Lab 02: PL Resolution

Courses `CSC14003`: Intro to Artificial Intelligence
`18CLC6`, `FIT - HCMUS`.
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This is an INDIVIDUAL assignment:

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About this assignment:

Problem description

Given a knowledge base (**KB**) and a query **α**, both are sets of propositional clauses in CNF. Refer to Textbook, Chapter 7, Figure 7.12, to **implement the PL-RESOLUTION function** to check whether **KB entails α** (KB ⊨ α).

Provide 5 non-trivial test cases, along with the submission, to validate your program

In the report, discuss the PL resolution algorithm's efficiency and suggest some solution(s) to address the limitations

Input specifications

The input file stores the KB and α, whose format is as follows:

- The first line contains a positive integer M, which is the number of clauses in query α
- M next lines represent the clauses in query α, one clause per line
- The M+2 line contains a positive integer N, which is the number of clauses in the KB
- N next lines represent the clauses in KB, one clause per line.

A (positive) literal is represented by one uppercase letter, i.e. A-Z. A negated literal is a literal associated with a minus symbol ('-') right ahead. Literals are connected by the OR keyword. There may be multiple white spaces between literals and keywords.

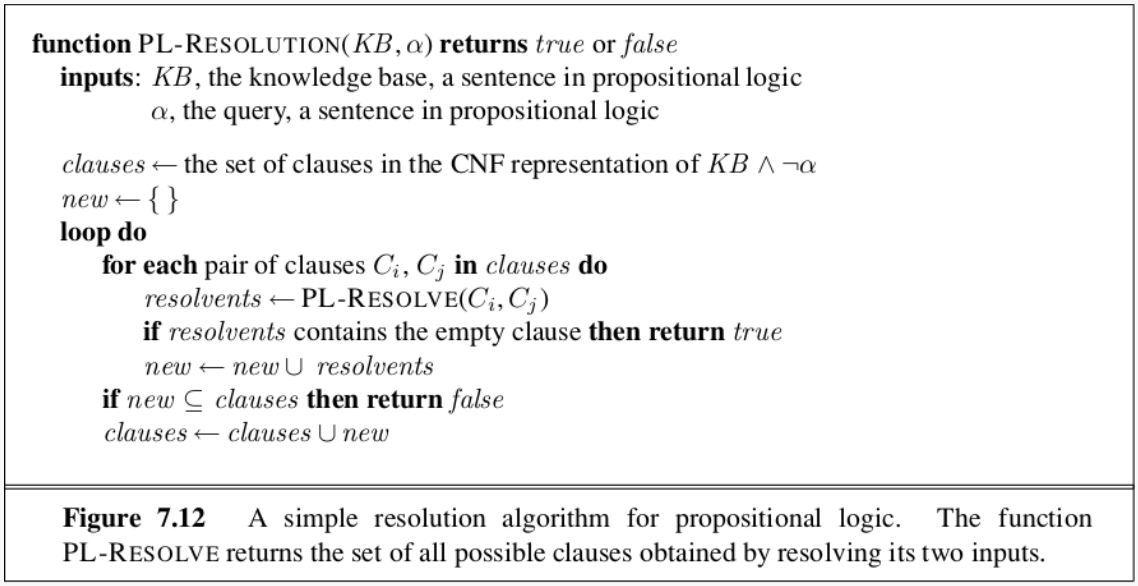
Output specifications

The output file stores the set of clauses generated during the resolution and the conclusion. The output format is as follows

- The first line contains a non-negative integer M_1, which indicates the number of clauses generated in the first loop.
- M_1 next lines represent the newly generated clauses in the first loop (including the empty clause). An empty clause is represented by the string "{}".
- Subsequent loops (of M_2, M_3,..., M_n clauses) are represented as stated above.
- The last line shows the conclusion, i.e. whether "KB entails α". Print YES if KB entails α. Otherwise, print NO.
- Duplicates, i.e., clauses that are identical to some clauses appeared previously (e.g., in the current/previous loop or in the original KB) are ignored.

