] B

(3)-178的补码是[填空3] H（short类型)

(4)-39的补码是[填空4] H （short类型)

(5)-26的补码是[填空5] H （short类型)

作答

下面给出了一组变量在计算机内的编码（补码），请写出他们的真值(16进制形式)。

(1)FF3C（short类型）的真值是 [填空1]
(2)FF86（short类型）的真值是 [填空2]
(3)char x =10110111b，x的真值是 [填空3]

作答

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Integral data type in C

- Signed type (for integer numbers)
  - char, short [int], int, long [int]
- Unsigned type (for nonnegative numbers)
  - unsigned char, unsigned short [int], unsigned [int], unsigned long [int]
- Java 没有无符号类型
  - Byte: signed char

Casting

- Casting among Signed and Unsigned in C
- C允许一种类型按照另一种类型的方式来理解
  - Type conversion (implicitly)
  - Type casting (explicitly)
- Signed和unsigned类型之间的显式转换
  - 即 U2T 和 T2U
    - int tx, ty;
    - unsigned ux, uy;
    - tx = (int) ux;
    - uy = (unsigned) ty;

Signed vs. Unsigned in C

- Conversion
  - 通过赋值语句或call语言（函数调用）来隐式转换
    - int tx, ty;
    - unsigned ux, uy;
    - tx = ux;
    - uy = ty;

    - int func () {…}
    - unsigned ux = func();

Mapping Signed ↔ Unsigned

Bits	Signed	Unsigned
0000	0	0
0001	1	1
0010	2	2
0011	3	3
0100	4	4
0101	5	5
0110	6	6
0111	7	7
1000	–8	8
1001	–7	9
1010	–6	10
1011	–5	11
1100	–4	12
1101	–3	13
1110	–2	14
1111	–1	15

$\xrightarrow{\text{T2U}}$

$\xleftarrow{\text{U2T}}$

Mapping Signed ↔ Unsigned

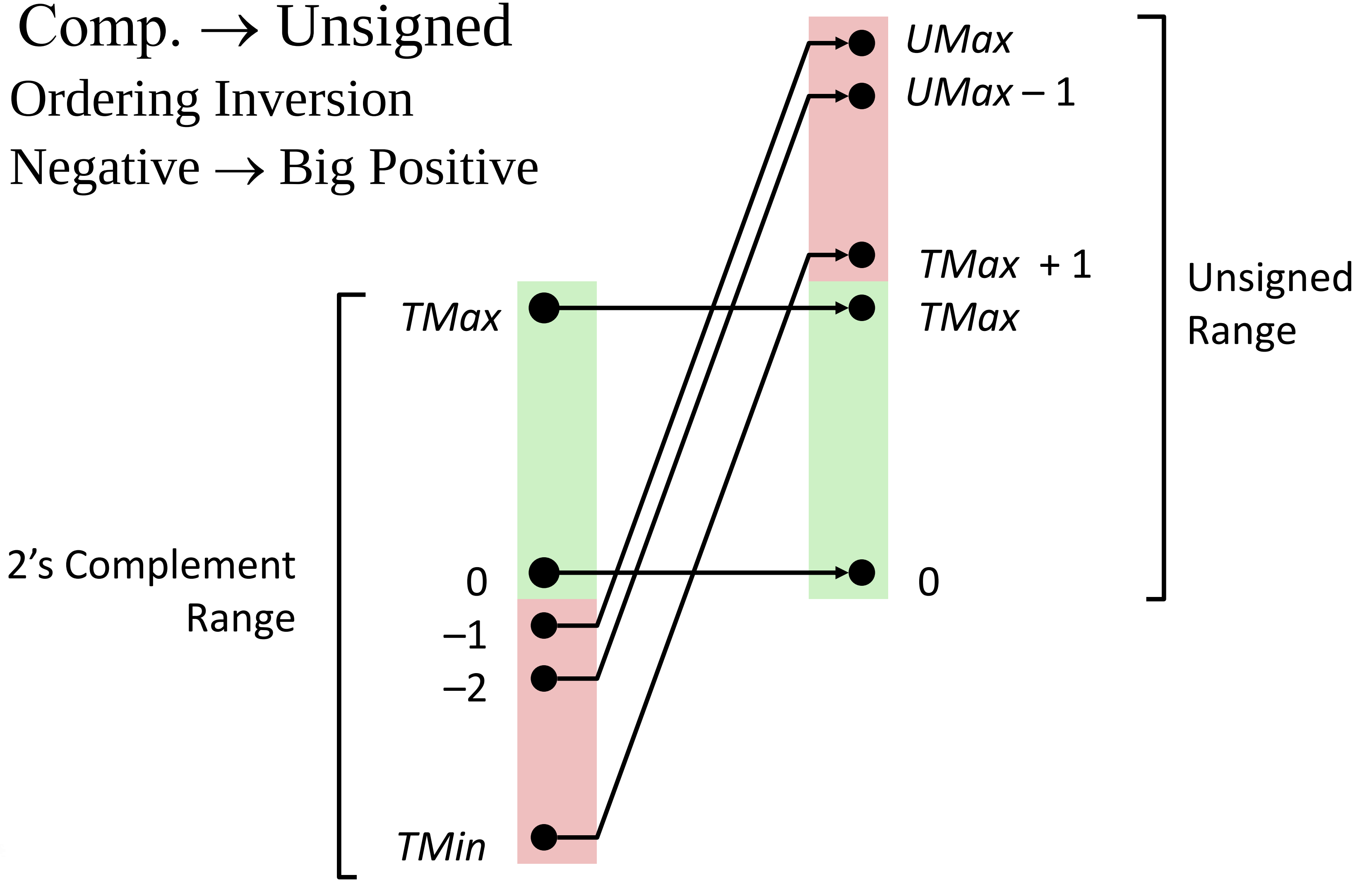
Bits	Signed	Unsigned
0000	0	0
0001	1	1
0010	2	2
0011	3	3
0100	4	4
0101	5	5
0110	6	6
0111	7	7
1000	-8	8
1001	-7	9
1010	-6	10
1011	-5	11
1100	-4	12
1101	-3	13
1110	-2	14
1111	-1	15

