MATH/DATA 412 Midterm 1 topics [1]

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1) **Orthogonality, inner products** (§2.1, 2.2)
   - Cauchy-Schwarz inequality
   - triangle inequality
   - Pythagorean theorem
2) **Orthogonal and orthonormal bases** (§2.3, 2.5, 2.6)
   - orthogonal bases
   - orthonormal bases
   - Gram-Schmidt
   - orthogonal complement
3) **Orthogonal and orthonormal bases** (§2.4)
   - orthogonal projection
   - closest point in a subspace
   - fitting line to data
4) **Matrix algebra** (§3.1, 3.2, 3.3)
   - matrix operations: sums, products, scalar product
   - matrix product and inner products
   - matrix product as sum of outer products
   - transposes and symmetric matrices
5) **Rank, nullity, and fundamental subspaces** (§3.4, 3.5, 3.6, 4.5)
   - row space, column space, nullspace
   - rank
   - nullity
   - Rank-Nullity Theorem
   - nonsingular matrices and invertibility
6) **Symmetric, symmetric positive definite, and symmetric positive semidefinite matrices** (§4.1)
   - Cholesky factorization
7) **Sample covariance** (§7.1.1)

- sample mean
- sample variance
- sample covariance
- covariance matrix

8) **Orthogonal matrices and QR factorization** (§4.5, 4.7.1)