Problem

The `PL-RESOLUTION` function mention above:



function PL-RESOLUTION(KB, α) **returns** *true* or *false*
  inputs: KB, the knowledge base, a sentence in propositional logic
    α, the query, a sentence in propositional logic

  clauses $\leftarrow$ the set of clauses in the CNF representation of $KB \wedge \neg\alpha$
  new $\leftarrow \{\}$
  loop do
    for each pair of clauses C_i, C_j **in** *clauses* **do**
      resolvents $\leftarrow$ PL-RESOLVE(C_i, C_j)
      if *resolvents* contains the empty clause **then return** *true*
      new $\leftarrow$ *new* $\cup$ *resolvents*
    if *new* $\subseteq$ *clauses* **then return** *false*
    clauses $\leftarrow$ *clauses* $\cup$ *new*

Figure 7.12 A simple resolution algorithm for propositional logic. The function PL-RESOLVE returns the set of all possible clauses obtained by resolving its two inputs.

PR-RESOLUTION pseudo code

There is some note for my implements of this function:

- The entailment is checked at the end of each loop, instead of after generating a clause. (In the `Important notes` of `statement` file).
- In my implementation, i merged the step write to file to `PL-RESOLUTION` function, due to assignment requirement. Because of if I split it into 2 functions, it's hard to construct the flow and deal with it (It lead to the situation that we need to call `PL-RESOLUTION` 2 times and the output write to file depend on the for loop of `PL-RESOLUTION` function). So, the function will write output to file and resolve the query α to `KB`.

An example of the given KB and query α in the input.txt file.

Input.txt	Output.txt	Note
1	3	
-A	-A	(-A OR B) resolves with (-B)
4	B	(-A OR B) resolves with (negative of -A)
-A OR B	-C	(B OR -C) resolves with (-B)
B OR -C	4	
A OR -B OR C	-B OR C	(A OR -B OR C) resolves with (-A)
-B	A OR C	(A OR -B OR C) resolves with (B)
	A OR -B	(A OR -B OR C) resolves with (-C)
	{}	(-B) resolves with (B)
	YES	KB entails α since an empty clause exists in KB.

- Another example of the same KB yet another query α.

1	2	
A	-C	
4	-B OR C	(A OR -B OR C) resolves with (-A)
-A OR B	2	
B OR -C	-A OR C	(-A OR B) resolves with (-B OR C)
A OR -B OR C	A OR -B	(A OR -B OR C) resolves with (-C)
-B	1	
	A OR -C	(B OR -C) resolves with (A OR -B)
	0	No new sentence found
	NO	KB does not entail α since no new clause is created and no empty clause is found.

Sample input and output

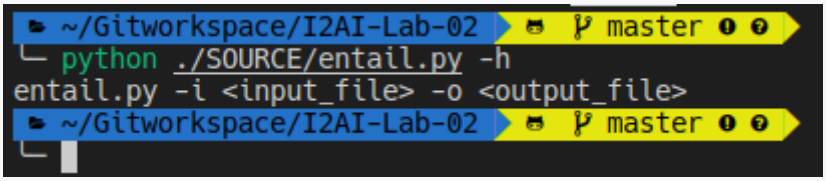
Progress Completeness

Criteria	Points	Completeness
Read the input data and successfully store it in some data structures	1.0pt	100%
The output file strictly follows the lab specifications	1.0pt	100%
Implement the propositional resolution algorithm	2.0pt	100%
Provide a complete set of clauses and exact conclusion	3.0pt	100%
Five test cases: both input and output files	1.0pt	100%
Discussion on the algorithm's efffiency and suggestions	2.0pt	100%

Assignment Plan

Read input data and stored & output:

Firstly, the `INPUT` and `OUTPUT` folder is concrete. It's more convenient if I have a `batch_test` for all test in `INPUT` folder and my program can get the input/output directory from program arguments.



The program take -i and -o as argv

Then I read the data and stored clauses to `list` as `clause data structure`. Each sentence is a list, so the `KB` and `alpha` is a `list` of sentences (`list`).

```
def read_file(input_file):
    """
    :param: input_file: input file's directory
    :return: dnf alpha: list, KB: list
    """
```

I read alpha as `dnf form` because for entailment, you need to resolution `CNF` representation of $KB \wedge \neg\alpha$.

Hence, I convert `dnf form` of `alpha` to `cnf form` of `not alpha`.

```
not_alpha = dnf_to_cnf(recursive_add([], alpha))
```

`recursive_add` is a `helper` function to convert `alpha` to set of clause in `dnf form`.

For the `input1.txt` in the sample, we get those printed lines in console:

```
[!] Finished read file ./INPUT/1-4-2.txt
[*] KB: [['-A', 'B'], ['B', '-C'], ['A', '-B', 'C'], ['-B']]
[*] NOT alpha: [['A']]
```

For `batch test`, I wrote a shell script:

```
for file in ./INPUT/*
do
  echo "---------------"
  python ./SOURCE/entail.py -i $file -o "./OUTPUT/out_`basename "$file"`"
  echo "---------------"
done
```

Taaaada, then the testing step is more convenient.