=

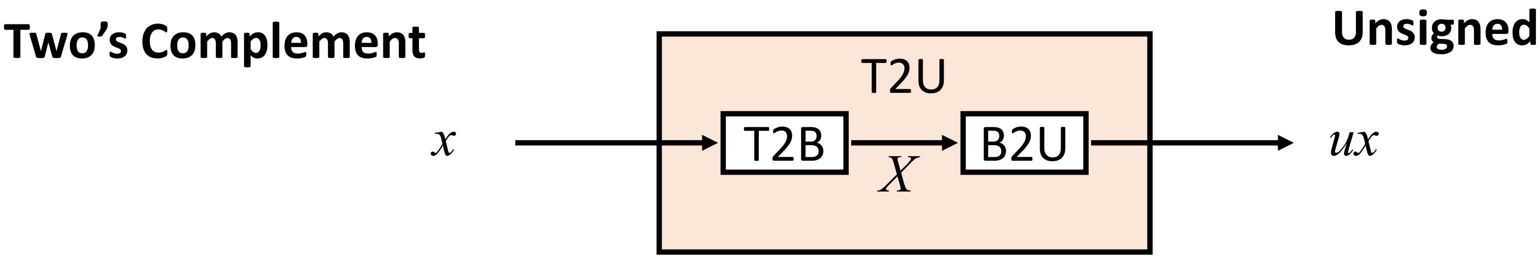
+/- 16

Conversion Visualized

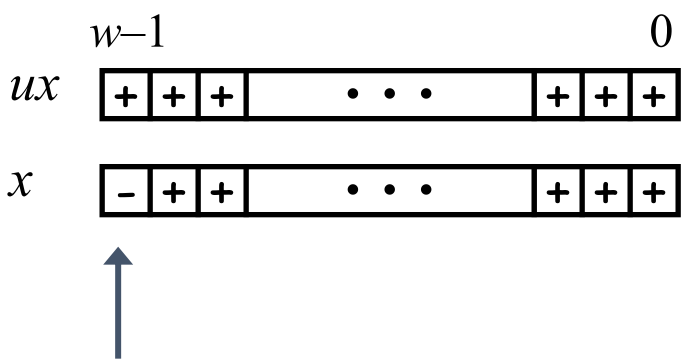
- 2's Comp. → Unsigned
  - Ordering Inversion
  - Negative → Big Positive



Relation between Signed & Unsigned



Maintain Same Bit Pattern

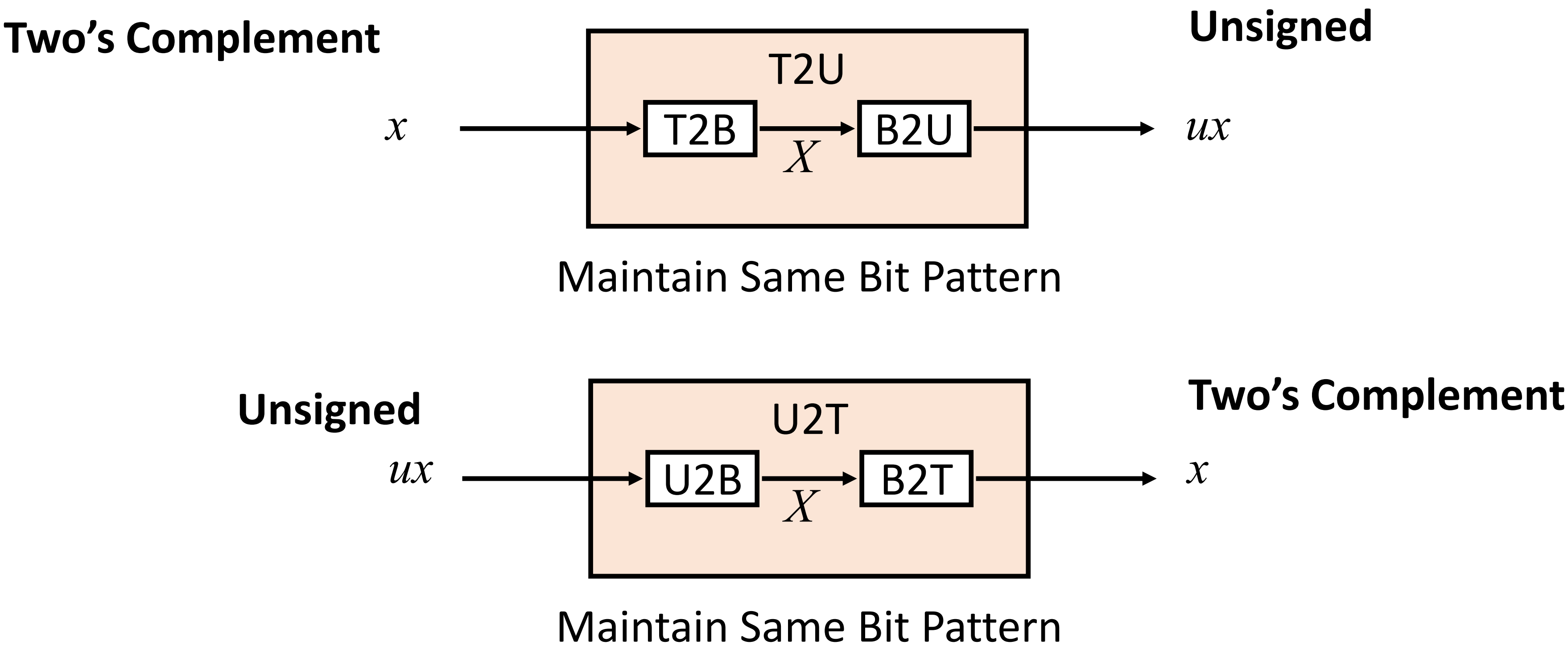


Large negative weight

becomes

Large positive weight

Mapping Between Signed & Unsigned



- Mappings between unsigned and two's complement numbers: keep bit representations and reinterpret

Casting from Signed to Unsigned

```
short int          x =  12345;
unsigned short int ux = (unsigned short) x;
short int          y  = -12345;
unsigned short int uy = (unsigned short) y;
```

- 转换的结果
  - 非负值不变
    - *ux* = 12345
  - 负值被转为(更大的)正值
    - *uy* = 53191

Weight	12,345		−12,345		53,191	
	Bit	Value	Bit	Value	Bit	Value
1	1	1	1	1	1	1
2	0	0	1	2	1	2
4	0	0	1	4	1	4
8	1	8	0	0	0	0
16	1	16	0	0	0	0
32	1	32	0	0	0	0
64	0	0	1	64	1	64
128	0	0	1	128	1	128
256	0	0	1	256	1	256
512	0	0	1	512	1	512
1,024	0	0	1	1,024	1	1,024
2,048	0	0	1	2,048	1	2,048
4,096	1	4096	0	0	0	0
8,192	1	8192	0	0	0	0
16,384	0	0	1	16,384	1	16,384
±32,768	0	0	1	−32,768	1	32,768
Total		12,345		−12,345		53,191

Unsigned Constants in C

- 默认情况
  - 常量被当作有符号数
- 如果希望是无符号数，在后面加上后缀U
  - 0U, 4294967259U

Casting Convention

- 表达式比较
  - 如果在一个表达式中混用unsigned和signed
    - 会隐式地把有符号数专为无符号数
  - 包括 <, >, ==, <=, >= 等比较符号

```
unsigned int i
for (i=n-1; i>= 0; i--)
```

```
for (i=n-1; i-sizeof (char) >= 0; i--)
```

```
int array[] = {1,2,3};
#define TOTAL sizeof(array) /* unsigned int */
void main() {
    int d = -1;
    if (d <= TOTAL)
        printf("small\n");
    else printf("large\n");
}
```

Casting Surprises

- Expression Evaluation
  - If there is a mix of unsigned and signed in single expression, signed values implicitly cast to unsigned
  - Including comparison operations <, >, ==, <=, >=
  - Examples for W = 32: TMIN = -2,147,483,648 , TMAX = 2,147,483,647

