COT 5600 Quantum Computing
Spring 2019

Homework 1

Problem 1 (Eigenvalues of Pauli operators)

https://github.com/t583046

Problem 2 (Trace inner product)

Proof:
Assuming $\mathrm{C}^{d\times d}$ is a real vector space

1) Using basic property of Trace: The matrix A and its Transpose have the same Trace. ie.

$$Tr(A) = Tr(A^T)$$

2)By definition of conjugate transpose, and assuming A has real entries, the conjugate transpose of A reduces to the transpose of A.

ie. from 2,

3)$\mathrm{A}^\dagger = Tr(A^T) = Tr(A)$. This is simply the trace of A.

4) Keeping the same expression on the LHS, now we are at $\langle A|B\rangle_{\mathrm{Tr}} = \mathrm{Tr}(AB)$.

$\langle A|B\rangle_{\mathrm{Tr}}$ is also by definition the inner product trace. This Definition states that for real square matrices of the same size, $\langle A|B\rangle = \mathrm{Tr}(AB^T)$.

5)From 1, $Tr(B) = Tr(B^T)$, thus $\langle A|B\rangle \Rightarrow \mathrm{Tr}(AB)$.

6) Therefore $\mathrm{Tr}(AB) = \mathrm{Tr}(AB)$.

Q.E.D. $\langle A|B\rangle_{\mathrm{Tr}} = \mathrm{Tr}(A^\dagger B)$.

Problem 3 (Unitary error basis)

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