Implement the propositional resolution algorithm:

I implemented it exactly the same as the pseudo code in textbook [1].

```
def PL_RESOLUTION(negative_alpha, KB, output_file):
    """
    Check if KB entails alpha and write to output_file
    """
    with open(output_file, "w") as fout:
        clauses = KB
        for c in negative_alpha:
            clauses.append(c)
        is_entailed = False
        while True:
            new_clauses = []
            num_of_res = 0
            string_write = ""
            for i in range(len(clauses)-1):
                for j in range(i+1,len(clauses)):
                    new_clause = resolve(clauses[i], clauses[j])
                    new_clause = trim(new_clause)
                    # print(num_of_res,clauses[i], "+", clauses[j],
"=",new_clause) #$
                    if new_clause == [] or (new_clause in clauses) or new_clause
in new_clauses or is_equivalent(new_clause):
                        continue
                    if new_clause == "{}":  # New clause is dump
                        is_entailed = True
                    string_write += to_string(new_clause) if new_clause != '{}'
else '{}\n'
                    print('[^] Resolve',clauses[i], "and", clauses[j],
"get",new_clause)
                    num_of_res += 1
                    new_clauses.append(new_clause)
            string_write = str(num_of_res) + "\n" + string_write
            fout.write(string_write)
            if new_clauses == []:  # Can not resolve new clause.
                string_write += "0\nNO"
                fout.write("NO")
                return False
            elif is_entailed:
                string_write += "\nYES"
                fout.write("YES")
                return True
            clauses += new_clauses  # Add new clause to KB
```

By resolve, we get this in console:

```
[^] Resolve ['-A', 'B'] and ['-B'] get ['-A']
[^] Resolve ['-A', 'B'] and ['A'] get ['B']
[^] Resolve ['B', '-C'] and ['-B'] get ['-C']
[^] Resolve ['A', '-B', 'C'] and ['-A'] get ['-B', 'C']
[^] Resolve ['A', '-B', 'C'] and ['B'] get ['A', 'C']
[^] Resolve ['A', '-B', 'C'] and ['-C'] get ['A', '-B']
[^] Resolve ['-B'] and ['B'] get {}
```

Then the conclusion:

```
[+] KB entails alpha.
```

Write to file:

```
[!] Finished write to ./OUTPUT/out_1-4-2.txt.
```

The hole message for a test look like that:

```
[!] Finished read file ./INPUT/1-4-2.txt
[*] KB: [['-A', 'B'], ['B', '-C'], ['A', '-B', 'C'], ['-B']]
[*] NOT alpha: [['A']]
[^] Resolve ['-A', 'B'] and ['-B'] get ['-A']
[^] Resolve ['-A', 'B'] and ['A'] get ['B']
[^] Resolve ['B', '-C'] and ['-B'] get ['-C']
[^] Resolve ['A', '-B', 'C'] and ['-A'] get ['-B', 'C']
[^] Resolve ['A', '-B', 'C'] and ['B'] get ['A', 'C']
[^] Resolve ['A', '-B', 'C'] and ['-C'] get ['A', '-B']
[^] Resolve ['-B'] and ['B'] get {}
[+] KB entails alpha.
[!] Finished write to ./OUTPUT/out_1-4-2.txt.
```

For `input2.txt`:

```
[!] Finished read file ./INPUT/1-4-3.txt
[*] KB: [['-A', 'B'], ['B', '-C'], ['A', '-B', 'C'], ['-B']]
[*] NOT alpha: [['-A']]
[^] Resolve ['B', '-C'] and ['-B'] get ['-C']
[^] Resolve ['A', '-B', 'C'] and ['-A'] get ['-B', 'C']
[^] Resolve ['-A', 'B'] and ['-B', 'C'] get ['-A', 'C']
[^] Resolve ['A', '-B', 'C'] and ['-C'] get ['A', '-B']
[^] Resolve ['B', '-C'] and ['A', '-B'] get ['A', '-C']
[-] KB does not entail alpha.
[!] Finished write to ./OUTPUT/out_1-4-3.txt.
```