Constant1	Constant2	Relation	Evaluation
0	0U		
-1	0		
-1	0U		
2147483647	-2147483647-1		
2147483647U	-2147483647-1		
-1	-2		
(unsigned)-1	-2		
2147483647	2147483648U		
2147483647	(int) 2147483648U		

填空题 9分

设置

W = 32:　TMIN = -2,147,483,648 ,　TMAX = 2,147,483,647

Constant1	Constant2	Relation
0	0U	[填空1]
-1	0	[填空2]
-1	0U	[填空3]
2147483647	-2147483647-1	[填空4]
2147483647U	-2147483647-1	[填空5]
-1	-2	[填空6]
(unsigned)-1	-2	[填空7]
2147483647	2147483648U	[填空8]
2147483647	(int) 2147483648U	[填空9]

作答

Casting Surprises

- Expression Evaluation
  - If there is a mix of unsigned and signed in single expression, ***signed values implicitly cast to unsigned***
  - Including comparison operations <, >, ==, <=, >=
  - Examples for W = 32: **TMIN = -2,147,483,648 ,** **TMAX = 2,147,483,647**

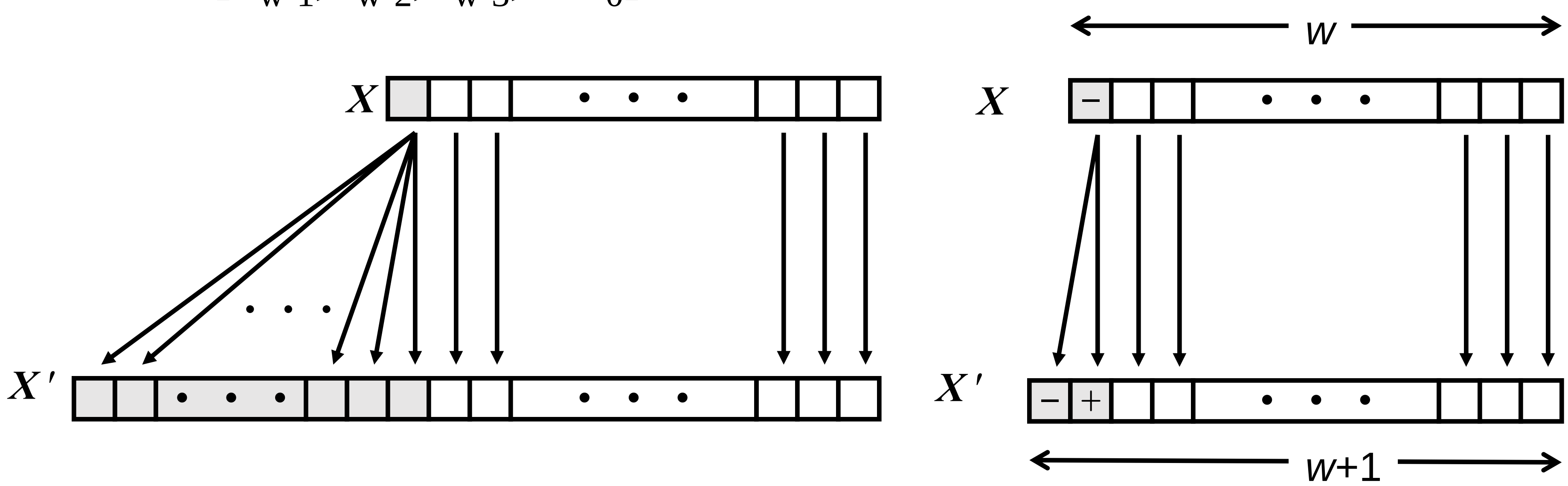
Constant$_1$	Constant$_2$	Relation	Evaluation
0	0U	**==**	**unsigned**
-1	0	**<**	**signed**
-1	0U	**>**	**unsigned**
2147483647	-2147483647-1	**>**	**signed**
2147483647U	-2147483647-1	**<**	**unsigned**
-1	-2	**>**	**signed**
(unsigned)-1	-2	**>**	**unsigned**
2147483647	2147483648U	**<**	**unsigned**
2147483647	(int) 2147483648U	**>**	**signed**

提纲

- 整数
  - 表示：有符号数和无符号数
  - 转换
  - **扩展和截断**
  - 计算
  - 总结

Expanding the Bit Representation

- Zero extension
  - 在前面添加0
- Sign extension
  - $[x_{w-1}, x_{w-2}, x_{w-3}, \ldots x_0]$



Sign Extension: Simple Example

Positive number

	-16	8	4	2	1
10 =	0	1	0	1	0

	-32	16	8	4	2	1
10 =	0	0	1	0	1	0

Negative number

	-16	8	4	2	1
-10 =	1	0	1	1	0

	-32	16	8	4	2	1
-10 =	1	1	0	1	1	0

From short to long

```
int fun1(unsigned word) {
        return (int) ((word << 24) >> 24);
}
int fun2(unsigned word) {
        return ((int) word << 24) >> 24;
}
```

w	fun1(w)	fun2(w)
0x00000076	**00000076**	**00000076**
0x87654321	**00000021**	**00000021**
0x000000C9	**000000C9**	**FFFFFFC9**
0xEDCBA987	**00000087**	**FFFFFF87**

练习：描述一下两个函数的行为区别

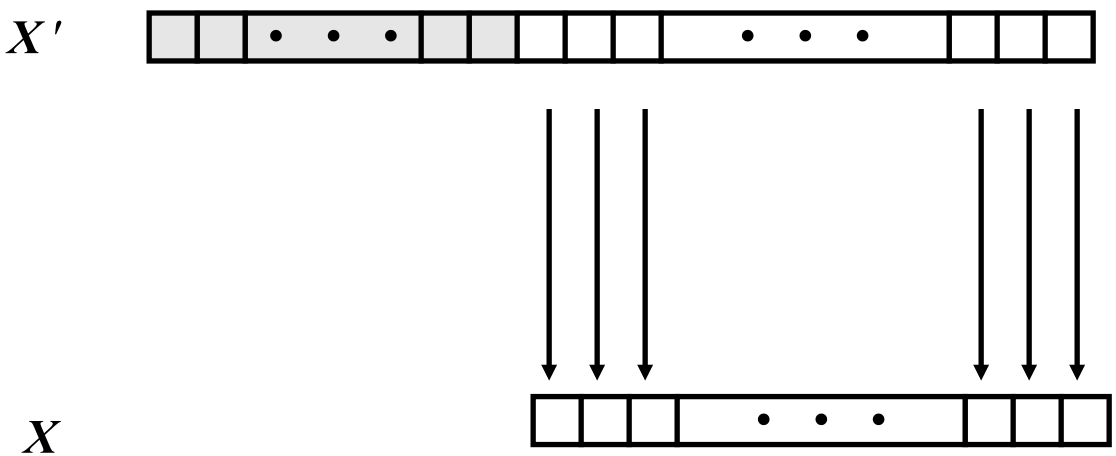
From long to short

```
int          x  = 53191;
short int sx = x;
int          y  = -12345;
Short int sy = y;
```

- 需要截断数据大小
- 从长类型到短类型转换需要特别注意

Truncating Numbers

	Decimal	Hex	Binary
x	53191	00 00 CF C7	00000000 00000000 11001111 11000111
sx	-12345	CF C7	11001111 11000111
y	-12345	FF FF CF C7	11111111 11111111 11001111 11000111
sy	-12345	CF C7	11001111 11000111



