Suppose you have a set of points

$$(x_1, y_1), (x_2, y_2), (x_3, y_3), \ldots, (x_n, y_n)$$

where the x_is are distinct. Consider the polynomials made from products of these as follows.

$$P_j(X) = \frac{(X - x_1)(X - x_2)\ldots(X - x_{j-1})(X - x_{j+1})\ldots,(X - x_n)}{(x_j - x_1)(x_j - x_2)\ldots(x_j - x_{j-1})(x_j - x_{j+1})\ldots,(x_j - x_n)}$$

Here the top is the product of all the terms $(X - x_i)$ except $(X - x_j)$ and the bottom is the same with the variable X replaced by x_j. (So they are polynomials in X of degree $n - 1$.)

If you evaluate $P(X)$ when $X = x_j$, then the top and bottom are the same, so $P_j(x_j) = 1$. If you use any other of the n x values, say x_i, then the top is zero. This means $P_j(x_i) = 0$ when $i \neq j$.

So now form the following sum.

$$P(X) = y_1 P_1(X) + y_2 P_2(X) + y_3 P_3(X) + \ldots + y_n P_n(X).$$

When $X = x_i$, all terms except $y_i P_i(X)$ are zero, and that one term is $y_i \cdot 1 = y_i$. This means $P(x_i) = y_i$ for each of our i values. Thus we have a polynomial $P(X)$ of degree (at most) $n - 1$ which goes exactly through the n points we started with.