The `trim` method for sort clause as alphabetical order and remove duplicated literals.

```
def trim(clause):
    """
    Bubble sort for clause - by alphabet
    :param clause:
    :return: sorted clause
    """
```

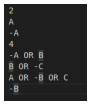
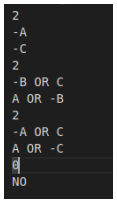
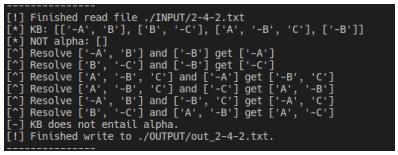
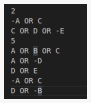
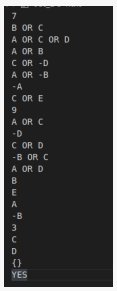
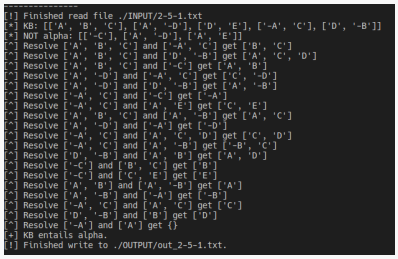
The `resolve` method take 2 clauses as parameters and return new clause (new clause can be null/empty):

```
def resolve(clause_A, clause_B):
    """
    :param clause_a: clause
    :param clause_b: clause
    :return: res as resolved clause of 2 param
    """
```

The `is_equivalent` method is a checker for this condition:

```
def is_equivalent(clause):
    """
    The clause [A,B,-B] is considered equivalent to [A,True] and hence equivalent
to True.
    Deducing that True is true is not very helpful.
    Therefore,any clause in which two complementary literals appear can be
discarded.
    :return: True if clause is useless
    """
```

Test scenarios:

INPUT	OUTPUT	LOG	COMMENT
			This test case show that alpha is False sentence, hence the KB can't resolve alpha.
			The not alpha contain a set is a subset of another set in clauses. So in `CNF` form, you need to reduce it.

And there are 12 test cases (ignored 2 samples) I've generated (few of them I looted from my friend scenarios/bugs) with. There are in `INPUT` and `OUTPUT` folder. There are non-trivial test, each of them has different scenario. For example: In test `1-6-1`

```
1
-R
6
P OR -Q
-P OR R OR S
Q
-P OR -R
Q OR P OR R
-Q OR S
```

```
9
-Q OR R OR S
P
-Q OR -R
P OR R
-P OR S
Q OR R OR S
S
-P
P OR R OR S
8
-Q
R OR S
-P OR -Q OR S
-R
-P OR Q OR S
Q OR R
{}
P OR -Q OR S
YES
```

Because of the rules:

> The entailment is checked at the end of each loop, instead of after generating a clause. (In the `Important notes` of `statement` file).

So we need to resolve more times till we have hit the condition to end.

At the main directory, just:

```
chmod +x ./batch_test.sh
./batch_test.sh
```

Then it will automatically run a batch of all input file in `INPUT` folder.

Discussion & suggestions:

In order to improve the reasoning efficiency of the resolution principle in propositional logic, in the purpose of reducing the size of clause set and eliminating the number of literal in clause set as soon as possible. `CNF` form is one of multiple solutions. Because of the input/output format rules of this assignment, I do not try to implement a different resolution algorithms.

PL-RESOLUTION end when it hit a complementary literals/empty clause ({}). Because the empty clause is get by a complementary pair, this complementary pair respectively are get by the addition of complementary pair, therefore, in turn, the rapid reduction of the literal in clause set is helpful to get the empty clause. And the first resolution of the literal of the least complementary pair, we can remove a literal by a relatively minimal resolution step. The less to the literal, the easier to get the empty clause. I've several document about efficiency of PL-Resolution, in this paper [2], it concludes that if you follow the strategies, priority resolve the related resolve of previous step, the empty set come faster than normal algorithms.

Hence, the book caption about completeness of PL-RESOLUTION because the clauses is finite. But there is no complexity bound for this algorithms [3], it's `NP`. So in a large test/clauses, it should be a different method.

References:

- Standford's logic course
- Cheng-Lin-Li's AIMA course note

Have a Great Day

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1. https://www.amazon.com/Artificial-Intelligence-Modern-Approach-3rd/dp/0136042597 ↩
2. https://ieeexplore.ieee.org/document/8004925 ↩
3. https://www.math.ucsd.edu/~sbuss/ResearchWeb/marktoberdorf97/paper.pdf ↩