Truncating Numbers

- Unsigned Truncating

$$B2U_w([x_w, x_{w-1}, \cdots, x_0]) \bmod 2^k \;=\; B2U_k([x_k, x_{k-1}, \cdots x_0])$$

- Signed Truncating
  - 先当作无符号数去截断，然后转换为有符号数

$$B2T_k([x_k, x_{k-1}, \cdots x_0]) \;=\; U2T_k(B2U_w([x_w, x_{w-1}, \cdots, x_0]) \bmod 2^k)$$

Truncation: Simple Example

No sign change

	-16	8	4	2	1
2 =	0	0	0	1	0

	-8	4	2	1
2 =	0	0	1	0

2 mod 16 = 2

	-16	8	4	2	1
-6 =	1	1	0	1	0

	-8	4	2	1
-6 =	1	0	1	0

-6 mod 16 = 26U mod 16 = 10U = -6

Sign change

	-16	8	4	2	1
10 =	0	1	0	1	0

	-8	4	2	1
-6 =	1	0	1	0

10 mod 16 = 10U mod 16 = 10U = -6

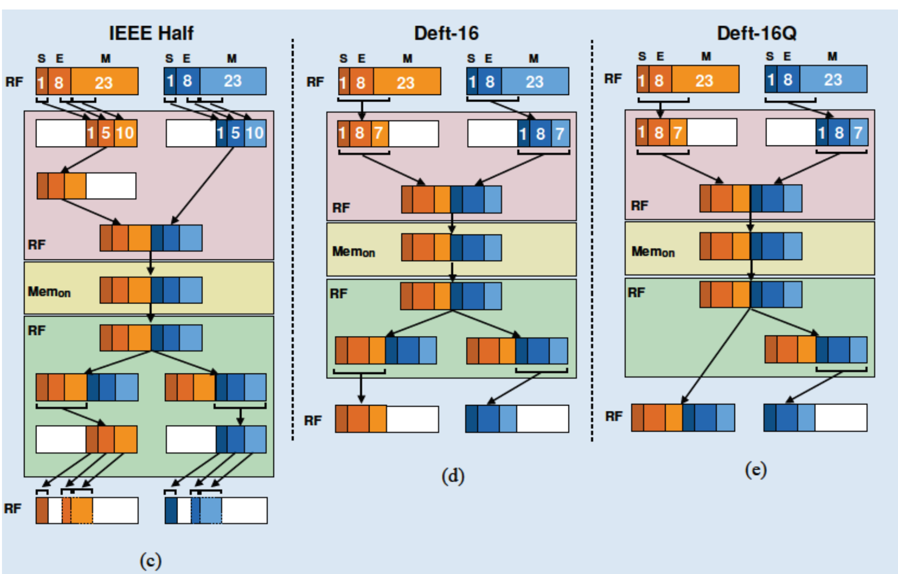
	-16	8	4	2	1
-10 =	1	0	1	1	0

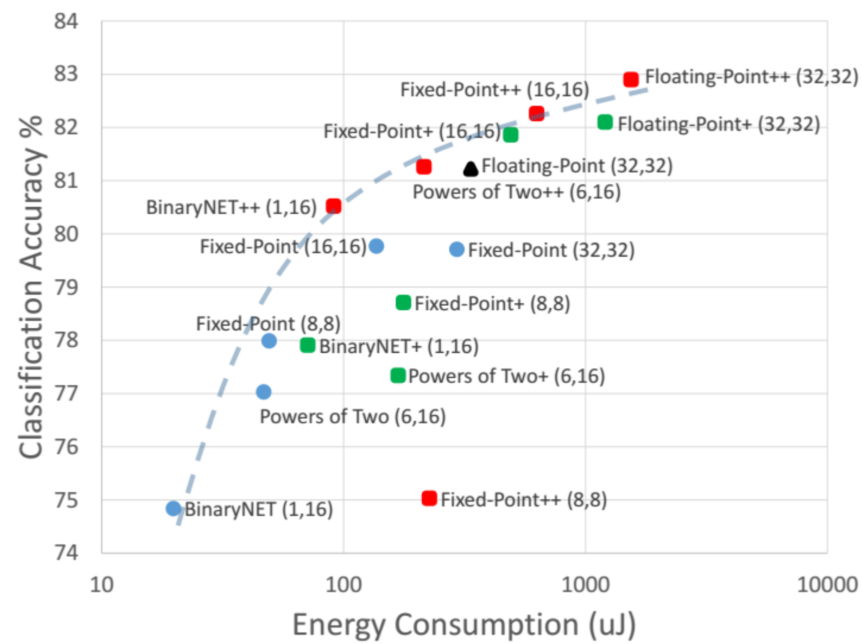
	-8	4	2	1
6 =	0	1	1	0

-10 mod 16 = 22U mod 16 = 6U = 6

截断在人工智能中的应用

- 精度缩放，压缩数据减少计算和存储开销
- 精度损失很小，几乎可以忽略不计
- 典型的深度学习模型用**8**位数据就可以达到**95%**的精度





Advice on Signed vs. Unsigned

```
float sum_elements ( float a[], unsigned length )
{
   int i;
   float result = 0;
   for ( i = 0; i <= length – 1; i++)
            result += a[i] ;
}
```

当length=0时，应该返回0.0，但实际会遇到内存错误，为什么？

Advice on Signed vs. Unsigned

```
/* Prototype for library function strlen */
size_t strlen(const char *s); /*size_t is unsigned */

/* Determine whether string s is longer than string t */
/* WARNING: This function is buggy */
int strlonger(char *s, char *t) {
  return strlen(s) - strlen(t) > 0;
}
```

FreeBSD's implementation of getpeername (…)

```
/* Declaration of library function memcpy */
void *memcpy(void *dest, void *src, size_t n);
/* Kernel memory region holding user-accessible data */
 #define KSIZE 1024u
char kbuf[KSIZE];
/* Copy at most maxlen bytes from kernel region to user buffer */
int copy_from_kernel (void *user_dest, int maxlen) {
/* Byte count len is minimum of buffer size and maxlen */
   int len = KSIZE < maxlen ? KSIZE : maxlen;
   memcpy(user_dest, kbuf, len); // 如果maxlen是个负值
   return len;  }
```

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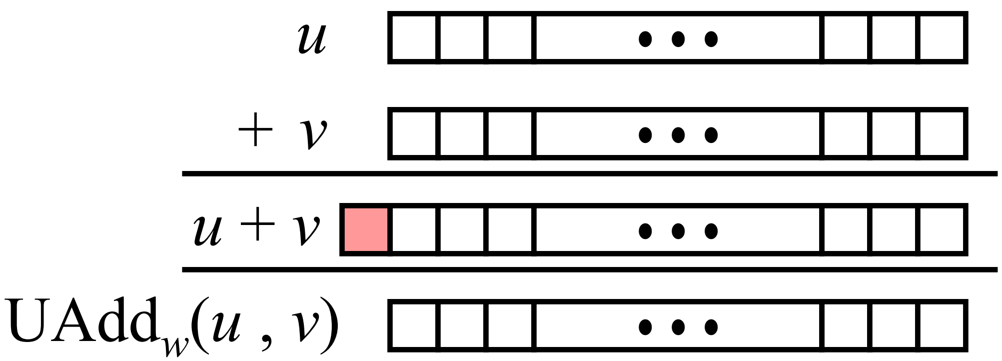
Unsigned Addition

Operands: *w* bits

True Sum: *w*+1 bits

Discard Carry: *w* bits



- Standard Addition Function
  - Ignores carry output
- Implements Modular Arithmetic

$$s \quad = \quad \mathrm{UAdd}_w(u\ ,\ v) \quad = \quad u + v \mod 2^w$$

```
unsigned char       1110 1001        E9        233
                  + 1101 0101      + D5      + 213
```

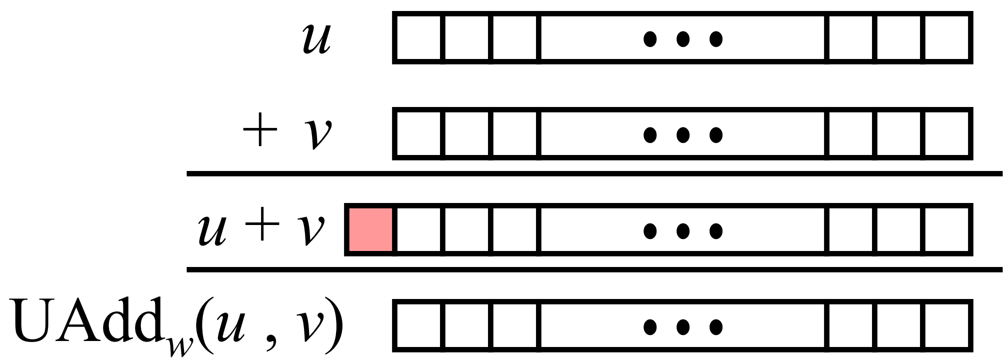
Hex	Decimal	Binary
0	0	0000
1	1	0001
2	2	0010
3	3	0011
4	4	0100
5	5	0101
6	6	0110
7	7	0111
8	8	1000
9	9	1001
A	10	1010
B	11	1011
C	12	1100
D	13	1101
E	14	1110
F	15	1111

Unsigned Addition

Operands: *w* bits

True Sum: *w*+1 bits

Discard Carry: *w* bits



- Standard Addition Function
  - Ignores carry output
- Implements Modular Arithmetic

$$s \quad = \quad \mathrm{UAdd}_w(u\ ,\ v) \quad = \quad u + v \mod 2^w$$

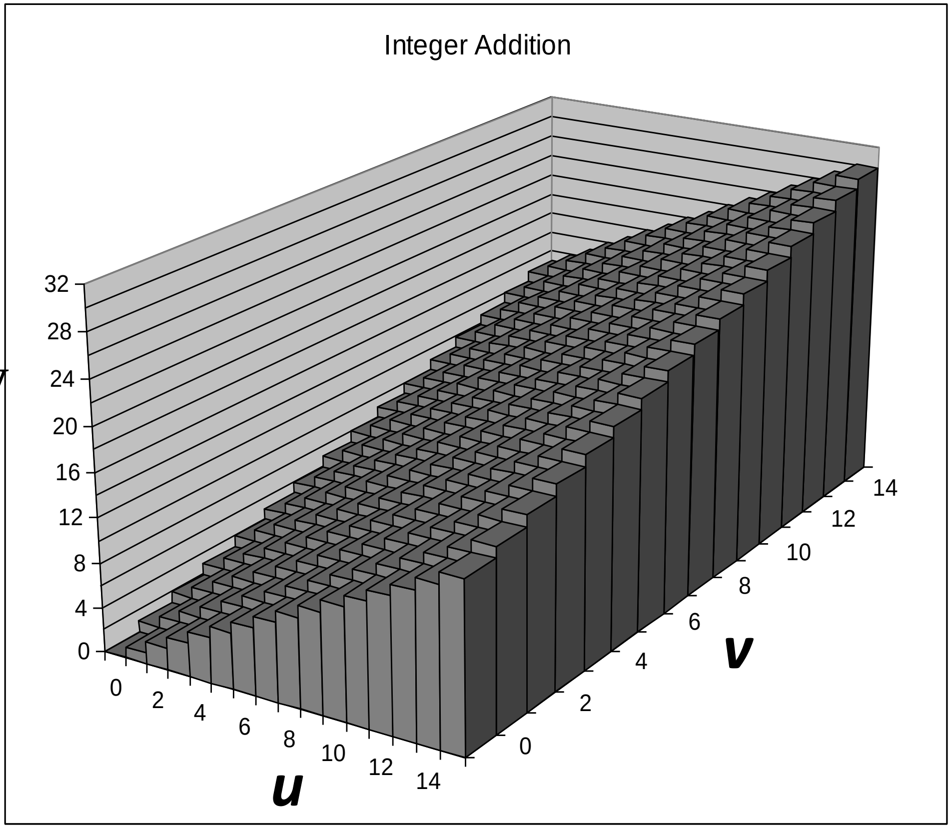
	Binary	Hex	Decimal
unsigned char	1110 1001	E9	233
	+ 1101 0101	+ D5	+ 213
	1 1011 1110	1BE	446
	1011 1110	BE	190

Hex	Decimal	Binary
0	0	0000
1	1	0001
2	2	0010
3	3	0011
4	4	0100
5	5	0101
6	6	0110
7	7	0111
8	8	1000
9	9	1001
A	10	1010
B	11	1011
C	12	1100
D	13	1101
E	14	1110
F	15	1111

Visualizing Integer Addition

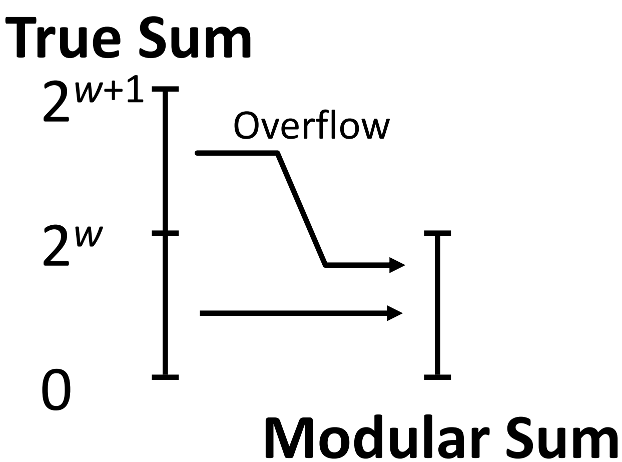
- Integer Addition
  - 4-bit integers u, v
  - Compute true sum $Add_4(u\ ,\ v)$
  - Values increase linearly with u and v
  - Forms planar surface

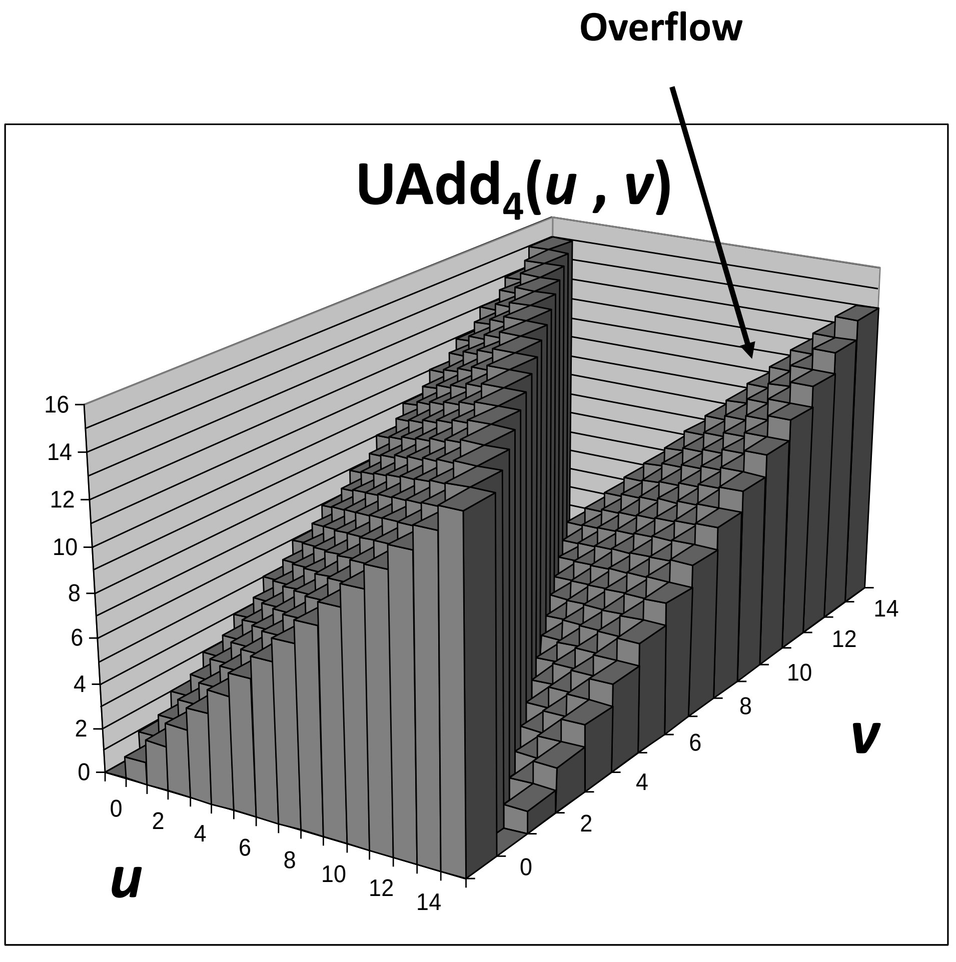
$Add_4(u\ ,\ v)$



Visualizing Unsigned Addition

- Wraps Around
  - If true sum $\geq 2^w$
  - At most once





Unsigned Addition

Write a function with the following prototype:

/* Determine whether arguments can be added without overflow */

int uadd_ok(unsigned x, unsigned y);

/*This function should return 1 if arguments x and y can be added without causing overflow*/

Overflow if (X+Y) < X

Unsigned Addition Forms an Abelian Group (阿贝尔群)

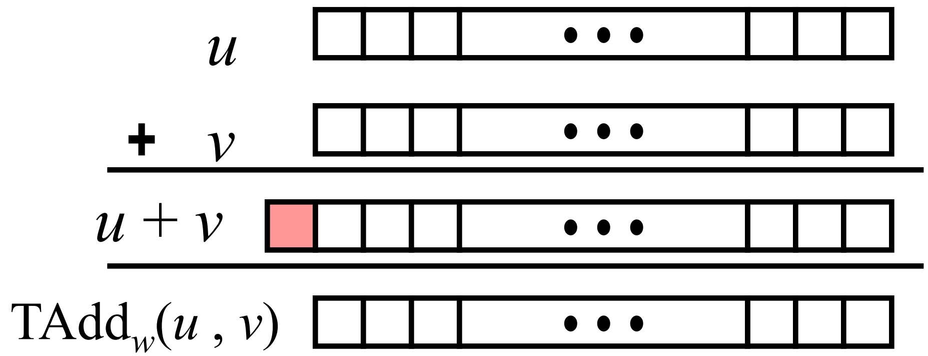
- 加法结果是封闭的（还在同样的数据范围内）
  - $0 \le UAdd_w(u, v) \le 2^w - 1$
- 交换律
  - $UAdd_w(t, UAdd_w(u,v)) = UAdd_w(UAdd_w(t, u), v)$
- 有一个单位元0
  - $UAdd_w(u, 0) = u$
- 每个元素都有一个加法逆元
  - Let $UComp_w(u) = 2^w - u$
  - $UAddw(u, UComp_w(u)) = 0$

Two's Complement Addition

Operands: *w* bits

True Sum: *w*+1 bits

Discard Carry: *w* bits



- TAdd and UAdd have Identical Bit-Level Behavior
  - Signed vs. unsigned addition in C:

```
int s, t, u, v;
s = (int) ((unsigned) u + (unsigned) v);
t = u + v
```

  - Will give `s == t`

```
      1110 1001       E9      -23
+     1101 0101     + D5    + -43
---------------    -----    -----
    1 1011 1110      1BE      -66
---------------    -----    -----
      1011 1110       BE      -66
```

Signed Addition- overflow

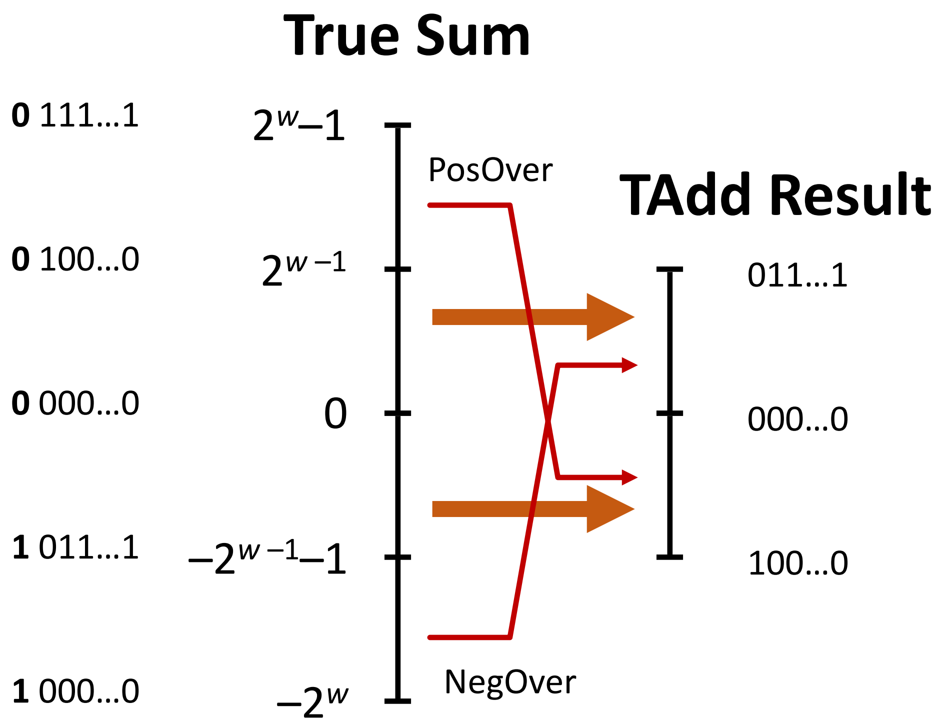
位宽是4，其中1位符号位，3位数值位

5	5	-5	-5
+ 4	+(-4)	+ 4	+(-4)
9	**1**	**-1**	**-9**

0101	**0101**	**1011**	**1011**
+ 0100	**+1100**	**+ 0100**	**+1100**
1001	**0001**	**1111**	**0111**
-7	**1**	**-1**	**7**

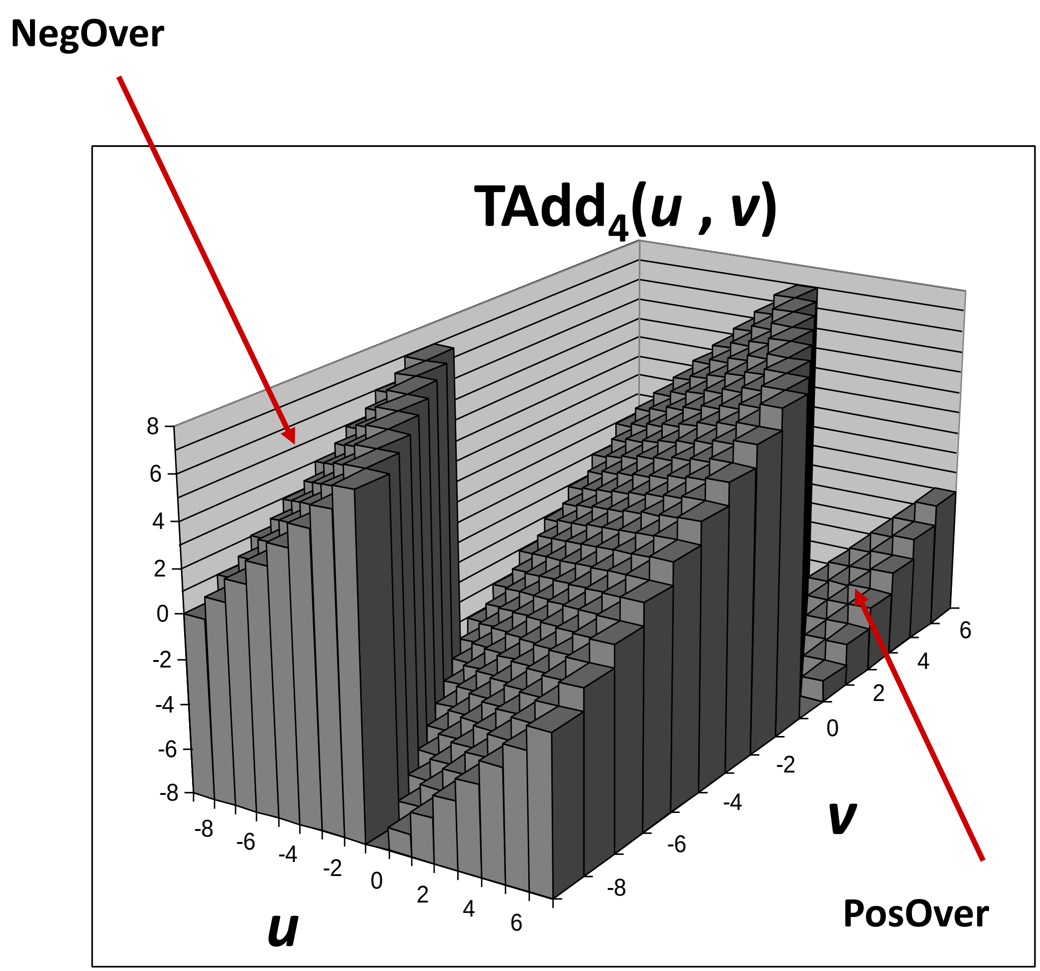
TAdd Overflow

- Functionality
  - True sum requires w+1 bits
  - Drop off MSB
  - Treat remaining bits as 2's comp. integer



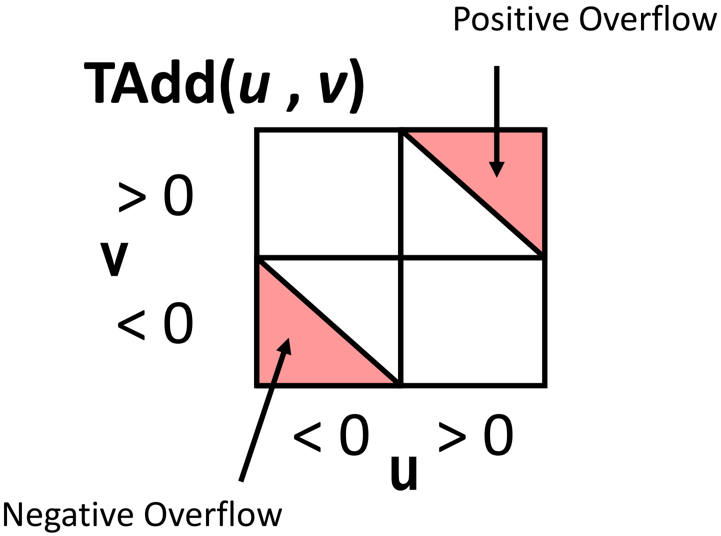
Visualizing 2's Complement Addition

- Values
  - 4-bit two's comp.
  - Range from -8 to +7
- Wraps Around
  - If sum $\geq 2^{w-1}$
    - Becomes negative
    - At most once
  - If sum $< -2^{w-1}$
    - Becomes positive
    - At most once



Characterizing TAdd

- Functionality
  - True sum requires w+1 bits
  - Drop off MSB
  - Treat remaining bits as 2's comp. integer

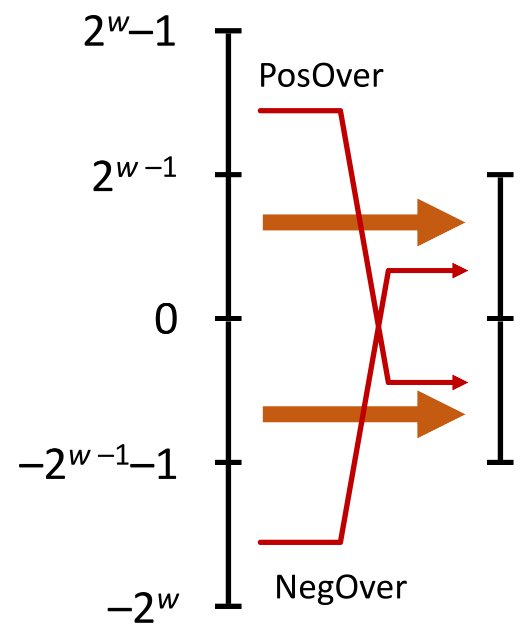


$$TAdd_w(u,v) = \begin{cases} u+v+2^w & u+v < TMin_w \quad \textbf{(NegOver)} \\ u+v & TMin_w \le u+v \le TMax_w \\ u+v-2^w & TMax_w < u+v \quad \textbf{(PosOver)} \end{cases}$$

Detecting Tadd Overflow

- Task
  - Given $s = \text{TAdd}_w(u, v)$
- Claim
  - Overflow iff either:
    - $u, v < 0, s \geq 0$ (NegOver)
    - $u, v \geq 0, s < 0$ (PosOver)

  - ovf = (u<0 == v<0) && (u<0 != s<0);



Detecting Tadd Overflow

```
int tadd_ok_bugy(int x, int y)
{
        int sum = x + y ;
        return (sum-x == y) && (sum-y == x)
} // 问题在哪？
```

e.g., [-8, 7), x=4, y=5

sum=9=-7. sum-x = -7-4=-11=5=y（-11还是会被拉回到5）

tadd是阿贝尔群，结果封闭（越界则加上或减少2^w），所以sum-x = x+y-x永远等于y（因为y就在合法区间，不会加减2^w）。此函数无法判定溢出。

//应该如何正确判断加法溢出？

Detecting Tadd Overflow

```
int tadd_ok (int x, int y)
{
        int sum = x + y ;
        return !((x>0&&y>0&&sum<0) ||
(x<0&&y<0&&sum>0))
}
```

Detecting Tsub Overflow

```
int tsub_ok(int x, int y)
{
        return tadd_ok(x, -y) ;
} // 问题在哪？
```

e.g., x>0, y=TMin; x<0, y=TMin; x=0, y=TMin

- 如果x是正数，判定是异号相加，肯定不溢出，但实际是正数-Tmin，肯定溢出。
- 如果x是负数，会判定溢出，实际不溢出。
- 如果x=0，会判定不溢出，实际溢出。

Detecting Tsub Overflow

```
int tsub_ok(int x, int y)
{
        int diff = x-y;
        return !(x>=0&&y<0&&diff<0 ||
x<0&&y>0&&diff>0) // 0-正数 不会溢出
}
```

Mathematical Properties of TAdd

- Two's Complement Under TAdd Forms a Group
  - Closed, Commutative, Associative, 0 is additive identity
  - Every element has additive inverse
    - Let
    - $\mathrm{TAdd}_w(u\ , \mathrm{TComp}_w\ (u\ ))\ =\ 0$

$$TComp_w(u) \quad = \quad \begin{cases} -u & u \neq TMin_w \\ TMin_w & u = TMin_w \end{cases}$$

Mathematical Properties of TAdd

- TAdd和Uadd：
  - $\mathrm{TAdd}_w(u, v) = \mathrm{U2T}(\mathrm{UAdd}_w(\mathrm{T2U}(u), \mathrm{T2U}(v)))$
    - 因为二者有相同的位级表示
  - $\mathrm{T2U}(\mathrm{TAdd}_w(u, v)) = \mathrm{UAdd}_w(\mathrm{T2U}(u), \mathrm{T2U}(v))$

补码：连同符号位一起运算

Negating with Complement & Increment

- In C
  - ~x + 1 == -x
- Complement
  - Observation: ~x + x == 1111…111 == -1
- Increment
  - ~x + 1
  - ==~x + x + (-x + 1)
  - ==-1 + (-x + 1)
  - == -x

```
      x   1 0 0 1 1 1 0 1
+    ~x   0 1 1 0 0 0 1 0
-------------------------
     -1   1 1 1 1 1 1 1 1
```

x为负数时:
x的补码是原码除符号位外,
每位取反，末位加1

-x的补码是原码连同符号位,
每位取反，末位加1

Arithmetic: Basic Rules

- 加法：
  - Unsigned/signed: 先正常加，再截断，二者在bit级的操作完全一样
  - Unsigned: 相加后模2^w
    - 正常加法结果 + 可能减去2^w
  - Signed:相加后模2^w
    - 正常加法结果 + 可能加上或减去2^w

课堂练习1

- 库函数calloc有如下声明：
- Void *calloc (size_t nmemb, size_t size);
- 该函数为一个数组分配内存，该数组有nmemb个元素，每个元素为size字节，内存设置为0。如果nmemb或size为0，则返回NULL。
- 具体实现时，通过malloc分配内存，并用memset将内存设置为0.
- 你的代码应该没有任何由算术溢出引发的漏洞
  - Void *malloc (size_t size);
  - Void *memset(void *s, int c, size_t n);

- void *calloc(size_t nmemb, size_t size) {
  - size_t asize = nmemb * size; /* Check for overflow */
  - if (nmemb == 0 || size == 0 || asize / nmemb != size) /* Error */
    - return NULL;
  - void *result = malloc(asize);
  - if (result != NULL) {
    - memset(result, 0, asize);
    - return result;
  - }
  - return NULL;
- }

课堂练习2

- Int x = random();
- Int y = random();
- Unsigned ux = (unsigned)x;
- Unsigned uy = (unsigned)y;
- Int为32位，对于下列每个C表达式，你要指出其是否总为1。如果是，请指出其数学原理；否则，举反例
    1. (x<y) == (-x>-y)
    2. ((x+y)<<4) + y –x == 17*y + 15*x
    3. ~x+~y+1 == -(x+y)
    4. (ux-uy) == -(unsigned)(y-x)
    5. ((x>>2)<<2) <= x

课堂练习2

1.(x<y) == (-x>-y)

否，当x=Tmin时不成立，-x=Tmin，还是小于-y

2. ((x+y)<<4) + y –x == 17*y + 15*x

是，补码的循环特性，即使一边溢出，或两边都溢出，最终左右还是会相等

3. ~x+~y+1 == ~(x+y)

是，~x+1+~y+1=–x-y

4. (ux-uy) == -(unsigned)(y-x)

是，有符号和无符号的加减法在binary层级是完全一样的，这个表达式相当于x==-(-x)是否成立？X=Tmin也是成立的。

5. ((x>>2)<<2) <= x

是，x为正数时，可能损失最后两位，变小；x为负数时，binary变小，值变小（绝对值变大）【补码映射是线性关系，增大变小的关系，正负数是一样的】

课堂练习3

你刚刚开始在一家公司工作，他们要用到一个数据结构，这个结构是将4个有符号字节封装为一个32位的**unsigned**。一个32位字中的字节从**0**（最低有效字节）编号到**3**（最高有效字节）。分配给你的任务是：编写一个函数，提取其中的按照指定的**bytenum**从中提取需要的有符号字节，将其符号扩展为一个**32**位**int**：

```
typedef unsigned packed_t;
/* Extract byte from word. Return as signed integer */
int xbyte (packed_t word, int bytenum);
```

你的前任（因为水平不够高被解雇了）编写了下面的代码：

```
/* Failed attempt at xbyte */
int xbyte (packed_t word, int bytenum) {
        return (word >> (bytenum << 3)) & 0xFF;
}
```

这段代码错在哪里？

((int)word<<((3-bytenum)<<3))>>24

给出函数的正确实现，除了赋值语句外，只能使用左右移位和一个减法。